**Digital Image Processing**

**Prof. P. K. Biswas**

**Department of Electronics and Electrical Communication Engineering**

**Indian Institute of Technology, Kharagpur**

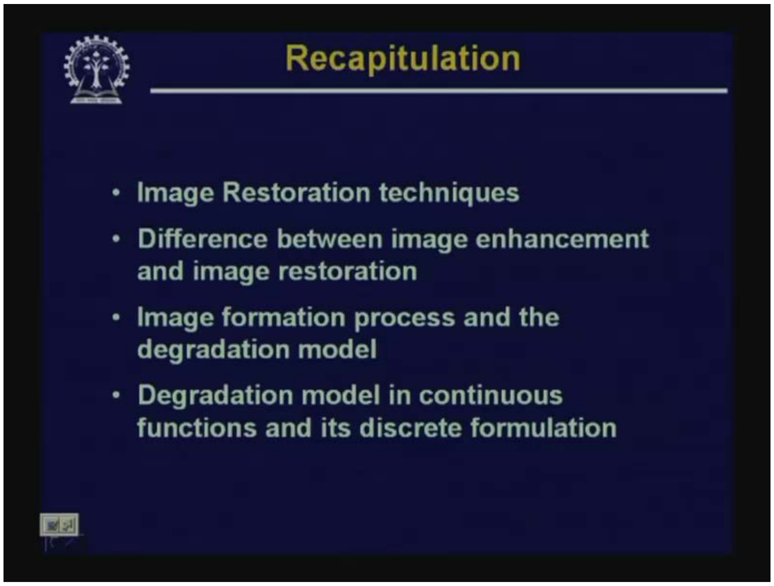
**Lecture - 23**

**Image Restoration - II**

Hello, welcome to the video lecture series on digital image processing. In the last class, we have started discussion on image restoration. We have said that there are certain cases where image restoration is necessary in the sense that in many cases while capturing the image or while acquiring the image, some distortions appear in the image. For example, if you want to capture a moving object with a camera; in that case, because of the movement of the camera, it is possible that the image that is captured will be blurred which is known as motion blurring.

There are many other situations say for example, if the camera is not properly focused, then also the image that you get is a distorted image. So, in such situations, what we have to go for is restoration of the image or recovery of the original image from the distorted image. Now, regarding this, in the last class we have talked what is image restoration technique.

(Refer Slide Time: 2:10)

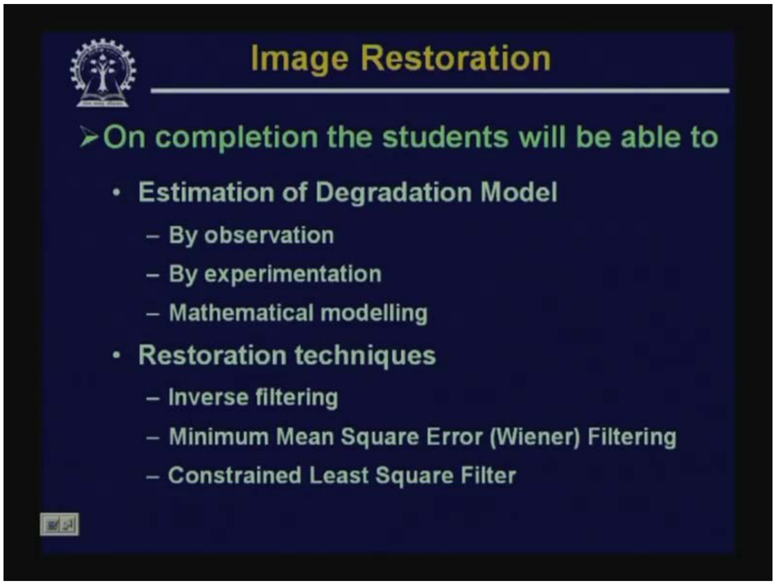



In previous classes, we have talked about image filtering. That is if the image is contaminated with noise, then we have talked about various types of filters both in spatial domain as well as in frequency domain to remove that noise and we just mentioned in our last class that this kind of noise removal is also a sort of restoration because there also we are trying to recover the original image from a noisy image.

But conventionally, this kind of simple filtering is not known as restoration. But what by restoration what I mean is that if we know a degradation model by which the image has been degraded and on that degradation model, on the degraded image, some noise has been added. So, recovery or restoration of the original image from a degraded image using the acquired knowledge of the degradation function of the model using which the image has been degraded; so, that kind of recovery is normally known as restoration process. So, this is the basic difference between restorations and image filtering or image enhancement.

Then, we have seen an image formation process where the degradation is involved and we have talked about the degradation model in continuous functions as well as its discrete formulation.

(Refer Slide Time: 3:50)

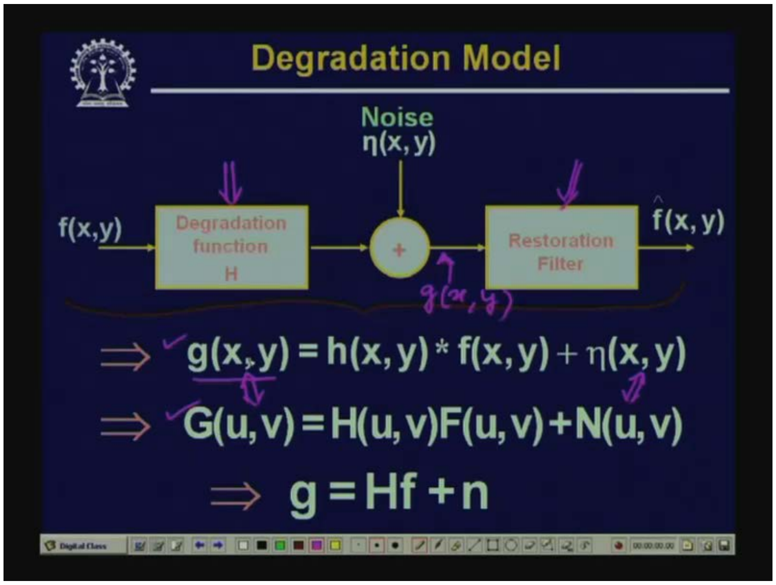



So, in today's lecture, we will talk about the estimation of degradation models and we will see that there are basically 3 different techniques for estimation of the degradation model. One is simply by observation that is by looking at the degraded image; we can estimate that what is the degradation function which is involved that has degraded the original image. The second approach is through experimentations. So, there you can estimate the degradation model by using some experimental setup and the third approach is by using mathematical modeling techniques.

Now, once we know the degradation model, I mean whichever way we estimate the degradation model, whether it is by observation or by estimation or by using the mathematical models; once we know the degradation model, then we can go for restoration of the original image from those degraded images. So, we will talk about various such restoration techniques.

The first one that we will see is what is called inverse filtering, the second one will be called minimum mean square error or wiener filtering and the third approach is called constrained least square filtering approach.

(Refer Slide Time: 5:17)




Now, in our last class, we have seen a diagram like this. So, in this diagram, you see that we have shown the degradation function. So here, we have an input image f (x, y) which is degraded by a degradation function H as has been shown in this diagram. So, H is the degradation function.

