

Transmission Lines and E.M. Waves
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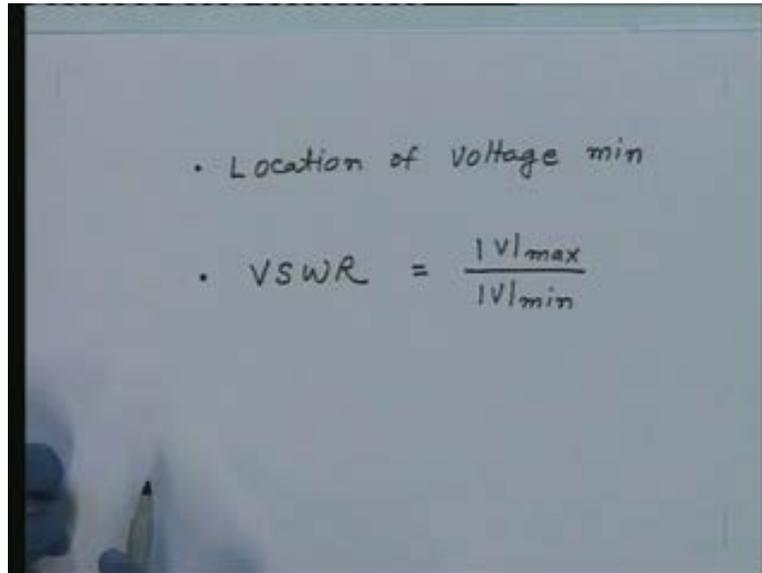
Lecture-9

Welcome, we have been discussing the Transmission Lines and its application for various things like identifying loads or measurement of load impedance and so on. In this lecture we will see if we know the standing wave pattern for a particular load termination on the Transmission Line then how quickly we should identify the type of load. We have seen that depending upon the load which is terminated on the line the standing wave pattern is set up on Transmission Line and standing wave pattern has two characteristic parameters, one is the location of the minimum from the load end of the Transmission Line and second one is the voltage standing wave ratio.

So essentially looking at the standing wave pattern and then using the parameters at the location of voltage or current minimum and the voltage standing wave ratio we can quickly identify the type of the load. So essentially without really calculating the quantity voltage standing wave ratio the idea here is to quickly identify the type of the load. We are not interested here in finding out what is the exact value of the load but just if I quickly glance at the standing wave pattern then how do we identify the type of the load.

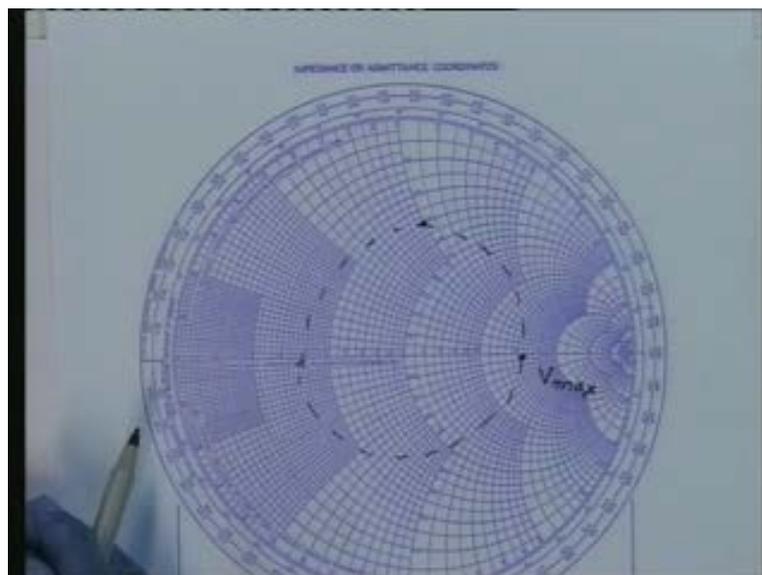
So here we will look at two quantities, one is the location of voltage minimum on the standing wave pattern and second one we will look at is the quantity called the VSWR. As we know the VSWR is nothing but the magnitude of voltage maximum divided by magnitude of voltage minimum. The smaller the value of voltage minimum the larger value of VSWR and when these two quantities are almost equal then the VSWR essentially approaches 1.

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Now let us look at the smith chart to identify the load and see how the impedance varies on the Transmission Line and then we will come back to the various standing wave patterns and then we will try to identify the loads. So if I look at the smith chart and this is the impedance smith chart essentially we are talking about.

