Signals and Systems
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Lecture - 37
Flat top Sampling

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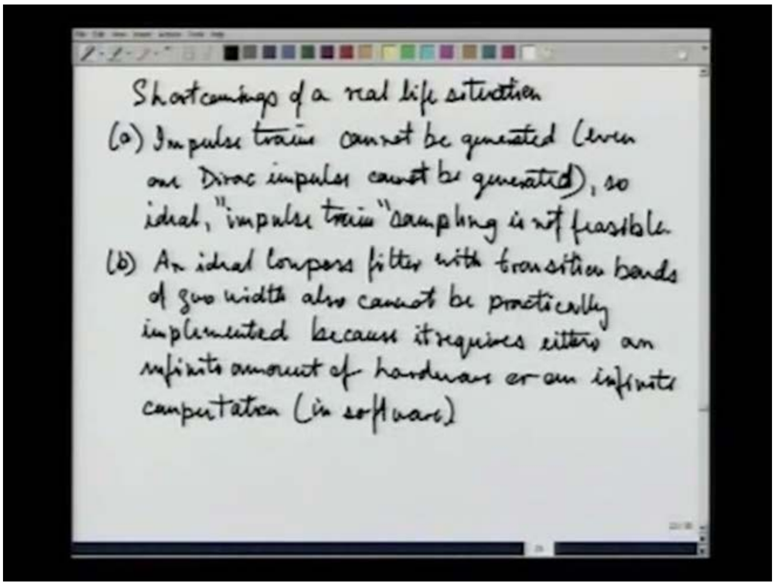

So let us see shortcomings of a real life situation. Impulse trains cannot be generated. In fact even one Dirac pulse cannot be generated impulse trains cannot be generated even 1 Dirac pulse cannot be generated. So, ideal sample is not be possible; impulse train sampling is not feasible. That is the first problem we face, there is also a second problem and that is an ideal low pass filter, low pass with transitions bands of 0 width of 0 width also cannot be practically implemented, be practically implemented. Because it requires either an infinite amount of hardware or an infinite amount of computation hardware or an infinite computation in software.

So, two things make it impossible to really do ideal sampling and then ideal recovery using an ideal low pass filter both these things are a problem. I need this theory was, were studying were what we read was not a waste of time, because it was so informative and it give us a picture between the discrete time word and the continuous time word. So, let us now address the possible imperfections the possible real world imperfections that will have an impact upon our theory of sampling a signal reconstruction.

We already know, in any case that the sampling theorem that we have just discovered must be respected, but apart from reading the sampling theorem there are other issues that complicate the picture. One of them is a reset the nonexistence or the impossibility of constructing an impulse train. What we can expect from that is a pulse train say a train of rectangular pulses of non zero width unlike the impulse train, which has each pulse of 0 width and finite height.

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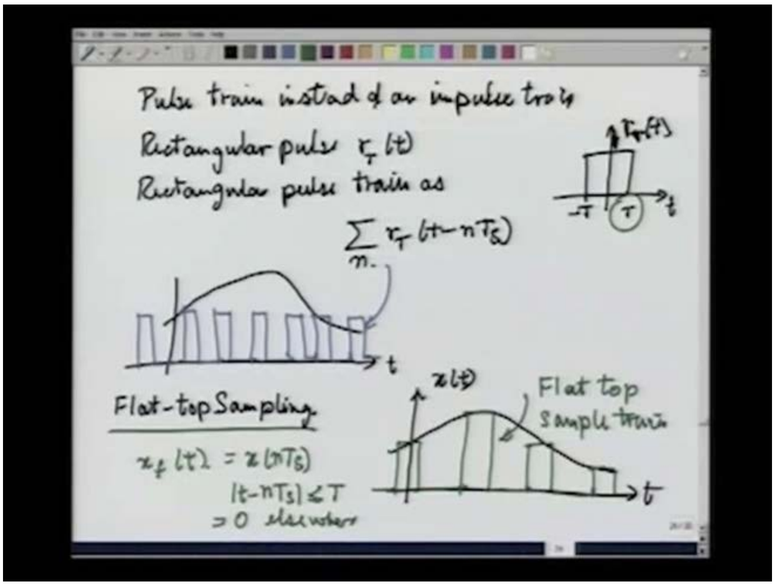

So, let us say we construct a pulse train instead of impulse train pulse train instead of a impulse train for this. Let us first define a standard rectangular pulse, I will denote this by r subscript t of t and sketch it over here. This is time it has a height of unity and width on either side of t and this is for minus t to t and it has a height of 1. This is r t of t, this is r t f t. So, if you have r t f t, then you will define a rectangular pulse train as a uniformly spaced sequence of r t f t. This will be of the form summation r t of t minus n t s. So, there placed at a distance of sampling time from each other.