So, once I degraded once we get the degraded image at the output of this degradation function H, then a noise eta (x, y), a random noise eta (x, y) is added to that degraded image and finally here we get what is our degraded image we call as g of x and y. So, this degraded image g (x, y) which is normally available to us and from this g (x, y), by using the knowledge of this degradation function H, we have to restore the original image and for that what we have to make use of is a kind of restoration filters and depending upon what kind of restoration filter we use, we have different types of restoration techniques.

Now, in our last class, based on this model, we have said that the degradation mathematical expression of this degraded operation can be written in one of these 3 forms. The first one is given by g (x, y) which is equal to h (x, y) convolution with f (x, y) plus eta (x, y) which is the random noise. So here, f (x, y) that is the original image and the degradation function h (x, y), they are specified in the spatial domain.

So, in spatial domain, the original image f (x, y) is convolved with the degradation function h (x, y) and then a random noise eta (x, y) is added to that to give you the observed image which in this case, we are calling as g (x, y). So, this is the operation that has to be done in the spatial domain and we have seen earlier that a convolution operation in spatial domain is equivalent to performing multiplication of their corresponding Fourier transformations.

So, if for spatial domain image f (x, y), the Fourier transformation is capital F (u, v) and for the degradation function h (x, y), its Fourier transformation is capital H (u, v); then if I multiply this capital H (u, v) and capital F (u, v) in the frequency domain and then take the inverse transform of it to obtain the corresponding function in the spatial domain, then I will get the same result. That is convolution in the spatial domain is equivalent to performing multiplication in the frequency domain and by applying that convolution theorem, this second mathematical expression of this degradation model which is given by G (u, v) is equal to H (u, v) into F (u, v) plus N (u, v) where this N (u, v) is nothing but Fourier transform of the random noise eta (x, y) and G (u, v) is the Fourier transform of the degraded image that is G (x, y).

So either, we can perform this operation in the frequency domain using the frequency coefficients or we can also perform the same operation directly in the spatial domain and in the last class, we have derived another mathematical expression for the same degradation operation. But there the mathematical expression was given in the form of a matrix and that matrix equation as has been shown here is given by g is equal to H into f plus n where this g is a column matrix or column vector of dimension m into n where the image is of dimension m by n. f is also column vector of the same dimension m into n.

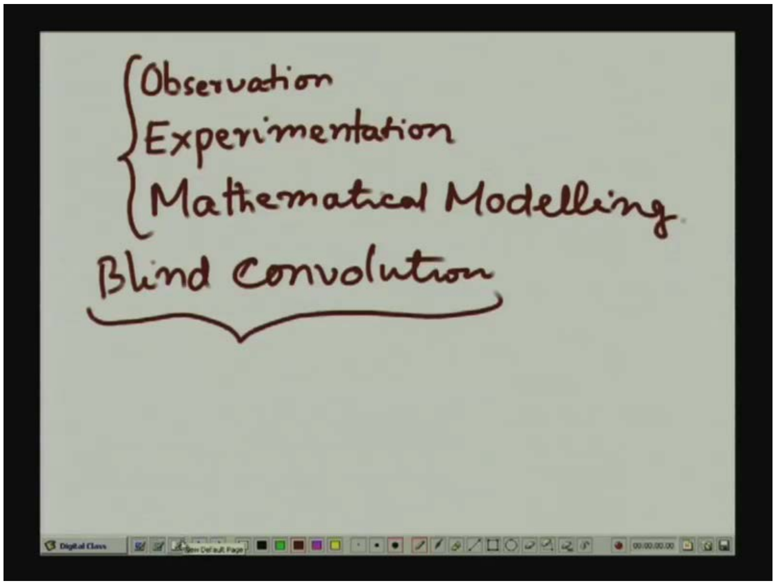
This degradation matrix H, this is of dimension m into n by m into n. So, there will be m into n number of rows and m into n number of columns. So, you find that the dimension of this degradation matrix h is quite high if our input image if is of dimension capital M into capital N and similarly this n is a noise term and these, all these 3 terms together gives you the degradation expression in within in the form of matrix equation.

Now, this particular expression that is matrix expression, direct solution using this matrix expression is not an easy task. So, we will talk about this matrix expression, the restoration using this matrix expression a bit later. But for the time being, we will talk about some other simpler expressions which are direct fall out of the mathematical expression which is given in the frequency domain.

Now here, you note one point that whether we are doing the operation in the frequency domain or we are doing the operation in the spatial domain or we make use of this matrix equation for restoration operation; in all of these cases, knowledge of the degradation function is essential because that is what is our restoration problem. That is we try to restore or recover the original image using acquired knowledge of the degradation function.

So accordingly, as we have said earlier that estimation of the degradation function which degrades which has degraded the image is very very essential and we have 3 different approaches using which we can estimate the degradation function.

(Refer Slide Time: 12:00)

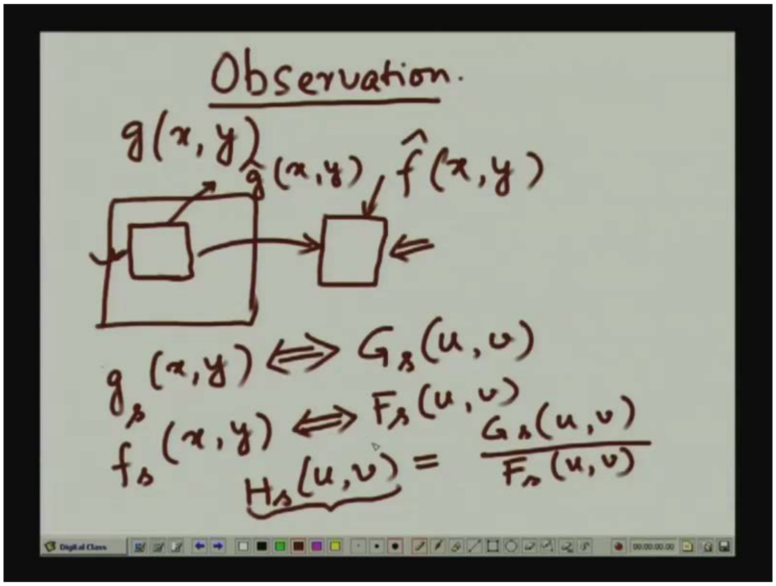



And those approaches, as we have said that there are 3 basic approaches; the first approach is by observation that is we observe a given image, a given degraded image and by observing the given degraded image, we can estimate, we can have an estimation of what is the degradation function. The second approach is by experimentation that means we will have experimental setup using which we can estimate what is the degradation function that has degraded the image and the third approach is by mathematical modeling.

So, we can estimate the degradation function using one of these 3 approaches and whichever degradation function or the degradation model we get, using that we try to restore our original image from the observed degraded image and the method of restoring the original image from the degraded image using the degradation function obtained by one of this 3 methods is what is called a blind convolution.

The reason it is called a blind convolution operation is that using one of these estimation techniques, the degradation model or the degradation function that we get is just an approximation. It is not the actual degradation that has taken place to get the degraded image. So, because it is not the actual degradation function, it is just an approximation, the method of getting the inverse process that is restored image using one of these degradation functions is known as blind convolution operation. So, we will talk about this degradation functions one by one.

(Refer Slide Time: 14:30)




The first one that we will talk about is estimation of the degradation function by observation. So, when we try to estimate a degradation function by observation when no acquired knowledge of the degradation function is given; so what we have is the degraded image g (x, y) and by looking at this degraded image g (x, y), we have to estimate what is the degradation function involved.

