Signal Processing Algorithms & Architecture

Dr. Anirban Dasgupta
Department of Electronics & Electrical Engineering
Indian Institute of Technology Guwahati

Lec 1: Introduction to Time-Domain Signal Processing

Hello everyone, so we are going to start a new session on the course on Signal Processing Algorithms and Architecture. I am Dr. Anirban Dasgupta, an Assistant Professor in the Department of Electronics and Electrical Engineering, IIT Guwahati. and let us start this first lecture on introduction to time domain signal processing. So first we will like to understand what is a signal? Now we have all come across the term signal. So what is a signal? Now technical definition of a signal is a manifestation of some physical quantity which usually varies with respect to some independent variable and that variable can be either space or time.

And the most important aspect of a signal is that it contains some information. So in this course, we will try to strict the independent variable as time. And mathematically a signal can be represented as a function of this independent variable. Now this may sound a bit complex but let us try to make this simple.

Suppose I want to record the ambient temperature of this room and plot it as a function of time. and say I have this equipment where I have a temperature sensor and this sensor sens the recorded temperature in an oscilloscope. And after some time say around 100 seconds, we get this kind of a plot. Now if you see this definition, so I am measuring a physical quantity which is the temperature and here the temperature is measured with respect to time. So that is my independent variable.

And the third thing is that it contains information. So from this plot, I can see that the temperature is somewhere around 32 degree centigrade. And the variation of this temperature is not much. It is within plus minus 0.2 degrees. So now this signal can be represented as a function-

$$s(t)$$

where this independent variable is a time. Now let us take some examples of real world signals which will give the importance of why signal processing is so important. Now this is a signal which is the speech signal and this is a measure of the air pressure as captured by the microphone. Now here some part of the signal we see there is a loud or high amplitude or variations which is the region where the person is speaking whereas there is regions whereas the person is silent. Another example of signal is This is an ECG signal and this is a record of our heartbeats.

So here the heart is beating. Another example is the seismograph signal or seismic signal which is a measure of the vibrations of the earth's surface. And similarly, we also have signals which are multichannel like our EEG signal or the brain signals which are capturing the information about our thought process, about our condition of the brain. And this is just the tip of the iceberg. So there are many examples of signals which makes signal processing a very useful topic, not only for electrical sciences, but across a lot of disciplines.

So now we know what is a signal. The next step is that how do we acquire a signal. And in the example we have seen that we use a sensor which have acquired the temperature as a signal. So in general we can have any physical quantity which we want to measure or which contains the information. This quantity is sensed by a sensor and what the sensor does is it creates a signal in electrical form , in the form of a time varying current or a voltage.

And this we called an analog signal because it is very much analogous to the physical quantity. Next we have something called a sampler. Now sampler is typically used for digital signals and in this course we are mainly focusing on digital or discrete time signals. So I would say that the analog signal is transferred to a sampler. and what the sampler will do? It will as the name suggests it will take samples of the analog signals at some specific time intervals and the result of the sampler is a discrete time signal.

Then this discrete time signal is sent to another operation which is called quantization and this element is called the quantizer. So sampler what it did? It sampled the time axis. Quantizer on the other hand will sample or will discretize the amplitude axis so that the amplitude values are at specific defined levels and why is it so? We will discuss soon and this quantized signal is now sent to something called an encoder Why encoder? Because finally since this is a digital signal it will be used in some digital hardware or digital memory. What is a digital memory? These are consisting of typical registers and these registers are made up of say flip-flops and each have a bit. So I would say that the encoder will encode this value, say some value is coming like 6. 1, it will encode this 6.1 using some definite amount of bits. And now what we get is a true digital signal. And then this digital signal is transferred to a digital hardware which is used for storage, processing or transmission of this signal. And if we combine the block sampler quantizer and encoder, this is basically combinedly called the ADC or analog to digital converter.

So now we know how to acquire a signal in digital form. Let us try to classify the types of signal. And the first classification is based on the capture, like if it is analogous to the physical quantity, we will call it a continuous signal. And if we take samples for processing in a digital hardware, it is a discrete time signal. Now discrete time signal is often confused with digital signal but the quantization and encoding step makes the signal actual digital.

