Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 19
Definition of Diversity Order

Hello welcome to another module in this massive open online course on the Principles of CDMA/MIMO/OFDM Wireless Communications Systems. So, what we have seen is basically we have seen the diversity aspect of a wireless communication system with multiple antennas. Now what we want to understand is we want to understand how to characterize the diversity of a system.

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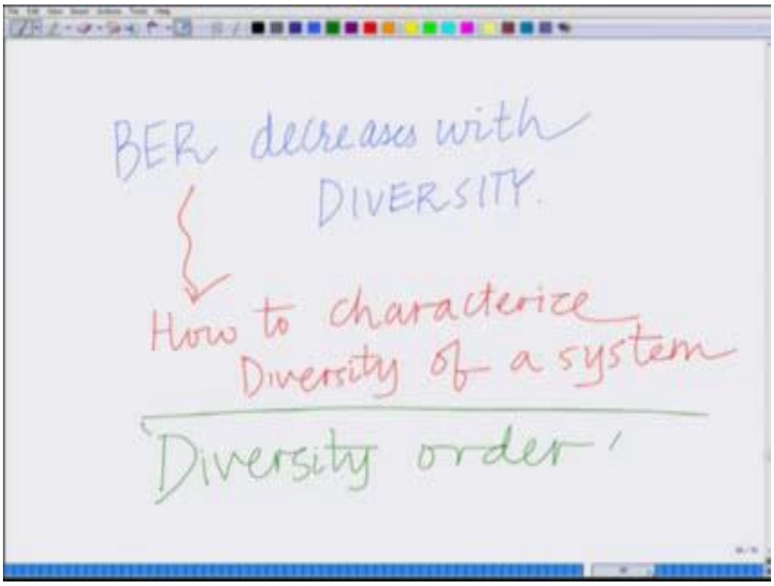

So, we have seen so far that the bit error rate decreases with diversity, but how to characterize diversity. So, as to how to characterize the diversity or how to measure the diversity of a system and this is basically given by the diversity order of the system and how is it defined? We are going to now define the diversity order which is basically a characterization of the diversity of a system.

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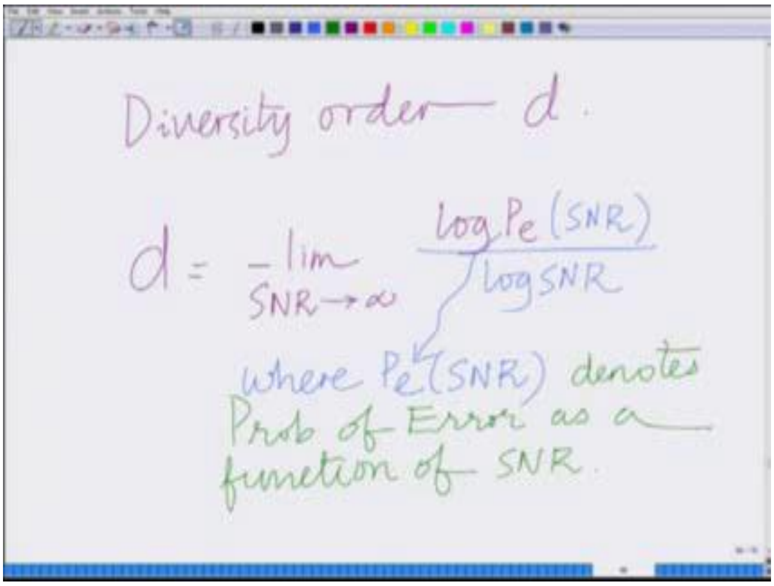

The diversity order d can be defined as follows;

d = - $lim_{SNR\rightarrow\infty} \frac{\log Pe(SNR)}{\log SNR}$

where P_e (SNR) denotes the probability of error as a function of SNR. So, the diversity order of the system is

d = - $lim_{SNR\rightarrow\infty} \frac{\log Pe(SNR)}{\log SNR}$

So, this is the definition of the diversity order of the system which characterizes the diversity of a system.