Now, for doing this, what we do is you look at the degraded image, then try to identify a region which is having some simpler structure. So, if we have a complete degraded image; in this complete degraded image, you identify a small region, the region which contains some simple structure. Say for example, it may be an object boundary where a part of the object as well as a part of the background is there.

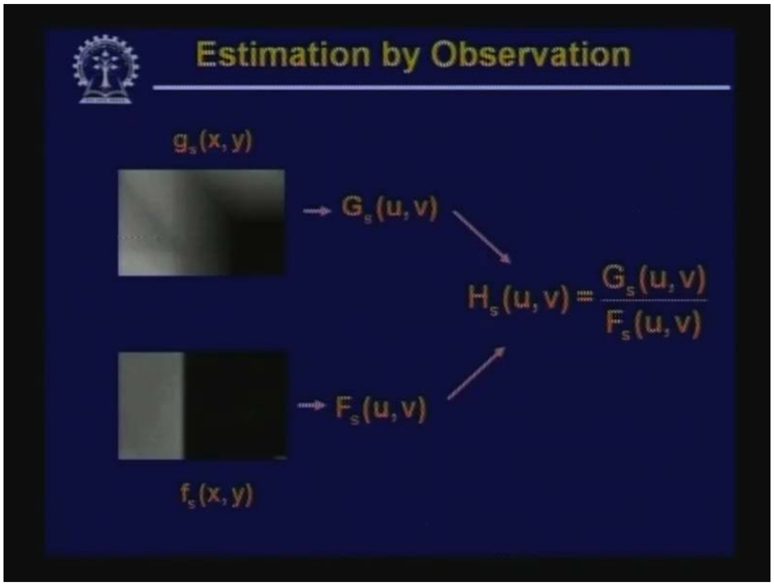
Now, after you identify such a region having simple structures, then what we do is we try to estimate an original image which should have been degraded to give you this degraded image and this original image should be of same size as the image that has been chosen from the sub image which has been chosen from the degraded image, their structure should be same and the gray level regions in this estimated image should be obtained by observing the gray levels in different regions of the image of the sub image sub degraded image that has been chosen and once I get this, this is my approximate reconstructed image say f hat (x, y) and this is my degraded image, let me call it as g hat (x, y).

And once I get this, then I take the Fourier transform of this g hat (x, y) or because it is sub image, instead of calling it as g hat, let me call it as $g_s$ $g_s$ (x, y); I take the Fourier transform of this to get capital $G_s$ (u, v). Similarly, the image that I have reconstructed that I have formed by observation; what should be the actual image? I call it $f_s$ (x, y) and from this, if I take the Fourier transform, I get the Fourier coefficients given by $F_s$ (u, v).

Now, our purpose is that we can have an estimation of the degradation function which is given by $H_s$ (u, v). That should be estimated as $G_s$ (u, v) upon $F_s$ (u, v). So, while doing this, you find that when we have got this particular expression, what we have done is we have neglected the noise term. Now, in order for this to be a logical, a logical one, this approach to be a logical one; when I choose a sub image in the original image of which the reconstructed image should have been this. This sub image should be in a region where the image content is very strong to minimize the effect of the noise in this particular estimation of $F_s$ (u, v) $H_s$ (u, v).

Now, this $H_s$ (u, v) has been approximated over a small sub region of the degraded image and then we have formed an approximation of the degraded image that what should have been the original image. So naturally, this $H_s$ (u, v) is of smaller size. But for the restoration purpose, we need H (u, v) to be of size m by n if my original image is of size capital M by capital N. So, the next operation will be that you extend this $H_s$ (u, v) to H (u, v) to encompass all the pixels, all the frequency components of that particular image.

(Refer Slide Time: 19:21)

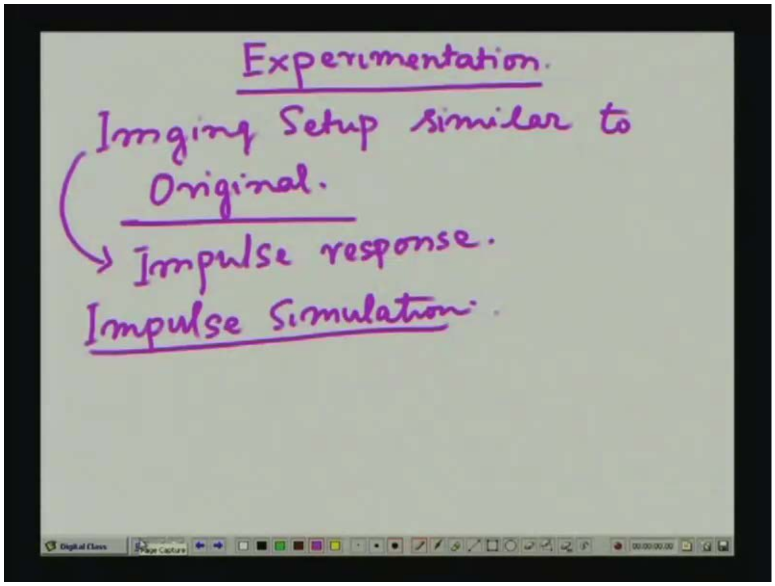



Now, let us just look at an example. Say here, we have shown a degraded image, say this is a degraded image which has been cut out from a bigger degraded image. So, this degraded image has been cut out from a bigger degraded image and by observation, we form original image like this. So, … if this is the degraded image, then the original image would have been something like this and while construction of this approximate original image, you find that in this region, the intensity value is maintained to be similar to the intensity value in this region.

Similarly in this region, the intensity value is maintained to be similar to the intensity value of this. So, this is my $f_s$ (x, y) and this one is my $g_s$ (x, y). So, from this, by taking Fourier transform, we will compute $F_s$ capital $F_s$ (u, v) and from here, by using the Fourier transformation, we will compute $G_s$ (u, v).

So, by combining this 2, from this 2; now, I can have an estimation of the degradation function which is given by $H_s$ (u, v) which is equal to $G_s$ (u, v) upon $F_s$ (u, v). So, this is the method that we can use for estimation by observation. The next technique, the other technique for estimation of the degraded function is by experimentation.

(Refer Slide Time: 21:49)




So, what we do in case of this experimentation? Here, we try to get an imaging setup which is similar to the imaging setup using which the degraded image has been obtained. So first, we have to get an imaging setup similar to the original imaging setup and the assumption is that using this imaging setup which is similar to the original imaging setup, if I can estimate what is the degradation function of this imaging setup which has been acquired which is similar to the original; then the same degradation function also applies to the original one.

