Digital Voice and Picture Communication

Prof. S. Sengupta

Department of Electronics and Communication Engineering

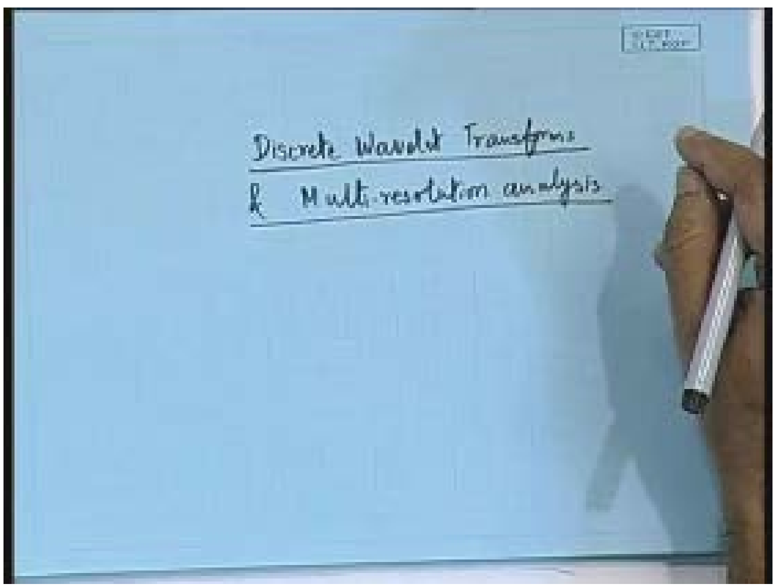
Indian Institute of Technology, Kharagpur

Lecture - 20

Discrete Wavelet Transforms and Multi-resolution Analysis

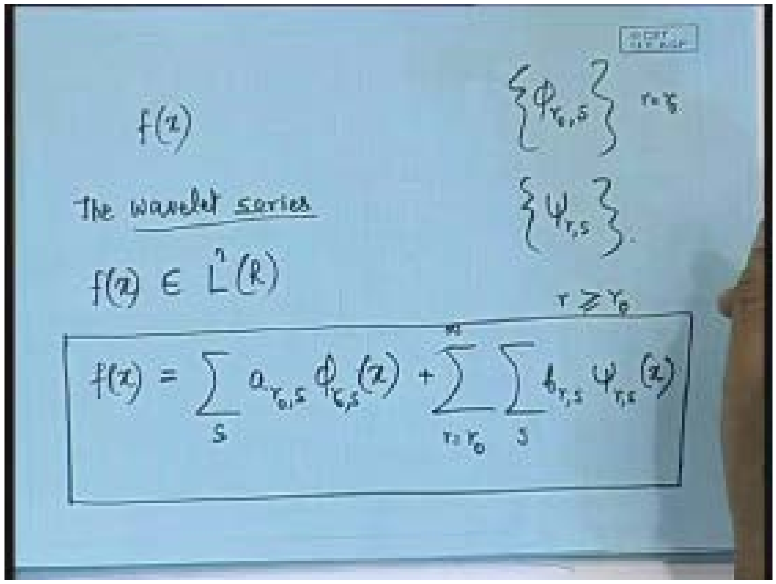
.....last class we......................discussed on the discrete wavelet transforms and also I will try to cover in this lecture that how to use the discrete wavelet transform in multi-resolution analysis.

(Refer Slide Time: 1:24)



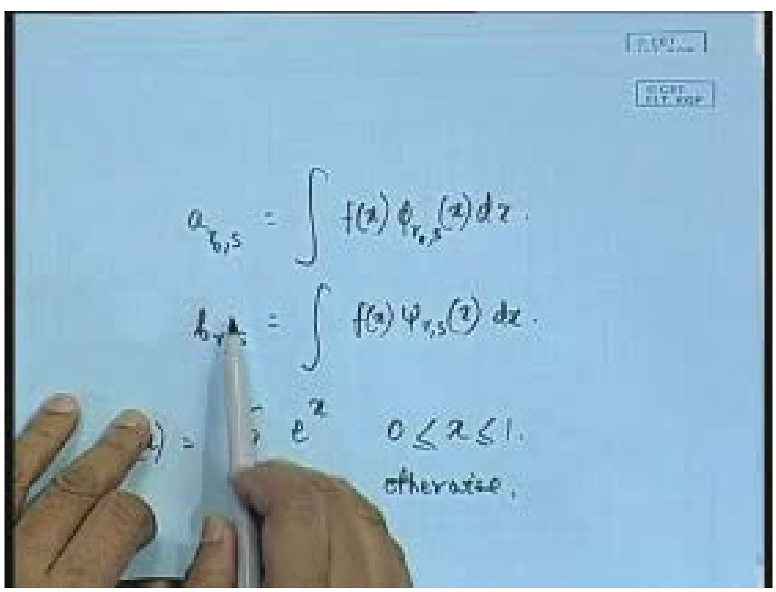
Now before we begin this topic let us have a look at what we did for the continuous wavelet functions and the continuous scaling functions. In fact this kind of a series that we had realized we are calling that as the wavelet series.

(Refer Slide Time: 1:45)



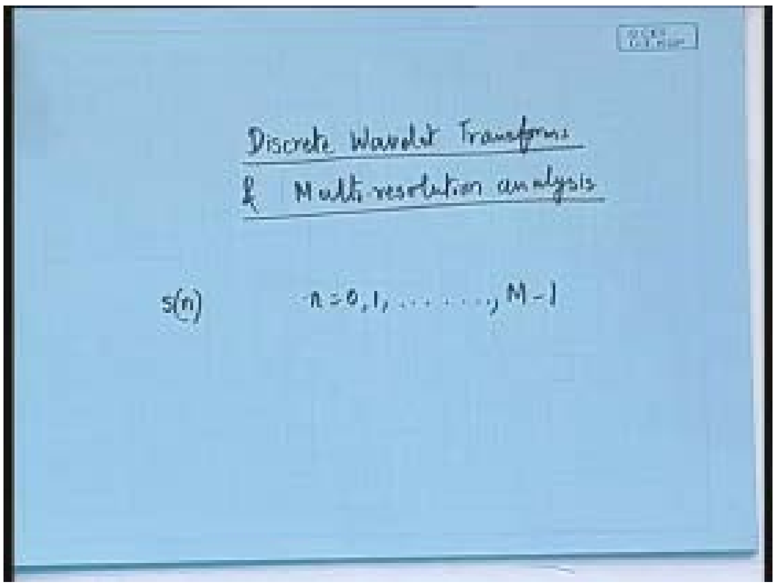
So essentially we are using this set of scaling functions (Refer Slide Time: 1:50) and the set of wavelet functions in order to approximate a continuous valued function f of x.

(Refer Slide Time: 02:05 min)



And what we did here is just how to compute the coefficients a r 0, s and b r, s where r happens to be greater than or equal to r 0 where the integral expressions are coming in. Now the integral expressions are okay as long as x is a continuous variable because it is integrated with respect to x. But when x is not a continuous variable, when we are considering the signal itself to be discrete in that case the functional form of this has to change. So what we will be doing now is that the definition of the signal now becomes that we write the signal as s of n where n is going be the samples of the signals and we are going to have n as 0, 1 up to M minus 1 so where we are considering that there are M number of samples in the signal, this is s of n. Then what we are going to do is to......... In fact although the image is a two dimensional signal for the time being just for the sake of simplicity I am considering one dimensional signal because whatever theory we can develop for one dimension would later on be easily applicable to two dimensions. In fact it is more so in the case of wavelets because wavelet can be realized wavelet 2 D wavelet filters can be realized as separable 1 D wavelet filters so essentially it means to say that as if to say cascading of two 1 D filters that is what we can do. So the signal is s (n) and the signal is discrete time signal where n is the parameter of the time and it is defined at some discrete points like 0, 1 up to M minus 1.