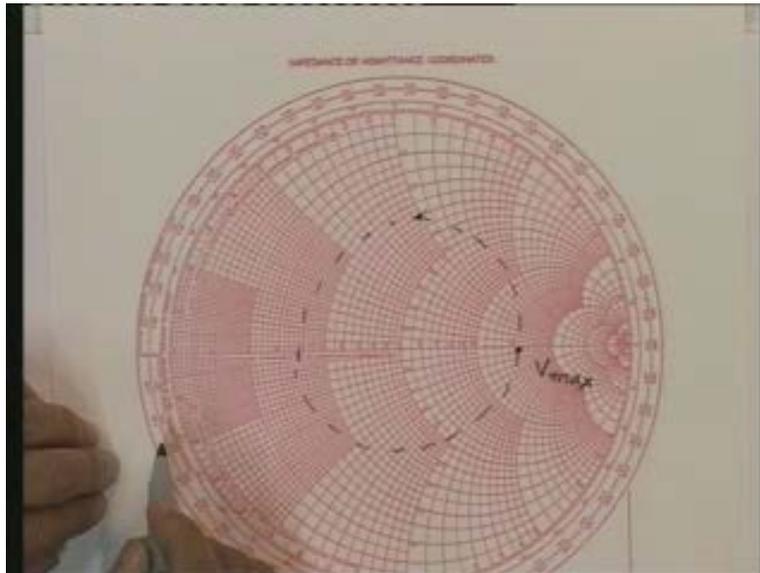
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As we have seen earlier the outermost circle essentially gives you the impedance which is purely reactive this line gives you the impedances which are purely resistive, on the right side of the center of the smith chart where a constant VSWR circle intersects this line is the location of the voltage maximum and where the constant VSWR circle intersects on the left side of the center on the horizontal line gives you the location of the voltage minimum.

Now let us say suppose the load impedance was some where lying here and this is the point which represents the load impedance so let us say this is the location which is the load impedance. Now if I draw a constant VSWR circle which is passing through the load impedance point then the VSWR circle would look something like this that means as I move towards the generator from the load point first we encounter this point here which is nothing but voltage maximum. Then if I move by a distance which is $\frac{\lambda}{4}$ then we will reach to the location here which is voltage minimum.

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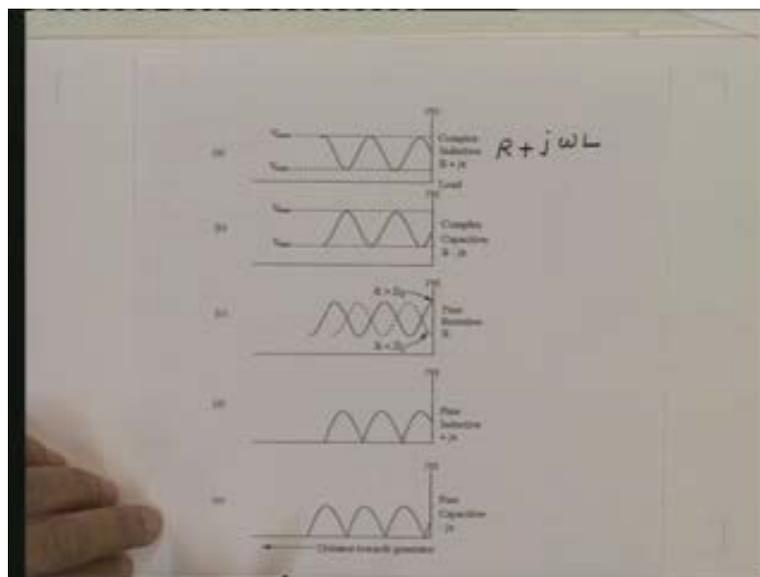
So since movement in the clockwise direction represent the movement on Transmission Line towards the generator for a load which is lying in the upper half of the smith chart

first we encounter the voltage maximum and then we encounter the voltage minimum. So if we consider this smith chart as the impedance smith chart then for all inductive loads as we move from the load towards the generator first we encounter voltage maximum, the exactly opposite will be true for a capacitive load if the point lies in the lower half of the smith chart then as we move towards the generator from the load first we encounter voltage minimum and then we will encounter voltage maximum.

So first thing is if the VSWR is not infinity that means if the voltage minimum is not zero then the load is not purely reactive so it has either inductive load or it is a capacitive load. If we move towards a generator and you encounter first the voltage maximum then we say that the load is inductive load if we encounter first the voltage minimum then we say the load is the capacitive load.

Here if I look at this plot there are various standing wave patterns which are given for different loads. If I consider the case is the first case here this is the load point this is the standing wave pattern the voltage minimum is not zero which means this load is not purely reactive load and as I move from the load towards the generator first we encounter a voltage maximum then voltage minimum.

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So with this understanding if we encounter first voltage maximum then the point lies in the upper half of the smith chart this impedance represents the inductive impedance, this load is a load of $R + j\omega L$. So we can say that as we move towards the generator if the voltage increases from load point towards generator then the load is an inductive load.

Similarly if I take this standing wave pattern here from the load when we move towards the generator first we encounter this point which is voltage minimum and then we will encounter point which is voltage maximum, that means this load must lie in the lower half of the smith chart because first we are encountering voltage minimum and that is the reason this load should correspond to the capacitive load. So then this one should correspond to a reactance which is $R - jx$.

