Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 21
RMS Delay Spread

Hello, welcome to another module in this massive open online course on the Principle of CDMA, MINO, OFDM Wireless Communication Systems, now in the last module we have seen metric to characterize the delay spread of a wireless communication system that is, the maximum delay spread.

Now, let us look another metrics, now the maximum delay spread might not be a very appropriate metric to characterize the delay spread of a wireless communication system for the following reason for instance, if you look at the wireless power profile there are some dominant paths with a large amount of power, but as you go towards the end there might be a large number of small paths rather spurious paths with a small amount of power.

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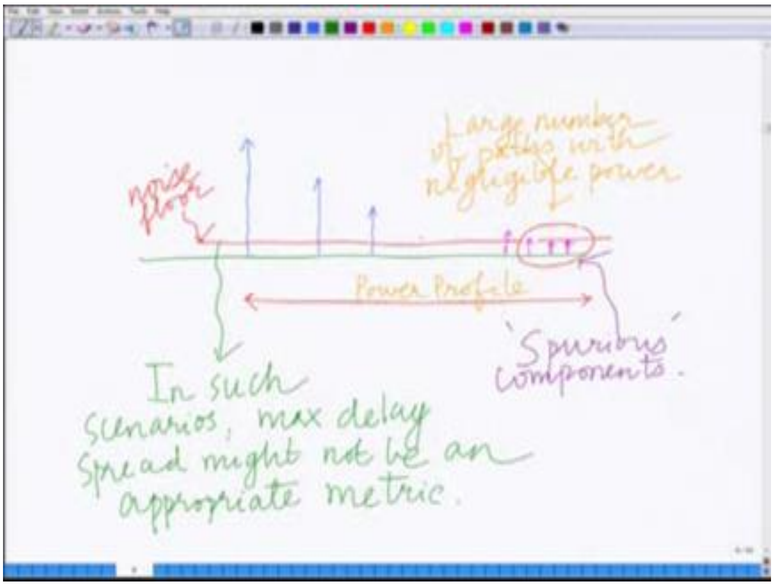

So, there might be in your power profile, if this is my profile or my multi path power profile, there are a large number of paths or spurious components with negligible power, there can be a large number of paths with negligible power and these are also known as Spurious Components, which might not really the signal components because, sometimes these small power path might, in fact, the below the noise threshold for instance let us say I have a noise threshold. So, this is let say my noise floor, these multipath components like below the noise floor. Therefore, very difficult to distinguish these multipath components the single components from the noise, so these can be spurious components this is might not be signal components at all right.

And therefore, the maximum delay spread which looks at the maximum difference between the first and the last arriving components might not be a metric reasonable metric because, it is affected by this spurious components which are arriving towards the end right, because the maximum delay spread does not consider weighing by the appropriate gain of each path. So, therefore, the maximum delay spread in such scenarios, with spurious components maximum delay spread might not be an appropriate metric. So, we have to consider something more appropriate and that is the, RMS or the root mean square delay spread.

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So, the metric that we would like consider is known as the RMS delay spread, this also stands for the root short for the root mean square that is the RMS. So, in this scenario, we would like to consider the RMS or the root mean square delay spread and this is calculated as fallows; for instance let us go back to our multipath power profile with L components.

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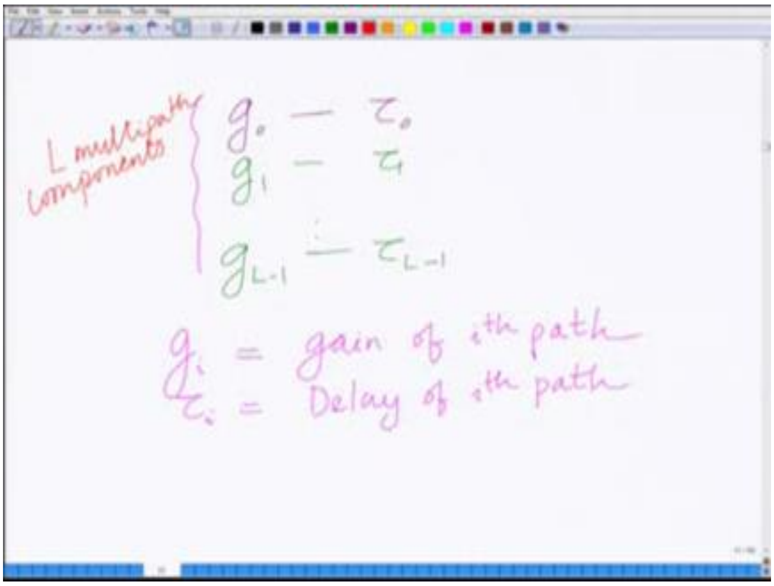

