Signals and Systems
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Lecture - 6
Frequently Used Discrete Time Signals

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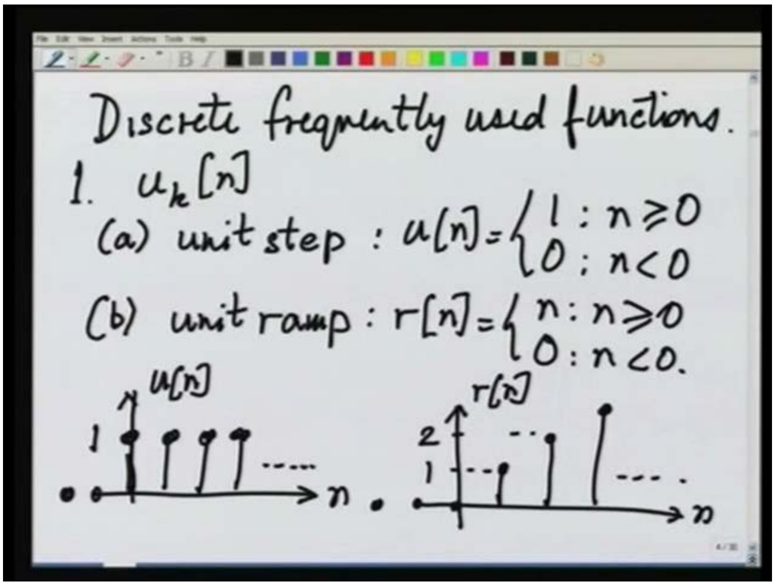

Now, go through the discrete counterparts of these functions. Starting with u k of n, the unit step is the first 1, it is defined as u n equal to 1, n greater than or equal to 0 for n less than 0, that is the unit step. The unit ramp is defined as r of n and is given as equal to n for n greater than equal to 0, and 0 for n less than 0; that is could be standard, if you would like me to plot them, here goes 0 0 1, r of n 0 0 0 1 2 3; this is 1 2 by the way this is 1 and so on, this is the unit ramp.

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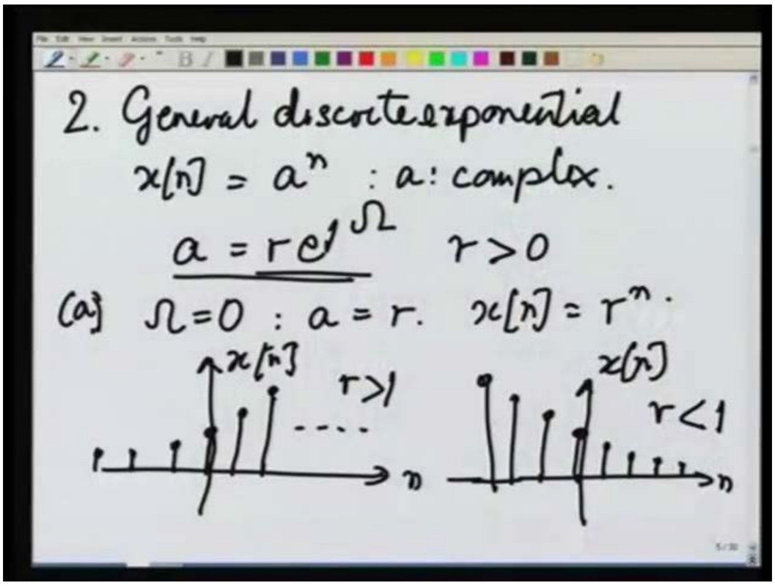

Going to the exponentials next. The general discrete exponential is denoted by x n equal to a to the power n, where a is generally complex. So, when a is complex, we would say that a can have the general form r e to the j theta or r e to the j omega. So, now when a is given by this, let us see what happens to different in senses of taking values for r different values for omega and so on and so forth. Let us start with omega equal to 0. So, that a is simply equal to r, in that case x n becomes nearly r to the power n. Now r to the power n will simply be a function which grows either to the right or to the left when does it grow to the right and when does it grow to the left. Suppose we have this as our axis, this is n and this is x n, we are concerned with the case a when I am sorry case a begins here, this is what we would call a real discrete exponential.

So, let us see what happens to the function? r to the n at n equal to 0, this will be equal to 1. For other values of n we will have to see what happens? Let us start with assuming that r is greater than 1; if r is greater than 1, than r square is greater than r, r cubed is greater than r square, and so on. While to for t less than for n less than 0, we get square root of r cube root of r and so on where we will of course, take the positive square root, the positive cube root and so on.

