Signals and Systems
Prof. K. S. Venkatesh
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 7
Transformations on Time and Range

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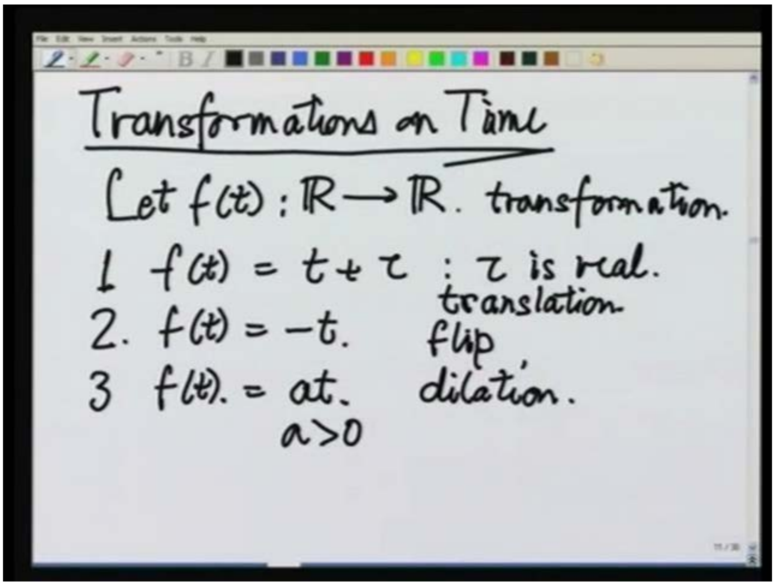

Is what we call the study of transformations on time. What we will now do is this, we have signals x t defined on time what we will do with time is to apply a function on time and rearrange the time axis. So, this will be called a time transformation, let f of t map the time axis which we will which is a set of real numbers to the time axis itself, this f of t will be called a transformation.

Principally, we will consider three kinds of transformations of time; the first is f of t of the form t plus tau, where tau is obviously real. The second kind is called is of the form f of t equal to minus t, a simple inversion of the time axis. The third shall be a scaling of the time axis f of t equal to a t. Now, there are names given to each of these; the first is called the translation, the second we will call a flip, and the third we will call a dilation; for convenience, we will assume for dilation that a is greater than 0. Now, our interest is in seeing what happens to any function x of t defined on time when the time axis is transformed by 1 of these three kinds of transformations.

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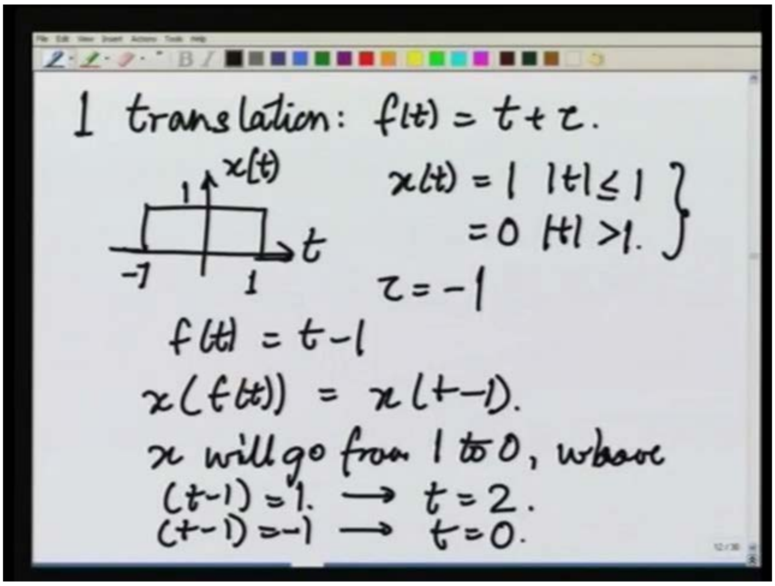

So, let us look at these cases 1 by 1. The first is of this translation, where f of t equals t plus tau or as we said earlier f its t plus tau, fine. Now, what happens is this? Suppose we have x t given as follows (( )). Let us take a example of x t sketched as follows, let us say that this is plus 1, this is minus 1, and let us say for simplicity that its value is 1 to minus 1 to 1, outside of minus 1 to 1 x t 0. So, we would put down the value of x t for different times as follows, x t equals 1 for mod t less than equal to 1 equals 0 for mod t greater than 1, this is our x t.