For instance, component g_0 corresponds to delay τ_0, gain g_1 corresponds to delay τ_1 gain g_{L-1} corresponds to delay τ_{L-1} that is, I hope everyone remember is the notation g_i equals gain of i-th path, and τ_i equals the delay of the i-th path, and I have L multipath components, where g0 is the gain of the 0-th path and τ_0 is the corresponding delay, g_1 is the gain of the first path τ_1 is the corresponding delay so on and so for, g_{L-1} is the gain of the L – 1 th path and τ_{L-1} is the corresponding delay.

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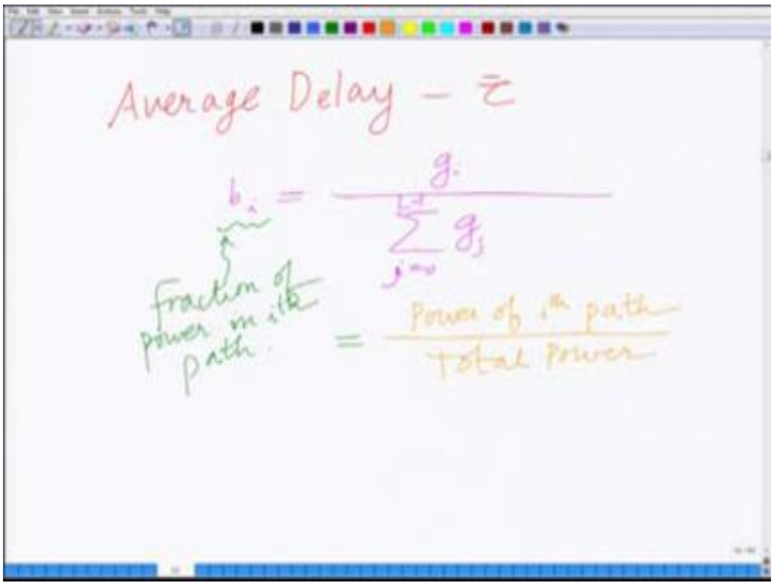

Now what we would do is, first we want to compute the average delay, which I am going to denote by $\bar{\tau}$ and this is given as follows; let us say the fraction of power in the i-th path i denote by b_i the fraction of power in the i-th path that is

$$b_i = \frac{g_i}{\sum_{j=0}^{L-1} g_j}$$

So, this is basically what I am doing is, the fraction in the i-th path is power of i-th path divided by the total power. And now we can compute the average delay by weighing by the fraction of the power, by the weighing each delay τ_i by the fraction of the power b_i in i-th path.

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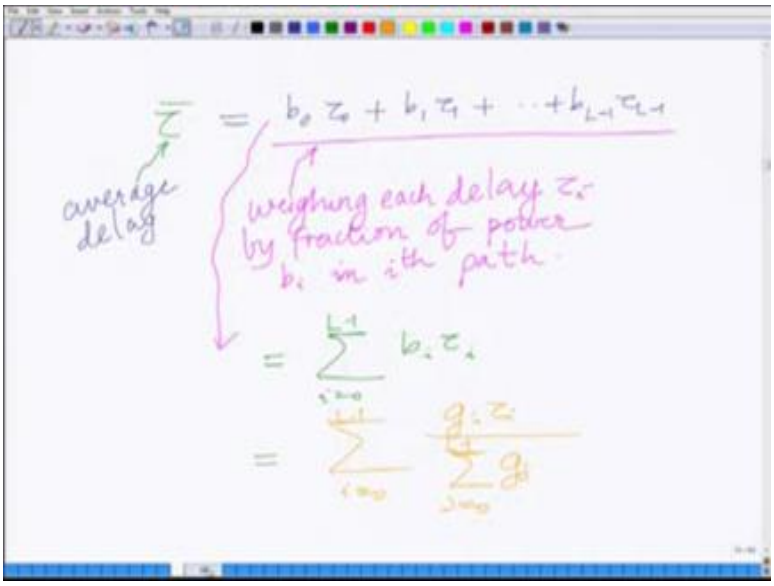