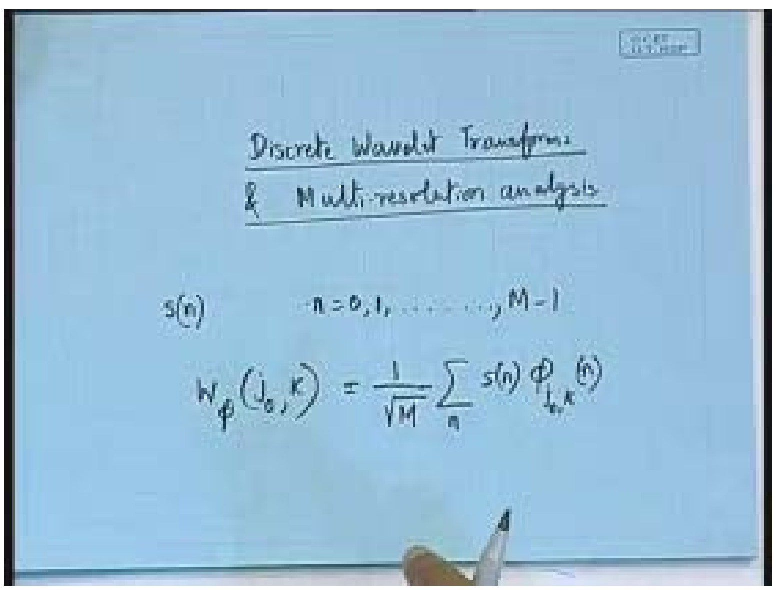
(Refer Slide Time: 4:16)



Now whenever we are trying to compute this a r 0, s and b r, s and we do not have the f(x) anymore and instead of f(x) we are having s of n in that case it will not be possible for us to integrate. So the integration will get replaced by a summation series and the manner in which we can write is like this that equivalently this a r 0, s........... what was a? a is the coefficients that is associated with the set of scaling functions (refer Slide Time: 4:53) with the shifted versions of the scaling function.

so now instead of writing this as a, the scaling function coefficients if we write as W and as suffix I use phi, phi means that it is it is associated with the scaling function and let us say some scaling parameter I take as j 0 just like the way we took r 0, in the equivalent discrete version form I take it as j 0 so W phi j 0, and the shifts that is s I am writing that as k. just be comfortable with the change of notation that I am applying in this lecture So j 0 is what our r 0 was and k is what our s was and W is what our (a) was. So everything was in the continuous domain and s of n is what our f of x was. So in the discrete form I can write W phi (j 0, k) as 1 upon root over M into a summation series over n s of n into phi j 0, k of n.

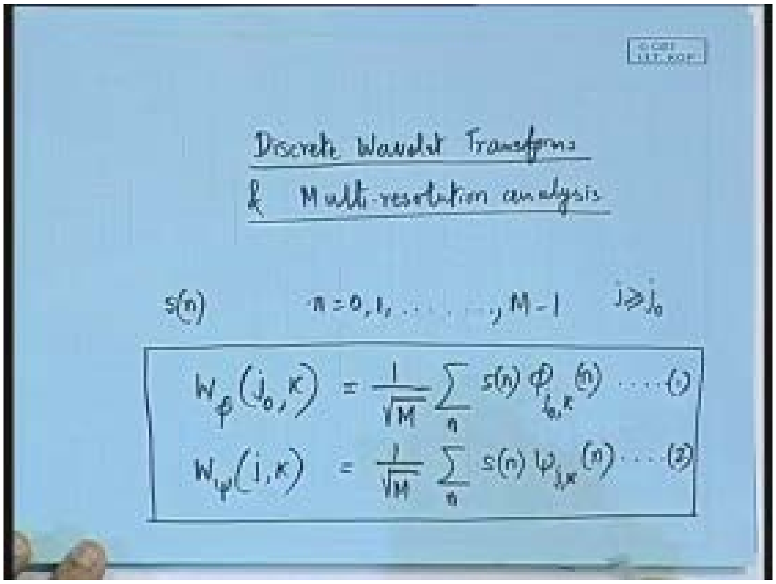
(Refer Slide Time: 6:26)



Now naturally the phi of x we cannot write anymore, we have to compute the value of the wavelet function at also those specific points n; n is equal to 0, 1, 2,............ etc so we have to compute the scaling function values and the wavelet function values at those specific discrete points. This is the equivalent form. So whatever I wrote for a r 0, s as an integral expression now I am writing the same thing W phi (j 0, k) as a summation expression and this 1 by root over M is actually a normalizing term that is coming in because what we are essentially doing is you can imagine that as if to say that from this s of n which is the special domain I am converting that to a new domain that is given by this W phi j, k. So again when I am transforming the signal to a new domain like this I should be able to recover the signal given some parameters of this domain or when these when the signals are equivalently transformed in this form then I should be able to get back. But of course I have not completed it because I have only written the scaling function term so similarly I have to write down the wavelet function term also.

So to write the wavelet function term I write it as W psi. it is what I am going to write as b r, s in the discrete form; b r, s I am writing as W psi and the parameters I will say as j, k now, where j is going to be greater than or equal to j 0 just like the way I had the condition that r is greater than or equal to r 0 that is to say these scaling functions are 0 (Refer Slide Time: 8:38), in this case I have to make j greater than or equal to j 0 and I can write W psi j, k as 1 by root over M into summation over n s (n) psi j, k (n). So I have got two equations let us call this as equation 1 and this as equation 2.

(Refer Slide Time: 9:10)



Any questions? Yes please..........

[Conversation between Student and Professor – Not audible ((00:09:08 min))]

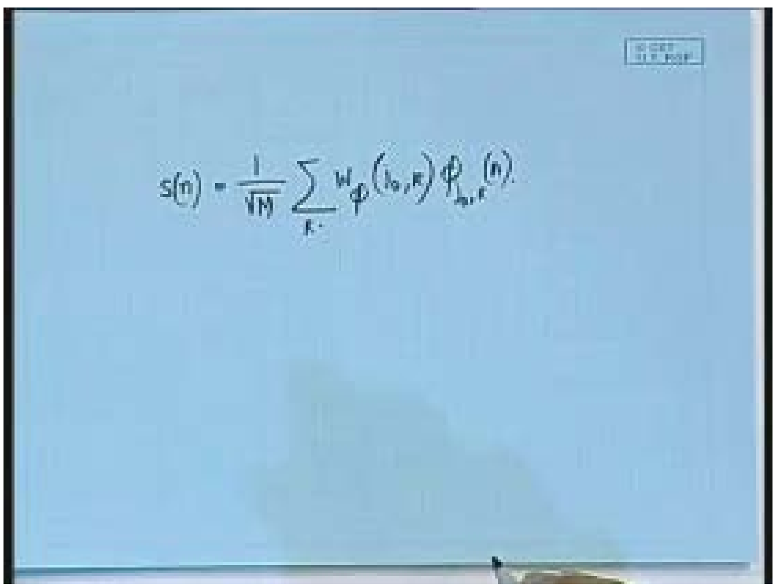
Why is this 1 by root over M? I will come to this very shortly. See, 1 by root over M is a normalizing term that is there because I have transformed s (n) to this space and I can get back s (n) from this space. Now when I want to apply this transformation and I want to get back the original signal from reverse transformation this process of transformation and reverse transformation that should not introduce any scaling coefficient. The energy of the signal in the s (n) domain and the energy of signal in this W domain they have to remain the same. So, to maintain that either in the forward transformation we have to apply a coefficient of 1 by M or in the inverse transformation we apply a coefficient of unity or otherwise an alternative form of doing it is that have in the direct transformation have a coefficient of 1 by root M, in the inverse transformation also have a coefficient of 1 by root M. The same thing is also applicable to the Fourier transforms. Fourier transforms also people are doing like this.

Even in continuous Fourier transform also, if you remember, that some books will show it as 1 by root over 2pi in the forward Fourier transform and in the inverse Fourier transform they will also show another 1 by root over 2pi. Some books mention 1 by 2pi in the forward and unity in the inverse. It is a very similar logic; this sort of normalization will be there. In our case n is nothing but the number of samples. So, this is conversion from the s (n) domain to a new transform domain and this new transform domain is also in the discrete space.

Now what we should do is to get back this s (n) given this W phi and W psi’s and how do we get back?

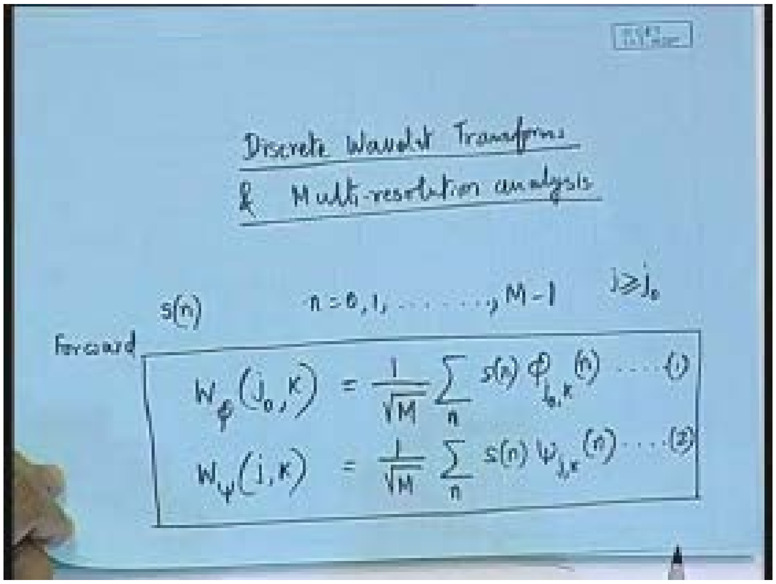
So again the functional form should be very similar to this because in this case we could obtain f of x from the coefficients a r 0, s and b r, s. Now what is a r 0, s that is our W phi and b r, s is nothing but our W psi’s. So given the W phi’s and the W psi’s we should be able to get not f(x) anymore but s of n in this case. So what we obtain is this: s of n can be expressed as 1 by root over M summation over k W phi j 0, k into phi of j 0, k (n).