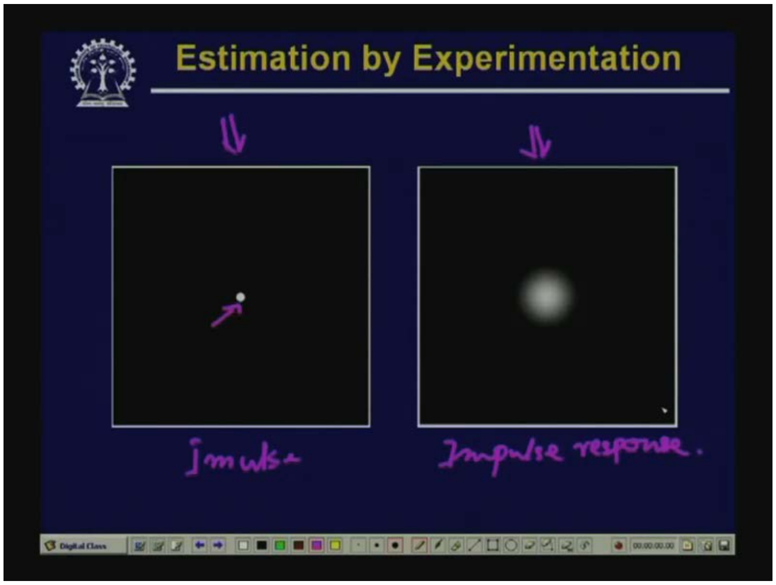
So here, our purpose will be to find out the point spread function or the impulse response of this imaging setup. So, our idea will be to obtain the impulse response of this imaging setup and as we have said earlier, during our earlier discussion that it is the impulse response which fully characterizes any particular system. So, once the impulse response is known, the response of that system to any arbitrary input can be computed from the impulse response.

So, our idea here is that we want to obtain the impulse response of this imaging setup and we assume because this imaging setup is similar to the original that the same impulse response is also valid for the original imaging setup. So here, the first operation that we have to do is we have to simulate an impulse. So, first requirement is impulse simulation.

Now, how do you simulate an impulse? An impulse can be simulated by a very bright spot of light and because our imaging setup is a camera, so we will have a bright spot as small as possible of light falling on the camera and this bright spot if it is very small, then it is equivalent to an impulse and using this bright spot of light as an input, whatever image that we get that is the response to that bright spot of light which in our case is an impulse.

So, the image gives you the impulse response to an impulse which is imparted in the form of bright spot of light and the intensity of light that you generate that tells you what is the strength of that particular impulse. So, by this simulated impulse and from the image that you get, I get the impulse response and this impulse response is the one which uniquely characterizes our imaging setup and in this case, we assume that this impulse response will also be valid for the original imaging setup.  So, now let us see that how this impulse response will look like.

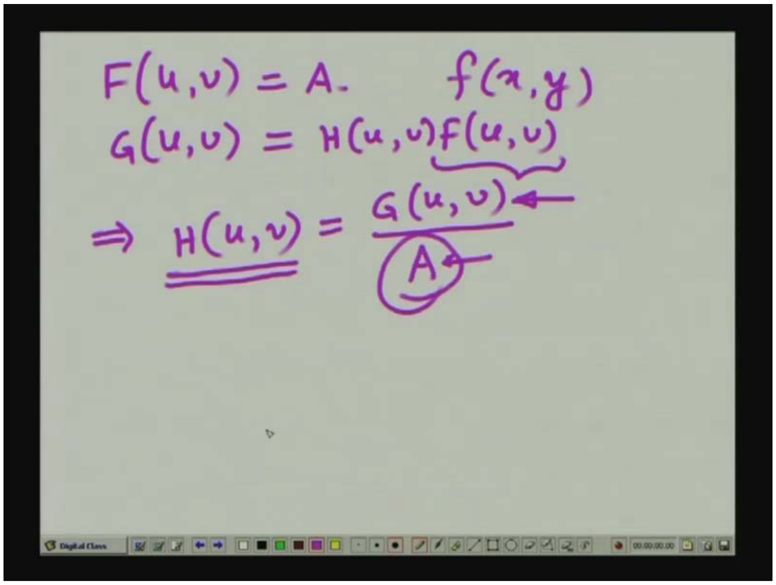
(Refer Slide Time: 25:35)




So, that is what has been shown in this particular slide. The left most image is the simulated impulse. Here you find that at the center, we have a bright spot of light. Of course, this spot is shown in a magnified form, in reality this spot will be even smaller than this and on the right hand side, the image that you have got, this is the image which is captured by the camera when this impulse falls on this camera lens.

So, this is my impulse, simulated impulse and this is what is my impulse response. So, once I have the impulse and this impulse response; then from this, I can find out what is the degradation function of this imaging system. Now, we know from our earlier discussion that for a very very narrow impulse, the Fourier transformation of an impulse is a constant.

(Refer Slide Time: 27:05)




That means F (u, v) where f (x, y) is the input image; in this particular case, it is the impulse. In that case, Fourier transform of f (x, y) which is F (u, v), this will be a constant say constant A and our relation is that the observed image G (u, v) which will be same as H (u, v) times F (u, v). Now, because this F (u, v) is now the impulse response in frequency domain; so from here, I straight away get H (u, v) that is the degradation function which is same as G (u, v) upon that same constant A.

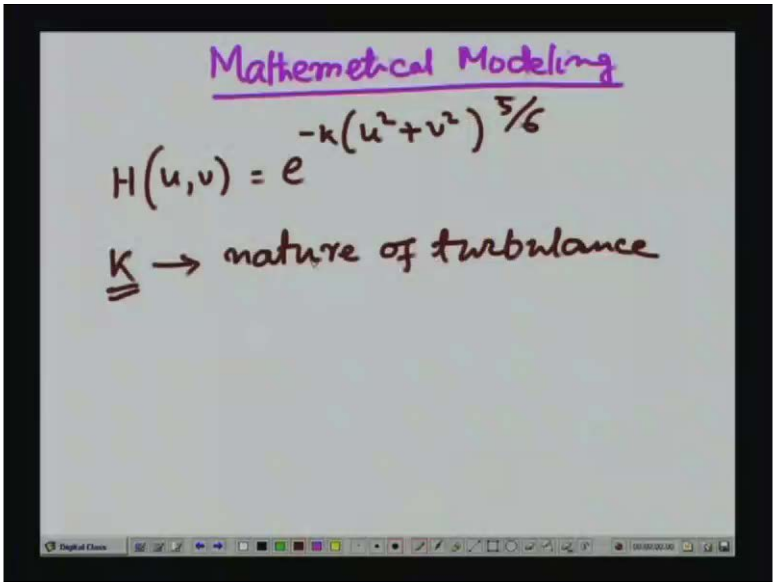
So, in this case, this G (u, v) is the Fourier transform of the observed image and here, this Fourier transform is nothing but the Fourier transform of the image that we have got which is response to the simulated impulse that has fallen on the camera. A is the Fourier transform of the impulse falling on the lens and the ratio of these 2 that is G (u, v) by this constant A that gives us what is the deformation or what is the degradation model of this particular imaging setup.

So here, we find that we have got the degradation function through an experiment or experimental setup is we have an imaging setup and we have a light source which can simulate an impulse. Using that impulse, we got an image which is the impulse response of this imaging system. We assume that the Fourier transform of the impulse or that is true is a constant A as has been shown here. We obtain the Fourier transform of the response which is G (u, v) and now this G (u, v) divided by A should be equal to the degradation function H (u, v) which is the degradation function of this particular imaging setup.

So, I get the degradation function and the same degradation function, we assume that it is also valid for the actual imaging system. Now, in this point, regarding this, one point should be kept in mind that the intensity of the light which is the simulated impulse should be very very high so that the effect of noise is reduced.

