Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 35
Examples of MIMO Systems

Hello, welcome to another module in this massive open online course on the principles of 3G, 4G wireless communication systems. In the previous module we have looked at a basic introduction to MIMO systems simple model of the MIMO system or the Transmit vector, the receive vector and the MIMO channel matrix.

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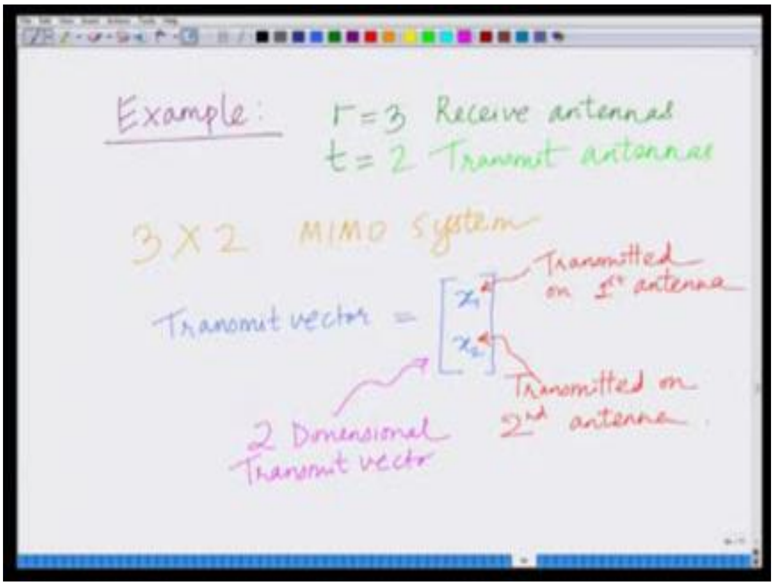

Lest us look at a simple example to understand this better; so, let us look at a simple example of a MIMO system. We will consider a MIMO system with r = 3 receive antennas and t = 2 transmit antennas. So, in effect this is a 3 x 2 MIMO system; we refer to the MIMO system as an r X t MIMO system since r = 3 t = 2; this is 3 x 2 MIMO system. Naturally the Transmit vector is equals x_1, x_2; x_1 is symbol transmitted on the first transmit antenna, x_2 is transmitted on the second antenna and therefore, naturally this is a 2 dimensional Transmit vector what we have is we have a 3 x 2 MIMO channel which means we have 3 receive antennas and 2 transmit antennas. Therefore on each transmit antenna we can transmit a symbol. So, x_1 is a symbol transmitted on the first transmit antenna, x_2 is the symbol transmitted on the second antenna x_1, x_2 is therefore the 2 dimensional Transmit vector which comprises of the symbols transmitted from both the transmit antennas.

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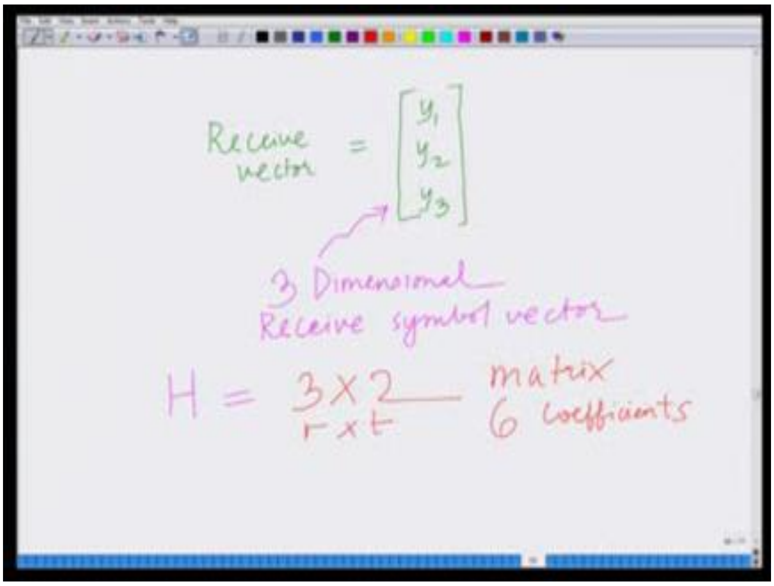