So, the average delay $\bar{\tau}$,

$$\bar{\tau} = b_0\tau_0 + b_1\tau_1 + \ldots + b_{L-1}\tau_{L-1}$$

Basically weighing each delay τ_i by fraction of power b_i

$$= \sum_{i=0}^{L-1} b_i\tau_i$$

$= \sum_{i=0}^{L-1} \tau_i \frac{g_i}{\sum_{j=0}^{L-1} g_j}$

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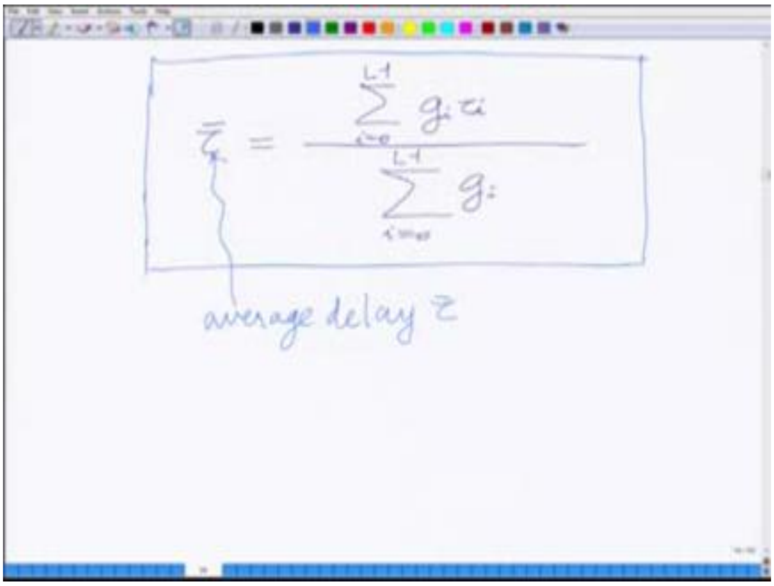

$= \frac{\sum_{i=0}^{L-1} g_i \tau_i}{\sum_{i=0}^{L-1} g_i}$

This is the average delay of the multipath power profile. Now, what we want to do, we would like to compute the RMS delay spread, and the RMS delay spread is given by the deviation of this multipath power profile about the average delay. So, how do I calculate the deviation ?

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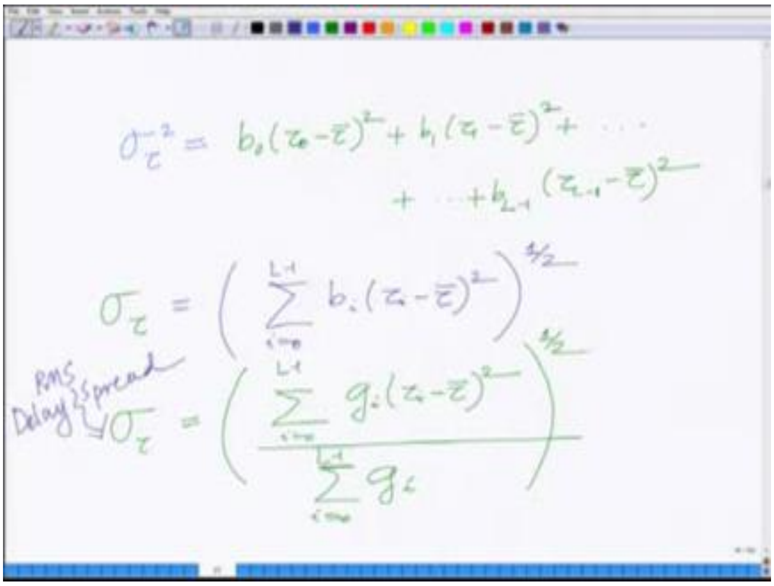

