Course Name: Power Electronics Applications in Power Systems

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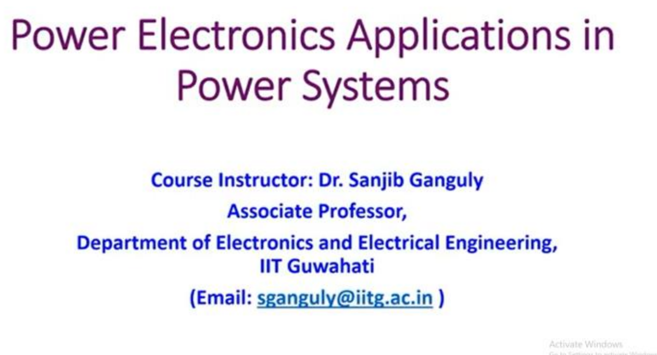


Lec 10: Plot of mid-point voltage vs line loading

So we derive this midpoint voltage, midpoint current, midpoint active power and midpoint reactive power expressions for a symmetrical lossless long transmission line. Now next what we will do, we will try to plot this midpoint voltage with respect to the ratio of active power, which is flowing through this, this midpoint to this P c, where P c is basically surge impedance rating. That is a surge impedance loading and P m is the midpoint active power. We will be interested to see this midpoint voltage with respect to the ratio of P m to P c. So, this derivation we will do right now. So, in order to do this derivation, let us again revisit this voltage expression that V r is equal to V m cos beta l by 2 minus I z c I m sin beta l by 2.

So, I will write this expression again, Vr is equal to Vm phasor cos beta l by 2 minus Im j Zc sin beta l by 2. Again, you understand that this Vr is basically this receiving end voltage, receiving end voltage and Vm is midpoint voltage. And you know that this Im is midpoint current and Zc as you know it is the surge impedance and beta is representing the phase constant. Now we will write this Im in terms of this Pm and Vm.

So we have a relationship over here that S m is equal to 3 V m I m star, which gives nothing but since V m and I m are in same phase. So, this gives nothing but the simple multiplication of V m and I m magnitudes. So, this is basically the real power multiplied by 3. So, this is what representing this real power flowing through this midpoint. Later on, we have seen that there is no, you know, imaginary part.

So, this represent P m. So, basically, this P m is representing here, if you look at this expression, it is equal to 3, V m magnitude, V m magnitude, this V m magnitude is this, P m magnitude is this multiplied by I m magnitude, this I m magnitude is this. So, I am just writing it over here since P m is equal to 3 V m I m magnitude. So, we can write that I m is equal to P m divided by 3 V and I will put it over here, but Im will have some angle as well. Now, here what we will do is in order to find out the expression for Vm, what we will do is that we will change the reference.