Before that we can say it is just a sampled version of the analog signal or the continuous time signal. The next is deterministic or stochastic signal. We will slowly discuss each of these classifications. We will slowly discuss each of these classifications. Let us just know what are the classification or the main classification exists.

The next is even versus odd signals. Another is periodic versus aperiodic signals and then power versus energy signals. Now again as I said that since this is a discrete time or a digital signal processing based course, so we will discuss all the four categorizations apart from continuous versus discrete specifically for discrete times or digital signals. So, coming to the basic classification of continuous versus discrete, So, continuous signal as I said that it is defined for every point in time. So, if you want the signal value at say

$t = 1.7$ or 1.733 or 1.7333 you get a value. And examples include are analog audio signal like when I am talking my voice is being recorded directly and converted into an electrical signal this audio signal is an analog signal. The next category of signal is the discrete signal where we basically take sampled values or the signal values at specific time intervals. And the example can be the temperature readings which we recorded or which I discussed if we take say the temperature value say after every minute that becomes a discrete signal. or putting it in simple words what is a discrete time signal.

So, it is a sequence which you can represent by $x[n]$ and here if you see the I have used the third brackets and this is by convention to separate it out from the symbol of continuous time where first bracket is used. And here the index is n which is kind of a representation of discrete time, but this is technically an integer which can vary from negative infinity to positive infinity. For example, say I represent $x[n]$ as a sequence. You can say it is a sequence, you can say this is a vector and I put an arrow here which is denoting my 0 index or $n = 0$. Which gives me that if I want to say $x[-1]$ this is 2,

$x[1]$ is 6, $x[3]$will be 1, 2, 3. So, it will be 7. Now, how do we obtain this discrete time signal? So, technically there are two ways. First is by sampling an analog signal which I have discussed and the second is you can directly generate that signal by hard coding or using some formula in some digital hardware like a computer. Like for example, this digital signal, this is not sampled from any analog signal, this is directly I have hard coded the signal for illustration purpose. So how do we discretize analog signal? I already said that the first step is sampling. But let us take an illustration.

So see this is a sine wave. Now this is not strictly a sine wave. This is just a part of a sine wave. You cannot ideally draw a sine wave on a hardware or any board because sine wave varies from negative infinity to a positive infinity. And this is just a part of a sine wave where we have kind of 4 periods. And I say this sine wave, so it has a frequency of 2 hertz which gives the formula

$$x(t)\ sin\ (4\ \pi\ t)$$

Now, what the sampler will do? It will take samples of this wave at specific intervals and here let us discuss uniform sampling and by uniform sampling means after every given time say this time is capital Ts. So, in this time interval it will pick every sample of this signal and the sample signal will look something like this. So this I will say $x[n\ T_s]$ is basically I obtain-

$$sin\ (4\ \pi n\ T_s)$$

where here n is an integer. Now although I get a sampled version of the signal but still my independent variable here is time because my T_s has units of time.

So $n\ T_s$ will be units of time. So the next step is to remove this unit of time or convert this time to the independent variable which is my sample index. And I do this by replacing n into Ts as just n. And here I get the true discrete time representation of my analog signal. So here if I say this is my $x[n]$. So here T_s will be there because T_s will have some value and that will impact the variable.

But I will say this is $x[n]$. So, now I have the sampled version of the signal as a function of sample index n. The next step is I want to discretize the amplitude. Now why I want to discretize the amplitude? You see this amplitude even at this discrete intervals can have any arbitrary value. Like if I say the value at n equals to 11, it will be some value which is a real number. And how do we find the sign? it will be some value which is a real number.

And how do we find the sign? it will be some value which is a real number. And how do we find the sign? And how do we find the sign? either do the Taylor series expansion or we can see a sign table. So we know that we cannot accommodate any number or any value on digital hardware. Why? Because digital hardware has a specific limit. I will explain this in a very simple way.

Suppose I have a single bit. So that 1 bit I can represent either 0 or 1. If I have 2 bits, I can represent 4 possible values. if there are 3 bits, I can represent 8 possible values. So, we can see that we can represent a signal into some l discrete levels, the simple signal amplitude and this L is usually some 2^b. So now we can see that by quantizing into L level, so in this case here my b is 3, so my L is basically 8, 2^3 is 8.

