Probability and Random Variables/Processes for Wireless Communication
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Module No. 2
Lecture 11
Mean, Variance of Random Variables

Hello, welcome to another module in this massive open online course on probability and random variables for wireless communications. So we have started looking at random variables. That is, random variables which can take values randomly on the real line or from a subset of the real line. We have defined the probability density function to characterize the behavior of these random variables, to characterize the probability of these random or the probability density rather of these random variables. Today, let us start looking at 2 other important parameters that are associated with any random variable. That is, the mean and the variance.

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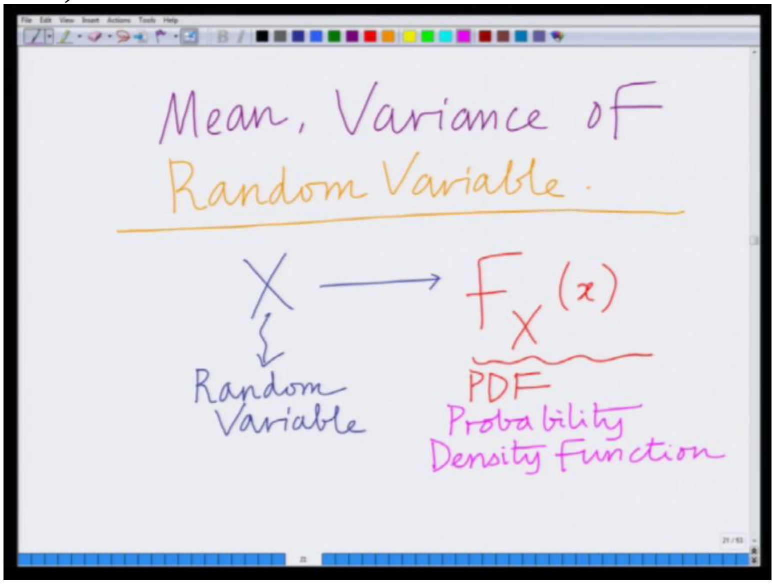

So in today's lecture, we will look at the mean and the variance of random variables. The mean, as you must already be familiar, is basically say I have a random variable, X. This is my random variable which is characterized by the probability density function $F_x(x)$. So this is a corresponding PDF that is basically our probability density function. So we are considering a random variable, X which is characterized by the pdf $F_x(x)$. Let us now define the mean of this random variable, X.

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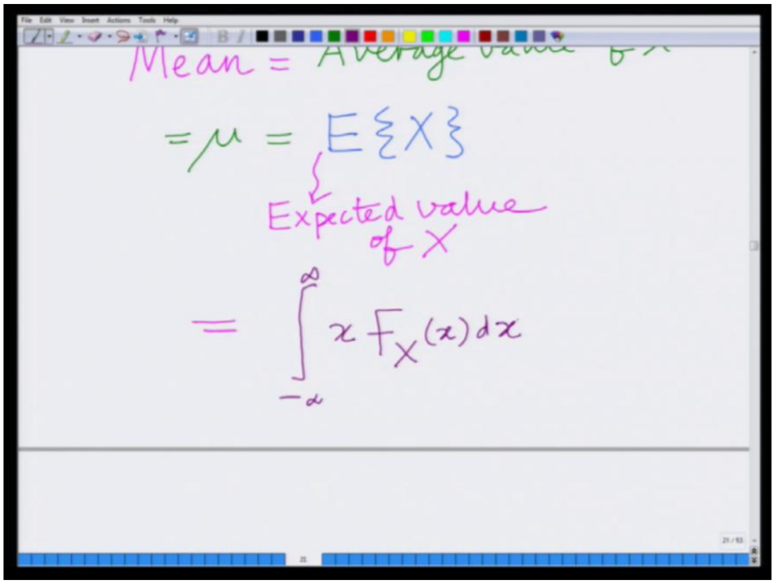

And the mean of this random variable, that is defined as,

$$\mu = E\{X\}$$

This basically stands for the expected value of the random variable X, and this is basically defined as,

$$\mu = \int_{-\infty}^{+\infty} x F_X(x) dx$$

This is the definition. Here $F_X(x)$ is the probability density function. Let us now look at what this means intuitively.