(Refer Slide Time: 12:38)



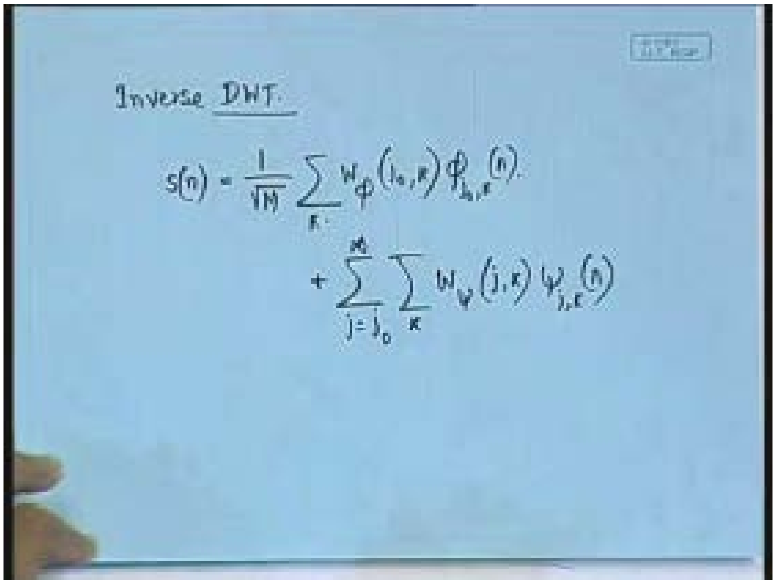
No major difference; f(x) is replaced by s (n), this s now we are calling it as k; a r 0, s; now we are calling it as W phi j 0, k and phi r 0, x we are now calling as phi j 0, k (n). So this is the scaling function quantity and the wavelet function quantity will be just like the way we had here a double summation; in this case also we will continue to have a double summation so it will be summation j is equal to j 0 to infinity summation over k the shift parameter W psi (j, k) psi j, k of n. This is how we get back s of n from W phi's and W psi's.

(Refer Slide Time: 14:03)



This one is our forward transformation or we can say this as forward wavelet transformation. we will not call it as wavelet transformation we will specifically call it as forward discrete wavelet transformation because the signal itself is discrete so we are using the wavelet functions and the scaling functions also in its discrete time form so this is forward discrete wavelet transform; the short form of discrete wavelet transform is DWT so we call this as the forward DWT.

(Refer Slide Time: 14:44)

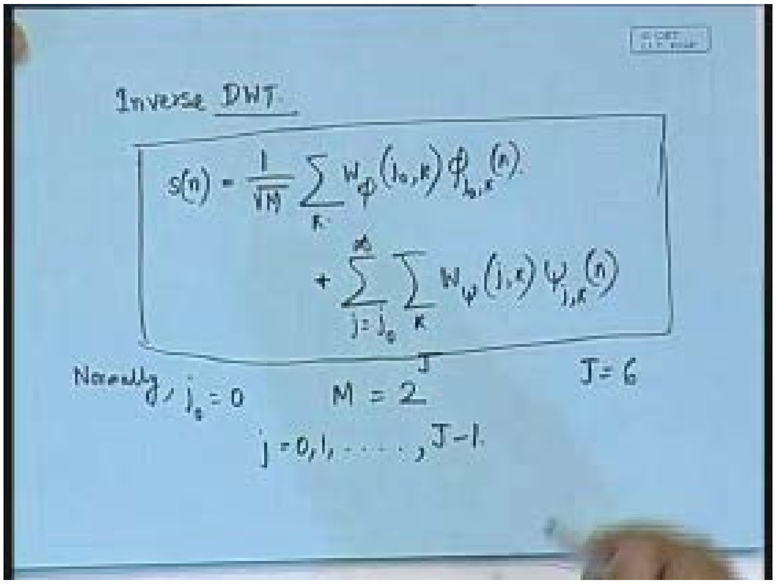


And this will be the expression for the corresponding inverse DWT.

So in this case........... okay, since you are familiar with different types of transformation now you can see that every transformation has got two things: the original signal, the transformed signal and also it has got a transformation kernel. Now in this case what is the transformation kernel? The transformation kernel is this phi j 0, k and this psi j, k's these are the transformation kernel and this is just the inverse transformation kernel so in this case the functional form of the inverse transformation kernels are also the same so using this it is possible for us to realize this s of n.

Now what we normally do is to choose j 0; j 0 is chosen to be 0; normally we can choose j 0 to be equal to 0 and select M as some power of 2. So say if I have this as M as M expressed as 2 to the power capital J where capital J is an integer in that case the summations are performed over this, j these summations are performed over j is equal to 0, 1 up to capital J minus 1 meaning that if I let us say start with 64 samples so say n is equal to 0, 1 up to 63 that means to say then M is equal to 64, when I have M is equal to 64, 64 is nothing but 2 to the power 6 so my J becomes equal to 6 and in that case in the summation quantity summation term associated with the wavelet function wavelet coefficient that would be having six terms because there can be six different wavelet six six k's of wavelet function that is what we can choose and that is why the summation will be in that case from j is equal to 0 to j is equal to 5e because capital J being equal to 6 it is 0 to 5 yes, that is what it becomes.

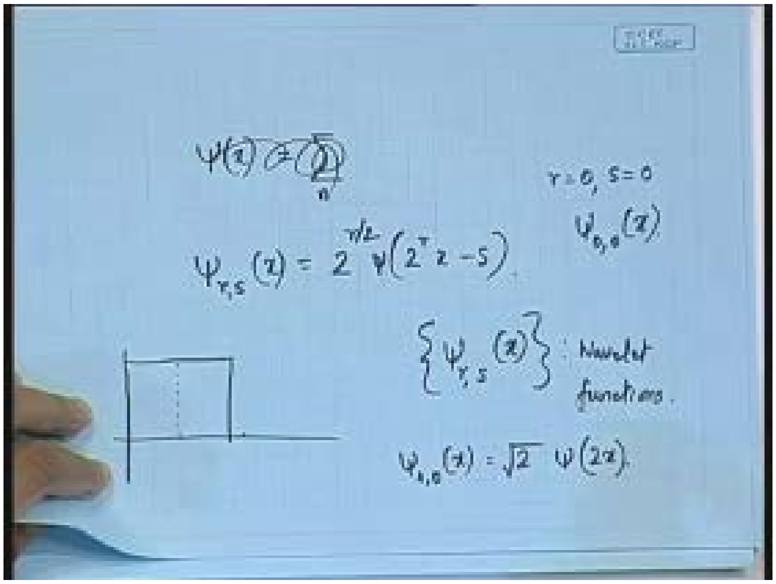
(Refer Slide Time: 17:26)



Although in the general expression the summation goes up to infinity but in the practical cases this is not going up to infinity this will be only up to the extent that you require. Now, given this functional form we want to just see what is the easy and convenient way of computing a discrete wavelet transform. You see that alright.............. you can say that we should be able to get different shifted versions of this and finally obtain this W phi of j, k but is there any easy way; is there any nice recursive way of computing the discrete wavelet transforms so let us have a look at that aspect.