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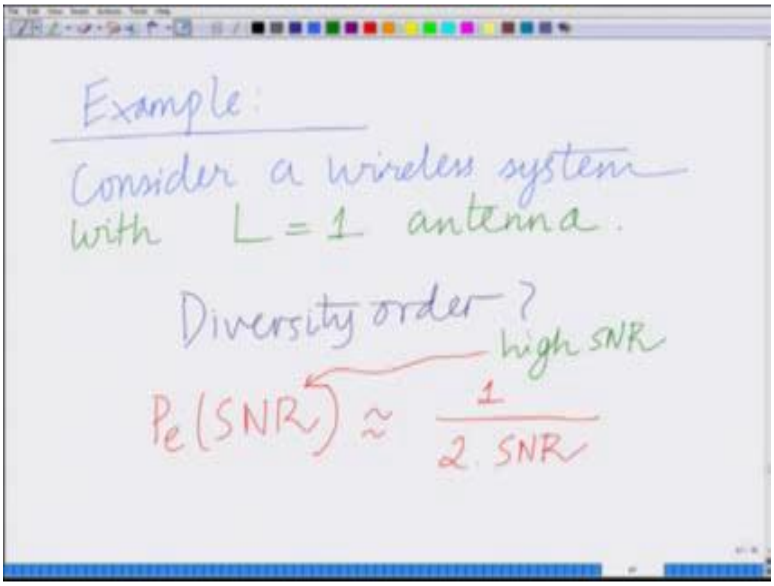

Let us for instance take a simple example, let us consider a wireless system with L = 1 antenna and we want to ask the question what is the diversity order of the system with L = 1 antenna and we can see that for this system the probability, the bit error rate; the probability of error we have already as

$$P_e\,(\text{SNR}) = \frac{1}{2\,\text{SNR}}$$

Remember this is at high SNR.

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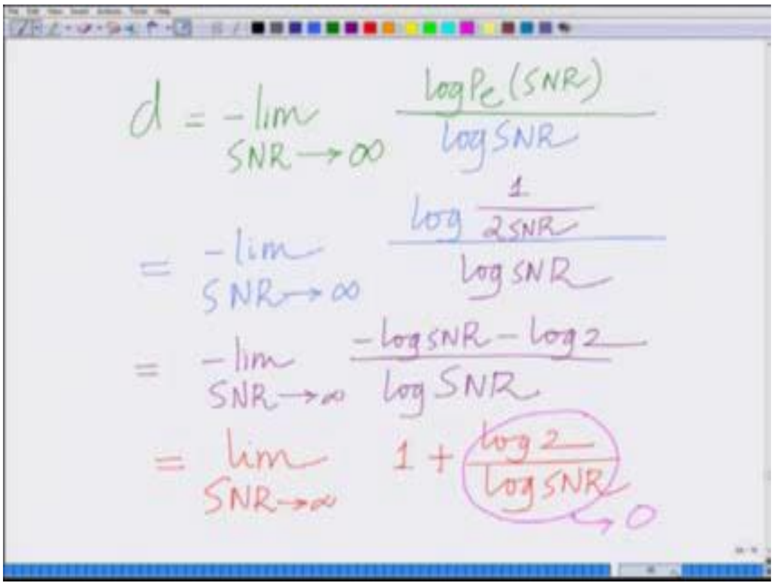

Therefore the diversity order of this system is

$$d = -\lim_{SNR\to\infty}\frac{\log Pe(SNR)}{\log SNR}$$

$$= -\lim_{SNR\to\infty}\frac{\log\frac{1}{2\,SNR}}{\log SNR}$$

$$= -\lim_{SNR\to\infty}\frac{-\log SNR-\log 2}{\log SNR}$$

$$= \lim_{SNR\to\infty} 1+\frac{\log 2}{\log SNR}$$

Now, you can see that $\log 2$ is a constant and therefore as SNR tends to infinity, $\log SNR$ tends to infinity therefore, $\frac{\log 2}{\log SNR} \to 0$ at high SNR as SNR $\to$ infinity.

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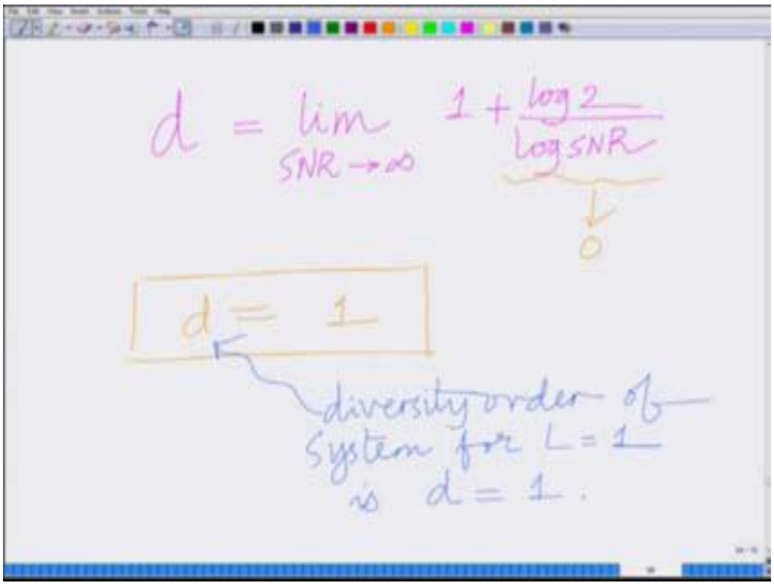