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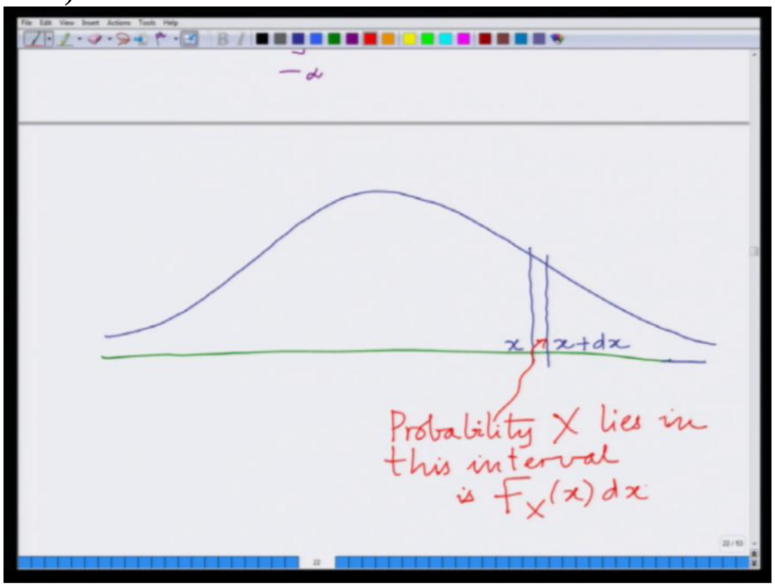

What this means intuitively is the following thing. Let us go back and look at, the schematic for our probability density function. Also let us look at a small interval around x, i.e. length dx around x. That is between x and x + dx.

Now the probability that x takes a value in this interval or x lies in this interval is as we have seen before, $F_x(x)$ xdx. So the probability the random variable X lies in this infinitesimally small interval of length dx around x is **F(x)** times dx where **F(x)** is the probability density function.

Now we take the probability that it lies in these interval and multiply it by the value x. So it takes the value x with the probability **F(x)** dx. Therefore, the mean, the average is basically the weighted average of all these values, that is

$$\int_{-\infty}^{+\infty} x F_X(x) dx$$

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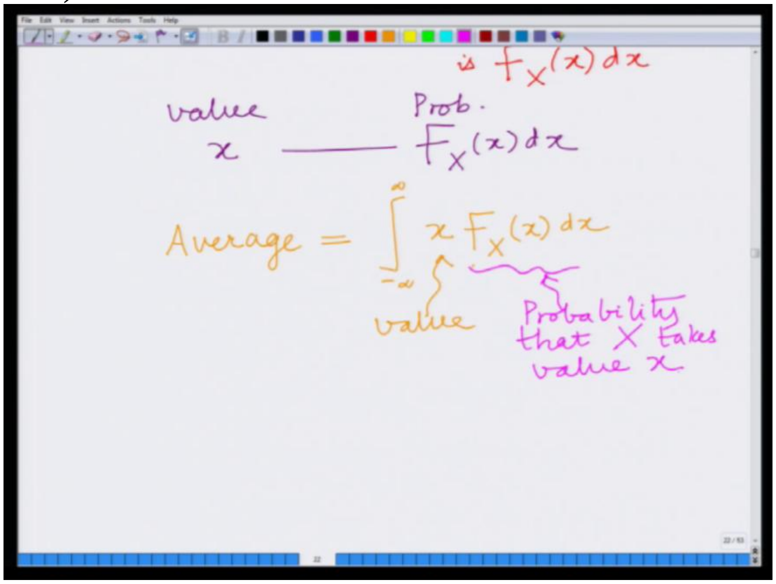

So this is the value x which is weighted by the corresponding probability. This is the corresponding probability. This is the probability that X takes the value of x. So what I am doing? I am taking the value x multiplying it by the corresponding probability, **F(x)** dx and I am integrating it from the interval between the limits, -∞ to ∞. So, I am weighing the values by the corresponding probability and that gives me the average value.

So that is the definition of mean of the random variable X or the expected value which is also denoted by, we said E(X).

Let us now look at another interesting statistic. Let us look at the variance. The variance of the random variable.