So, the deviation σ_τ^2, the square of the deviation is basically, from statistics you know

$$\sigma_\tau^2 = b_0\,(\tau_0 - \bar{\tau})^2 + b_1\,(\tau_1 - \bar{\tau})^2 + \ldots + b_{L-1}\,(\tau_{L-1} - \bar{\tau})^2$$

Now we have the σ_τ which I can call the RMS.

$$\sigma_\tau = \left(\sum_{i=0}^{L-1} b_i(\tau_0 - \bar{\tau})^2\right)^{0.5}$$

$$= \left(\frac{\sum_{i=0}^{L-1} g_i(\tau_0 - \bar{\tau})^2}{\sum_{i=0}^{L-1} g_i}\right)^{0.5}$$

This is the RMS delay spread.

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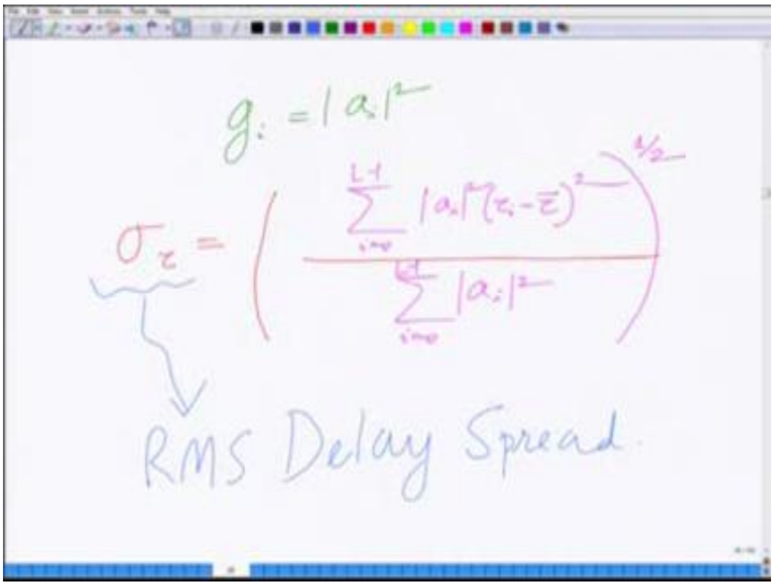

And I can also now substitute $g_i = |a_i|^2$

$$\sigma_\tau = \left(\frac{\sum_{i=0}^{L-1} |a_i|^2 (\tau_0 - \bar{\tau})^2}{\sum_{i=0}^{L-1} g_i} \right)^{0.5}$$

which is the basically, the RMS delay spread.

Let me write this clearly, this is the RMS delay spread of the multipath power profile this is the expression for the root mean square delay spread or the RMS delay spread of the multipath power profile. Let us now do simple example to understand this better. So, let us do a simple example for instance, let us consider again scenario with $L = 4$ multipath components.