Therefore we have the diversity order basically equal to

$$d = \lim_{SNR\to\infty} 1+\frac{\log 2}{\log SNR} = 1$$

So, the diversity order of a system with a single received antenna, so this is the diversity order of a system for $L = 1$ is d equals that is the diversity order of the system, of a wireless communication system in which the bit error rate at high SNR decreases at $\frac{1}{2\ \text{SNR}}$ is 1; the diversity order of SNR system with single received antenna is 1.

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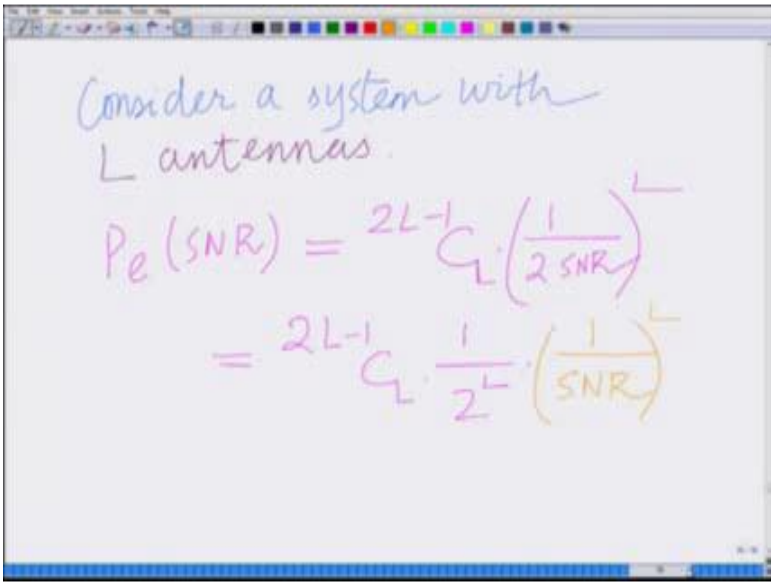

Now, let us see what happens as the number of antennas increases for a system with L antennas, consider a system or a wireless system with L antennas and in this case we know that the probability of error

$$P_e\,(\text{SNR}) = {}^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$$

$$= {}^{2L-1}C_L \frac{1}{2^L}\left(\frac{1}{\text{SNR}}\right)^L$$

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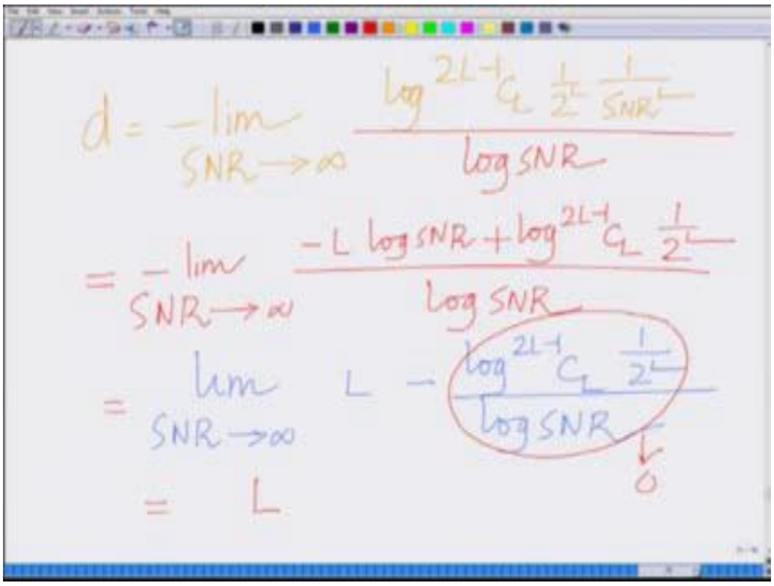

But

$$d = -\lim_{SNR \to \infty} \frac{\log {}^{2L-1}C_L \frac{1}{2^L}\left(\frac{1}{\mathrm{SNR}}\right)^L}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{-L\log SNR - \log {}^{2L-1}C_L \frac{1}{2^L}}{\log SNR}$$

$$= \lim_{SNR \to \infty} L - \frac{\log {}^{2L-1}C_L \frac{1}{2^L}}{\log SNR}$$

$$= L$$

Therefore, we are left with this quantity L, so the diversity order equals L as $SNR \to \infty$; therefore, diversity order of the system with L antennas is basically equal to L.