If the intensity of light is not very high, if the light is very feeble; in that case, it is the noise component which will be very very dominant and using that whatever estimation of this H (u, v), we get, that estimation will not be a correct estimation or in any case, we will not get a correct estimation. But it will be very far from the reality.

(Refer Slide Time: 30:10)



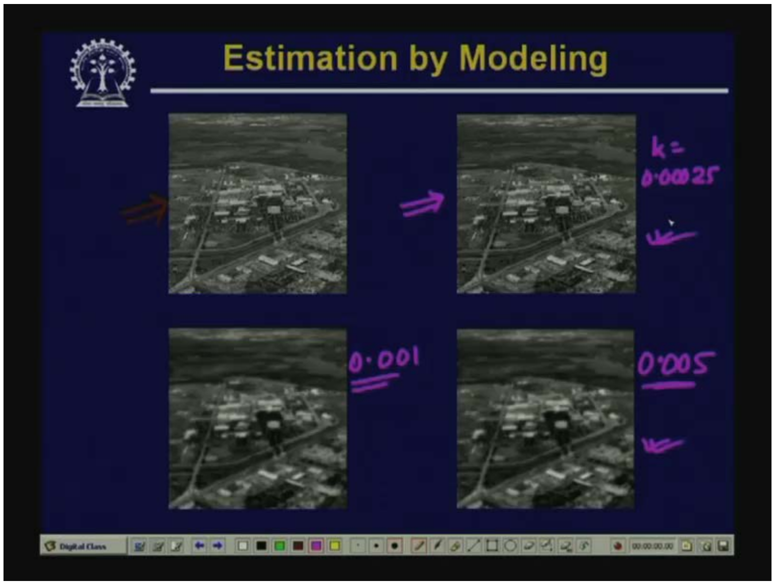

Now, the third approach of this estimation technique as we said that is estimation by mathematical modeling. Now, this mathematical modeling approach for estimation of the degradation function has been used for many many years. There are some strong reasons for using this mathematical approach.

The first one is it provides an insight into the degradation process. Once I have a mathematical model for degradation, I can have an insight into the degradation process. The second reason is such a mathematical model can model even the atmospheric disturbance which leads to degradation of the image. Now, once such mathematical model which is used to model the degradation and this also can model the atmospheric turbulence which leads to degradation of the degradation of the image is given by this expression - H (u, v) is equal to e to the power minus k into u square plus v square to the power 5 by 6.

So, this is one of the mathematical models of degradation which is capable of modeling the turbulence, the atmospheric turbulence that also leads to degradation in the observed image and here, this particular constant K, this gives you what is the nature of the turbulence.

So, if the value of K is large, that means the turbulence is very strong whereas if the value of K is very low, it says that the turbulence is not that strong, it is a mild turbulence. So, by varying the value of k, we can have the intensity of the turbulence that is to be moderate. Now, using this, we can have a number of degraded images as has been shown in this particular slide.

(Refer Slide Time: 32:50)

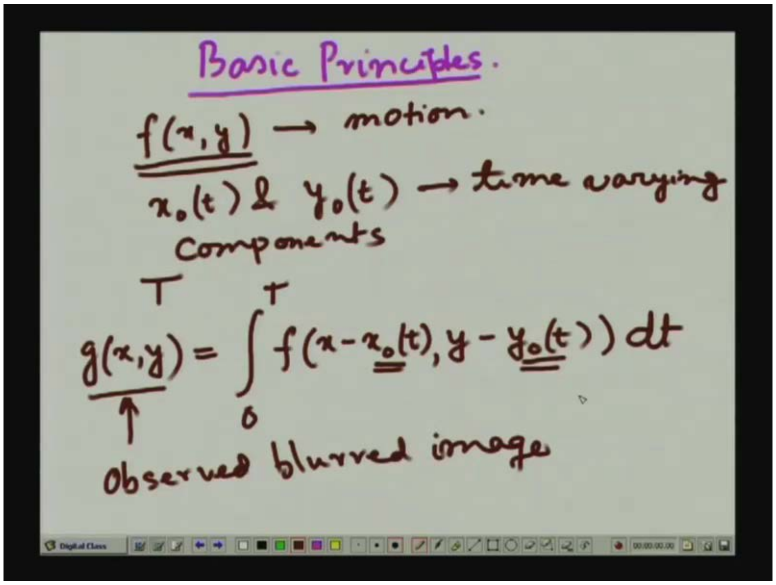



So here, you find that on the top left, we have this original image. This shows an original image. This is a degraded image where the value of k was something like 0.00025; this is the value of k in this particular case. Here, the value of k was something like 0.005 and in this case, the value of k was something like 0.001 sorry here it was 0.001 and in this case, it was 0.005.

So, the first image, this particular image; here the turbulence is very poor. So, this has been degraded using the same model as we have just said which models mild turbulence. Here, the turbulence is medium and here the turbulence is strong and if you closely look at these images, you find that all these 3 images are degraded to some extent. In this particular case, the degradation is maximum, here the degradation is minimum.

So, this is the one which gives you modeling of degradation which occurs because of turbulence. Now, there are other approaches of degradation mathematical model to estimate the degradation which are obtained by fundamental principles. So, from the basic principles also, we can obtain what should be the degradation function.

(Refer Slide Time: 35:01)




So, one such case, so here, we will discuss the basic principle, the degradation model estimation from basic principles and I try to find out what will be the degradation model, degradation function where the image is degraded by linear motion and this is a very very common situation that if we try to estimate or if we try to image a fast moving object; in many cases, we find that the image that we get is degraded. There is some sort of blurring which is known as motion blurring and this motion blurring occurs due to the fact that whenever we take the snap of the scene, the shutter of the camera is open for certain duration of time and during this period, during which the shutter is open, the object is not stationary, the object is moving.

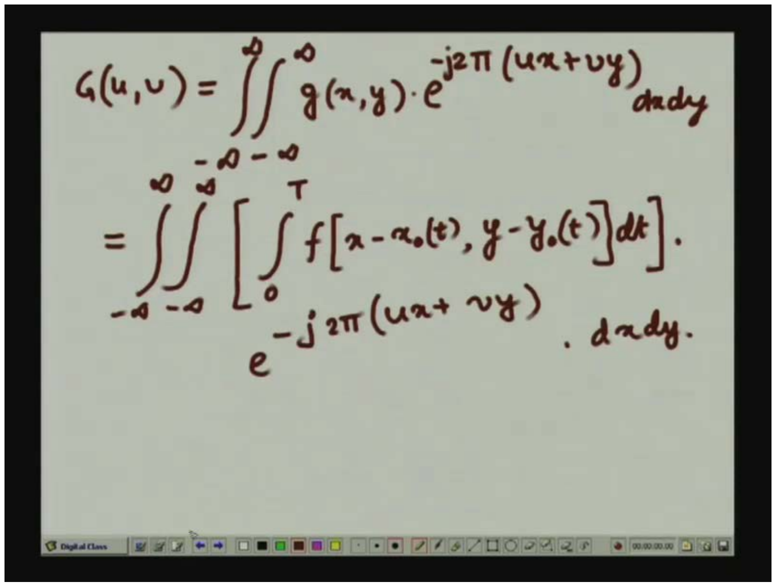
So, considering any particular point in the imaging plane, here the light which arrives from the scene does not come from a single point. But the light you get at a particular point on the imaging sensor is the aggregation of the reflected light from various points in the scene. So, that tells us that what should be the basic approach to model to estimate the degradation model in case of motion of the scene with respect to the camera.