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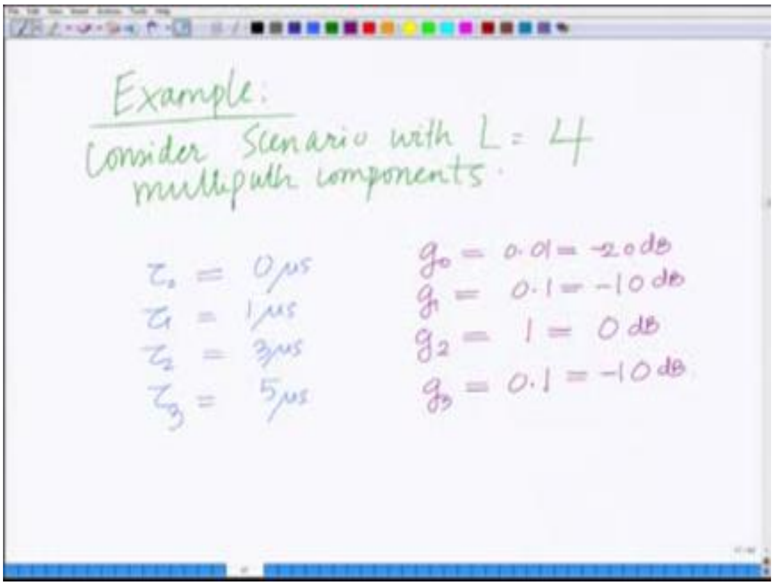

Let us consider a scenario with $L = 4$ multipath components and this scenario is as follows; let us say the delays are given as follows

$\tau_0 = 0\ \mu s \qquad g_0 = 0.01 = -20dB$

$\tau_1 = 1\ \mu s \qquad g_0 = 0.1 = -10dB$

$\tau_2 = 3\ \mu s \qquad g_0 = 1 = 0dB$

$\tau_3 = 5\ \mu s \qquad g_0 = 0.1 = -10dB$

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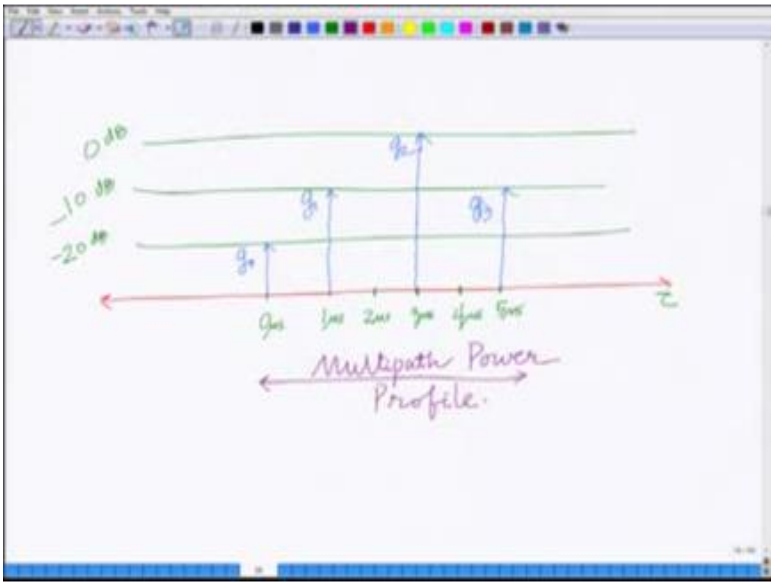

And now, what we would like to do is, let me first draw a picture to sort of illustrate what is going on in this channel, I have a multipath power delay profile with 4 components. So, I have 4 component multipath power delay profile and basically the first components. So, I have different levels let me draw different levels here, let us call this as -20dB, -10dB and 0 dB and let us have 0 μs that is intervals of 1 μs.

So, this my multipath power profile drawn on the dB square scale. This is basically the multipath power profile I have which is drawn on the dB scale let me also write at in a tabular form.