Now just in order to do that let us again go back to some of the continuous wavelet transform expressions that we had derived in the last lesson. Let us see this expression

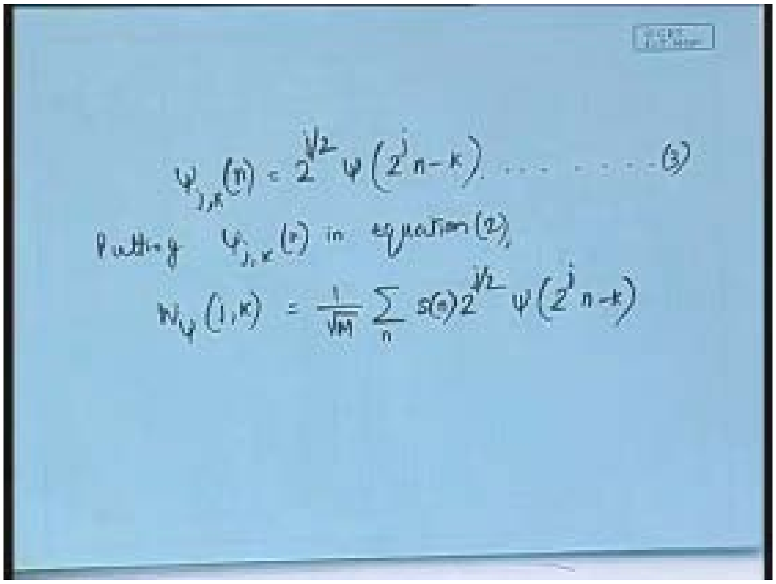
(Refer Slide Time: 18:48)



Have a look at this expression: psi r, s; we had said psi r, s of x was equal to 2 to the power r by 2 psi of this 2 to the power r, x minus s. Now in this case the nomenclatures have changed; we are calling this s as k and this x as n the discrete variable we are calling it as n; this is as k and r we are calling as j (Refer Slide Time: 19:18) so this same expression it is possible for us to write down this. In fact what we do is that this psi r of s instead of psi r, s if I now start writing psi j, k of n in that case this equation 2 what I have written as equation 2 W psi j, k is equal to this the equation 2 can be rewritten by substituting this psi j, k expression as we would obtain from this. So let us first write down the discrete form psi j, k so psi j, k first we write down and then we will put that into equation number 2.

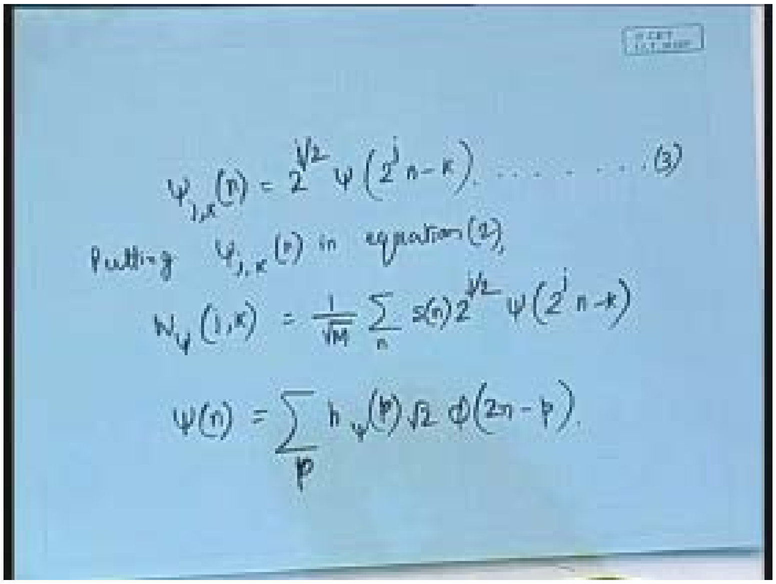
So psi j, k I can write psi j, k n as 2 to the power j by 2 into psi. Here I have to write 2 the power j n minus k. now if I call this as equation number 3 I will be applying equation number 3 into equation number 2 means this I will be substituting psi of j, k term. So putting psi j, k in equation 2 what we obtain; we will obtain this W psi j, k expression so we can write W psi j, k as 1 by root over M summation n s(n) 2 to the power j by 2 psi (2 to power j n minus k).

(Refer Slide Time: 21:31)



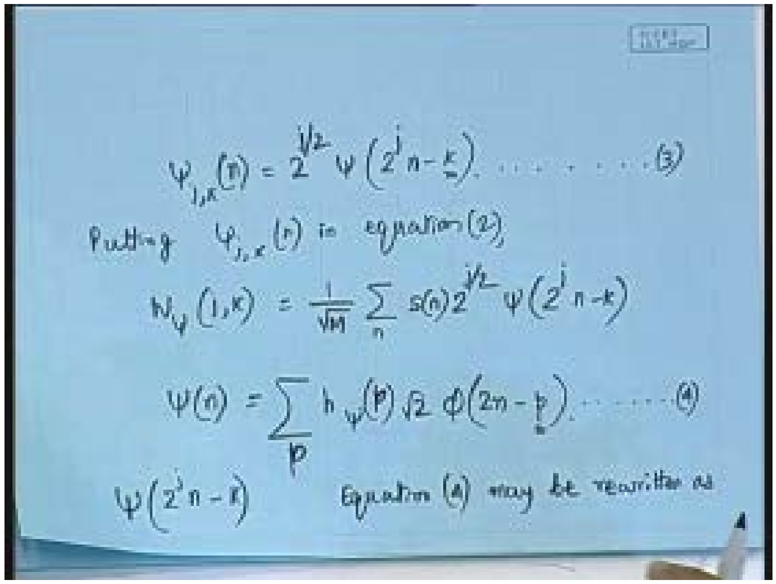
Now, again just see, we have got another very interesting relationship and what was that where we had expressed the psi function as a series summation of phi function. We had an expression like this and this in the new form in the in the in a in a in the new notation we would like to write it like this: psi of n I should be writing as the summation over........... I cannot call this as n anymore (Refer Slide Time: 22:13) because n I am reserving for the sample number so n is replacing this x so please do not get confused with the earlier notation and this notation I do not call it as n anymore but the shift parameter I call that as r; so summation over r h psi of r again do not confuse earlier I was using r for the scaling term, now the scaling term is j; shift parameter is k but k I am writing in this case I am writing k as r; if you are confused popular defend is to have it as p okay let me write it as p. So h psi of p and then I have to write it as root 2 phi what am I going to write? 2 n minus you have introduced p so I write as 2 n minus p so this is psi of n that I now have.

(Refer Slide Time: 23:38)



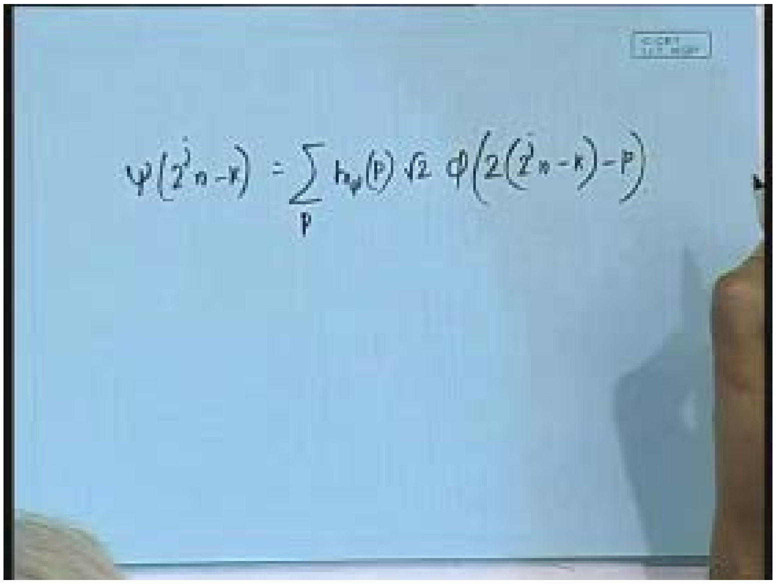
Now in this equation if I multiply this n by a factor 2 to the power j that means to say that if I scale it up by a factor 2 to the power j and I give it a shift of k units; because k already I am using as a shift parameter that is why I wanted to give a different shift parameter over here so that you do not get confused with the 2 k's. Fine, you have already told that it should be p. So now what I want to do is that I want to get psi (2 to the power j n minus k). So if I want to do that what I have to simply do is to replace this n by 2 to the power j n minus k and put it in this equation.

(Refer Slide Time: 25:01)



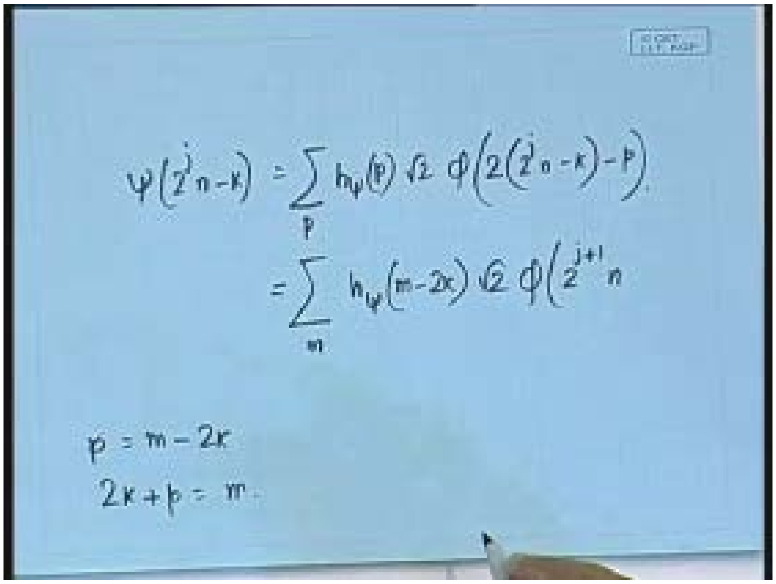
So what do we obtain? So if I say that this is 4 then equation 4 can be rewritten as; equation 4 may be rewritten as: let me write down: psi (2 to the power j n minus k)) that will be equal to summation over P h psi of P root 2 phi 2 into this is not n anymore n is replaced by (2 to the power j n minus k)) so 2 into 2 to the power j n minus k and this whole thing minus this was in our expression this was 2 n minus P so this this will be, n is this and P remains as before.