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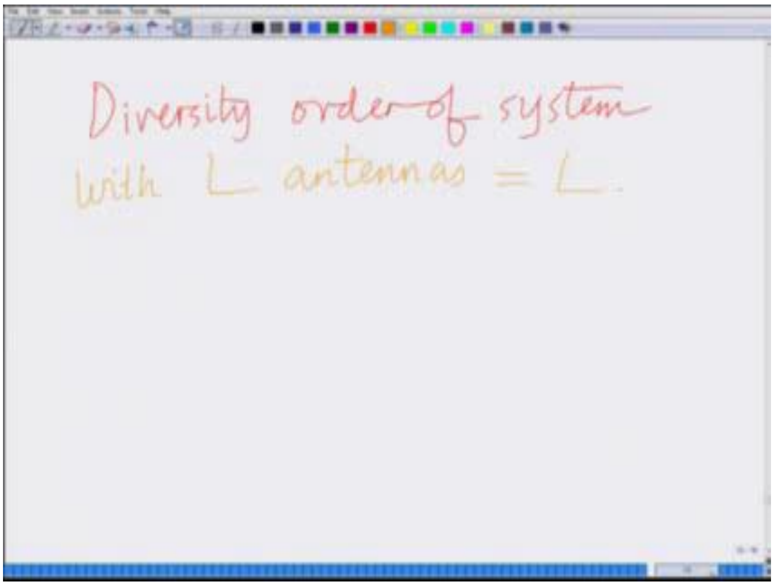

Now, let us look at another interesting scenario and let us look at the diversity order of the; AWGN or the wire line channel, what is the diversity order of the wire line system?

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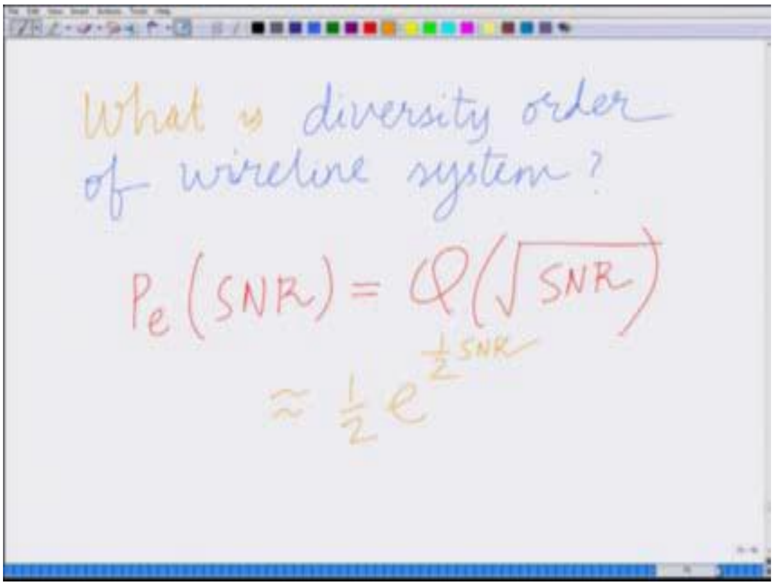

Let us ask this question what is the diversity order of the wire line system or so, of the simple AWGN channel. We know that the probability of error for the wire line system,