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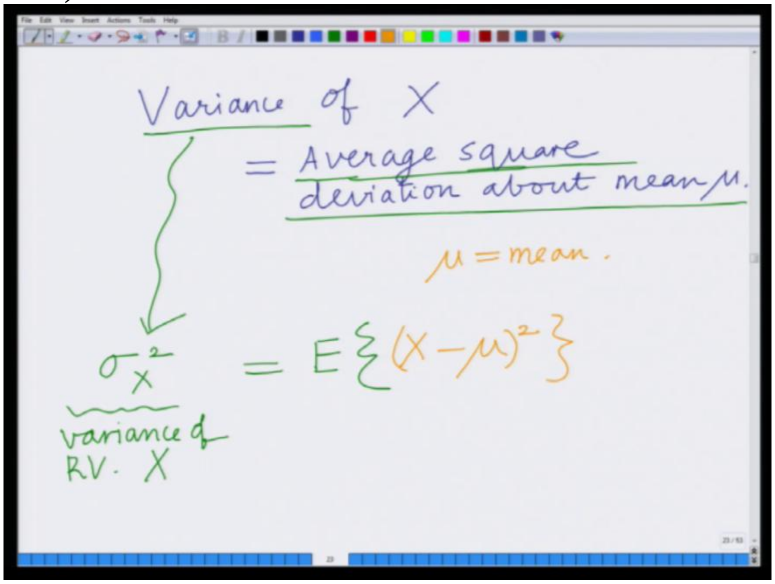

Variance of X is basically the average square deviation about the mean μ. So the variance is basically,

$$\sigma_X^2 = E\{(X-\mu)^2\}$$

This is the variance of random variable X and it is given as the expected value or the average value of $(X-\mu)^2$. Remember,

$\mu = mean\ of\ X,$

$X - \mu = deviation\ from\ the\ mean$

$(X-\mu)^2 = Square\ of\ deviation\ from\ the\ mean$

And we are taking the average of this square of the deviation about the mean.

So as I said, I have to take it for every point small X and I have to wait for the corresponding probability, **F(x)** dx.

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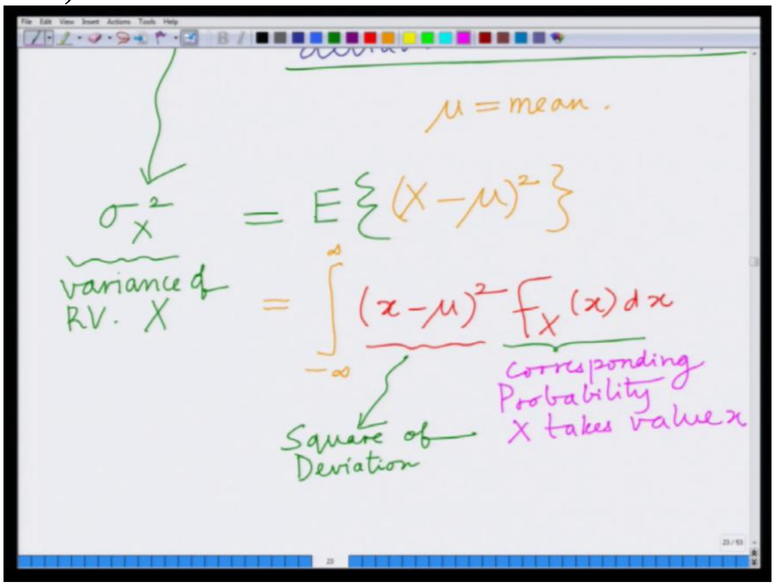

So the average of the square of the deviation is nothing but,

$$\sigma_X^2 = \int_{-\infty}^{+\infty} (X-\mu)^2 F_X(x)dx$$

Basically what we are doing is we are looking at the square of the deviation at any point x and we are weighing it with the corresponding probability. That gives me the average square of the deviation of this random variable.

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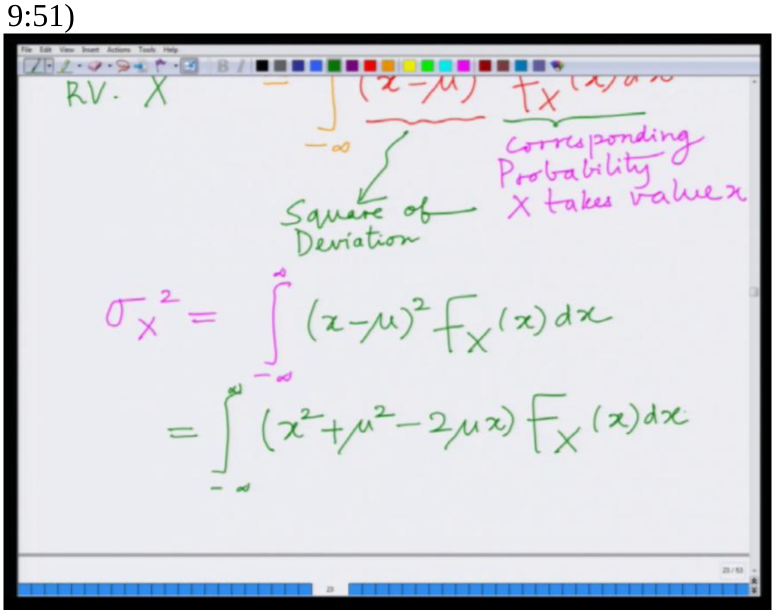