Now if the load is neither capacitive nor inductive then the load must lie on this horizontal line here. And since the voltage maximum and the voltage minimum correspond to the intersection of the constant VSWR circle with the horizontal line the location of the load itself would be the location of the voltage maximum or voltage minimum. So if the load is purely resistive then at the load end either the voltage maximum would lie or the voltage minimum would lie.

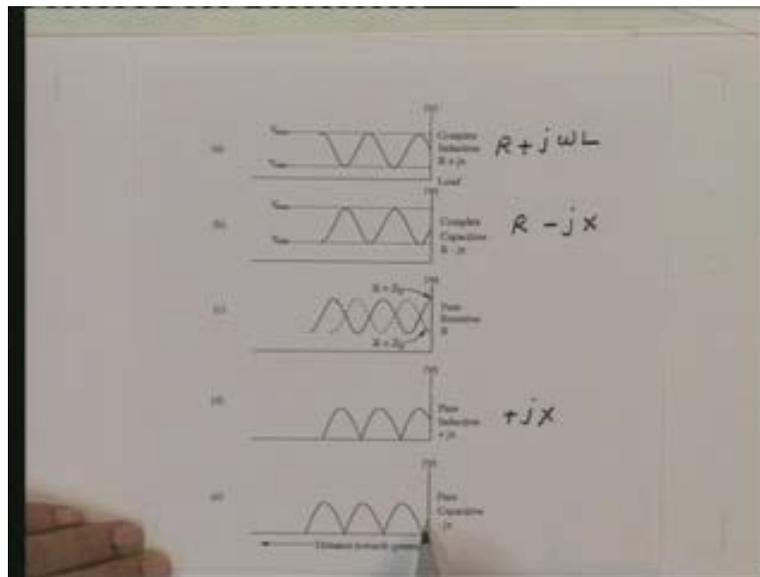
If I look at a pattern like this and if you have the voltage maximum at the load end then we know that the impedance now is purely resistive and since there is a voltage maximum the impedance will be greater than the characteristic impedance. Similarly if there is a voltage minimum at the load end then the load is purely resistive but the resistance will be greater than or less than the characteristic impedance. So the dotted line here essentially shows the standing wave pattern for resistance less than characteristic impedance and the thick line here essentially shows standing wave pattern for a resistance greater than the characteristic impedance.

If this standing wave pattern looks like this then first thing we note from here is that now the voltage minimum is zero that means the standing wave ratio is infinite which again means this load is either a purely reactive load or a short circuit or open circuit.

But if it is a short circuit or open circuit then the point would lie here or here or the voltage will be maximum at the load end or minimum at the load end but here the voltage is neither maximum nor minimum at the load point but the standing wave ratio is infinite.

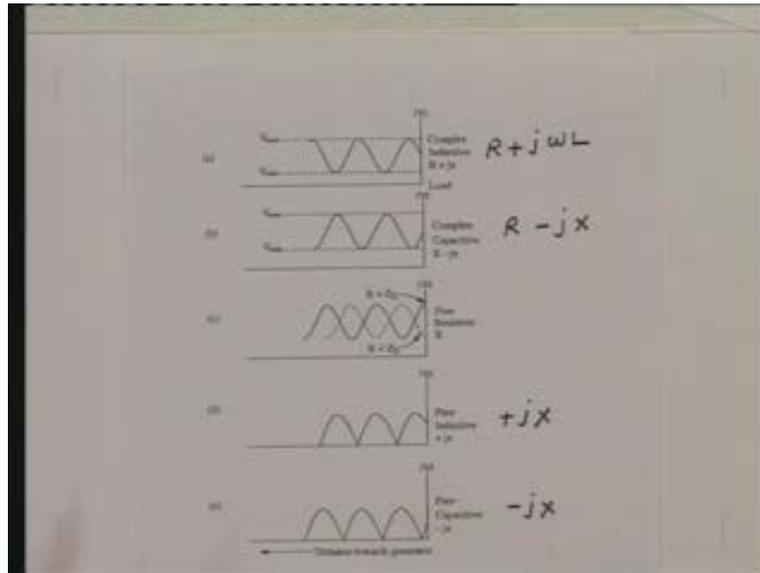
But as we move from the load towards the generator first we encounter the voltage maximum that means this load is purely reactive load and it lies on the upper half of the smith chart that mean this is representing purely inductive load. So if I have standing wave pattern something like this then the load is a purely inductive load so this load is $+jx$ type of load.

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Similarly if I take a standing wave patterns something like this again say the voltage minimum is zero at this location I get a VSWR which is infinite but now the voltage is dropping as I move towards the generator from the load point so first we encounter the voltage minimum and then we encounter voltage maximum that means this load point must lie in the lower half of the smith chart or in other words this represents a capacitive reactance so this gives you a $-jx$ impedance.