So, here this process is called quantization. I am quantizing or I am dividing any signal value to one of the nearest L levels. It can be either nearest neighbor quantization, it can be floor quantization, it can be seal quantization, but any value will be mapped to either of these L levels. Now, let us take some examples of important discrete time signals.

And the first is a unit impulse. which is having a value of 1 when n is 0 and otherwise its value is 0. And here n varies from negative infinity to positive infinity. Another is my unit step function which has a value of unity for n greater than equals to 0 and this is again up to positive infinity and from minus 1 to minus infinity the signal value will be 0. Now there is a relation between the unit impulse and the unit step function. So, what is the relation? So, my unit impulse function is u n minus u of n minus 1 and this is basically you see that if I shift my unit step by 1 and then I subtract I only one sample at n equals to 0 will remain and remaining will be subtracted out.

So, this gives this relation. And similarly, if I sum all my shifted impulses, I get my un back. So, this is the relation of un getting from $\delta[n]$. The next is the ramp function where the signal values are of course 0 at the negative values of n but for positive values of n it is basically n and which means this is n times $u[n]$. Here we see that at given value of n the signal value is the value of n if the n is positive. Another signal is the exponential decay signal where we have a power n, $u[n]$ where mod a is less than 1.

So, this is typically the signal which is decaying exponentially. And probably the most important signal in discrete time is the complex exponential signal and this is very important for analysis like Fourier analysis. So, typically this is the sequence and again I have just shown a part of the signal. The signal will vary from negative infinity to positive infinity, and this specific waveform is for omega naught equals to 0.1 with a $\varphi[0]$. Now if I have a phi which is positive, there is a shift in the signal. Even if it is a negative phi, then also the signal will shift. Now unlike the analog or continuous part complex exponential which is always periodic with periods of multiples of 2 π, the discrete time complex signal is not guaranteed to be always periodic. So, the condition for periodicity is that my omega not should be 2 π and ratio of two interiors. So, now we have a good understanding of discrete time signals.

Let us try to understand the other categorization of signals with respect to discrete time signals. And the first categorization is deterministic and stochastic. So, well a signal is said to be deterministic if there is no uncertainty with respect to its value at any sample index. So, suppose if I have a signal, so at any given value of n, say I want to have the value of the signal at n equals to 20, I have the value as 1.00 which will always hold true. this is in contrast to a stochastic signal where if you take different realisation you will get different values at a given value of n. So in this case I have taken three realization, each realization depicted by a different color and we see that at a given value of n we have three different values of the signal or in other words where in deterministic my given n will have a specific value of the signal, my stochastic signal will have a distribution instead of having a specific value. And in reality, you can say all signals are technically stochastic in nature. The next is even and odd signals.

The next is even and odd signals. So even signal is defined by the relation

$$x[n] = x[-n],$$

where n varies from negative infinity to positive infinity. And this means that the signal is symmetric about the amplitude axis or the y axis. This is an example. Similarly, an odd signal is defined as

$$x[n] = -x[-n],$$

where n varies from again minus infinity to positive infinity. And you can say this is anti-symmetric about the amplitude axis, something like this.

So, all signals need not be either even or odd. Like there are some signals which are neither odd nor even. But any signal can be decomposed into an even component and an odd component. So even and odd signals are often useful in analysis of transforms like cosine transform and sine transforms. The next is periodic and aperiodic discrete time signals. And a discrete time signal or sequence is periodic if it repeats after a value n, like

$$x[n] = x[n + N], N \in \mathbb{Z}, \forall\, n \in (-\infty, \infty)$$

and the minimum value or the minimum positive value of n is called the period because if it is periodic by n, this will also hold for n equals to 2n or n equals to 3n or n equals to any integer multiple of n.

So, the minimum positive value is selected as its period. And again, I said that periodic signals have to be of infinite length from negative infinity to positive infinity. This is just a reflection that this is a section where three periods are shown, but this extends from negative infinity to positive infinity. If the signal is a finite duration, it cannot be a periodic signal. So, any other signal which does not obey this relationship is an aperiodic signal like this. But any signal if you have finite length will be definitely aperiodic.