Let us now simplify this expression further. Now if I simplify this expression further,

$$\sigma_X^2 = E[\int_{-\infty}^{+\infty} (x-\mu)^2 F_X(x) dx]$$

$$= E[\int_{-\infty}^{+\infty} (x^2 + \mu^2 - 2\mu x) F_X(x) dx]$$

$$= E[\int_{-\infty}^{+\infty} x^2 F_X(x) dx \ + \ \int_{-\infty}^{+\infty} \mu^2 F_X(x) dx \ - \ \int_{-\infty}^{+\infty} 2\mu x \, F_X(x) dx]$$

$$= E[\int_{-\infty}^{+\infty} x^2 F_X(x) dx] \ + \ \mu^2 E[\int_{-\infty}^{+\infty} F_X(x) dx \,] - \ 2\mu \, E[\int_{-\infty}^{+\infty} x \, F_X(x) dx]$$

as integral from $-\infty$ *to* ∞ *of* ***F(x)*** *dx is 1 because that is the total probability.*

Thus the middle integral term is equal to 1.

$$= E[\int_{-\infty}^{+\infty} x^2 F_X(x) dx] \ + \ \mu^2 . 1 - 2\,\mu . \mu$$

$$= E[\int_{-\infty}^{+\infty} x^2 F_X(x) dx] - \ \mu^2$$

$$= E[\,X^2] - \ \mu^2$$

$$= E[\,X^2] - (E[X])^2$$

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$$= \int_{-\infty}^{\infty} x^2 f_X(x)\,dx + \mu^2 \underbrace{\int_{-\infty}^{\infty} f_X(x)\,dx}_{1} - 2\mu \int_{-\infty}^{\infty} x f_X(x)\,dx$$

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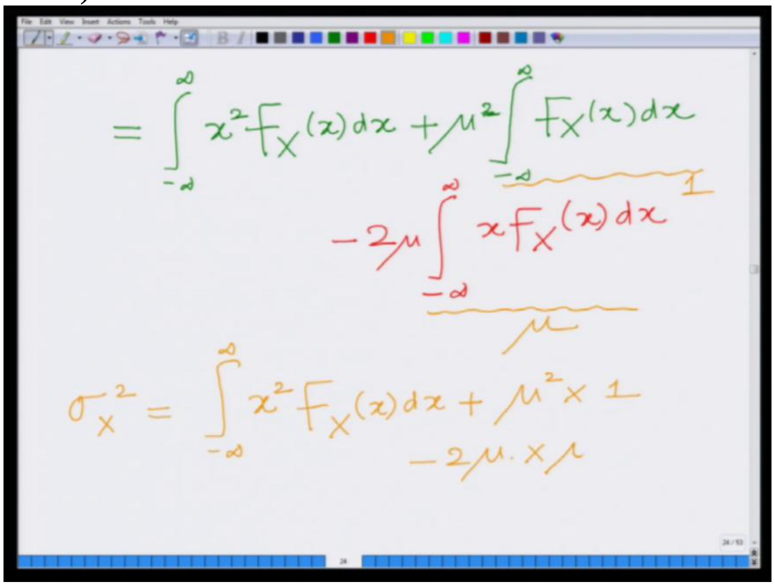

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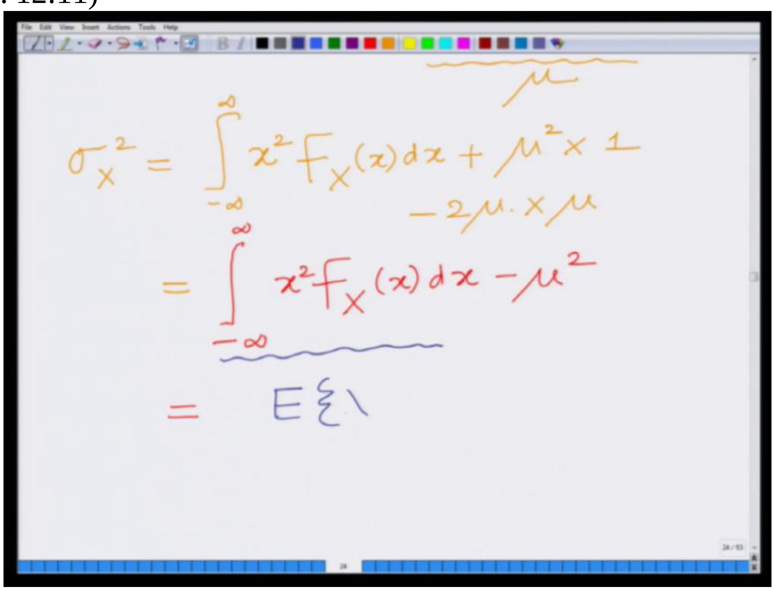

Therefore what we have is something very interesting.