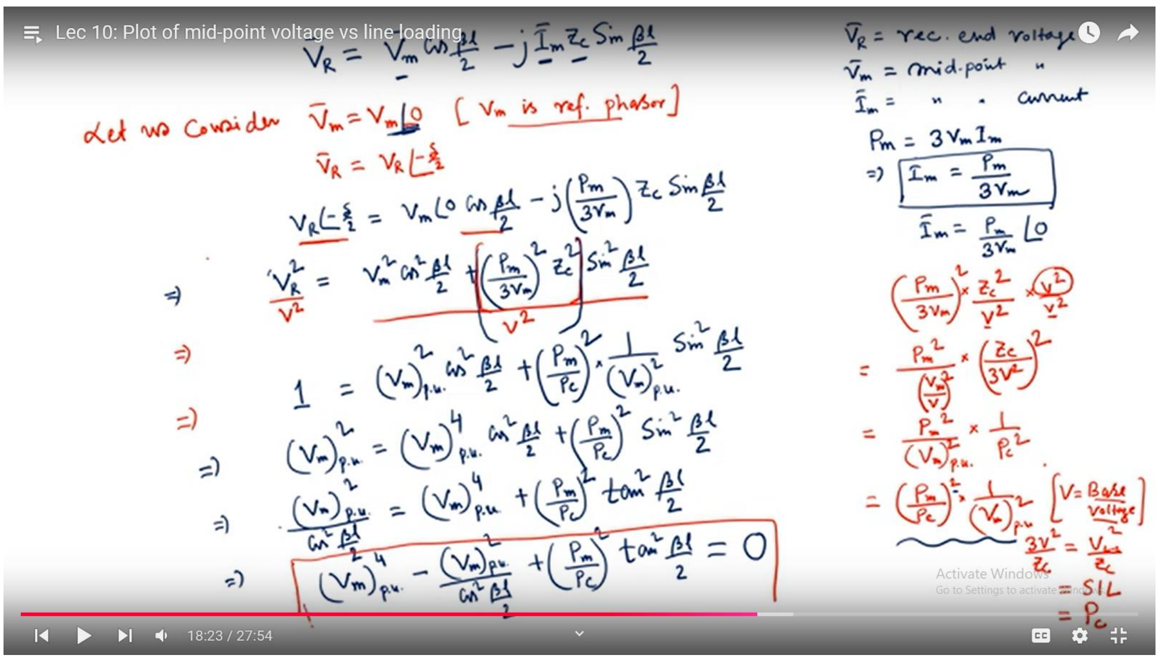
Now, here in this particular problem, we consider that if you look at, we consider that the reference was the sending end voltage that is Vs. And, according to this reference, we determine the angle at the midpoint, which is coming out to be lagging with respect to the sending end voltage at an angle of minus delta by 2. And this Vr, which is lagging with respect to the sending end voltage at an angle of minus delta. So, here for the sake of derivation, what we will do, we will consider, let us consider V m is our reference. So, V m is our reference.

So, we will consider V m is reference phasor So, if it is so, then Vr will be equal to Vr magnitude at an angle minus delta by 2, because you know that Vr is lagging delta by 2 angle with respect to the midpoint. This is what we have seen. already. Here you can see this V r which is considered minus delta. So, this is considered to be V r at an angle minus delta and V m is V m at an angle minus delta by 2.

If you compare these two, then you will see there is a phase difference between the midpoint voltage and the receiving end voltage of an angle of delta by 2. So same thing is we used by changing the reference. So here I have changed the reference and we consider that the Vm is our reference point instead of the sending end voltage. So here we lag with respect to this midpoint voltage at an angle of delta y2 and I will put it there. So if I put it there then I will get. V r at an angle of minus delta by 2 is equal to V m at an angle 0 cos beta L by 2 minus j Im. Now, magnitude is basically replaced by this, Im magnitude is basically replaced by this, but Im angle will be similar to this Vm angle. So, we can write this Im is equal to its magnitude is Pm divided by 3 Vm. And angle is similar to this midpoint voltage, which is 0, because we consider this midpoint voltage as a reference phasor. And we also have seen that midpoint voltage and midpoint current are in same phase that we established in this expression.

So, we can write that if this midpoint voltage, we consider as a reference voltage and it is at an angle 0. So, midpoint current will be also some magnitude with a phase angle at an angle 0. So, I am just replacing this. This is P m divided by 3 V m Z c sin beta L by 2. Now, from this equation, what we can write, if we take the magnitude of this right-hand side and the magnitude of the left-hand side and equate, then what we will get? We will get a left-hand side v r square, right-hand side v m square cos square beta l by 2 plus p m by 3 Vm whole square Zc square sin square beta L by 2.

So, this we got by equating the left-hand side magnitude with the right-hand side magnitude. Now what we will do, we will consider this particular term, and we will further simplify this, and then whatever we will get, we will put it over here. So let us write this, P m divided by 3 V m square Z c square, and what we will do? we will do another thing that we will divide this with this voltage at the sending end and the receiving end side, which is considered to be same. So, if we go back and see that since the line is symmetrical, we consider that V s and V r are equal and we further consider that this V s and V r are equal and equal to V. So, what we can do is, before we simplify this, let us do one thing that we will divide the whole left-hand side and right-hand side with the v square.