Now, what happens to x t, if we shift the time axis as we have done over here. Let us take tau equal to minus 1, if we take tau equal to minus 1 let us see what happens to this function, and let us also see how we compute the effect of incorporating a time transformation on the values of x t on the transformed time axis. Now, we have x t given by some expression over here, fine. Now, if tau equals minus 1 we will have f of t equals t minus 1. So, that the new function we have to deal with is x of f of t, which is equal to x of t minus 1 this is the function we now require to plot.

Now, how do we understand the manner in which x of t minus 1 is to be plotted, the answer is this. Look at the original x t, we have the figure available here, look at some salient points of x t; the salient points are t equal to minus 1 and t equal to 1, these are the places where x t undergoes a transition at t equals to minus 1 undergoes a transition from a value 0 to value 1 and t equal to 1 it transits from 1 to 0. Now let us take one of these points, let us say t equal to 1 x has undergone a transition from 1 to 0, when its argument was equal to 1.

Now, x t minus 1 will do the same thing when its argument t minus 1 takes on a value of 1, in short x will go from 1 to 0. Now, at the point where t minus 1 equals one; that means at t equal to 2, that is where it undergoes the transition from 1 to 0 where does it undergo the other transition; the other transition is from value 0 to value 1 and it underwent the transition when its argument was minus 1. Now, it will happen where t minus 1 equals minus 1 which corresponds to the point t equal to 0. So, let us sketch the transformed function x of t minus 1 in the next screen.

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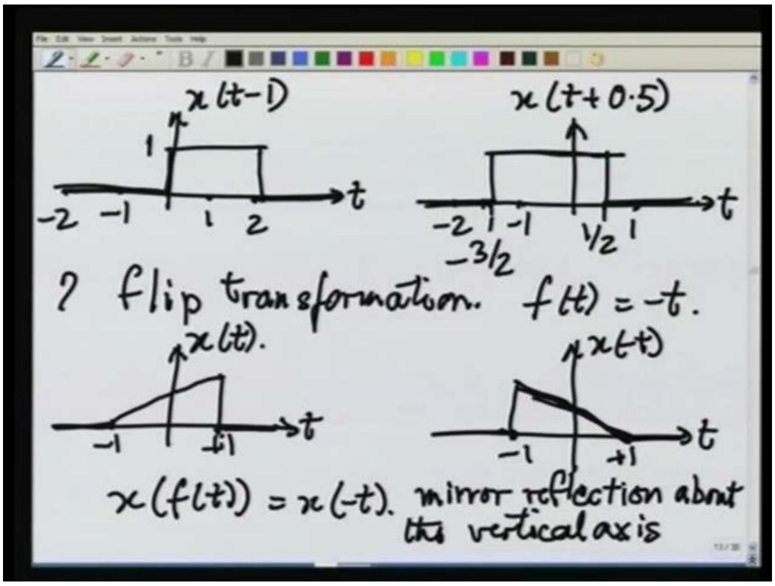

x of t minus 1 x of t minus 1 would be 0 until the first transition, the transition which occurred at minus 1 now occurs at 0, then it transits to a value of unity, stays there until the second transition which occurs at t equal to 2, and then goes back to 0 and stays there for all time, this is x of t minus 1. With this in hand we can see what happens for any arbitrary x of t minus tau, the graph of x t simply shifts to the right for negative tau and to the left for positive tau, and it will shifts by an amount equal to the value of tau. Thus for example, x of t plus half can be immediately plotted, x of t plus half could be plotted like this. The amount of shift is by the value of mod tau the direction of shift is as follows for negative tau it goes to the right, and for positive tau it goes to the left.