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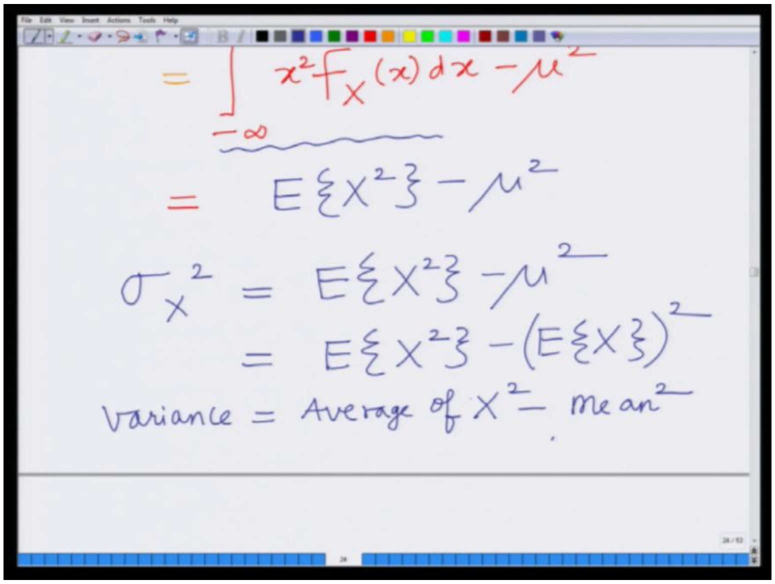

We have the variance of the random variable equals

$$Variance = Average\ of\ X^2 - Square\ of\ Average\ of\ X$$

This is also written as,

$$\sigma_X^2 = E[\,X^2] - (E[X])^2$$

So the variance is basically the average value of the square of the random variable - the square of the average value of the random variable. This is the definition of the variance.

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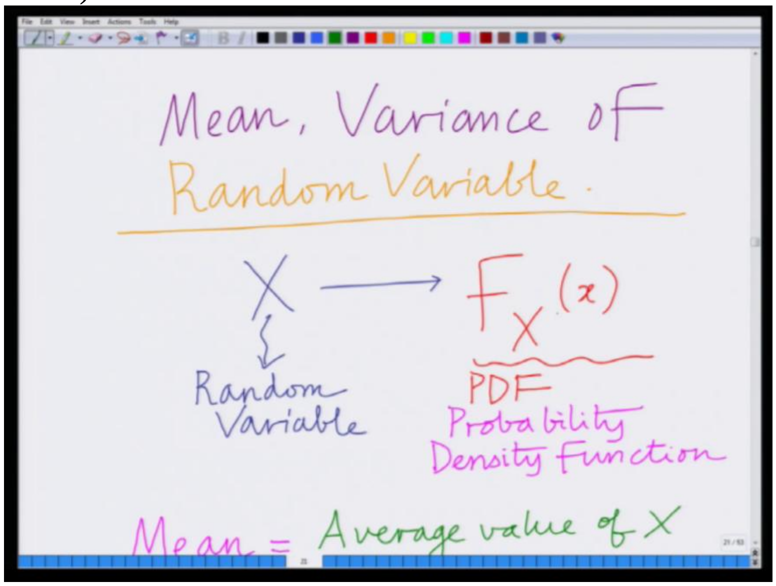