Since we have 3 receive antennas my receive vector equals y_1, y_2, y_3 where y_1 is a symbol received in the first receive antenna, y_2 is the symbol received in the second receive antenna, y_3 is the symbol received on the third receive antenna and therefore, we have a 3 dimensional receive symbol vector. We have 3 dimensional receive symbol vector and naturally the channel matrix H is equal to r X t; so h is a 3 x 2 dimensional basically this is r cross t. So, channel matrix H is basically a 3 x 2 matrix containing a total of 3 times 2 that is 6 channel coefficients. Therefore, I can write this system now as follows.

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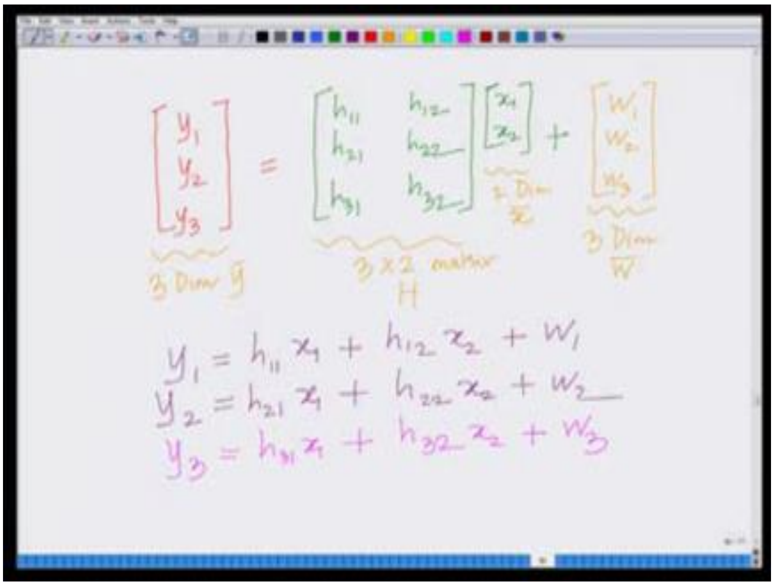

I have the 3 dimensional received vector

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

so, this is my 3 dimensional received vector $\bar{y}$, this is my 3 x 2 matrix which is H, this is my 2 dimensional Transmit vector $\bar{x}$ and this is the 3 dimensional received vector or 3 dimensional noise vector $\bar{w}$.

We have the 3 received symbols y_1, y_2, y_3, the transmit symbols x_1, x_2 and now we have these relations or we have this model that is $\bar{y} = H\,\bar{x} + \bar{w}$. In general when we have an r X t MIMO system for a general MIMO system; for a general r X t MIMO system I can write the symbol y_1 where y_1 is symbol received on the l th receive antenna.

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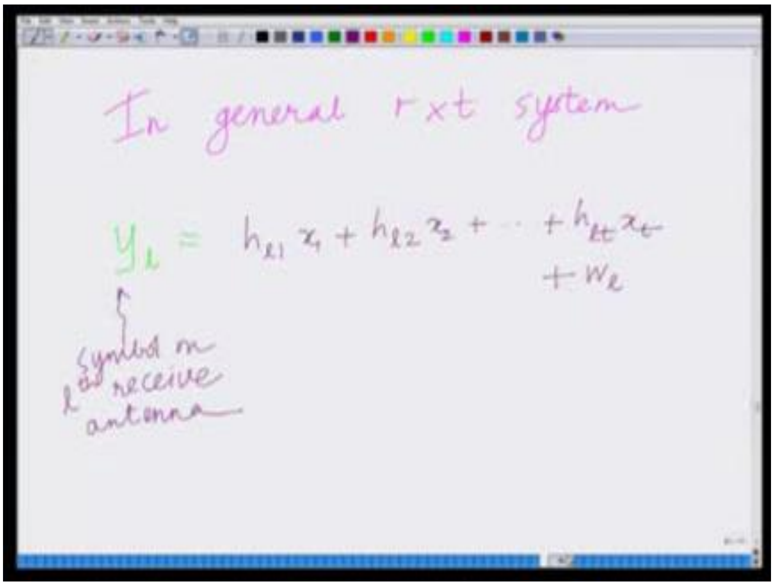

I can write this as

$$y_l = h_{l1}x_1 + h_{l2}x_2 + \ldots + h_{lt}x_t + w_l$$

So, this is the general expression for the symbol received on the l th receive it.