$P_e\,(SNR) = Q\,(\sqrt{SNR}\,)$

$$= \frac{1}{2} e^{-\frac{1}{2}SNR}$$

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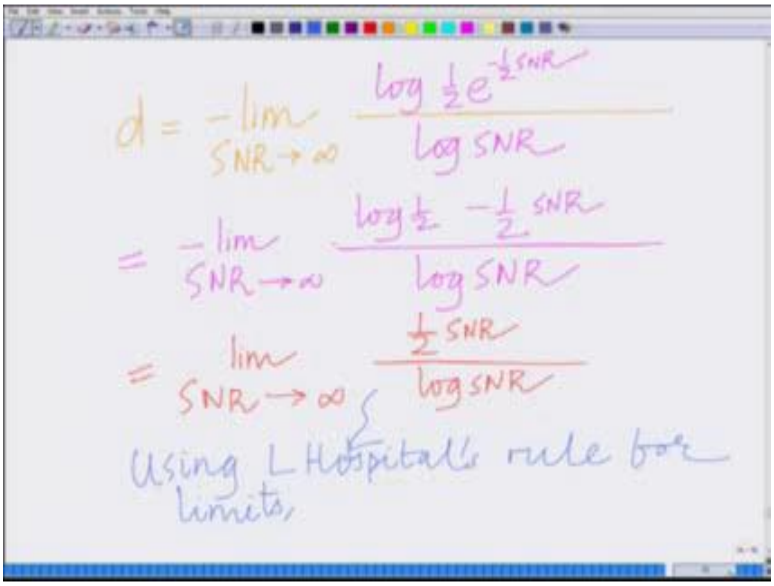

Therefore the diversity order d equals

$$d = -\, lim_{SNR\to\infty} \frac{\log \frac{1}{2} e^{-\frac{1}{2}SNR}}{\log SNR}$$

$$= -\, lim_{SNR\to\infty} \frac{\log\frac{1}{2} - \frac{1}{2}\log SNR}{\log SNR}$$

$$= lim_{SNR\to\infty} \frac{\frac{1}{2}SNR}{\log SNR}$$

Therefore, this is an indeterminate form, so to compute the limit I can use the L'Hopital's rule, so you that is basically differentiate the numerator and the denominator.

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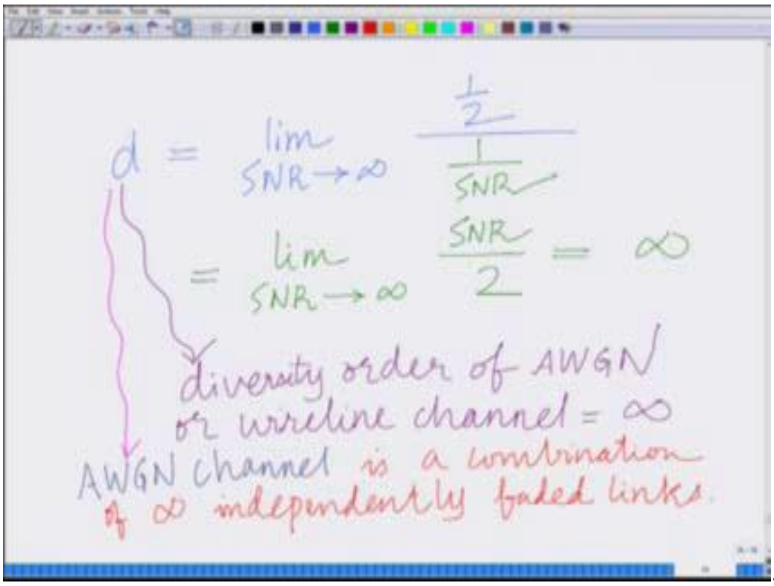

So, basically using the L'Hopital's rule, what we have is the diversity order

$$= lim_{SNR\to\infty}\frac{\frac{1}{2}}{\frac{1}{\mathrm{SNR}}}$$

$$= lim_{SNR\to\infty}\frac{SNR}{2} = \infty$$

So, therefore the diversity order, so this says diversity order of my simple AWGN channel or wire line is equal to ∞; that is the wire line channel can be thought of as comprising of an infinite number of independently fading links, that is the diversity order which is the number of basically related to the number independently fading links is ∞ for the wire line channel. So, it can be thought of as the combination of an infinite number of independently fading links.

So this basically means AWGN channel is a combination of infinite independently fading links. So what this tells us is that, the diversity order of the AWGN channel or the simple wire line communication system with no fading is equal to infinity, therefore this can be thought of as or this can be thought of intuitively as a channel with an infinite number of independently faded links which gives rise to a diversity order of infinity alright.

So, that basically includes this module on diversity order and we have comprehensively looked at several aspects of multi antenna wireless communication systems such as; what is optimal combining which is given by the maximal ratio combiner, the bit error rate performance, the deep probability of deep fade in the system with multiple antennas and finally, now we have also derived the diversity characterized the diversity order of different wireless communication systems. So, we will conclude this module here and we will explore other topics in subsequent modules.

Thank you very much.