By the way in this representation here it is implicitly understood that r is greater than 0, in all our considerations, r is like the magnitude because this is after all the polar form. So, r is the magnitude of the complex exponential and omega you can say is the angle, coming back here when r is greater than 1 what we get is a growing exponential towards the right, this would grow higher and higher as we grow to the right as and towards the left it would get smaller, and smaller, and still smaller. This is for the case of r greater than 1, for r less than 1 we would start with unity at n equal to 0 as before, but now this sequence should diminish towards increasing n, and increase towards decreasing n, towards negative n. This is what would we get? r greater than 1 and this is for r less than 1 perhaps I should erase this put it a little closer, these are the 2 graphs that indicate what happens?

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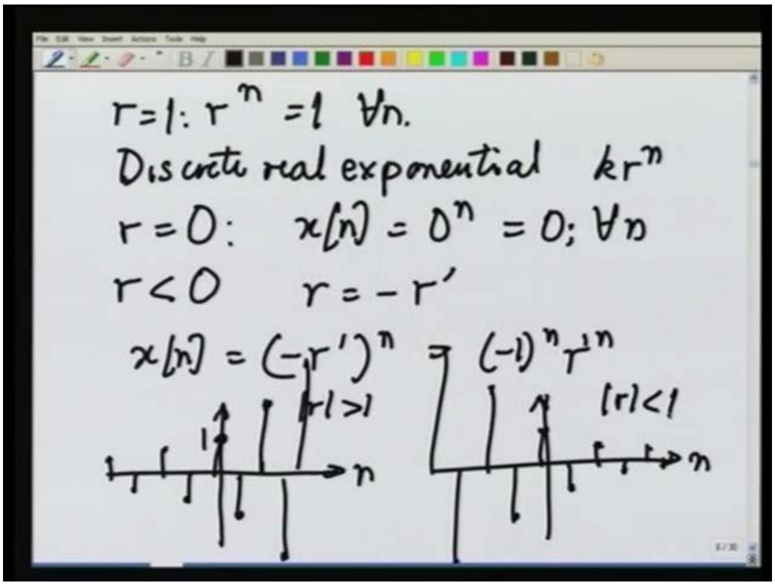

Now, let us go on to what happens when r equals 1, when r equals 1 clearly r to the n is simply equal to 1 for all n. So, it is the constant function you get a d c sequence. More generally perhaps, we could phrase the discrete real exponential in the form k times r to the n. So, that this entire curves that the entire curve that we drew last time just a minute earlier would get scaled appropriately depending on the value of k, k could be positive or negative, but for the time being lets confine k to be a real number. Next, let us take a look at what happens, if r equals 0 if r equals 0, then we have the sequence x n equals 0 to the power n which its identically 0 for all n what happens? When r is less than 0, when r is less than 0 which we do not consider there is no problem, but theoretically there is no problem considering it.

So, let us go on and see what happens when r is less than 0, if r is less than 0 we can write it is as say a r equals minus r prime, and therefore express x n as equal to minus r prime sorry minus r prime to the power n which we rewritten as minus 1 to the n r to the n r prime to the n, where r prime now is greater than 0. So, all that happens is that our sequence gets multiplied by minus 1 to the n, the effect of multiplying by minus 1 to the n is to invert alternate samples. In fact, invert all the odd number samples, thus you would get when mod r is greater than 1, you would get as before an exponential that expands towards the right and shrinks towards the left, it is shrinks for shrinks for negative end and expand for positive end for increasing n, but which has alternate samples inverted.

This remains equal to unity, this is what we would get? Towards the right and towards the left, this is what we would get for mod r greater than 1, for mod r less than 1 that is for r lying between minus 1 and 0, we get; and for the right side we would have this is for mod r less than 1. This covers the case of all real exponentials, discrete real exponentials, let us now see what happens? When we have omega not equal to 0.

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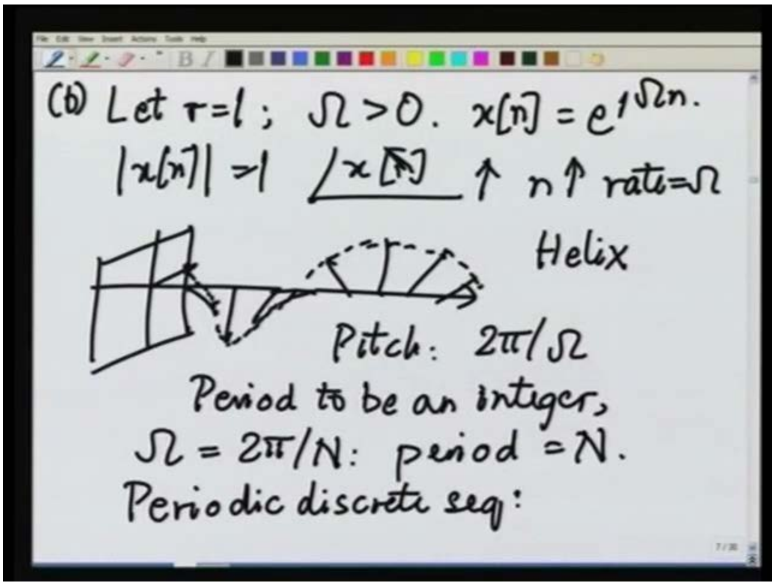