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So, one can quickly identify the type of loads just looking at the standing wave pattern because the information about the load is completely available in the standing wave pattern. And as we said the two parameters just the location of the voltage maximum or voltage minimum and the lowest value of voltage which you measure on the Transmission Line which again is related to the voltage standing wave ratio can help you in identifying the loads very quickly.

Now let us go to the laboratory and see some experimental setups for Transmission Line measurements. Here we are seeing a Co-axial connector the center conductor which is yellow and the outer conductor is the outer shell of the coaxial cable.

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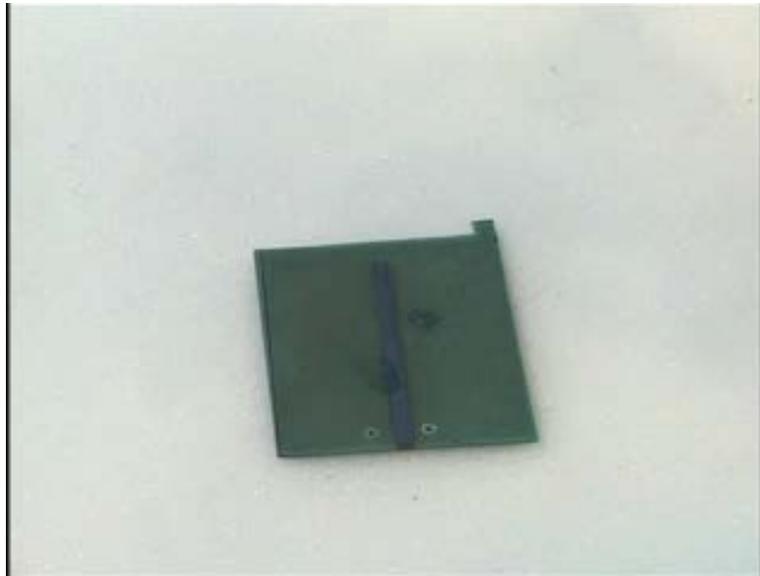
Here you are seeing a bare cable and again you see a center conductor the white one is the dielectric which is separating two conductors and then you are having an outer conductor.

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Then you are having a Transmission Line which is micro strip kind of Transmission Line. So you see a upper conductor there and on the bottom side you are having a ground plate which is separated by a dielectric.

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Now we want to do some measurements of impedance by using the Slaughtered Transmission Lines.



So here we have a set up for making measurements of VSWR reflection coefficient and measuring the unknown impedances. What you see here on the table are the different impedances which are kept there and a setup which called a Slaughtered Transmission Line.

This is a fifty ohm termination which is open termination.



This is twenty five ohm termination.

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And this is a slaughtered section of a Transmission Line.

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So you see this is a Co-axial cable, this is the connector on the one end of the line so you have a center conductor and you have a outer shell. But now if you look at the length of

the section of the Transmission Line there is a cut which is along the length of this section in which a voltage probe can be moved and we can measure the voltage along the Transmission Line.

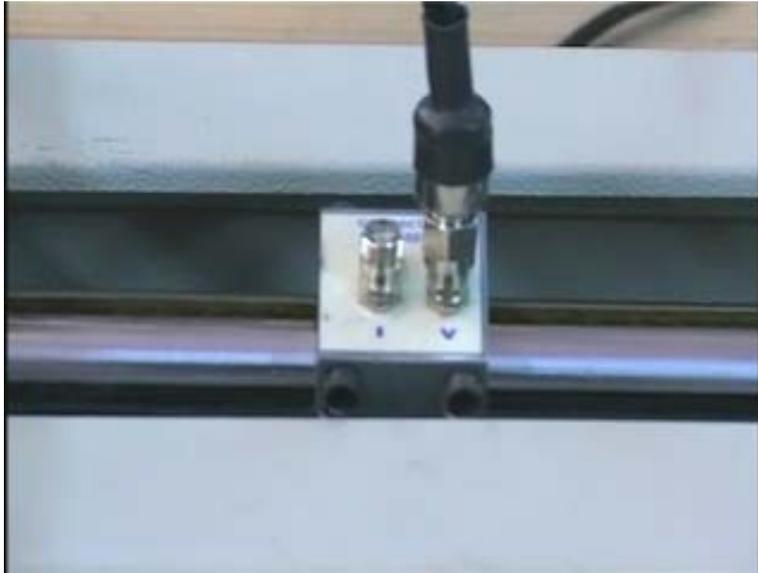
Here is the set up for measuring the voltage along the length of the Transmission Line when it is terminated with different impedances.

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Here what you are seeing is the voltage probe which is moving with the help of a motor along the length of the Transmission Line. The voltage is sensed and it is given to the computer so the computer registers the location of the probe and the voltage measured at that location. Then computer uses this information to get a plot called the voltage standing wave pattern which is nothing but the variation of magnitude of voltage along the length of the Transmission Line when it is terminated in a load.