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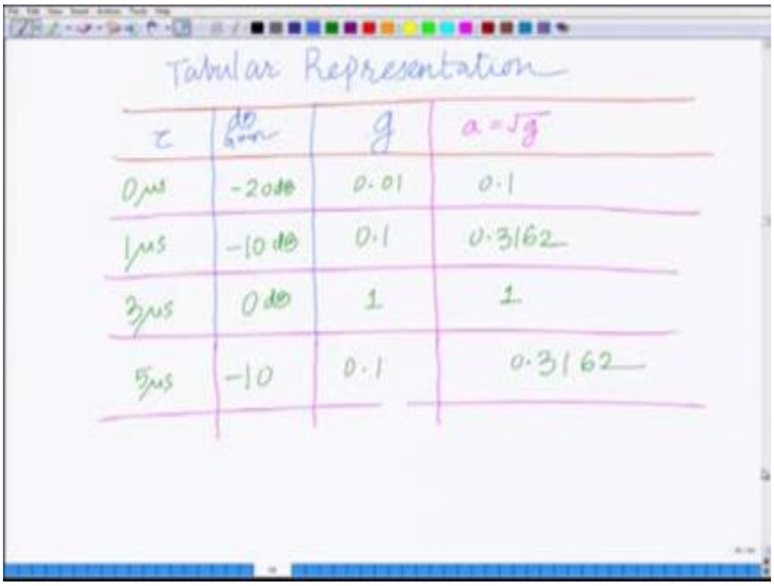

Tabular Representation

τ	dB gain	g	a = √g
0μs	-20dB	0.01	0.1
1μs	-10 dB	0.1	0.3162
3μs	0 dB	1	1
5μs	-10	0.1	0.3162

So, we can also write in a tabular form. The table is representation of the various multipath components the gains in dB, the gains in normal and the amplitude, which is the square root of the power gain and also the corresponding delay. So, this is the very convenient table that gives us a snapshot of the multipath power profile. And now what we want to do is, want to compute the RMS delay spread and therefore, we will start by first computing the average delay, and the average delay as you know is given as tau bar.

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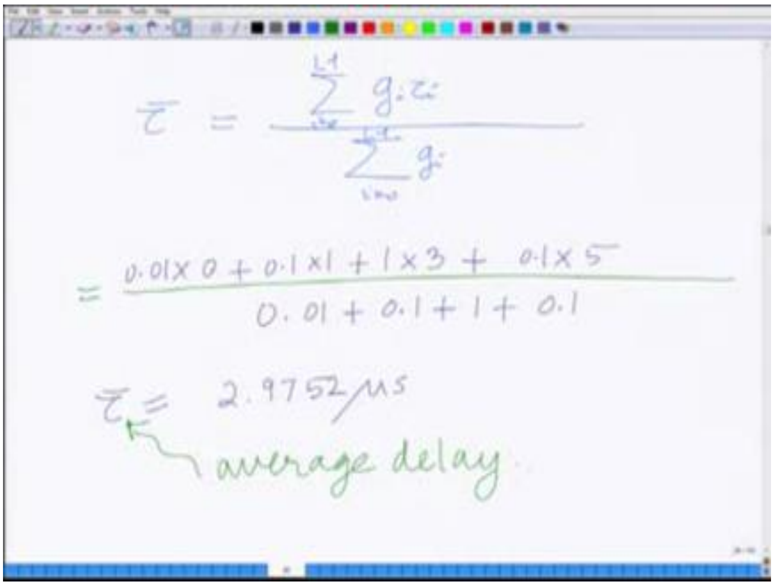

So, we want to compute the average delay

$$\bar{\tau} = \frac{\sum_{i=0}^{L-1} g_i \tau_i}{\sum_{i=0}^{L-1} g_i}$$

$$= \frac{0.01x0+0.1x1+1x3+0.1x5}{0.01+0.1+1+0.1}$$

$$= 2.9752\ \mu s$$

So, the average delay tau bar is equal to 2.9752 μs. So, this is the average delay of this multipath of the given multipath power profile under consideration. So, we have calculated this quantity $\bar{\tau}$ which is the average delay and now, using this $\bar{\tau}$ we want to compute the RMS delay spread.