So, that is what we are trying to estimate here. So here, we assume that the image f (x, y), this undergoes motion and when f (x, y) under goes motion, then there will be some moving component. So, I assume 2 components $x_0$ (t) and $y_0$ (t) which is the moving components or time varying components. So, these are the time varying components along x direction and y direction respectively.

So, once the object is moving, then the intensity, the total exposure at any point in the imaging plane can be obtained by aggregation operation or integration operation where the integration has to be done over the period during which the shutter remains open. So, if I assume the shuttered image open for a time duration given by capital T; in that case, the total exposure at any point which is the observation at point (x, y) given by g (x, y) will be of this form - f (x minus $x_0$ (t), y minus $y_0$ (t) dt and integration of this from 0 to capital T.

So here, the capital T is the duration of time during which the shutter of the camera remains on and $x_0$ (t) and $y_0$ (t), these 2 terms, they are the time varying components along x direction and y direction respectively and this g (x, y) gives us the observed blurred image. Now from this, we have to estimate what is the degradation function or the blurring function.

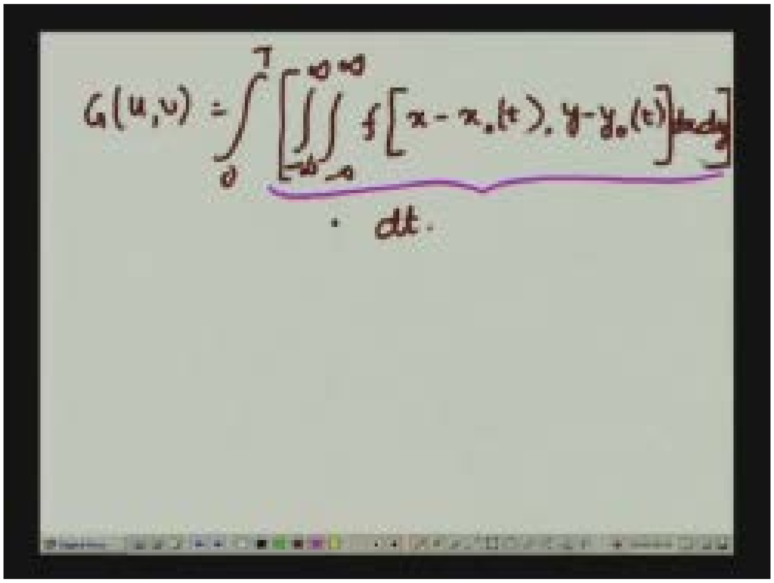
(Refer Slide Time: 39:02)




So, once we get g (x, y), then our purpose is to get the Fourier transform of this that means we are interested in the Fourier transformation G (u, v) of g (x, y) and this G (u, v) as we know from the Fourier transformation equations is given by g (x, y) into e to the power minus j 2 phi (ux plus vy) dx dy and take the integration, double integration from minus infinity to infinity over both x and y.

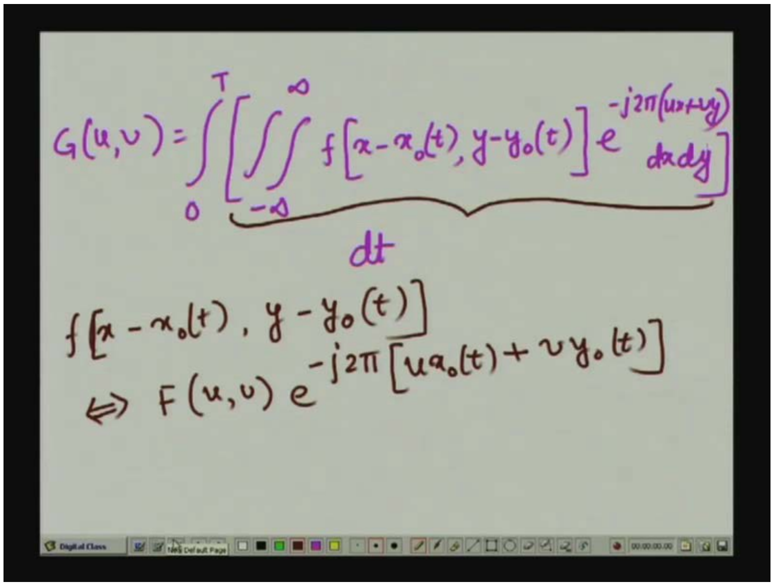
So, it is this expression, using this Fourier transformation expression, we can find out what will be G (u, v) that is Fourier transformation of the degraded image g (x, y) and if I derive this, it will be of this form; so, minus infinity to infinity, again minus infinity to infinity. Now, this g (x, y) is to be replaced by 0 to T, the expression that we have got earlier f [x minus $x_0$ (t), y minus $y_0$ (t)] dt into e to the power minus j 2 phi (ux plus vy) into dx dy. So, if we just do some re organization of this particular integral equation, we can write G (u, v) in the form.

(Refer Slide Time: 41:13)



G (u, v) equal to integral 0 to T, double integral minus infinity to infinity, minus infinity to infinity f [x minus $x_0$ (t), y minus $y_0$ (t)] dx dy into dt. Now, from this particular expression, you find that the expression within this bracket sorry there will be some more addition, so the final expression will be like this - G (u, v) will be equal to 0 to capital T. Then within bracket we have to have this double integral varying from minus infinity to infinity f [x minus $x_0$ (t), y minus $y_0$ (t)] e to the power minus j 2 phi (ux plus vy) dt and then dx dy and then dt. So, this will be the final expression.

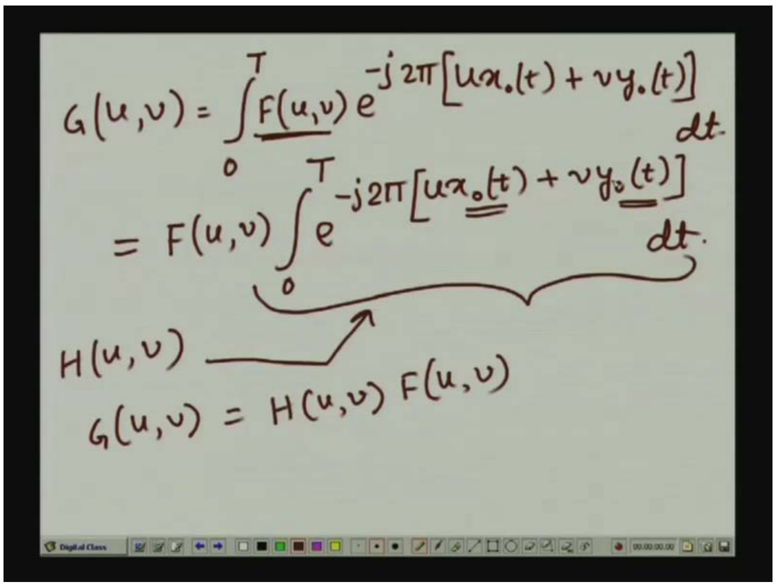
(Refer Slide Time: 42:19)



Now, in this, if you look at this inner part, this is nothing but the Fourier transformation of shifted f (x, y) where the shift in the x direction is by $x_0$ (t) and shift in the y direction is by $y_0$ (t). And, from the property of Fourier transformation we know that the Fourier transformation is shift invariant in the sense that the Fourier transforms magnitude will remain the same, only it will introduce some.