This is over all n, so this a rectangular pulse train, now we can use the rectangular pulse train to sample the sequence instead of being rectangular and the ideal impulse train. So, if we, suppose this let us understand what happens to the result of sampling even before we do this. Let us consider another example of sampling now. Suppose, we do this over here, suppose we carry out a rectangular pulse train sampling, then let us see what we get? We can get two kinds of signals. I am going to make a sketch now of this is t and let us say x t is this let us say x t is this.

You have first of all a train of rectangular pulses, which we can sketch over here. So, this is a train of rectangular pulse equally this expression that we have over here that is the train of rectangular pulses. Now, there are two different ways in which one can construct using rectangular pulses rather than an ideal impulses a rectangular pulse train instead of an ideal impulse train. We sample train one in one case we samples we consists of rectangular pulses of height given by the value of x t at the point of occurrence of the centre of the impulse and the top of the pulse will be flat.

So, if you have tops of the pulses which are flat and heights of pulses given by x an n t s, then you have what is called, what I call flat top sampling. We have what we called flattop sampling now what is flattop sampling. Let us make a sketch as a separate sketch for this. This is x t in black and flat top sampling will make will find samples of appropriate heights at which plus, so let us we just do flat top sampling here and green. So, you have this is the flat top sample train which I will denoted by x f of t. Now, what is the expression for x of of t mathematically, how does it represent x t? Now, we will say that x of of t equals x t at certain places and it is equal to in fact is not equal to x t.

It is equal to x of n t s x of of t equals x of n t s over a width of time that goes from n to s minus p to n p s plus capital T, fine? So, is equal to this when mod t minus n t s is less than equal to t t is half the pulse width remember t is given according to this equals 0 when elsewhere. So, this is what define x f of t, this expression is very nice to look at, but it does not being really tells us what is the sequence of mathematical operations. That will give us x f of t; that is what we shall explore next x f of t is a sequence of flat top samples. Now, the best way to generate a sequence of flat top samples where the height of each sample is determined by the local value of x t at the point n t s, it just follows you first generate x s of t that is the original ideal sampled train.

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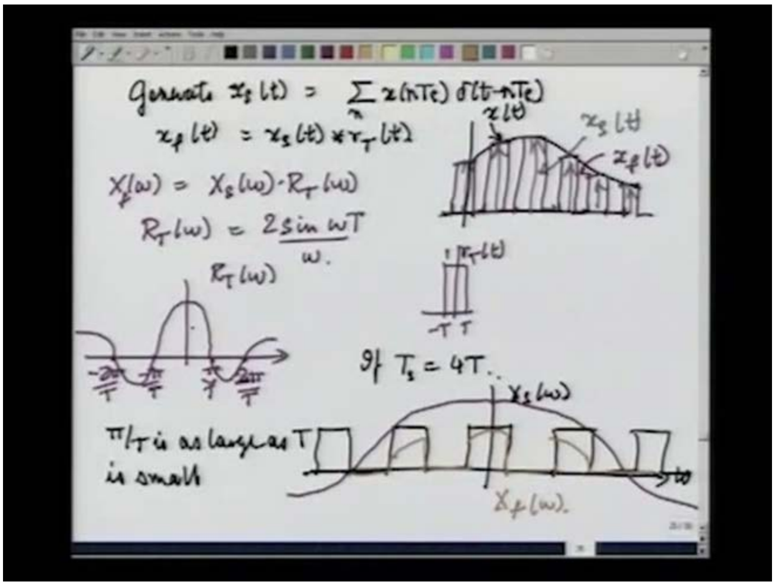

So, first step generate x s of t equal to summation over all n x n t s delta t minus n t s first generate this. Now, having generated this x s of t will be convolved by R T f t. It is to say that x f of t is nothing but the convolution of x s of t with x sorry, with R T of t. If you do not understand this graphically, you just do this. You first carry out an ideal sampling, let us do that the ideal samples are these we have this ideal samples. So, this sample train is nothing but x s of t which is been drawn in grey this is x s of t when x s of t is convolved with R T f t where R T f t is as we know nearly sorry R T f t is just this. This is t and this is minus t and this is R T of t of unit height.

So, when a convolve R T of t with x s of t at each place where there is a sampling pulse it will be replaced by a rectangular pulse of height proportional to the sampling pulse. In fact equal to the height of the sampling pulse and this will replace everywhere and you will finally get this. The last one, the last coloured function that we have here is this is this is x of t sorry. In black you have x of t and in this colour you have x f of t is in purple. So, you have x s x of t in black, x s s of t in grey and x f of t in purple.