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So for measurement of unknown impedance first we terminate the line with a short circuit load and then find out the variation of voltage as a function of distance on Transmission Line.

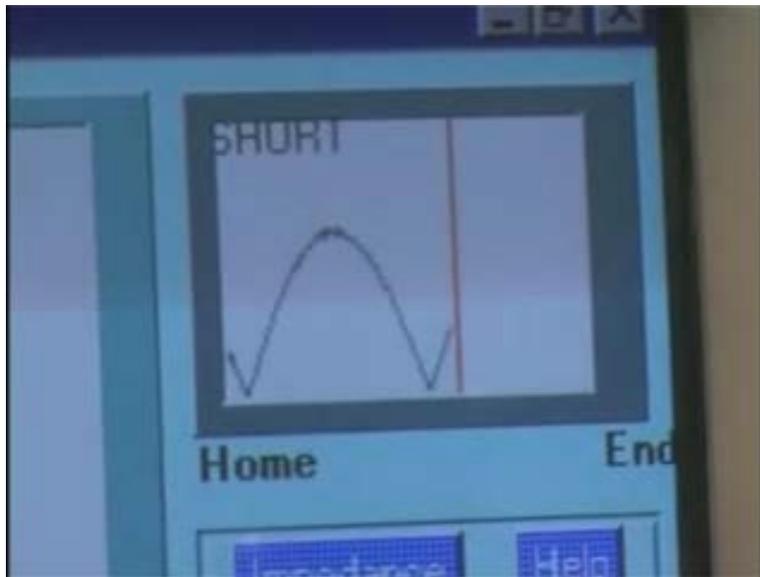
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So we get the voltage standing wave pattern for the short circuit load. So the load which is connected here is short circuit the voltage probe is moved along the slaughtered section of Transmission Line and the voltage is measured and is given to the computer.

This is the display which gives you as the voltage is measured along the length of the Transmission Line so you see here the load which is connected is short circuit.

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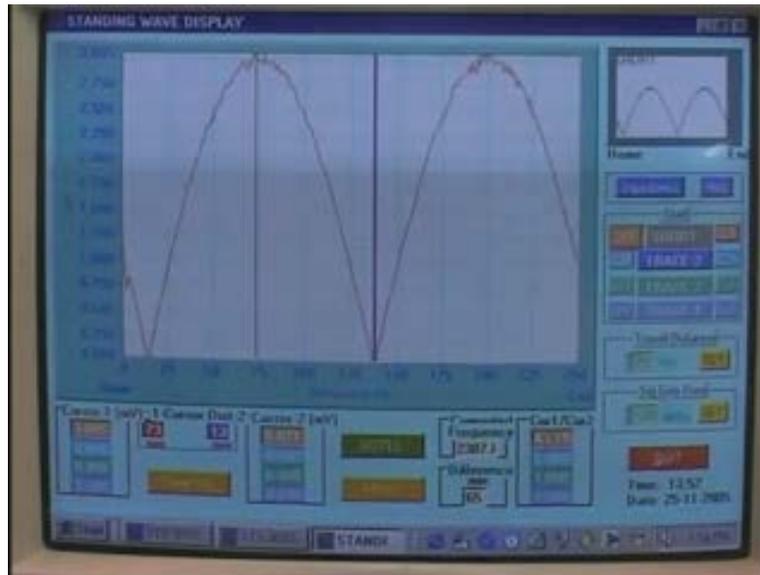


see it is written there as short and then you are having a voltage variation as a function of distance so essentially we are having the voltage standing wave pattern for the short circuit load.

As we know for the short circuit load the minimum voltage is zero you see from this standing wave pattern the minimum almost touches to the bottom line, that mean the voltage is almost zero and the voltage goes to maximum then again zero and then maximum and then so on.

Now the same data is displayed on a bigger screen so you can see that voltage pattern which we were seen in the small window now essentially is available on the full screen.

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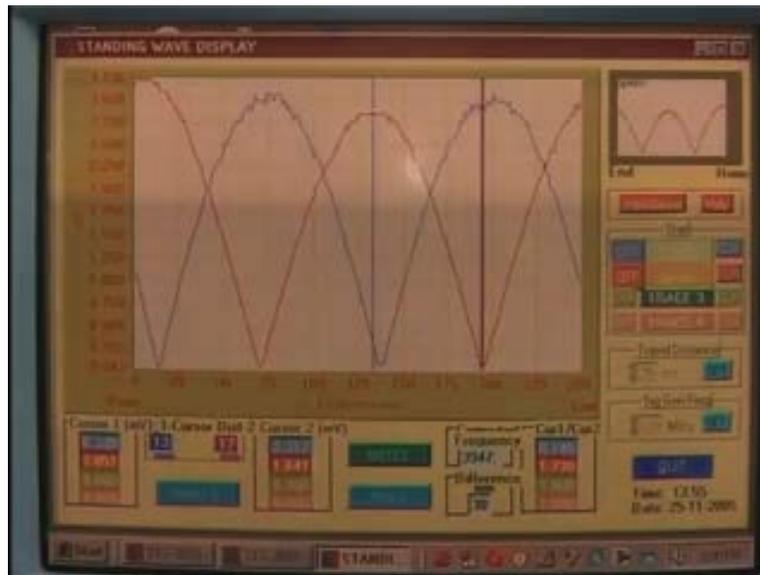


So this is the voltage standing wave pattern for the short circuit load the blue line gives you the location of the voltage minimum and the red line gives you the location of the voltage maximum.