Now, we have 0.5 as the amount of shifts to be given towards the left. So, compared to the original x t we will take this as a minus 1, this is a minus 2, this is 1 we have to go to 0.5, the transition at 1 will occur at 0.5 and the transition from 0 to 1 which occurred at minus 1 is also shifted towards the left by half. So, we come to this place and make this plot, for x of t plus half is shifted by half this point is actually 1 by 2, this point is actually, sorry this point is actually minus 3 by 2.

So, it is its transitions have been shifted by half towards the left, because it is x of t plus half here it is shifted by 1 towards right, because it is x of t minus 1. So, this explains the behaviour of the translation transformation on the time axis, let us move to the flip transformation; the flip transformation is given as f of t equal to minus t. So, if we take the same x t as before we would face some problems, because we would really see no change occurring in x t at all. So, let me take a new x t, now let me take x t sketched as the following minus 1 plus 1, this is t let us say 0 to minus 1 then increases in the straight line up to 1 and then 0 again.

How would we form an expression for this function that is a little complicated, that is stay with the graph of the function for time being it would be easier that way. So, let us say that this is x t, we want to plot x of f of t which comes out to be x of minus t, that is what we want to plot. Let us plot that here, how do we find out what is x of minus t? The process the procedure is exactly same as before. We look at salient points on this graph, one salient point is minus 1 again, where x has for the first time seems to be 0 which it was since t equal to minus infinity and has started increasing in a positive direction. The other salient point is t equal to plus 1 where it drops to 0, and remains 0 for all future time. So, the salient points are again minus 1 and plus 1, we have to see what happens to these salient points under the time transformation x of f of t equal to x of minus t as f of t equal to minus t.

Now, the argument was plus 1 when x underwent this abrupt transition, if the same happens with the new function as well; that means, minus t must be equal to plus 1 for this transition to occur. The other interesting point was t equal to minus 1 and this is the value at which it will rise from 0, and therefore that will now correspond to minus of minus 1 equal to plus 1. Essentially we then get the following curve minus 1 plus 1 the function is all 0 up to here, it goes up and falls in a linear manner this place, this is what happens with x of minus t. We observe that the flip operation is the mirror reflection about the vertical axis. So, this is the effect of a flip all it does is to mirror reflect whatever function x t about the vertical axis. So, that whatever was to the right comes to the left, and whatever was to the left comes to the right.

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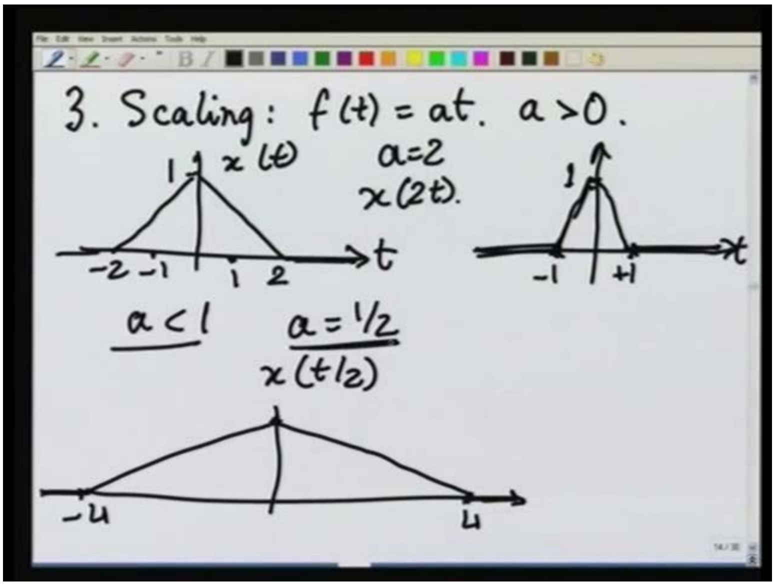