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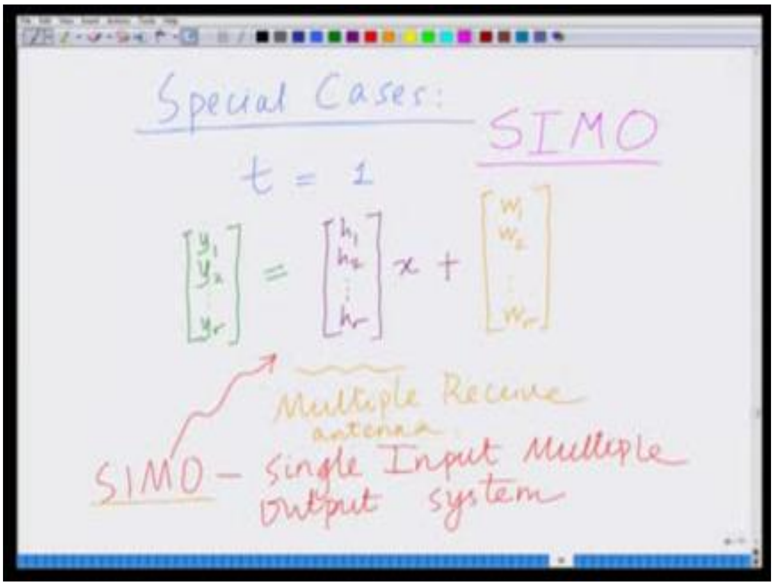

Let us now look at some special cases in this MIMO system. One is something that we are already familiar with when t = 1 when the number of transmit antennas equal to 1; we have an r cross t MIMO channel, but t equals 1. So, we have an r x 1 MIMO channel, but r x 1 MIMO channel is nothing, but a single vector because r x 1 dimensional matrix is nothing, but a vector of r dimensions; therefore the model for this MIMO system can be expressed as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_r \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_r \end{bmatrix} x + \begin{bmatrix} w_1 \\ w_2 \\ w_r \end{bmatrix}$$

have already seen this model this is nothing, but a model for multiple receive antenna.

This is a model for a wireless communication system with multiple receive antennas and single transmit antenna; we have already seen this model in the context of receive diversity and we also demonstrated that maximal ratio combining is the optimal reception technique and this is therefore, also known as a single input multiple output wireless communication system model because we have a single transmit antenna and multiple receive antennas. This is also known as a single input this special case is known as a single input multiple output wireless system or a SIMO wireless system. So, this wireless system which has a single transmit antenna and multiple receive antennas is also known as a SIMO or Single Input Multiple Output system. Let me write this down over here this also known as a SIMO system or basically a Single Input Multiple Output wireless communication system.