So, this is we can do. So, we divide both left-hand side and right-hand side expressions with V square. Then we will have another V square over here. We will take this part only. Now, we will simplify this. Let us see what will happen. What we will do is, so this gives us P m square, this gives us 9 V m square. Now, what we will do is that we again multiply and divide V square to this particular term, then what we will get that this numerator V square will bring over here and we will keep V m inside that and this V square will multiply with this and what we will get it Z c V square whole square. Now, remember this V is basically representing the per phase voltage of the sending end side as well as the receiving end side. Now, what we will do is, this will keep it as it is P m square.

$$\boldsymbol{S_m = 3V_m\bar{I}_m^*}$$

$$= \mathbf{3}\left(\frac{V cos\frac{\delta}{2}}{\cos\frac{\beta l}{2}}\right)\left(\frac{V sin\frac{\delta}{2}}{z_c\sin\frac{\beta l}{2}}\right)\angle-\frac{\delta}{2}+\frac{\delta}{2}$$

$$= \frac{3V^2\left(2\cos\frac{\delta}{2}\sin\frac{\delta}{2}\right)}{z_c\cos\frac{\beta l}{2}\sin\frac{\beta l}{2}} = \frac{3V^2\sin\delta}{z_c\sin\beta l} = \frac{V_{L-L}^2\sin\delta}{z_c\sin\beta l} \quad (1)$$

$\boldsymbol{V_{L-L}: Line - line\ voltage}$

Mid-point active power

$$P_m = \frac{V_{L-L}^2\sin\delta}{z_c\sin\beta l} = P_S = P_R \quad (2)$$

[We considered lossless line]

$Q_m = 0$ $\Leftarrow$ This happens due to the fact that $\overrightarrow{V_m}$ &$\overrightarrow{I_m}$ are in same phase.

Plot $(\boldsymbol{V_m})_{\boldsymbol{p.u.}}$ Vs $\left(\frac{\boldsymbol{P_m}}{\boldsymbol{P_c}}\right)$

$P_c = Surge\ impedence\ loading$

$P_m = Mid - point\ active\ power$

From equation $\vec{V}_R = \vec{V}_m \cos\frac{\beta l}{2} - \vec{I}_m j\, z_c\sin\frac{\beta l}{2}$ (3)

Let us consider $\overrightarrow{V_m} = V_m\angle 0$ $[V_m\ is\ reference\ phasor]$

Now, $P_m = 3V_mI_m$

$$\Rightarrow I_m = \frac{P_m}{3V_m} \quad (4)$$

Now, substituting the value of equation (3) in equation (4)

$$\overrightarrow{V_R} = V_R\angle-\frac{\delta}{2}$$

$$V_R\angle-\frac{\delta}{2} = V_m\angle 0\cos\frac{\beta l}{2} - \left(\frac{P_m}{3V_m}\right)j\, z_c\sin\frac{\beta l}{2}$$

$$V_R^2 = V_m^2\, cos^2\frac{\beta l}{2} + \left(\frac{P_m}{3V_m}\right)^2 z_c^2 sin^2\frac{\beta l}{2} \quad (5)$$

Since it is a symmetrical line, $V_s = V_R = V(base\ voltage)$

Thus, equation (5) can be written as $V_R^2 = V_m^2 \, cos^2 \frac{\beta l}{2} + \left(\frac{P_m}{3V_m}\right)^2 z_c^2 sin^2 \frac{\beta l}{2}$

$$\Rightarrow 1 = \frac{V_m^2}{V^2} \, cos^2 \frac{\beta l}{2} + \frac{\left(\frac{P_m}{3V_m}\right)^2 z_c^2 sin^2 \frac{\beta l}{2}}{V^2}$$

$$\Rightarrow 1 = (V_m)_{\,p.u.}{}^2 cos^2 \frac{\beta l}{2} + \left(\frac{P_m}{P_c}\right)^2 \times \frac{1}{(V_m)_{p.u.}^2} sin^2 \frac{\beta l}{2}$$

$$\Rightarrow (V_m)_{p.u.}^2 = (V_m)_{p.u.}^4 cos^2 \frac{\beta l}{2} + \left(\frac{P_m}{P_c}\right)^2 sin^2 \frac{\beta l}{2} \qquad (6)$$