(Refer Slide Time: 25:52)



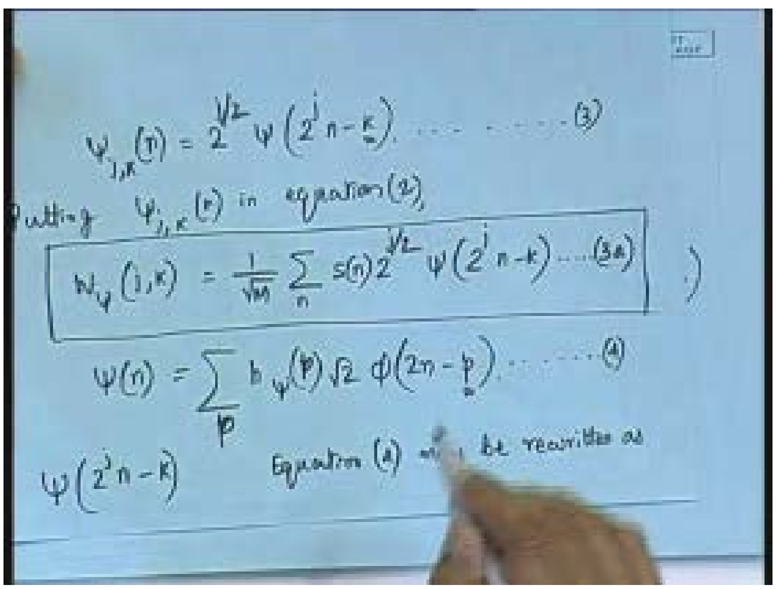
Now this can be represented as............ we can now have a change of variable let's say that h psi if I have a have a change of variable........... just this P if it is changed to (m minus 2k) so I introduce a new variable m so that (m minus 2k) becomes equal to P so I sum up now over m and with this new substitution what results is root 2 phi and in this case it is (2 to the power j plus 1 n (2 to the power j plus 1 into n and here what I have is minus 2k minus P and according to our substitution P is equal to m minus 2k so that 2k plus P is equal to m. So in this case I am just going to write (2 to the power j plus 1 n minus m).

(Refer Slide Time: 27:16)



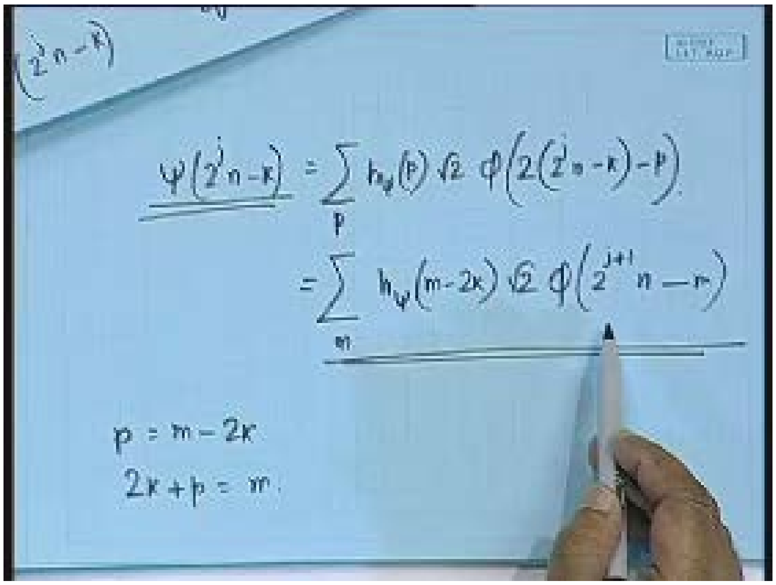
So now what I want to do is that I want to substitute this h of......... okay, this psi of (2 to the power j n minus k) is what we actually required in this equation W psi j, k just in this equation, yeah.....................

(Refer Slide Time: 28:00)



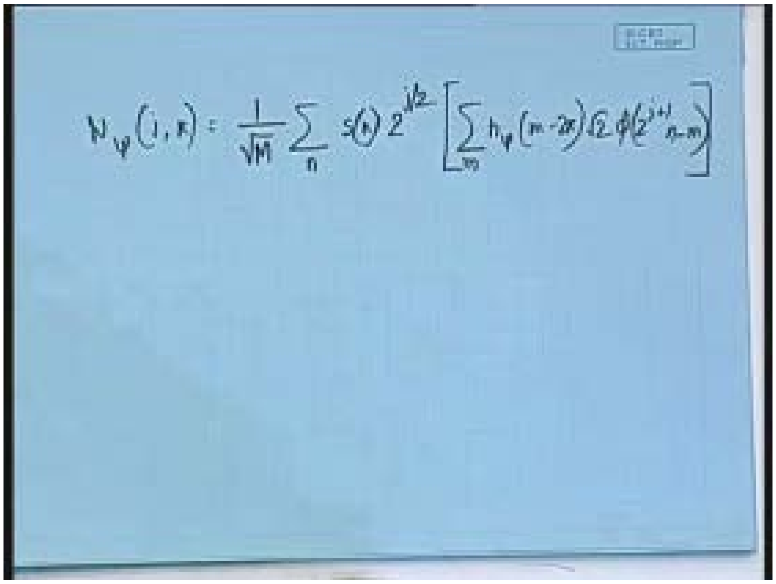
So let's call this as............ oh I did not put any number let's call it as (3a). Now in this equation (3a) what I want to do is that whatever value of psi (2 to the power j n minus k) I have got over here this value I will substitute in equation number (3a) in order to express this W psi j k in terms of this phi's.

(Refer Slide Time: 28:35)



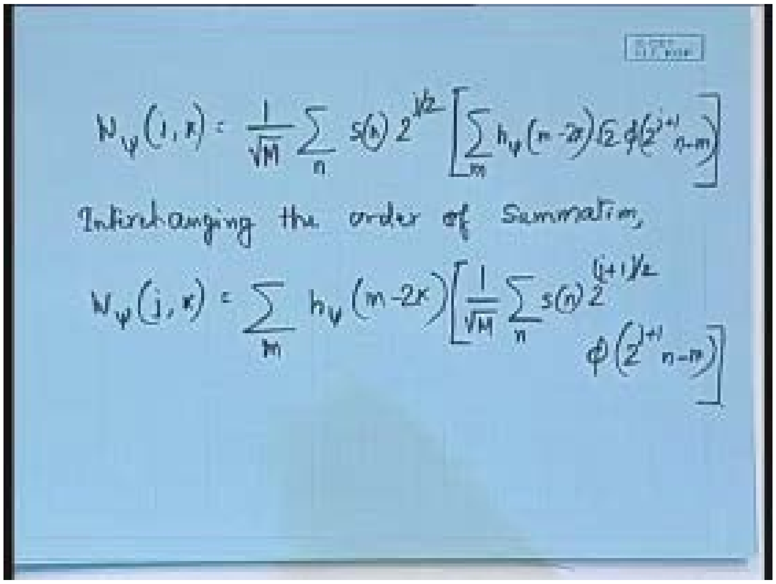
So what I am doing is that the wavelet coefficients I am expressing in terms of the different versions of the scaling functions. So let us see that what results whether any interesting thing results when I substitute psi (2 to the power j n minus k) is equal to this into equation (3a) so what we get is this: W psi j, k should be obtained as 1 by root over M summation n into s(n) 2 to the power j by 2 and now this expression that is to say instead of psi 2 to the power j m just the new form that we wrote that is to say [summation over m h psi (m minus 2k) into root over 2 phi (2 to the power j plus 1 n minus m)].

(Refer Slide Time: 30:02)



Now, interchanging the order of the summation; if I interchange the order of the summation what we should obtain is W psi (j, k) will be given by summation over m into h psi (m minus 2k) and then I am going to write [1 by root over M summation over n s(n) 2 to the power (j plus 1) by 2 because here it is 2 to the power j by 2; here there is a 2 to the power half (Refer Slide Time: 31:07) so it makes 2 to the power (j plus 1) by 2 so 2 to the power (j plus 1) by 2 into phi of (2 to the power j plus 1 into n minus m) the quantity which we wrote over here.