This every sequence of operation, first do ideal sampling that is calculate d t times x t and on the result of this with this result of this you convolve R T of t. With all this information in hand let us try to understand what is x f of omega? x f of omega will be by our knowledge of convolution property x s of omega multiplied by R T of omega. Now, we have solved some examples earlier of what R T omega would be, this was done when we were discussing examples of the continuous time Fourier transform R T of omega is given by 2 sine omega t by omega, fine?

Now, let us examine this function 2 sine omega t by omega. Let us just plot this first it will have zeros at certain points at certain point where omega t equals k pi. That is at t at omega equal to k pi by t k times pi by t. So, if this is pi by t 2 pi minus pi by t minus 2 pi by t, then it will have zeros at these places. Otherwise, it will have the general shape of the sinc function, so let us plot it, so this is the shape of R T of omega. Now, remember that pi by t is as large as t is small. Suppose, t equals 4 times t s if t equals 4 times t s, then pi by t will be correspondingly that much larger, sorry. Now, t is 4 times t s t s is 4 times t. Suppose, t s is 4 times t that means the first minimum will occur at pi by t, which is pi by t s by 4 that is pi 4 pi by t s.

Considerably far of point in short, let us now make a spectrum of both the x s of omega as well as R T of omega together, because we have to multiply the 2 to finally get x f of omega. So, suppose x s of omega looks like this, x s of omega. Let us say if like, this is let us say is x s of omega. Now, as we know R T of omega is going to be resembling this with t considerably larger than t s. So, considerably smaller than t s, so we get a copy of this looks like this.

This is what we get and when we multiply these two we will end up with a product that perhaps looks like this. This is x f of omega in orange. So, you have now x f of omega and you can clearly see that the fact that all t of omega is not flat in the vicinity of omega equal to 0. More particularly over the interval minus b to b ensures that x f of omega is not faithful to excess of omega in the interval minus b to b.

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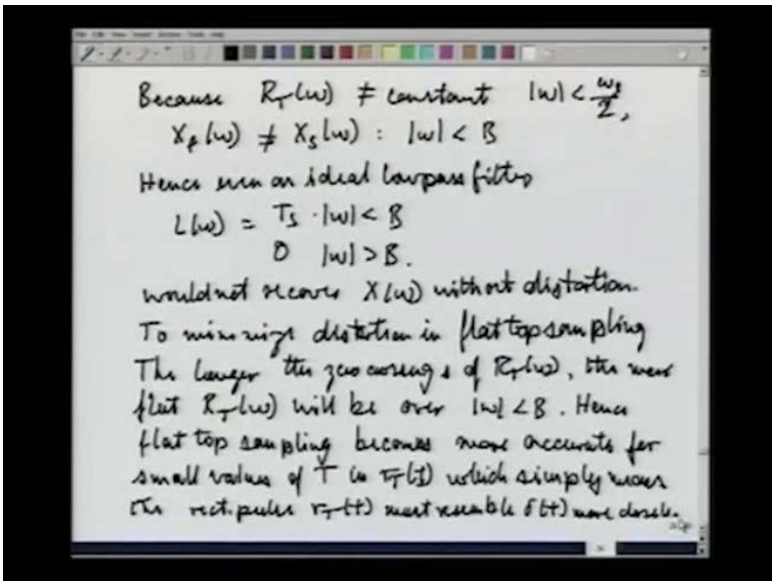

That is to say, because R T of omega is not is not equal to a constant in the interval mod omega less than b or at least if you have been more generous we will say mod omega less than omega s by 2; x f of omega is not equal to x s of omega over the interval mod omega less than the less than b, yes that is how let matter. Hence, even an ideal low pass filter with its specification of l omega equal to t s for mod omega less than b equal to 0 for mod omega sorry, greater than b would not recover x omega without distortion fine. Because as you can see from the diagram, we just drew x f of omega is irreparably by the non flat nature of R T f omega.

Let us write that the here that this is R T of omega. This curve is R T f omega. This is x f of omega and this thing in black of course, is x s of omega. Thus, flat top sampling falls flat is not particularly convenient to use, because it causes distortion or let us just see how to minimise distortion in flat top sampling going? Back to the diagram, we have, we find that the 0 crossings of R T omega occur at pi by t. Now the farther way this zero crossings are the more flat will R t f omega be in the interval mod omega less than b the larger the zero crossings of R t of omega the more flat R T of omega will be over mod omega less than b given the bandwidth of the original signal.

Hence, flat top sampling becomes more accurate small values of t of t in R T f t, which is simply saying its simply means that the rectangular pulse R T f t must resemble delta t more closely. So, this is why this is difficult this is at least reassuring because it tells us that when flat top sampling is carried out with many narrow rectangular pulses then inevitably the recovered signal using an ideal low pass filter will be pretty close to the original signal. So, we know how R T f t must be improved in order to make flat top sampling improve in performance.