Now let us replace the short circuit load by another load and again measure the voltage along the length of the Transmission Line.

As you can see the voltage standing wave pattern for this load is in blue the standing wave pattern for short circuit load is in red and as you can see very clearly that the minima of the short circuit coincides with maxima of this load also the voltage minimum is zero that means this load is nothing but the open circuit load.

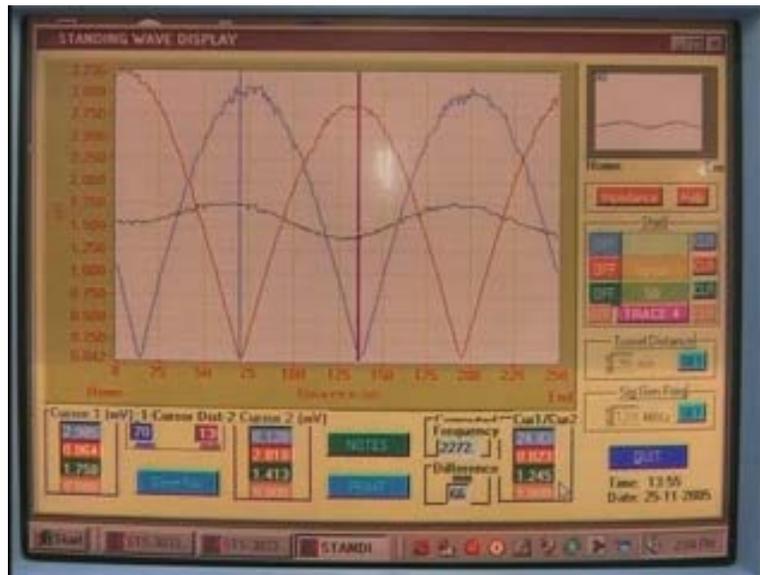
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Now we have put an unknown load which we want to measure so the unknown load is connected to the slaughtered section of Transmission Line and again would do the same exercise we measure the voltage along the length of Transmission Line and plot that voltage standing wave pattern with respect to the open circuit load and the short circuit load.

So this is the voltage standing wave pattern for the unknown impedance. What you can notice is the voltage maximum for this pattern more or less aligns with the minimum for the open circuit and the maximum for the short circuit

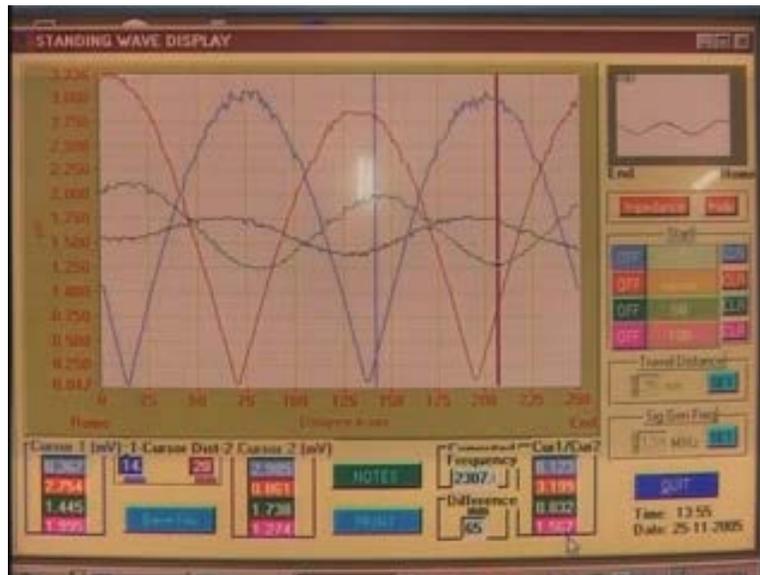
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That means this impedance is almost resistive impedance and since the maxima of this pattern aligns with the maxima of the short circuit the impedance value is less than the characteristic impedance.

Let us say one more load and get the standing wave pattern for this load. Then now you are having this new data which we have got for the voltage maxima or the voltage minima do not coincide with voltage maxima or voltage minima or by the short circuit or by the open circuit impedance. Then in general this a general load and then knowing the VSWR and the location of the voltage maxima or minima we can find the unknown impedance connected to the Transmission Line.