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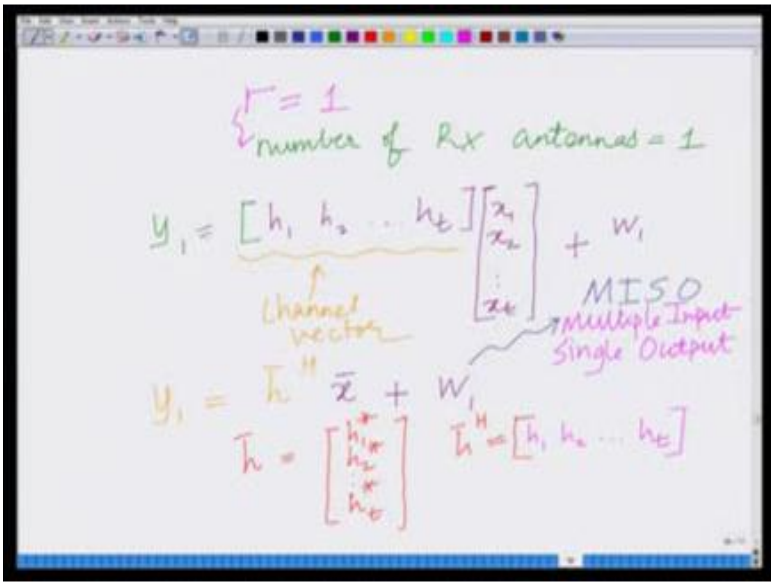

Let us look at now the other case when r = 1; number of receive antennas is equal to 1 that is basically number of receive antennas is equal to 1 and the number of transmit antennas is arbitrary; therefore, we have an r x t channel matrix, but r = 1. Therefore, I have a 1 X t channel matrix that is which means I have a row vector and this is a very interesting scenario where we if you not seen before which where we have multiple transmit antennas and a single receive antenna and therefore, I can represent this as y_1 is equal to

$$y_1 = \begin{bmatrix} h_1 & h_2 & h_t \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_t \end{bmatrix} + w_1$$

This is the channel vector and therefore, I can denote this as

$$y_1 = \bar{h}^H \bar{x} + w_1$$

where $\bar{x}$ is the Transmit vector and where

$$\bar{h} = \begin{bmatrix} h_1{}^* \\ h_2{}^* \\ h_t{}^* \end{bmatrix}.$$

Therefore, you can see that $\bar{h}^H = \begin{bmatrix} h_1 & h_2 & h_t \end{bmatrix}$

This is a very interesting scenario of a single of single receive antenna and multiple transmit antennas which you not seen before this is an instance of what we have what we know what we call transmit diversity because there are multiple transmit antennas and this is also called as a multiple input because there are multiple transmit antennas; multiple input single output because this is single receive antenna. So, this is a Multiple Input Single Output wireless communication system or a MISO wireless communication system. This is a MISO wireless system or a Multiple Input Single Output wireless communication system and of course, now the most special case where r = 1 and t = 1 that is we have single transmit antenna and a single receive antenna reduces to the standard scenario of a relay fading channel.

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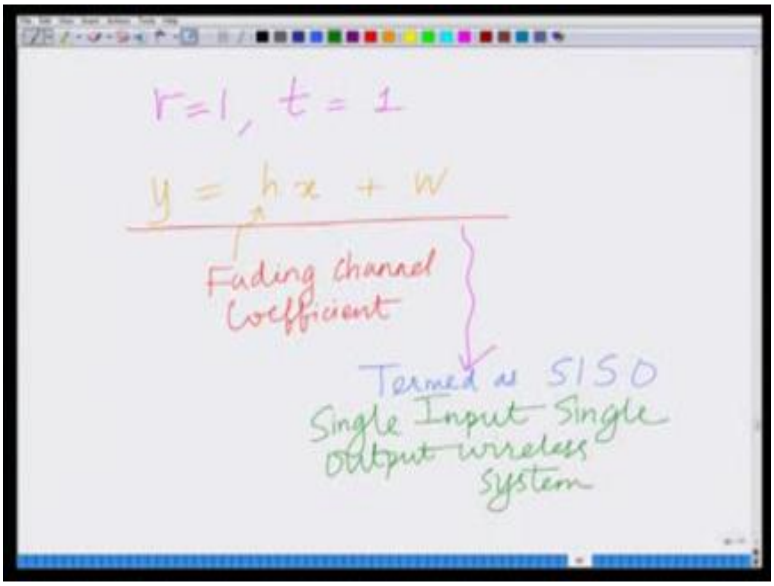

When we have a single transmit antenna r = 1 and t = 1; we have the scenario that we have seen at the very beginning that is

$$y = h x + w$$

This is the fading channel coefficient and this system model is termed as SISO that is Single Input Single Output. When we have r = 1 and t = 1; we have go back to the standard scenario which is basically a Single Input Single Output wireless communication system or also known as a SISO wireless communication system. These are the various special cases of MIMO system.