The third kind of time transformation is scaling, f of t equals a t a greater than 0 what does this do in order to understand this, let us again take an example. Let x t mutual and thus follows, let us say that this is 2, this is minus 2 minus 1 and 1, 0 outside all this. Let us again not try to find the mathematical expression of this is of course, can be done, but it is a little cumber sum would waste of few moments. Instead let see what happens to the case when a equals 2. We therefore, have to find x of 2 t, again go by the salient points; the salient points are 2 0 and minus 2, each of these points has now got to be transformed. Therefore, going about the transformation in the conventional manner, we note the following, the original function x undergoes a transition when its argument is equal to 2, as a salient point at the value of its argument is equal to 2. Now that salient point will occur when 2 t equals 2.

So, when 2 t equals 2 t is equal to 1 fine. So, t is equal to 1, let us say that the amplitude of this function is 1 and I want to sketch it over here, its amplitude is not going to be effected in any manner, because these are only a time transformation. Similarly, the salient point at minus 2 will now occur at 2 t equal to minus 2 or t equal to minus 1. So, this is the second salient point, this is the first salient point, now placed at plus 1 this placed at minus 1 shifted from their earlier places of minus 2 and plus 2. The salient point at 0, where its slope changes from positive to negative, now becomes the point 0.2 t equal to 0. So, it remains in the same place t equals to 0. So, now let us sketch this point before the salient point which was at minus 2 which is now at minus 1 x t has to be always 0. So, also for points greater than plus 1 earlier plus 2 between these 2 points the function is a straight line plotted in this manner.

So, what has happened if we take a look at the graph we find that the function seems to have got compressed along the horizontal direction. It has been compressed by a factor of a, that is factor of 2; if a had been larger, it would have got more compressed, if a had been it smaller it would have got less compressed and if in fact we choose a to be less than 1, it would result in an expansion of this function a stretching of this function. So, let us consider say a equal to half, then we would get x of t by 2 as follows, the salient point at 2 would shift to the point t by 2 equal to 2, that is t equal to 4. So, we would have it at 4 salient point at minus 2 would go to minus 4 the point at 0 would remain at 0. So, we would have a function that is 0 up to here, then increases linearly up to this point, decreases linearly up to this point and then stays at 0, this is for a less than 1 in particular a equal to half.

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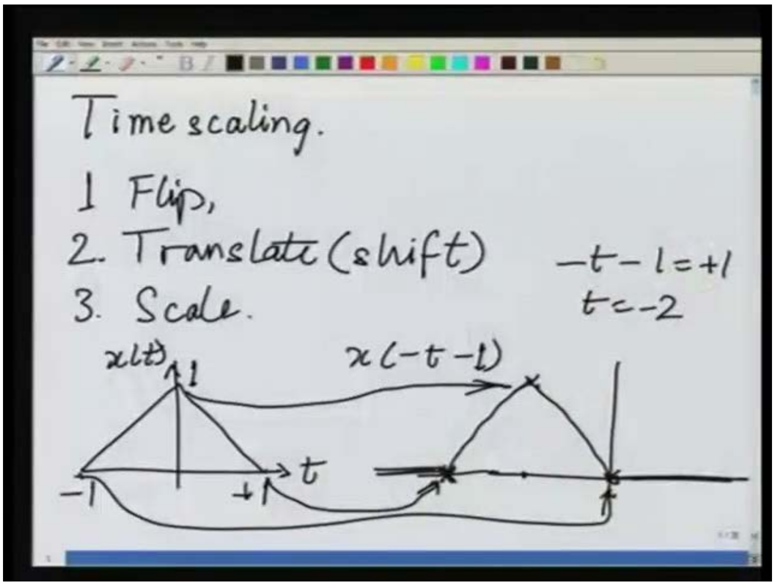

Dimension is also sometimes called time scaling, dimension is also sometimes called time scaling, because it involves a scaling of the time axes. Now one thing that we should understand by now is that of the three kinds of domain transformation operations or time axes transformations that we have discussed, namely flip, translate, and scale. One can generate further transformations out of these by combining them 1 or 2 at a time or even all 3 at a time. For example, let us consider a simple signal such as the previous one, let us say the amplitude is one and that is stretches from minus 1 to plus 1, this is the time axis and this is the function x t. Let us consider what happens, if we plot x of minus t minus 1 what is the right procedure to understand what x minus t minus 1 looks like, we should look at x t identify specific points at which interesting events occur in x t and use these to identify the corresponding points in the new graph, this is the thumb rule method of doing it.