Dividing both sides of equation (6) by $cos^2 \frac{\beta l}{2}$

$$\Rightarrow \frac{(V_m)_{p.u.}^2}{cos^2 \frac{\beta l}{2}} = (V_m)_{p.u.}^4 + \left(\frac{P_m}{P_c}\right)^2 tan^2 \frac{\beta l}{2}$$

$$\Rightarrow (V_m)_{p.u.}^4 - \frac{(V_m)_{p.u.}^2}{cos^2 \frac{\beta l}{2}} + \left(\frac{P_m}{P_c}\right)^2 tan^2 \frac{\beta l}{2} = 0 \qquad (7)$$

Let $(V_m)_{p.u.}^2 = x$

Then equation (7) becomes $x^2 - \frac{x}{cos^2 \frac{\beta l}{2}} + \left(\frac{P_m}{P_c}\right)^2 tan^2 \frac{\beta l}{2} = 0$ (8)

Now solutions to equation (8) can be obtained by solving the quadratic equation

$$(V_m)_{p.u.}^2 = \frac{1}{2cos^2 \frac{\beta l}{2}} \pm \frac{1}{2} \sqrt{\frac{1}{cos^4 \frac{\beta l}{2}} - 4 \left(\frac{P_m}{P_c}\right)^2 tan^2 \frac{\beta l}{2}} \qquad (9)$$

$$(V_m)_{p.u} = \sqrt{\frac{1}{2cos^2 \frac{\beta l}{2}} \pm \frac{1}{2} \sqrt{\frac{1}{cos^4 \frac{\beta l}{2}} - 4 \left(\frac{P_m}{P_c}\right)^2 tan^2 \frac{\beta l}{2}}} \qquad (10)$$

This 9, we will put inside this and so if we put this 9 inside this, then it will be 3 square. Now, this V m divided by V, it represents a ratio which means that that V m is divided by the voltage of the sending end side and receiving end side. So, the voltage of the sending end side and receiving end side are considered to be the same. So, if we consider this is a base voltage, then this V m divided by V represents V m per unit, because in power system we know that if any quantity is divided by its base value, then it is converted to per unit. It becomes unit less and it represents a per unit quantity.

So, we consider that V is our base value, considering V is our base voltage. So, the ratio of Vm to V is representing Vm per unit. Now, what this 3 V square Zc will We know that V is the base voltage. So, 3 V square divided by Zc is nothing but V line to line square divided by Zc. This is nothing but the surge impedance loading or SIL of the network.

So, this is represented by Pc. So, then this will be equal to 1 upon Pc square. So, the whole this part would be P m divided by P c whole square multiplied by 1 upon V m per unit whole square. So, that is what we get, and I will put it over here. Again, if we consider so, then the ratio of Vr to V is equal to 1, because already we consider that this voltage at the receiving end side and the sending end side are equal. You can see over here, we consider that voltage at the receiving end side and the sending end side are equal. Because, if the line is symmetrical, so the ratio Vr to V is 1. Now, this is 1. Now, here when we put this Vm to V ratio, this will be also Vm per unit square cos square beta l by 2. Now, this part is being converted to this.

So, I am just writing over here. So, this is P m divided by P c multiplied by 1 upon V m per unit square sin square beta l by 2. So, as you can see that this part, we already derived over here and we got this, this same thing we have put over here. Now, what we will do is, we will multiply this V m square p u both in the left hand side and the right hand side. So, what we will get? We will get V m per unit square is equal to V m per unit.