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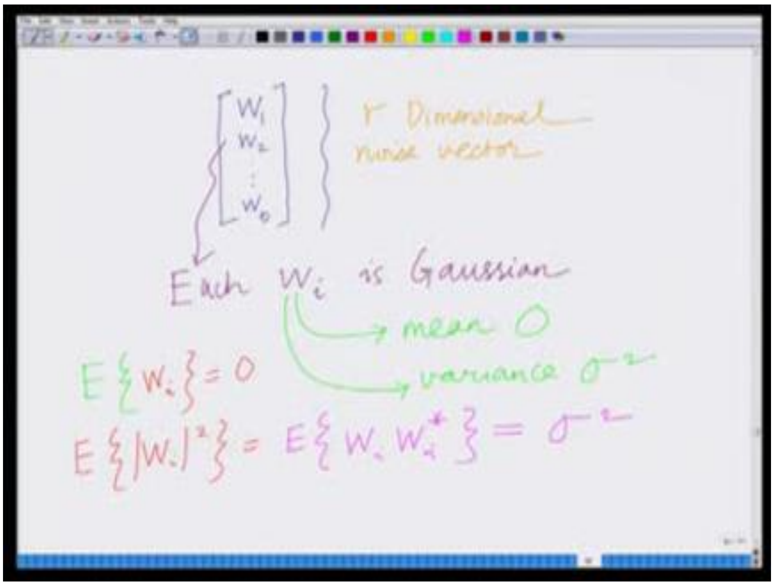

Now, a little bit about the noise vector because we have not said anything about the noise the noise vector w_1 the noise vector if I look at the noise vector w_1; what is this? This is my r dimensional noise vector where each w_i is Gaussian with mean 0, variance σ^2 that is to say that $E\{w_i\} = 0$ and

$$E\{|w_i|^2\} = E\{w_i\, w_i{}^*\} = \sigma^2$$

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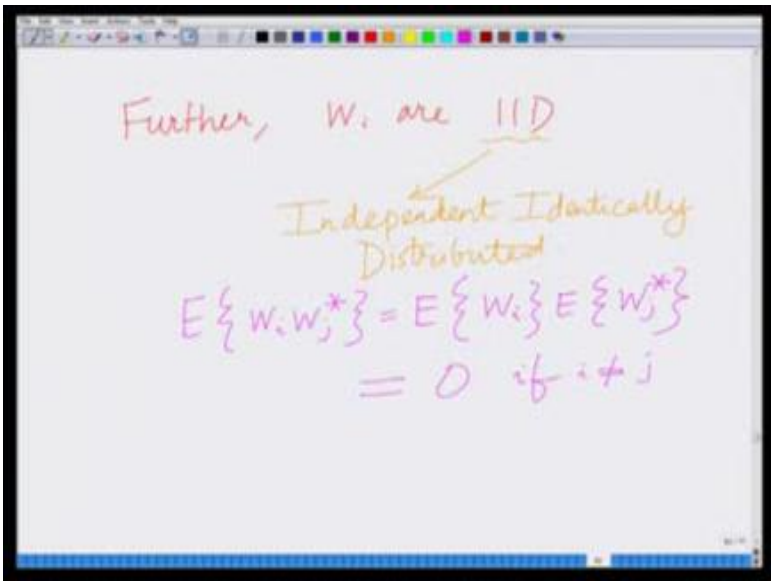

Further we are going to assume that the different noise elements further standard assumption is that different noise elements that is w_i are IID that is these different noise elements are Independent and Identically Distributed and since these are Independent as we have already seen

$$E\{w_i\, w_j^{\,*}\} = E\{w_i\}\; E\{w_j^{\,*}\} = 0 \quad \text{if } i \neq j$$