The next categorization is power and energy discrete time signals. So, first let us try to

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

understand what signal energy is. Signal energy is obtained by taking the absolute value of the signal and squaring it and summing it over negative infinity to positive infinity. i,e

So, if this value E or signal energy is finite, then the signal is called an energy signal, something like this. So, this is a finite length signal. So, you can definitely calculate the energy, and it will be a finite value. Signal power on the other hand is defined by-

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

where this is summed from minus n to n and if this power is finite, then the signal is called power signal. and typically by finite I mean finite non-zero value. Again, this is just a sample or a window of a sine wave. This sine wave if considering that this is expanding from negative infinity to the positive infinity, this is a power signal and all periodic signal is a power signal.

And one more important thing you should remember is that the power of an energy signal is 0 while the energy of a power signal is infinite. So there can be some signal which are neither energy signal nor power signals as well. So we had a lot of discussion about digital or discrete time signal and the categorization of them. Now let us try to understand the approach of digital signal processing. So while I am speaking or recording this lecture, I am generating a voice signal and there is some noise possibly from some fan or some AC and what my sensor is recording is the total effect of my voice as well as the ambient noise.

And this is converted into an electrical signal as I said and this signal which is now captured is an analog signal that passes through an ADC we get a digital signal, this digital signal now goes to a digital signal processor. And maybe the task of this processor here is to reduce this ambient noise such that my voice is crystal clear. And once that processing is done then this output signal will go to a digital to analog converter which will then go to a speaker where the speaker will give you or give us the original physical quantity in the form of sound waves. And this is typically what I expect in a classroom setting where I will use a microphone and there will be a DSP processor which will actually remove all the background noise and in the speaker the student should be able to listen to a very crystal clear audio. So, this process is signal processing where we analyze, manipulate, transform signal and why We want to extract useful information, improve quality and we want to prepare them for further analysis or transmission.

So why DSP? Why not analog signal processing? So, there are some obvious reasons which make digital signal processing advantageous over analog signal processing. Like for example DSP operations can be performed by inexpensive hardware. You can just write pro codes in the processor or DSP or the microcontrollers, and you can Now you have used one filter for a specific operation. Suppose you want to change the filter. So, for analog you have to change the components like your resistor, capacitor, your op amp.

But in DSP just to change the program you get a fresh algorithm or a fresh filter. It is easy to store and transmit digital data. Now in this age we can easily send a huge amount of data via internet or even with or even with wired communication we can easily send them.

Storage is also so simple. and memory is so inexpensive nowadays. But earlier days I used to remember these cassettes were recorded and each cassette costing around 35 to 40 rupees and that could only store around 90 minutes of audio. So, we can see that why analog signal processing or analog signals are difficult or expensive to store and transmit. And digital signals can also be processed offline. Say I want to analyze the signals after recording it much ahead of time or much after a long time, then it is not at all a challenge. But with analog signals, it is very challenging to like record and process it later.

But there are limitations of DSPs which are the present areas of research in signal processing like one is the extra requirement of some anti-aliasing filters, your ADCs and DACs are required and then since these are active devices there can be some power dissipations which can limit their usage in like cases where the power constraint is there and also there is a restriction on the operating frequency. And this is given by the Nyquist sampling rate where you can use at least sample twice the bandwidth or twice the maximum frequency content of the signal. So you cannot have any arbitrary frequency range but analog signals there are no restrictions. So, some typical DSP problems include in extracting of information like from finding the peaks and zero crossings, noise reduction as I said that I can I have an actual sine wave say but it is contaminated with noise and I want to filter it or clean it. I even can want to compress the signal say I have this signal and I want to represent it with a very few number of points and this can be useful for transmission.

Artifact removal is another signal processing application where artifacts are kind of noise which are signal to some other problems but it is noise for some other problem. So removing this kind of artifacts is a signal processing problem. So, in this course, we will typically have three ways of observing signals. One is in the time domain like what we introduced, then in the frequency domain and then we will also discuss something on the time frequency domain.

So, these are two books which I want to recommend. Of course, there are lot of books and lot of material available but to me I think these two books are really very good books. One is by Proakis and Manolakis and the second is by Oppenheim. So, thank you for the first lecture. We will start soon with the second lecture.