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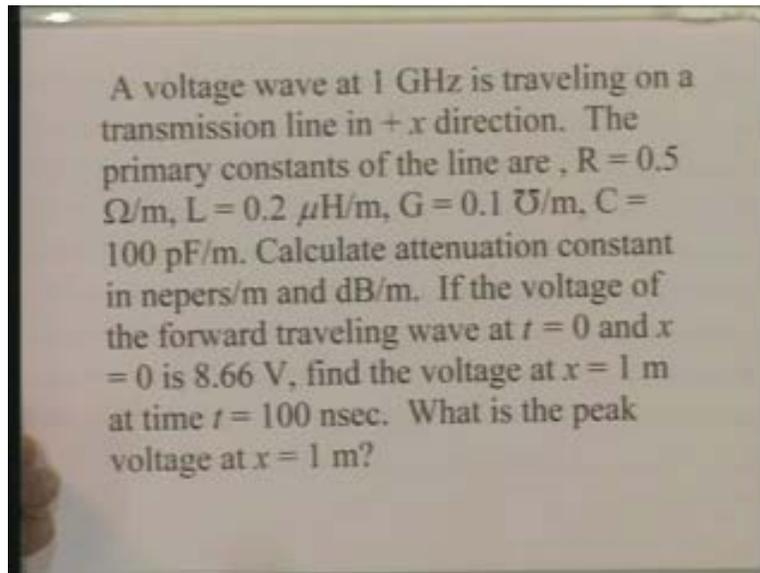


We have developed the basic theory of Transmission Line, also we saw the experimental setup which is used for carrying out the voltage standing wave pattern on a Transmission Line.

Now let us solve some very simple problems based on the theory which we have developed for the Transmission Line.

So here we have a problem which says A voltage wave at 1GHz is traveling on a Transmission Line in + x direction. The primary constants of the line are given that is R which is a resistance per unit length is $0.5 \Omega/m$, inductance is $0.2 \mu H/m$, the conductance is $0.1 \text{ } \square /m$ and the capacitance is $100 \text{ pF}/m$. And we have to calculate the attenuation constant of the Transmission Line in nepers/m and also in dB/m. Later we have to find out the voltage so if the voltage of the forward traveling wave is given at $t = 0$ and at $x = 0$ which is 8.66 volts and we have to find the voltage at $x = 1\text{meter}$ and at a time $t = 100$ nano second, also we have to find out what is the peak voltage at $x = 1\text{m}$.

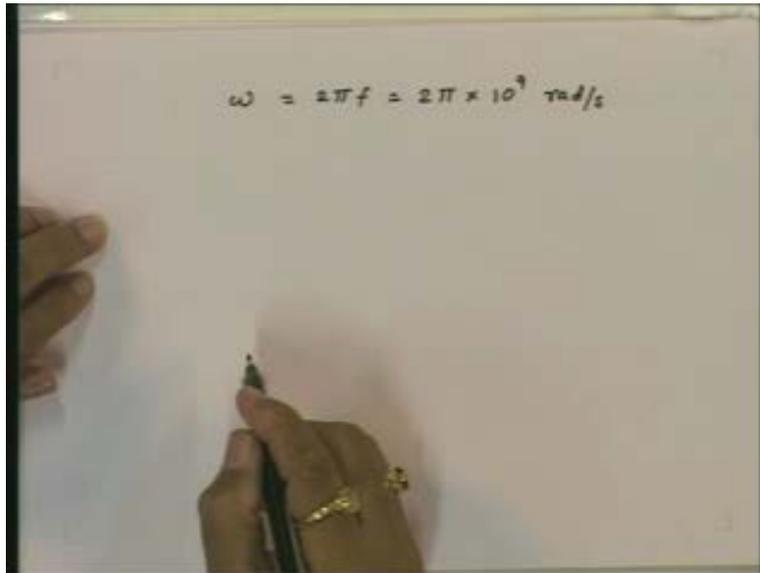
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So we have given the frequency which is 1GHz so from here we can first find out what is the angular frequency and then we find out what is the propagation constant which will be complex using the primary constants so from there we can find out the attenuation constant and the phase constant. And once we get that then by using the voltage equations of the Transmission Line we can find out the voltage at $x = 1$ meter and at time $t = 100$ nano seconds.

Since the frequency is given as 1GHz the angular frequency ω which is equal to 2π into frequency which again is equal to 2π into 1GHz which is 10^9 Hertz so this is radians per second

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The image shows a whiteboard with a hand writing the equation $\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s}$. The hand is holding a black marker and is positioned at the bottom center of the whiteboard. The equation is written in black ink and is centered on the whiteboard.