Let us said r equal to 1 for the present exercise, let this is our case be r be equal to 1 and omega we let us say greater than 0, then we have a sequence of complex numbers x n equal to e to the j omega n, the thing to note about this complex numbers is that mod x n stays at unity, whereas the angle of x n which I will simply denote by this, increases as n increases. Thus we have a complex number whose phase or angle keeps increasing with n at a study rate equal to omega, the rate is omega. And therefore, what we get is a sequence of phrases represented in polar coordinates which of which each successive number of the sequence is rotated counter clock wise, a little more than the previous number of the sequence. So, if we tried to plot it on the usual frame we use for a complex valued function, then we would start with the case of n equal to 0, where it would just be equal to unity, so we will just get something like, this is the value.

Then we would have something that goes downwards, then something going for further down, then going this way, going this way, sorry we have plotted effectively one cycle of this complex exponential roughly one cycle. Now, if I just connect the various tops of these samples with a dotted line, I would get the same old helix is the helix, just as it was with the continuous case for the exponential with imaginary exponent, except now that this is a discrete helix, it is a sequence of samples of the continuous helix that we do earlier. Otherwise it is still a helix, the pitch of the helix is the period of this sequence, if we can call in period, but let us keep that discussion for a little later, because the period has to handled properly here.

The pitch of the helix would be the distance from a particular point, where it takes a specific value to the next point where it takes the same value where the helix takes the same value where the phase is the same as at the previous point, that is the pitch. How does it relates to omega? The pitch is 2 pi by omega. And therefore, we see what is the required condition for the pitch to be an integer, for the pitch or the period to be an integer, omega has to be an irrational number of the sort 2 pi by n.

Only when this is satisfied, does the pitch or does the period come out to be capital N. If omega is anything other than of the form 2 pi by N, then we will not have an integer period and it is necessary by definition of the period of a periodic discrete sequence which is actually given as follows, if well. Let me just give you the definition of a periodic discrete sequence.

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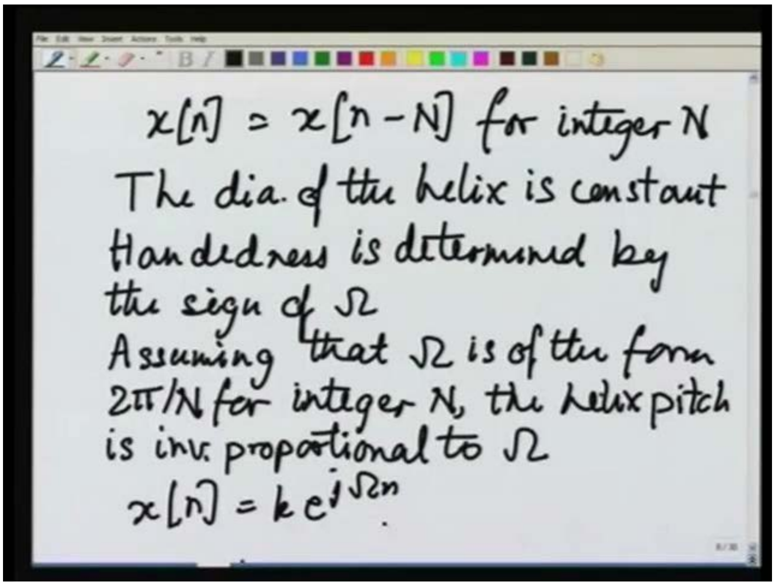

x n is to be is considered periodic, if it is equal to x n minus N for integer N, of this will certainly not happen, if you do not choose omega properly in the example that we are currently dealing with. N has to be an integer for x n to be called a periodic signal, in the present case if omega equals to 2 pi by N as we said, then x n will indeed repeat itself every n samples. And we would have a periodic helix a proper periodic helix, in any case this much is evident.

The diameter of the helix is constant and handedness is determined by the sign of omega, and assuming that omega is of the form 2 pi by N for integer N, then the pitch, the helix pitch is inversely proportional to omega. So, that takes care of the various properties of the helix, and their relation with the values of omega the sign of omega etcetera, etcetera. If we append a multiplicative real constant to the helix, and make a more general expressions such as x n equals k times e to b j omega n, then k would have their role of expanding their diameter of the helix approximately. The diameter would be equal to mod k, moving finally to the general discrete complex exponential case c.