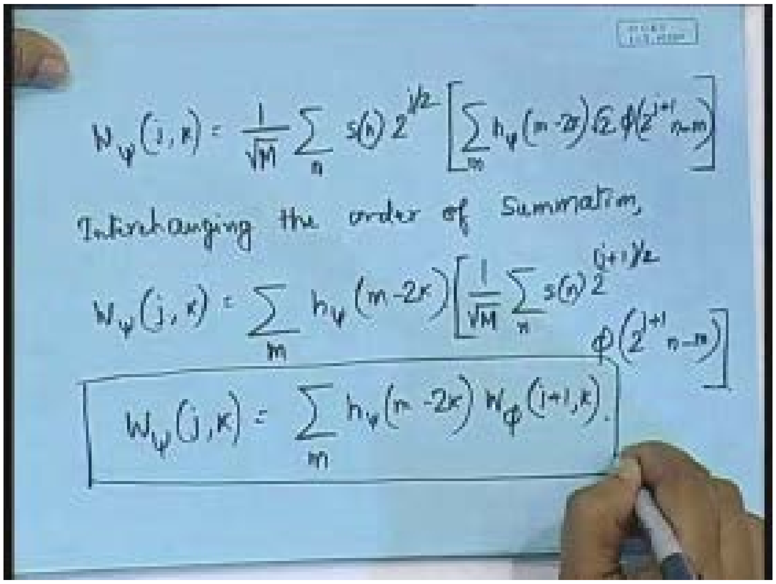
(Refer Slide Time: 31:26)



Now can you make out anything interesting out of this; the term that I have written within the square bracket?

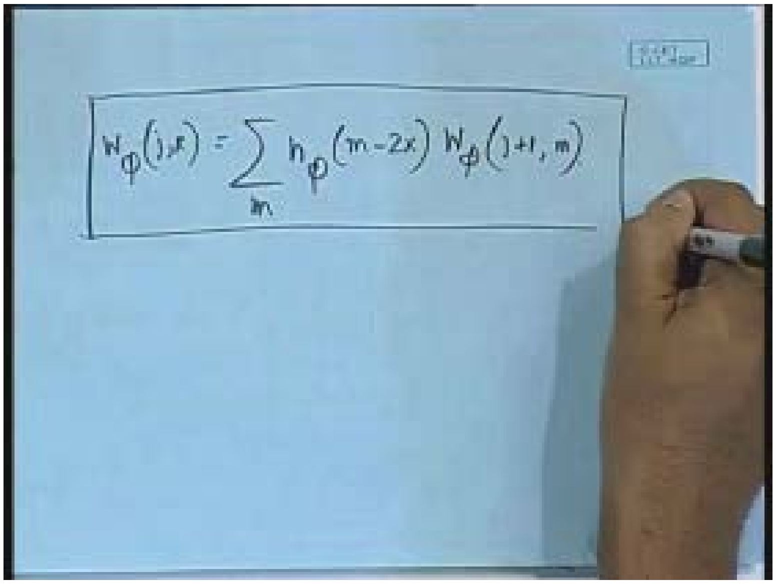
[Conversation between Student and Professor – Not audible ((00:31:40 min))] W phi j plus 1m; this is very interesting, yes. because if you just have a look at W phi j 0, k W phi j 0, k makes the form of s (n) phi j 0 k this expression........ no, W yes W phi is like this and in this case it becomes instead of j 0 it becomes j plus 1 so what results is W psi (j, k) becomes equal to summation over m h psi (m minus 2k) into W phi (j plus 1, k) this is a very interesting relationship.

(Refer Slide Time: 32:44)



[Conversation between Student and Professor – Not audible ((00:32:47 min))] M? yeah, that is true because in the index we are using as m; yeah. so this is one very interesting relation that we have obtained for W psi (j, k) and in a very similar way we can obtain the expression for W phi (j, k) also and W phi (j, k) W phi (j, k) in a very similar way can be obtained as summation over m h in this case phi of (m minus 2k) into W phi (j plus 1, m) this is also a very interesting expression.

(Refer Slide Time: 33:43)



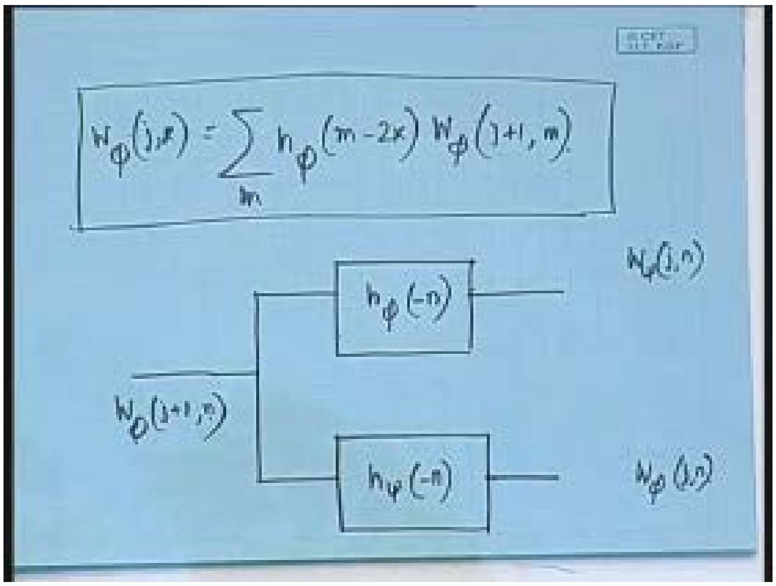
which means to say what In fact you should not be bothering about this m and the k; why I am telling this is (Refer Slide Time: 34:03) here k is a shift parameter that we are considering in this case so essentially whenever you are considering the summation series from minus infinity to plus infinity from m this is also what you will be obtaining for the summation series over k because the transformation is that m is equal to 2k plus P.

[Conversation between Student and Professor – Not audible ((00:34:30 – 34:47))] Okay, I think things will become clear when we talk about the further interpretation of this equation. After all what is this; what is the interpretation of this equation?

That means to say that if W phi (j plus 1, m) is available with you, what you are essential doing is you are convolving that with h psi. It is a convolution with h psi function that is what you are considering and in this case it is convolution with h phi function (Refer Slide Time: 35:32). So if you have two filters one with impulse response h phi of m and the other with h psi of m, at the input of the filter what you are giving is that you are you are giving this quantity and you are getting this quantity out of it because it is a convolved from.

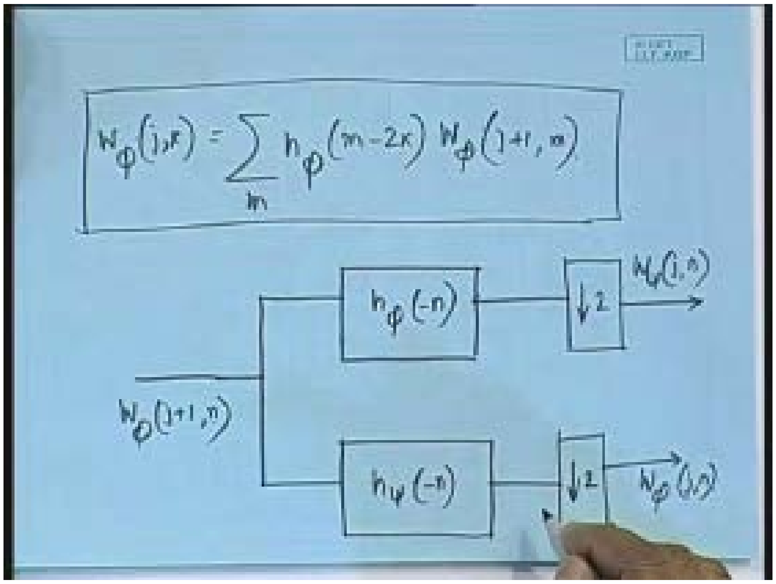
Therefore, in the block diagram form what you can do is like this that W phi (j plus 1, n) if you take this as (j plus 1, n) then as if to say that you can pass it through two filters. You can call them as the analysis filter, this one is given by h phi of minus n because if you are using this as n then this becomes (minus n) h phi of minus n and this as h psi of minus n. So this one will be the scaling analysis filter and this one will be the wavelet analysis filter. In that case from this you should be able to obtain this W psi (j, n) and W phi (j, n)

(Refer Slide Time: 37:12)



[Conversation between Student and Professor – Not audible ((00:37:12 min))] there is a two factor.................... I think, the point what we have to solve is just some substitution of variables that is what we might be missing or I will............. [Conversation between Student and Professor – Not audible ((00:37:54 min))] Decimation by 2 is anyway needed, yes; decimation part I have not yet included; decimation part I mean since I have not explained this I have not come to the decimation part; yes, decimation part takes care of that two term. And what we have to do is yes, so this is two filters so this W phi (j plus 1, n) if you are having as an input then you can get W psi (j, n) and W phi (j, n) out of this but essentially and as mathematically also evolves that there is a decimation by 2 and I did not talk about this decimation because I want you to interpret the decimation in a little different way and I am coming to that point soon.