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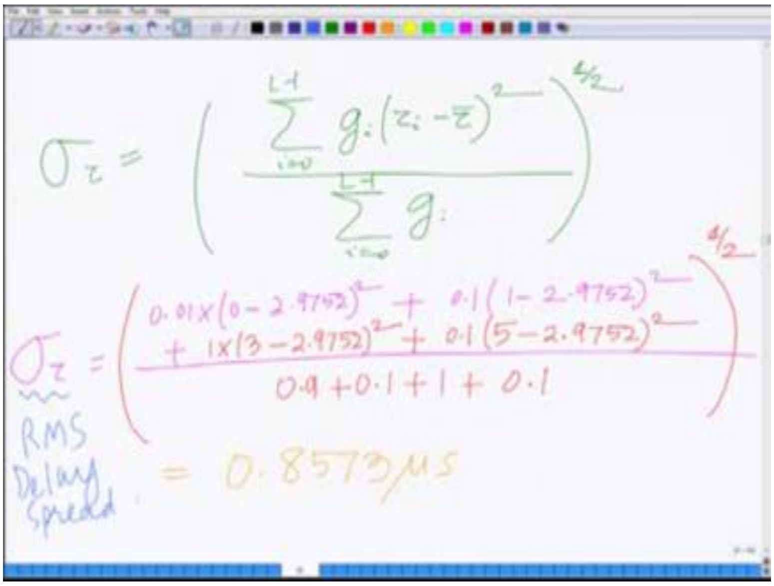

And remember the RMS delay spread that is

$$\sigma_\tau = \left(\frac{\sum_{i=0}^{L-1} g_i(\tau_0 - \bar{\tau})^2}{\sum_{i=0}^{L-1} g_i} \right)^{0.5}$$

$$= \left(\frac{0.01(0-2.9752)^2 + 0.1(1-2.9752)^2 + 1(3-2.9752)^2 + 0.1(5-2.9752)^2}{0.01+0.1+1+0.1} \right)^{0.5}$$

$$= 0.8573\ \mu s$$

This is given as the maximum this is given as the RMS delay spread and remember this is the RMS.

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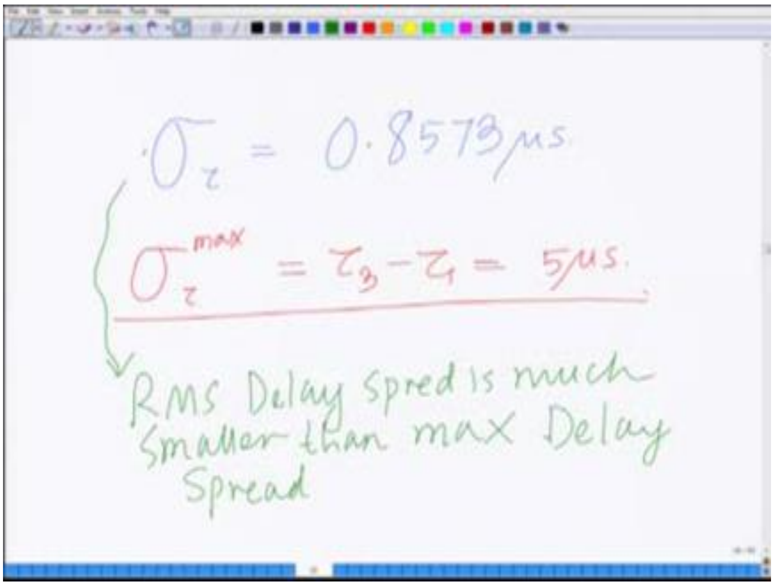

So, the RMS delay spreads σ_τ for this channel, under consideration is 0.8573 μs. Now, if you go back and look at the maximum delay spread, the maximum delay spread is simply

$\sigma_\tau^{max} = \tau_3 - \tau_0 = 5 - 0 = 5$ micro seconds

that is the delay between the first and last arriving component. and therefore, what you can see is that the RMS delay spread is much smaller than the maximum delay spread, otherwise; delay spread is, 0.8573 μs , maximum delay spread is 5 microseconds and this is arising because, the RMS delay spread is actually weighing like, delay of each component by the corresponding fraction.

So, it is a weighted delay spread rather than looking at the maximum delay spread, which is simply the delay between the first and last arriving components which might not be an appropriate metric. Therefore, the RMS delay spread which weighs the delay by the corresponding power is much more appropriate metric and we have seen in this example, how to computed this is given as 0.8573 microseconds. So, we will conclude this module here, and we will explore other topics in subsequent modules.

Thank you very much.