So, by doing that we can say that this f [x minus $x_0$ (t), y minus $y_0$ (t), this will have a Fourier transformation which is nothing but F (u, v) e to the power minus j 2 phi into [u$x_0$ (t) plus v$y_0$ (t)]. So, this is from the translation invariant property of the Fourier transformation.

(Refer Slide Time: 44:46)

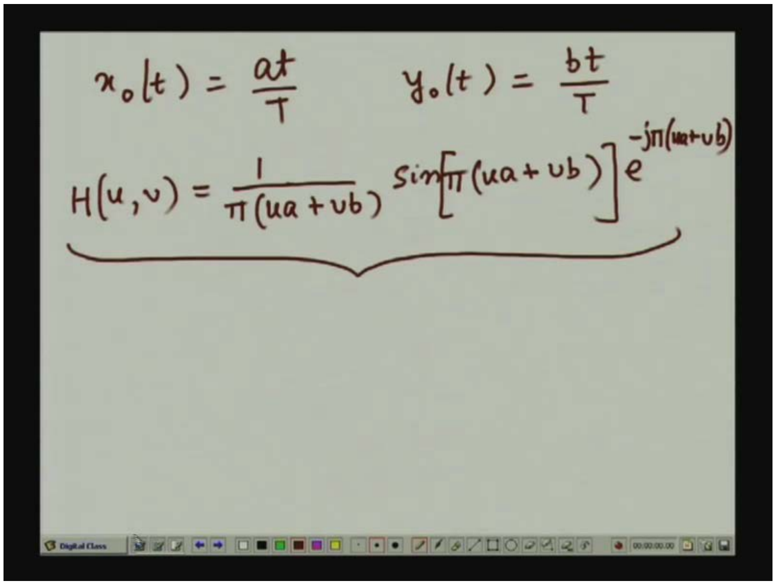



So, using this expression, now the expression for G (u, v) can be written as G (u, v) will be equal to integral 0 to capital T F (u, v) e to the power minus j 2 phi [u$x_0$ (t) plus v$y_0$ (t)] dt and because this term F (u, v) is independent of T, so you can take this term F (u, v) outside the integration. So, the final expression that we get is F (u, v) into integral 0 to capital T e to the power minus j phi j 2 phi [u$x_0$ (t) plus v $_0$ (t)] dt.

So from this, you find that now if I define my degradation function H (u, v) to be this particular integration, so if I define H (u, v) to be this; then I get expression for G (u, v) is equal to H (u, v) into F (u, v). So here, this motion term, the degradation function is given by integration of this particular expression and in this expression, this $x_0$ (t) and $y_0$ (t), they are the motion variables which are known.

So, if the motion variables are known; then using those motion variables, the values of the motion variables, I can find out what will be the degradation function and using that degradation function, I can go for the degradation model. So, I know H (u, v), I know G (u, v) and from that I can find out the restored image F (u, v).

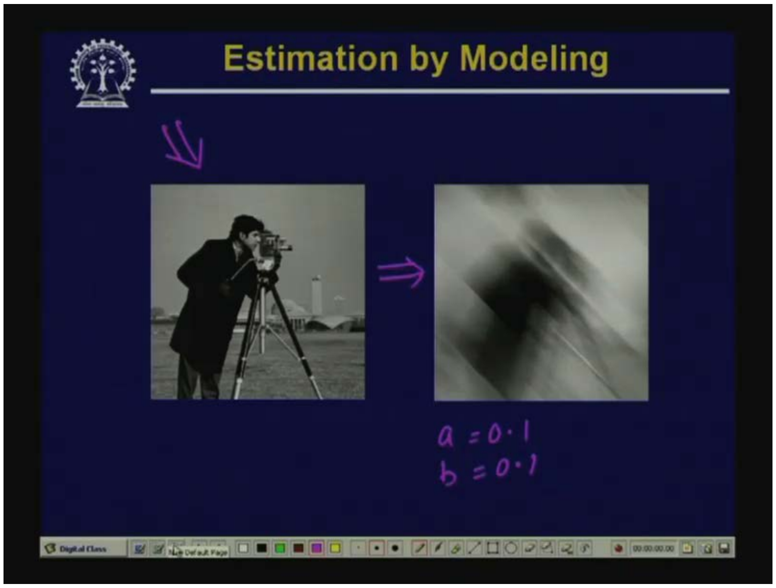
(Refer Slide Time: 47:15)




Now, in this particular case if I assume that $x_0$ (t) is equal to at upon T and similarly $y_0$ (t), I assume this also to be some constant bt upon capital T. That means over a period of capital T, during which the camera shutter is open; in the x direction, the movement is by an amount a and in the y direction, the movement is by an amount b.

So by using, by assuming this, we can find that H (u, v) by using that integration, by computing that integration will given by 1 upon phi (ua plus vb) into sin phi (ua plus vb) into e to the power minus j phi into (ua plus vb). So, this is what is the degradation function or the blurring function. So now, let us see that using this degradation function what is the kind of degradation that is actually obtained.

(Refer Slide Time: 48:40)

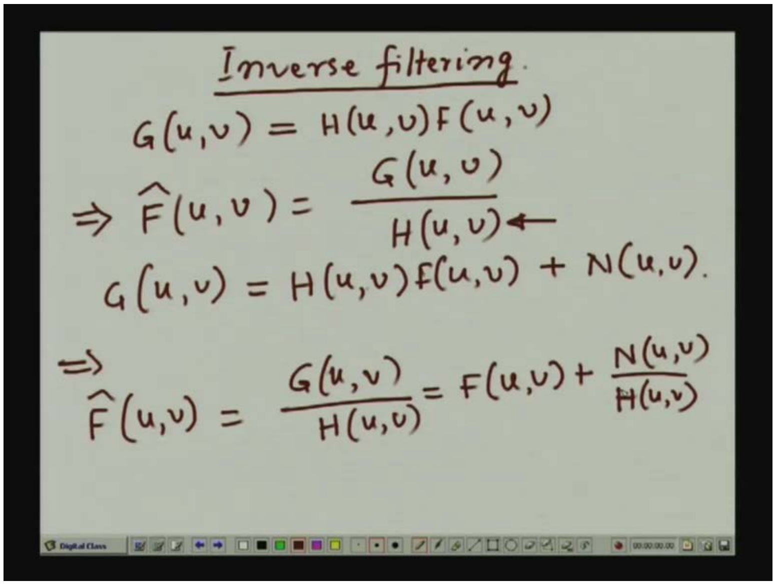



So, here again on the left hand side, we have an original image and on the right hand side, this is the corresponding blurred image where the blurring is introduced assuming uniform linear motion and for obtaining this particular blurring, here we have assumed a is equal to 0.1 and b is also equal to 0.1.

So, using this values of a and b, we have obtained this, we have obtained a blurring function or degradation function and using this degradation function, we have obtained this type of degraded model and you find that this is quite a common scene whenever you take the image of a very fast moving object the kind of degradation that you obtain in the image is similar to this.