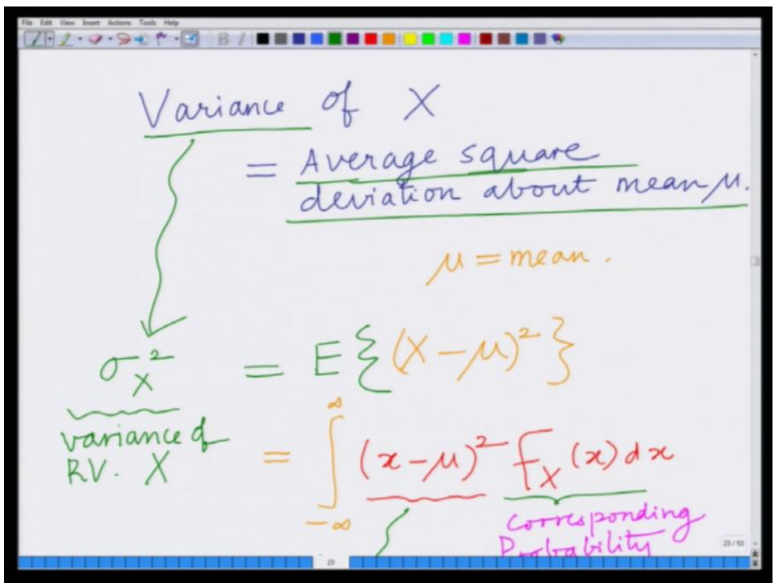

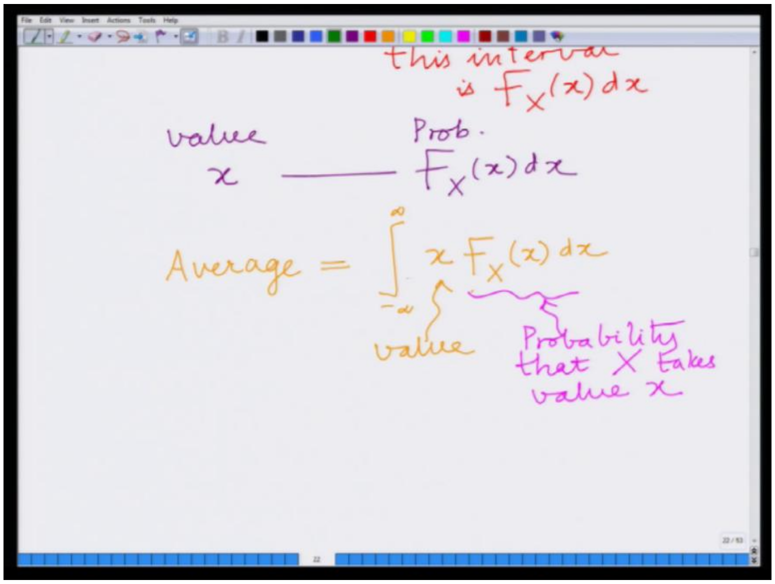

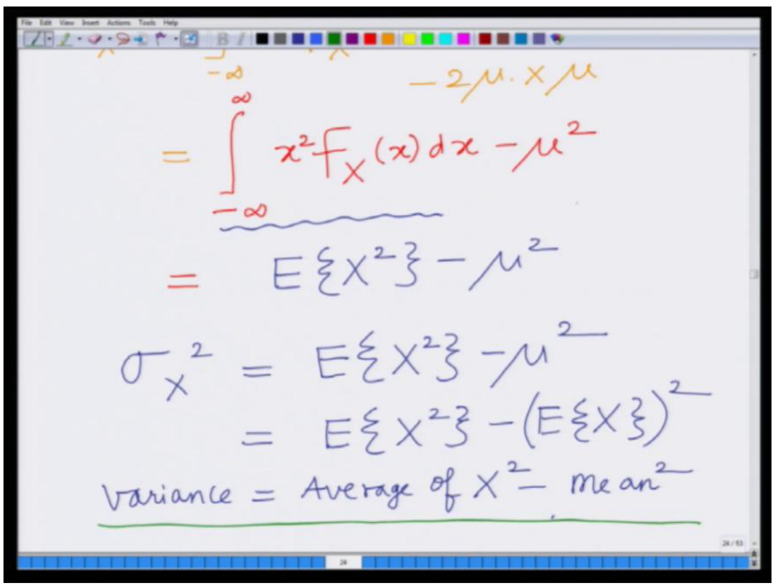

So what we have done so far is if you look at this, basically we have started with the random variable, X, the probability density function, **F(x)** and basically we have defined the mean of this random variable, the mean of this random variable is the average value which is basically integral $-\infty$ *to* ∞ x times **F(x)** dx and we have also looked at the standard deviation of the random variable. We have said that the standard deviation of the random variable is basically the average value of the square of the deviation of the random variable about the mean and we said, this is denoted by Sigma X square, the variance which is equal to basically the expected value of X square, the average value of the square of the random variable - myu where myu is the mean of the random variable. All right? So these are the 2 important parameters for the random variable X. We will stop this module here and look at applications of these in the subsequent modules. Thank you very much.