Therefore, what we have is we have

$$E\{w_i\, w_j^{\,*}\} = 0 \quad \text{if } i \neq j \text{ and}$$

$$E\{w_i\, w_j^{\,*}\} = \sigma^2 \quad \text{if } i = j$$

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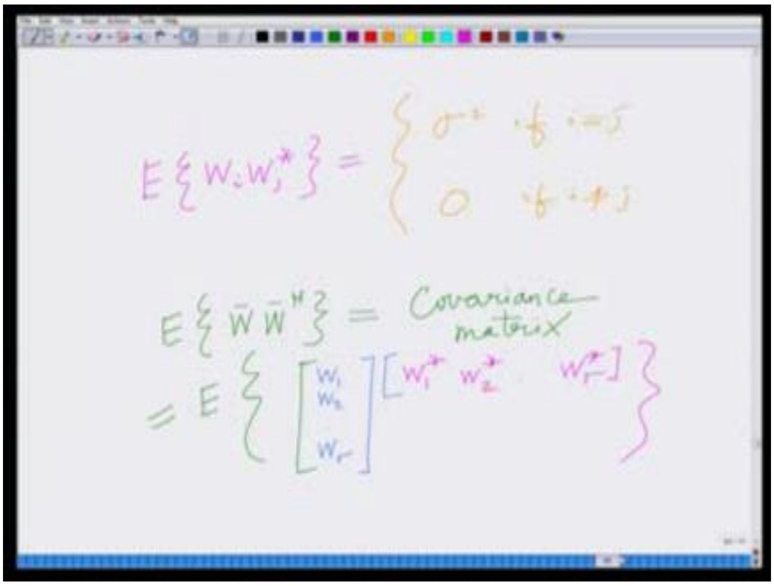

So, this is independent. So, these noise samples are Independent Identically Distributed across the receive antennas and the one last thing we would like to do is we would like to characterise the covariance matrix of this noise that is

$$E\{\bar{w}\;\bar{w}^{H}\} = E\left\{\begin{bmatrix} w_1 \\ w_2 \\ w_r \end{bmatrix}\left[\, w_1^{\,*} \quad w_2^{\,*} \quad w_r^{\,*}\,\right]\right\}$$

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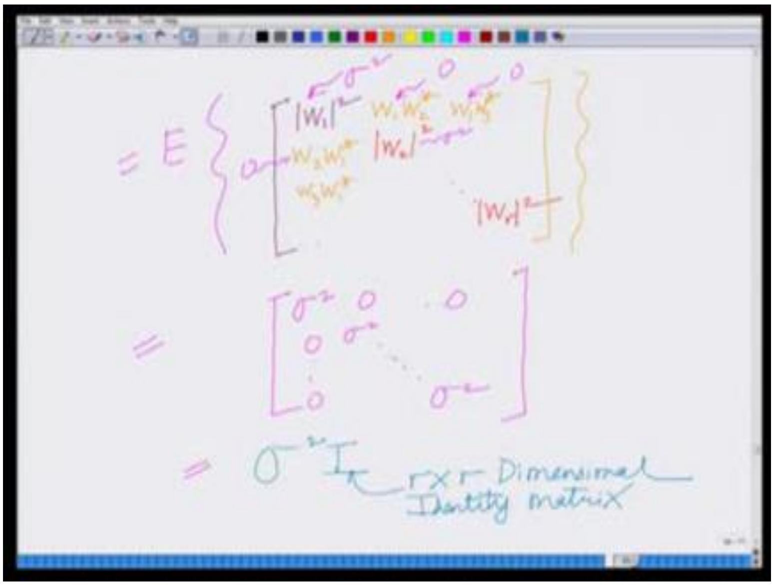

And if I expand this; I am going to have

$$= \begin{pmatrix} |w_1|^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |w_1|^2 \end{pmatrix}$$

$$= \begin{pmatrix} \sigma^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma^2 \end{pmatrix}$$

$$= \sigma^2$$

So, when the different noise elements are 0 mean Independent Identically Distributed with variance σ^2 the noise covariance is basically σ^2 times the identity matrix which is r x r dimensional. So, this is the r x r Dimensional Identity Matrix.