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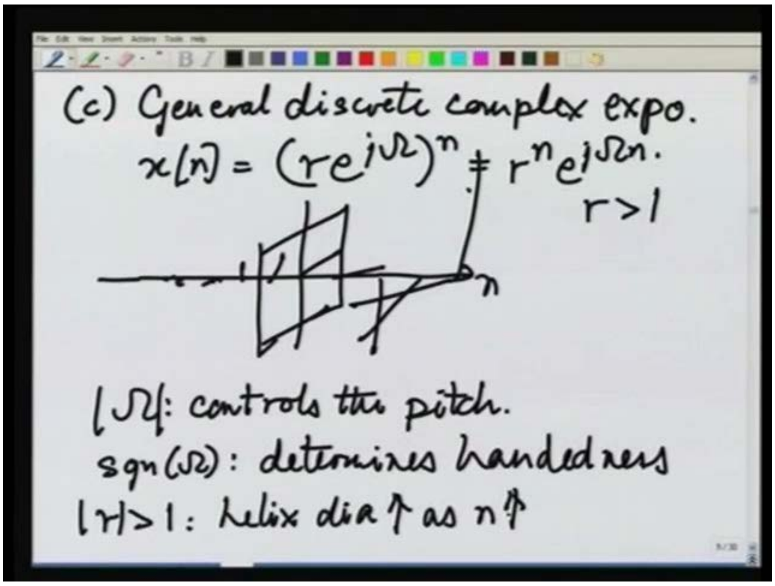

Let us put this down as x n equal to r e to the j omega multiply to the power n, we can proceed to discuss this along line similar to the manner in which we discuss the general continuous time complex exponential. There we showed that this general complex exponential can be expressed as a product of a real complex a real exponential and an imaginary exponent exponential.

Let us do the same thing over here and rewrite this as r to the n times e to the j omega n, immediately it becomes clear, that what we have on hand is an expanding helix, it is a helix whose pitch remains the same, but whose diameter increases with time for mod r greater than 1, and decreases with increase in time for mod r less than 1. Furthermore this sign of r itself incase r is not positive is has the effect of inverting alternative samples. So, let us put that all down, and lets first make a little plot the last of the time being, we have let us consider a case where r is greater than 1. That means, r is positive and also has a magnitude greater than unity.

So, it would be a right ward expanding helix, this is n and you have a helix here a point there then something that is going down there, something that is even longer here, even longer here, even longer here, still longer there. Does not look very clear from the graph, but that is how it is? That is the best we can plot towards negative time it would get smaller and smaller. So, we would have something over here, something over here, something over here, something over here, something still time here, and so on, diminishing for negative n.

Now, let us just put down the various properties of this general case, first the effect of omega, controls the pitch rather mod omega controls the pitch, there is of course the proposal that the omega has to be of the form 2 pi by n for having an integer as the period of this exponential helix, sign omega determines handedness. So, much for omega, coming to r we can say the following; if r is greater than 1, let us say if mod r is greater than 1 helix diameter increases as n increases.

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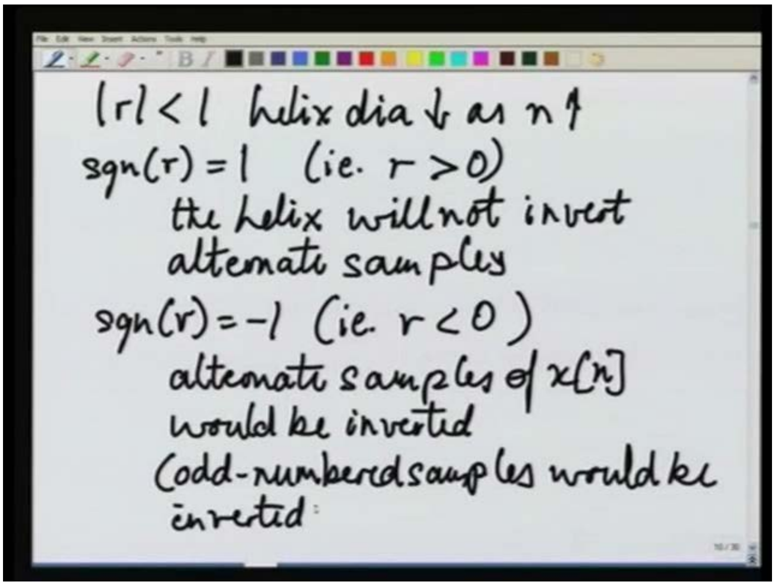

If mod r is less than 1 then helix diameter decreases as n increases. Finally, if sign r is equal to 1; that is if r is greater than 0 then the helix will not invert alternate samples. If sign r is minus 1 that is to say r is less than 0, then alternate samples of x n would be inverted. What I mean to say is that odd numbered samples would be inverted.