(Refer Slide Time: 38:59)



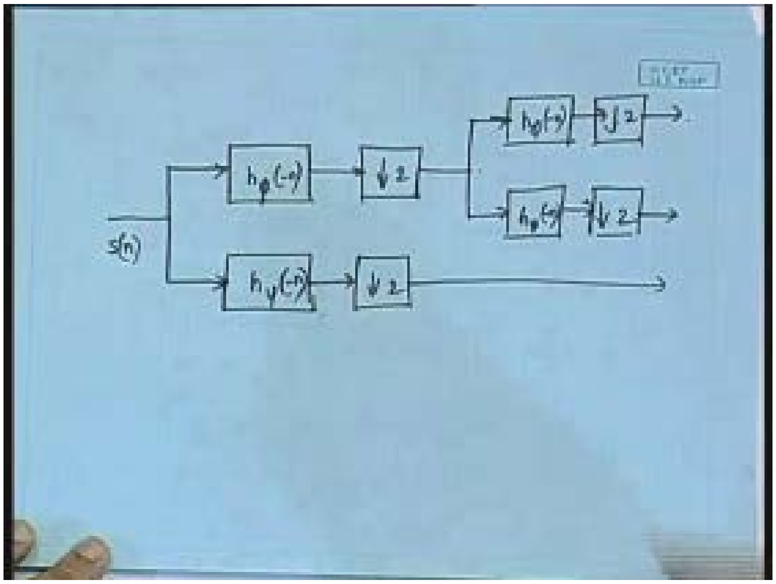
But if it is decimated by a factor of 2 decimated by a factor of 2 means that you just take the alternate samples and you drop the alternate samples because what happens is that if you are starting with m number of samples for n is equal to 0 to m minus 1 so you are starting with m number of samples in that case after the low pass filtering also you get n number of samples, after the high pass filtering also you get m number of samples so your number of samples are that way increasing.

In fact, on the other hand, your bandwidth that gets halved because if you are realizing the filter as something like this; see, h phi of minus n is a low pass filter; h psi of minus n is a high pass filter. So if I have the filter responses like this, now I cannot have an ideal filter that is not practically realizable so if my LPF the low pass filter is like this (Refer Slide Time: 40:09) and if I am able to construct the high pass filter in a manner that this becomes my high pass filter and in that case the overall filter characteristic should be................. if this is equal to 1 then it should be able to pass all the frequencies; I mean, together the overall response should be this. So this is what I want to achieve and here since the bandwidth has become halved so if this is my total so if this is my maximum frequency then individually this band has got half of the maximum frequency, this band has got another half of this so the bandwidth is halved and that is why we can reduce the sampling rate also by half in the individual channels. So that is what we achieve by the decimation process.

Therefore, from W phi (j plus 1) it is possible for us to get W psi (j, n) and W phi (j, n) which means to say that I can do this further. Using this W phi (j, n) it should be possible for me to analyze it further and get W phi (j minus 1, n) and W psi (j minus 1, n). So, if I start with the highest scale the highest scale means that where............. now what is the maximum value of small j? The maximum value of small j is equal to capital J minus 1. So, if I start with W phi capital J, n which is nothing but our original image; if you start with W phi or original signal; if I start with W phi (j, n) which is our original signal we can put it into this two kinds of filter banks and we can obtain a low pass filter version, we can obtain a high pass filter. By the way I have written totally wrong this should be W phi (Refer Slide Time: 42:27) this should be W phi, this should be W psi so this is the low pass filter version and this is the high pass filtered version and I can apply it iteratively. So in that case what we need to do is to analyze this low pass further.

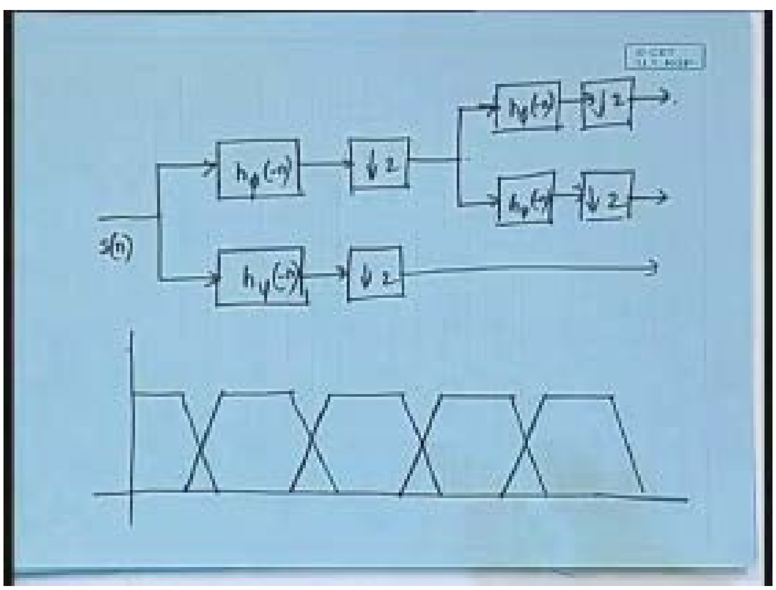
Therefore, what it means to say is that, given the original signal given the original signal let us say that original signal is s of n we are passing this s of n into two filters: one is this h phi filter, the other is the h psi filter and then we are decimating by 2 here, decimating by 2 here, this is our low pass filtered version and this I am again further analyzing h phi minus n, h psi minus n again decimating by 2, decimating by 2 like this.

(Refer Slide Time: 43:53)



So it is possible for us to obtain such bank of analysis filters. And in terms of the frequency response, the frequency response plots would be like this that if this is the low pass filter, this is the next band, this is the next band so the low pass filter and a bank of band pass filters that can be realized using a system like this.

(Refer Slide Time: 44:28)



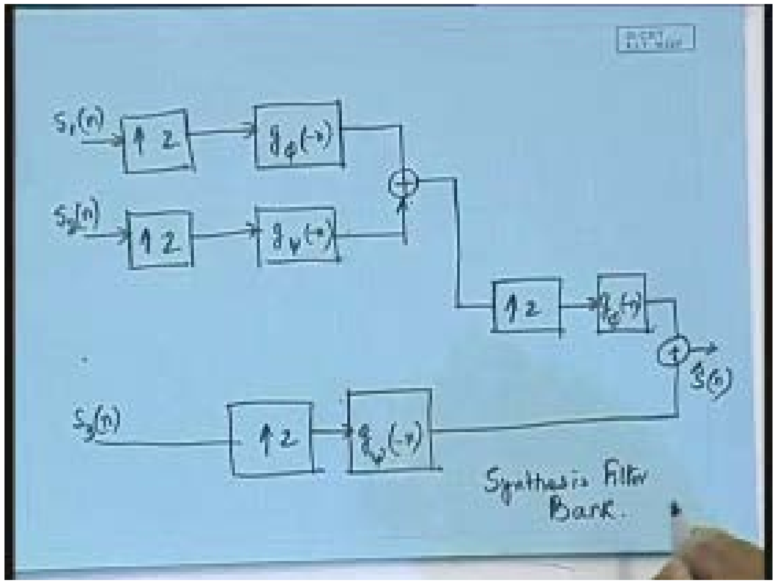
Yes please; [Conversation between Student and Professor – Not audible ((00:44:34 min))] Yes, yes, banks will not be having the same bandwidth. In fact what happens is the low pass filter bandwidth............ because you see that the ultimate low pass filter is this which will have the one fourth of the original bandwidth. In this case it will be one fourth of the original bandwidth this will be having one fourth of the original bandwidth, this will be having half of the original bandwidth. So the picture should actually de drawn like this that if I want to draw only these three in that case I should draw it like this that this should be the low pass filter, this should be the next band pass filter which is this one (Refer Slide Time: 45:30) and then this should be two times; I mean this should be equal to this which means to say that this band should be this one. That means to say that wavelet essentially permits us to realize a bank of analysis filters which means to say that we can obtain the original signal back if we apply a corresponding bank of synthesis filters.

Just if you analyze the signal, from s of n you analyze it into three bands like this how to get back the s of n?
You can obtain s of n back, you can call that as s cap of n because you are not too sure that whether you will get back the signal exactly or not but you can obtain this s cap of n by a process of synthesis filters. So the synthesis filter would be just the opposite of this.