For example, x t reaches its maximum of unity at t equal to 0; that means, when its argument is equal to 0, that peak would now occur at the point where minus t minus 1 equals 0. So, if minus t minus 1 equals 0, then t must be equal to minus 1 and so at minus 1, we would put this maximum point of x t; once we have this it is not very hard to locate the remaining points for example, it reaches its 0 a 0 at t equal to plus 1 at the argument equal to plus 1. So, when minus t minus 1 equals plus 1 it will reach zero; that means, when t equals minus 2. So, at minus 2, it will reach 0 the other point is when its argument is minus 1 when minus t minus 1 equals minus 1 it will reach the other 0, and therefore that corresponds to t equal to 0, in the new axes.

So, these are the 3 points, and once we have the 3 points we can plot this curve like this; this is how the new function, the transformed function looks in the relation to the old function, but it is important to recognise one thing over here which has been lost due to the symmetry of x t. This point has carried over to this point, this point has carried over to this place, and this point has carried over to this place. So, the function has actually got inverted in time, and shifted backwards by 1 second, this is the complete story of this particular transformation.

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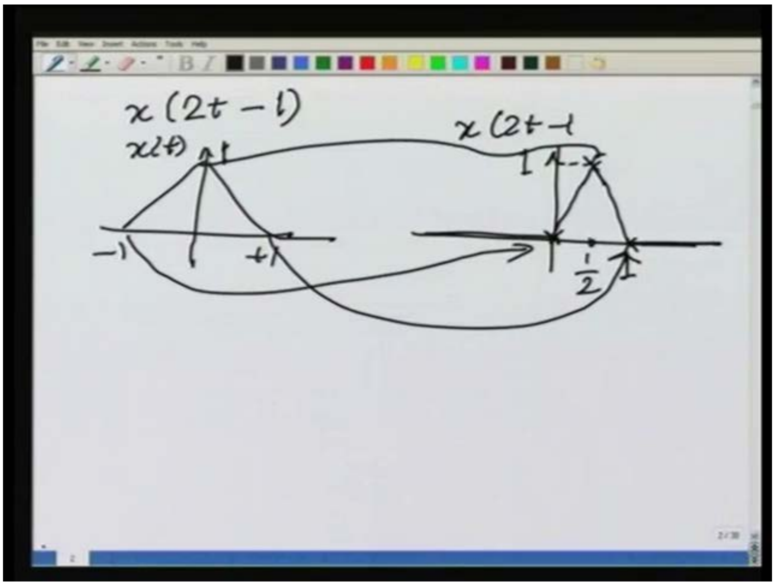

Another example what happens to x 2 t minus 1 for the same function x t, let us just redraw x t on the side for convenience, and make an attempt to plot x of 2 t minus 1 over here, it reaches its maximum at the origin that is when its argument is 0. So, when is 2 t minus 1 equal to 0, when 2 t minus 1 is 0 when t equals half. So, at t equal to half it reaches its maximum, when 2 t minus 1 is plus 1 it reaches 1 of its points of attaining 0. So, 2 t minus 1 equals plus 1 2 t equals 2 t equals 1. So, at this point there is the right side 0 cross, when 2 t minus 1 equals minus 1 t equals 0 is the third point.

So, we get this shape, where this is the point one on the time axes, this has an amplitude of unity, and this is 0 and this is of course, half. Again if we wish to note the correspondences between the respective points, here we see that the maximum has come to the maximum as usual, this point has come to this point, and this point has been brought to this point. In the earlier example seen over here, there is an exchange of the points on the time axes, the points on the left has come to the right and the…