So, this square and this square will cause this to the power 4, then cos square beta l by 2 plus P m divided by P c. So this is the whole square which was missing, here you can see this is the whole square, so this is the whole square, then sin square beta l by 2. Now, again what we will do is, we will divide both right-hand side and left-hand side with cos square beta l by 2, so what we will get and we will simplify it, so what we will get is V m per unit square divided by cos square beta l by 2 is equal to V m per unit to the power 4 plus P m to P c ratios square. Then if you divide sin square cos sin square beta l by 2 to cos square beta l by 2, it will be tan square beta l by 2. Again, we will simplify the alignment and make it a quadratic equation form. So, this will give V m per unit 4 minus this V m per unit square divided by cos square beta l by 2 plus P m to P c whole square tan square beta l by 2 is equal to 0. I just simply put this left hand side part to right hand side and equate with 0. So, I will get a equation over here. This forms a quadratic equation. And if we consider the variable is Vm per unit square, then it gives a perfect quadratic equation similar to a form of ax square plus bx plus c is equal to 0.

And we can find out this value of x from that. So, from this we can find out or rather here we will find out this V m square per unit is basically representing the variable over here, it is equal to the square root of 1 upon 2 cos square beta l by 2 plus minus 1 by 2 root over 1 upon cos to the power 4 beta l by 2 minus 4 P m by P c square multiplied by tan square beta l by 2. So, that is what we get from this and that is what our solution is. So, this is the solution of V m per unit, not V m per unit square. This is the solution of V m per unit, but we got it from this particular equation.

Let us consider that V m per unit square is equal to x and then put it over here, then we will get the coefficient in the form of a x square plus b x plus c and then is equal to 0, then solve what is x. Similar way we solve it and we get it like this. So, this equation itself will give you the mathematical foundation to plot this. Here we will not plot only V m, rather we will plot V m per unit. Because remember, in power system we always represent this voltage in the form of per unit.

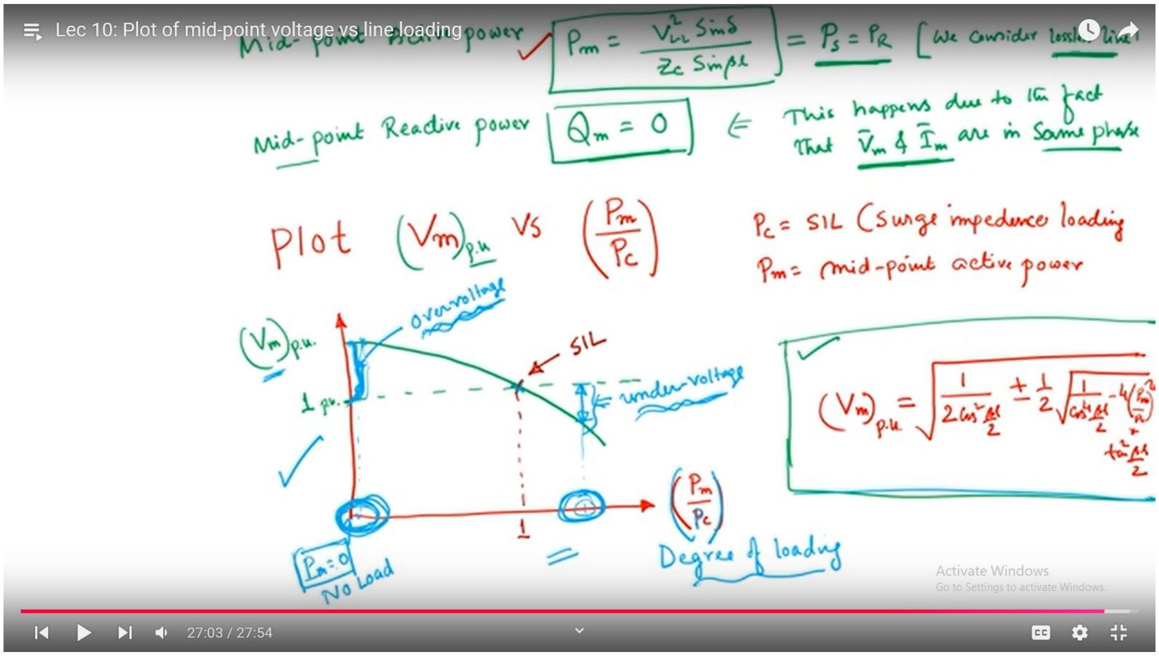
This will facilitate us to compare the voltage easily between two buses. Now, if this is your expression, anybody can use this particular expression in MATLAB or any other software you know, and then you can plot this, Vm per unit versus Pm to Pc. Now, if you plot this, then the plot would be something like that. If you plot this, then plot will be something like that. If you consider V m per unit is 1, if you consider that this is 1, 1 per unit, then this plot would be something like this.