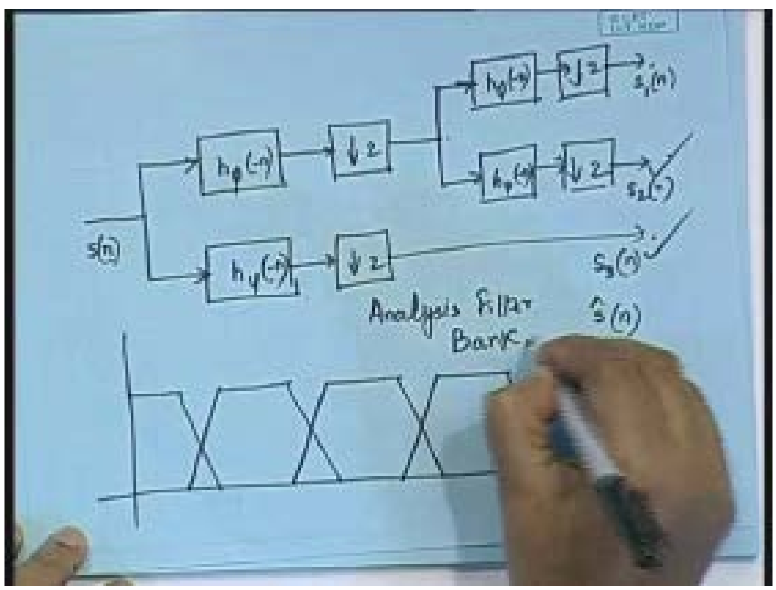
Now since you have down-sampled by 2 you have to correspondingly up sample by a factor of 2 over here. So this is up-sampled by 2, this is up-sampled by 2 and then you have to apply a synthesis filter so call that as g phi of minus n, this you call as g psi of minus n and then these two responses you just add up and then what you do is that you up-sample this by 2, apply again g phi of minus n and add up with this.

am talking of if you If you say that this is s 1 of n, this is s 2 of n and this is s 3 of n (Refer Slide Time: 48:06) so this one is s 1 of n, this one is s 2 of n and this is s 3 of n. Now s 3 of n this we will pass through g psi of minus n; add it up and this will realize s cap of n. So we can realize s cap of n using this. This is the synthesis filter bank and this is the analysis filter bank.

(Refer Slide Time: 49:06)

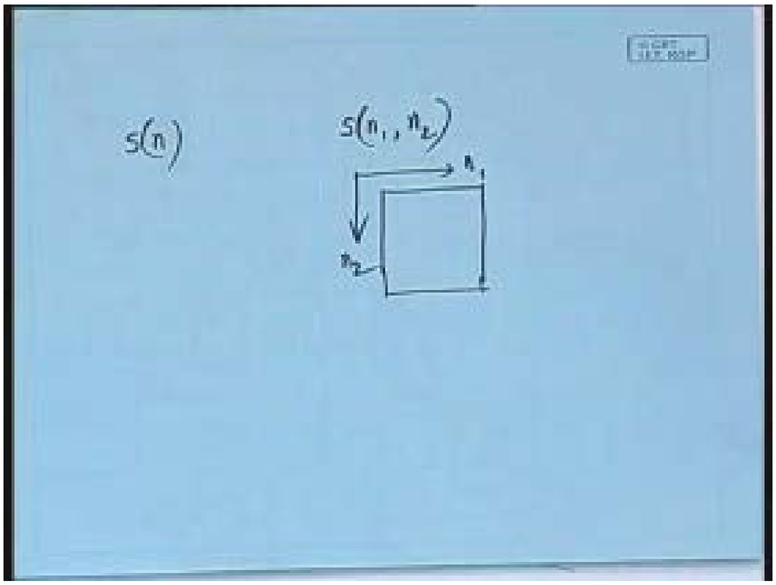


(Refer Slide Time: 49:09)



Now in the case of images what we have to do is that we have to apply the wavelet transforms in both the directions because then our signal will not be represented as s of n but rather it will be represented as s of (n 1, n 2) so we have two directions (n 1) direction and (n 2) direction and let us say that we choose our convention like this that (n 1) is the horizontal direction and (n 2) is the vertical direction.

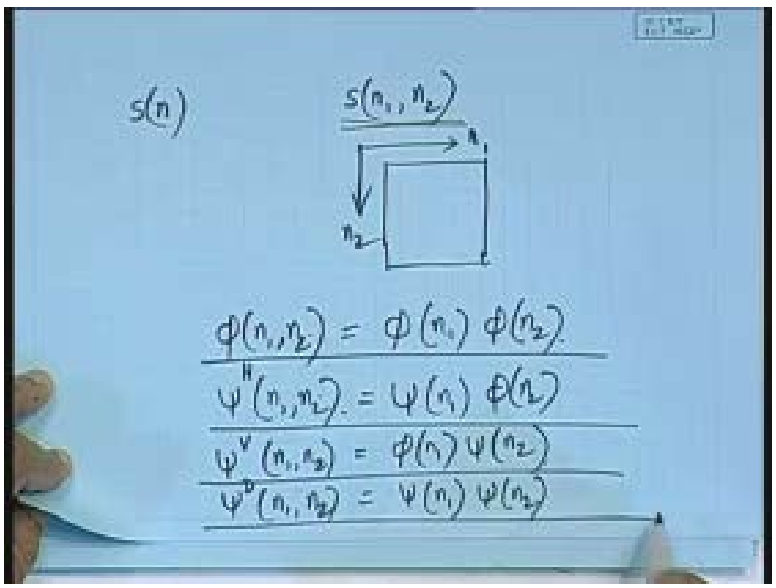
(Refer Slide Time: 49:51)



By having it like this we can define four different filters which would be like this: phi (n 1, n 2)............. so we define a filter called phi (n 1, n 2) which is nothing but a product of phi (n 1) into phi (n 2) and then we define another filter psi (n 1, n 2) but call this as psi with a superscript H, what the superscript H means I will define it shortly that becomes psi (n 1) into phi (n 2) which means to say that along the (n 1) direction that is along the horizontal direction it is high pass filtered but along the low pass filter but along the (n 2) direction it is low pass filtered; phi is the scaling function which is essentially a low pass filter and psi is a wavelet function which is essentially a high pass filter. So it is along (n 1) it is high pass filtered so we write it as s psi H along horizontal so horizontally it is high pass and then I define another filter psi v (n 1, n 2) and what does it and what it should be; it should be phi (n 1) into psi (n 2) and I should have a fourth filter also possible which will be just the product of psi (n 1) and psi (n 2) so psi (n 1, n 2) I can write as a product of psi (n 1) psi (n 2). That means to say, what is the physical significance of psi (n 1) and psi (n 2)'s product; that means to say that it is high pass filtered in both horizontal and vertical direction. That means to say that, in diagonal direction also it is having a high pass filtering so we call this as psi diagonal. So there are four filters that we can have for the case of two dimensional signals. So, when our signal, instead of one dimensional is two dimensional s (n 1, n 2) then we can use four different filters.

And you see, whenever I am applying phi (n 1) phi (n 2)'s products or any such product, with every filter I have do a decimation by a factor of 2. That means to say that in overall I am doing a decimation by a factor of 4; decimation by a factor of 2 in horizontal direction and decimation by another factor of 2 in the vertical direction so totally a decimation by a factor of 4.

(Refer Slide Time: 53:11)



So if I start with an image; if this is my image (Refer Slide Time: 52:58) then I can split this image. Now when I apply the four different wavelet filters, when I apply filter 1, filter 2, filter 3 and filter 4 these individual filter responses I can represent in my image space like this that I can split the image space into four quadrants and in this top left quadrant I represent the image which is low pass filtered horizontally and low pass filtered vertically. Hence, I call this as low pass filtered horizontally low pass filtered vertically I call it as LL. So LL will occupy only one fourth of the image space because it is decimated by a factor of 2 in both horizontal and vertical direction.

Here what happens?

Here (Refer Slide Time: 54:00) it is horizontally high passed but vertically it is low passed. So I call it as horizontally high passed, vertically low passed.

What is this one?

This one (Refer Slide Time: 54:13) is horizontally low passed but vertically high passed LH and this one, the last filter is horizontal high passed vertically also high passed so LL HL LH HH, so four different filtered versions I am obtaining. And as I was showing you, that it should be possible for us to apply some further sub-divisions like I can take this LL band and I can analyze the signal further; just like the way I was showing you that the low pass filtered version signal I can further analyze I can further split, there is no limitation on the on how many analysis filter banks I can use. We will discuss more about this in the coming lecture, thank you.