Now, the problem is we have obtained a degradation function. Now, once I obtained a degradation function or an estimated degradation function, now given a blurred image; how to restore the original image or how to recover the original image? So, as we have mentioned that there are different types of filtering techniques for obtaining or for restoring the original image from a degraded image. The simplest kind of filtering technique is what is known as inverse filtering.

(Refer Slide Time: 50:20)




Now, the concept of inverse filtering is very simple. Our expression is that G (u, v) that is the Fourier transform of the degraded image is given by H (u, v) into F (u, v) where H (u, v) is the degradation function in the frequency domain and F (u, v) is the Fourier transform of the original image, G (u, v) is the Fourier transform of the degraded image

Now, because this H (u, v) into F (u, v), this is a point by point multiplication. That is for every value u and v, the corresponding F component and the corresponding H component will be multiplied together to give you the final matrix which is again in the frequency domain.

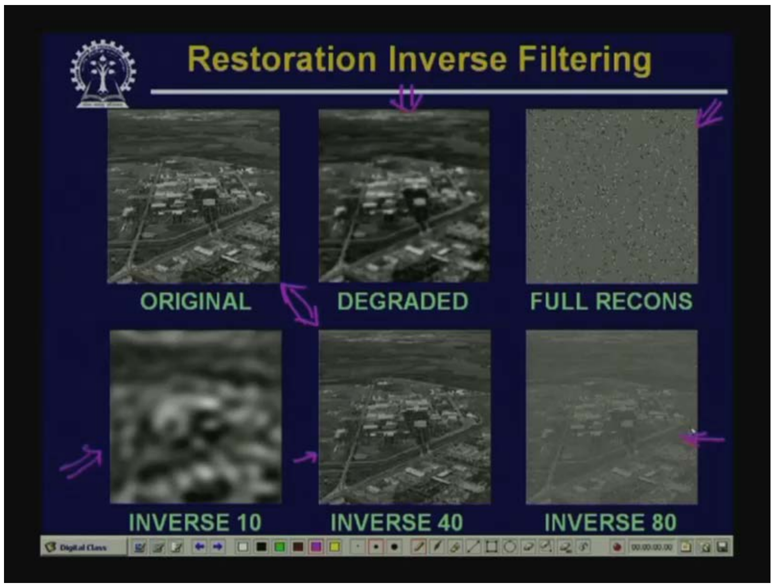
Now, from this expression, it is quite obvious that I can have F (u, v) which is given by G (u, v) upon H (u, v) where this H (u, v) is our degradation function in the frequency domain and G (u, v), I can always compute by taking the Fourier transformation of the degraded image that is obtained. So, if I divide the Fourier transformation of the degraded image by the degradation function in frequency domain; what I get is the Fourier transformation of the original image and as I said that when I compute this H (u, v), this is just an estimated H (u, v). It will never be exact.

So, the reconstruction of the recovered image that we get is not the actual image but it is an approximate image, approximate original image which we represent by H (u, v) F hat (u, v). Now here, as we have already said that G (u, v) if I consider the noise term is given by H (u, v) into F (u, v) plus the noise term N (u, v).

Now from here, if I compute the Fourier transform of the reconstructed image that will be F hat u v which is equal to G (u, v) upon H (u, v) and from this expression, this is nothing but F (u, v) plus N (u, v) upon H (u, v). So, this expression says that even if H (u, v) is known exactly, the perfect reconstruction may not be possible because we have seen earlier that in most of the cases, the Fourier transformation coefficients are very very small when the value of u and v is very large.

So, that means for those cases, N (u, v) by H (u, v), this term will be very high that means the reconstructed image will be dominated by noise and that is what is obtained practically also. So, to avoid this problem what will be what we have to do is for reconstruction purpose, instead of considering the entire frequency plane, we have to restrict our reconstruction to a component of the frequencies in the frequency plane which are nearer to 0. So, if I do that kind of reconstruction, that limited reconstruction; in that case, the dominance of noise can be avoided. Now, let us see that what kind of result we can obtain using this inverse filtering.

(Refer Slide Time: 54:46)



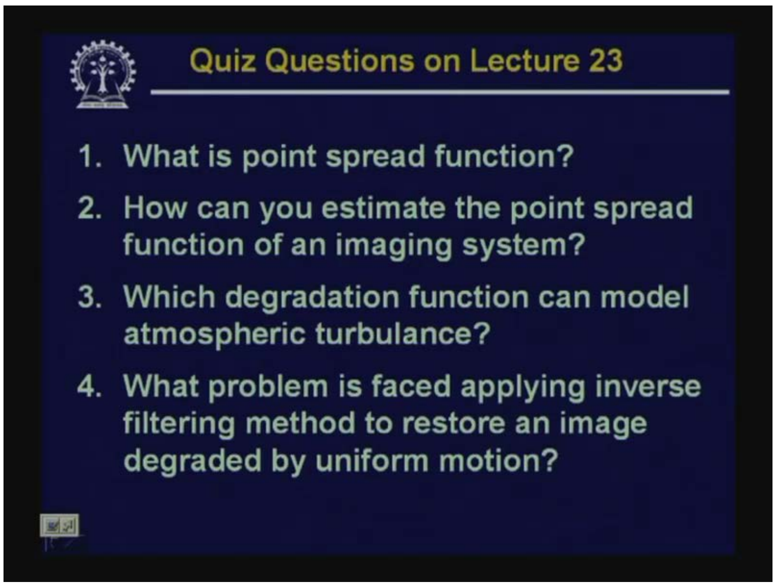

So, this shows an inverse filtering result here. We have the original image, in the middle we have degraded image. So, this degraded image we had already shown, so this is the degraded image and you find that on the right hand side, this is the reconstructed image using the inverse filtering when for reconstruction all the frequency coefficients are considered and as we have said that as you go away from (0, 0) component in the frequency domain that is as you go away from the origin, H (u, v) term becomes very very negligible. So, it is the noise term which tends to dominate.

So, you find that in this reconstructed image, nothing is available whereas if we go for restricted reconstruction that is we consider only few frequency components near the origin as has been shown here that we have considered only those frequency terms within a radius of 10 from the origin. So, this is the reconstructed image and as it is obvious because our domain of reconstruction or the frequency components that we have considered as very very limited, so the image becomes reconstructed image becomes very blurred and that is the property of the low pass transform. This is nothing but a low pass filter and that is the property of low pass filter. If the cut off frequency is very low, then the reconstructed image has to be very very blurred.

In the middle of the bottom row, again we have shown the reconstructed image but in this case, we have increased the cut off frequency; the cut off frequency instead of using 10, now we have used cut off frequency equal to 40 and here you find that if you compare the original image with this reconstructed image, you find that the reconstruction is quite accurate. If I increase the cut off frequency further as we said that it is the noise term which is going to dominate; so, on the right most, here we have increased the cut off frequency to 40.

So here, you find that we can observe the reconstructed image but as if the objects are behind a curtain of noise. That means it is the noise term which is going to dominate as we increase the cut off frequency of the filter. So, with this, we complete our today's discussion. Now, let us come to the questions on today's lecture.

(Refer Slide Time: 57:17)




So, the first one is what is point spread function? The second one is how can you estimate the point spread function of an imaging system? Third question, which degradation function can model atmospheric turbulence? And the fourth question, what problem is faced applying inverse filtering method to restore an image degraded by uniform motion?

Thank you.