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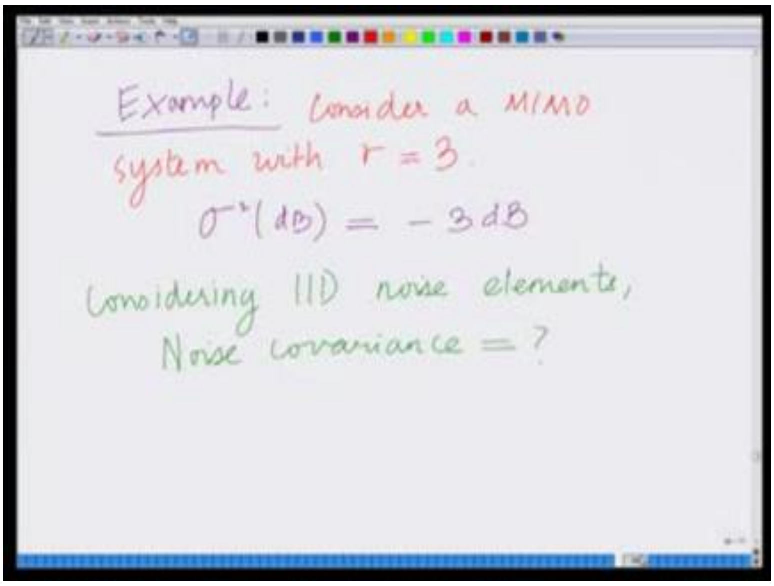

To understand this let us do a simple example. Let us consider a MIMO system again with $r = 3$ receive antennas and the noise variance σ^2 in dB = - 3 dB considering IID noise elements; what is the noise covariance?

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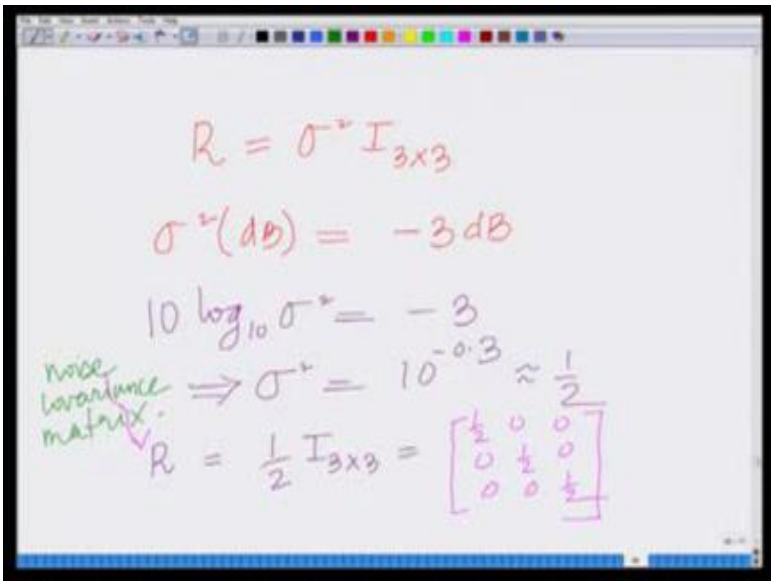

We already seen before the noise covariance R is equal to

$R = \sigma^2 I_{3x3}$

σ^2 in dB = - 3 dB

$$10 \log_{10} \sigma^2 = -3$$

$$\sigma^2 = 10^{-0.3} \sim \frac{1}{2}$$

$$R = \frac{1}{2} I_{3x3} = \begin{pmatrix} \frac{1}{2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{2} \end{pmatrix}$$

So, with this module has introduced us to basically a simple example of a MIMO system some special cases of MIMO system such as SIMO systems, MISO systems and the standard SISO system and also we have seen the properties of the noise elements when the noise elements are Independent Identically Distributed across the various receive antennas. So, let us stop this module here and we will continue with other aspects in subsequent modules.

Thank you very much.