So, this point is of our interest, this point corresponds to the point where the ratio of P m by P c would be equal to 1, which means that this point corresponds to the point when the line will be loaded with the search impedance. So, when the line is loaded with the search impedance, then only we will be having the V m per unit equal to 1. As you know that in the last lecture, I have shown you with a derivation that when this line would be loaded with surge impedance, then this voltage profile will be flat. Other than that, anywhere you will get a over voltage or under voltage condition. So this condition, this point stands for when p m is equal to 0, which means it is operating at no-load condition.

So, when there is a no load condition, this voltage at the midpoint is would be significantly higher than the one per unit. So, this much of over voltage over voltage the midpoint will experience during no load condition and even the neighborhood of this also there would be some amount of over voltage. And this will cause this stress of this mid of the line insulators near to the midpoint because they have to withstand significantly higher voltage. This could be 20 percent of higher voltage, this could be 10 percent of higher voltage and so on. So, the insulators which will be located near to the midpoint of the transmission line will experience some sort of over voltage and there would be chance that they will fail.

And so, there should be an over-voltage mitigation required there either at the midpoint or in the neighborhood. Now, you see that when this PM to PC is, the ratio is higher than 1, then the system will experience or the midpoint will experience some sort of undervoltage. So, that is also a problem if there is an under voltage in a line. Even though this both end voltages are regulated to 1 per unit, but the midpoint will suffer from some amount of over voltage and some amount of under voltage as well.

So, this is what the outcome of this analysis. So, we have some amount of over voltage at no load or light load condition. Even there is a small amount of load in the network, then also there would be some amount of over voltage at the midpoint and its neighborhood. Similarly, if there is a, you know, line is loaded with somewhere near to its rated load, there would be some amount of under voltage there. So, both under voltage and over voltage need mitigation and that is possible through some compensator. So, to mitigate this under voltage and over voltage, we need compensator and a compensator can mitigate this condition.



And that is what our goal of understanding different types of compensator which can mitigate this over voltage and under voltage conditions. This will happen in steady-state conditions, but as I said that at the very beginning, compensators are not only built for over-voltage and under-voltage mitigation, it may help in some other aspects also, specifically, during dynamic loading conditions, specifically during contingency conditions. But in steady-state, the primary goal of the compensator should be to mitigate this over-voltage and under-voltage, which will be caused by this light loading condition and this rated loading conditions. So, this is what I intend to discuss in this particular lecture. So, in this particular lecture, if I summarize what I discussed, first we derived the expression for different parameters in the midpoint, which includes voltage at the midpoint, current at the midpoint, active power at the midpoint and the reactive power at the midpoint of a symmetrical, lossless, long transmission line.

Then, I have shown you the expression of this midpoint voltage in as a function of this loading. So, this P m by 2 P c ratio is basically representing the degree of loading; the degree of loading to the transmission line with respect to this SIL that is surge impedance loading. And when it happens this characteristic shows you that there would be some sort of over voltage or under voltage specifically near to this no load or light load condition and near to the rated load condition which is obvious at the midpoint, even though the both end voltages are regulated. So, we need to have a mitigation of this overvoltage and undervoltage. Now, how do we mitigate this overvoltage and undervoltage? This will be the lesson of the next lecture.

So, this is all about this lecture. Thank you for joining. We will meet in the next lecture. Thank you.