Then we can find out the propagation constant on Transmission Line so we get propagation constant γ which is equal to square root of $\sqrt{(R + j\omega L)(G + j\omega C)}$ and now we can substitute the value of R, G, L and C so we can write here as this is equal to square root of $0.5 \Omega/\text{m}$ that is the value of R plus $j2\pi$ into 10^9 which is the angular frequency ω into L which is $0.2 \mu\text{H}/\text{m}$ into 10^6 multiplied by $(G + j\omega C)$ so G which is $0.1 \square /\text{m}$ plus $j2\pi$ into 10^9 and C which is $100 \text{ pF}/\text{m}$ so this is hundred into 10^{-12} .

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The image shows a whiteboard with handwritten mathematical equations. At the top, the angular frequency is given as $\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s}$. Below this, the propagation constant is defined as $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$. The final step shows the substitution of values: $\gamma = \sqrt{(0.5 + j2\pi \times 10^9 \times 0.2 \times 10^{-6})(0.1 + j2\pi \times 10^9 \times 100 \times 10^{-12})}$. A hand holding a pen is visible at the bottom of the whiteboard.

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s}$$

Propagation const.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \sqrt{(0.5 + j2\pi \times 10^9 \times 0.2 \times 10^{-6})(0.1 + j2\pi \times 10^9 \times 100 \times 10^{-12})}$$

If I simplify this and take a square root we get the propagation constant γ for the Transmission Line that is $2.23 + j 28.2$ per meter.

So as I mentioned once you know the complex propagation constant then this real part of this propagation constant gives you the attenuation constant and this quantity here the imaginary part gives you the phase constant. So from here we have α which is attenuation constant that is equal to 2.23 Nepers per meter and the phase constant β which will be equal to 28.2 radians per meter.

It is also asked here that you have to find out the attenuation constant now in Nepers/meter and also we have to find this quantity in dB/meter. If you recall we had mentioned that from Neper to dB conversion is multiplying this quantity by 8.68 so if I multiply this quantity 2.23 by 8.68 that will be the attenuation constant in dBs per meter so it will be 19.35 dB per meter.

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The image shows a handwritten derivation for the propagation constant γ . It starts with the angular frequency $\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s}$. Then, it identifies the propagation constant as $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$. The values for R , L , G , and C are substituted: $R = 0.5$, $L = 0.2 \times 10^{-6}$, $G = 0.1$, and $C = 100 \times 10^{-12}$. The result is $\gamma = 2.23 + j28.2 \text{ per meter}$. From this, the attenuation constant $\alpha = 2.23 \text{ Np/m} = 2.23 \times 8.68 = 19.35 \text{ dB/m}$ and the phase constant $\beta = 28.2 \text{ rad/m}$ are derived.

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s}$$

Propagation const.

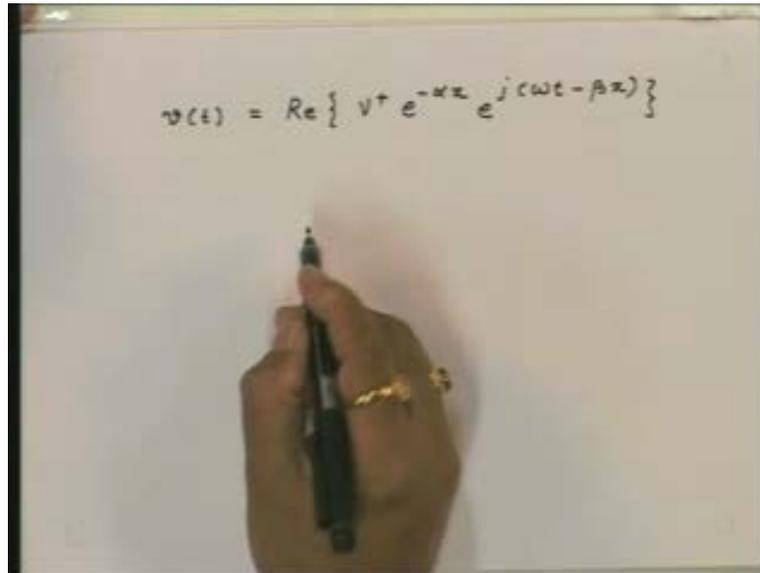
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \sqrt{(0.5 + j2\pi \times 10^9 \times 0.2 \times 10^{-6})(0.1 + j2\pi \times 10^9 \times 100 \times 10^{-12})}$$
$$\gamma = 2.23 + j28.2 \text{ per meter}$$
$$\alpha = 2.23 \text{ Np/m} = 2.23 \times 8.68 = 19.35 \text{ dB/m}$$
$$\beta = 28.2 \text{ rad/m}$$

So we have completed the first part of the problem where you have to find out the attenuation constant on the Transmission Line in nepers per meter and dB per meter.

Now in the second part we now have to find out the voltage at this location that is $x = 1\text{m}$ and at time $t = 100$ nano second if the initial voltage is given at $t = 0$ second and $x = 0$ meters. so now we can use the voltage equation first you find out the quantity which is the amplitude of the forward traveling wave so by using the initial condition essentially we calculate the arbitrary constant V^+ and then we can find out the voltage at any location on Transmission Line.

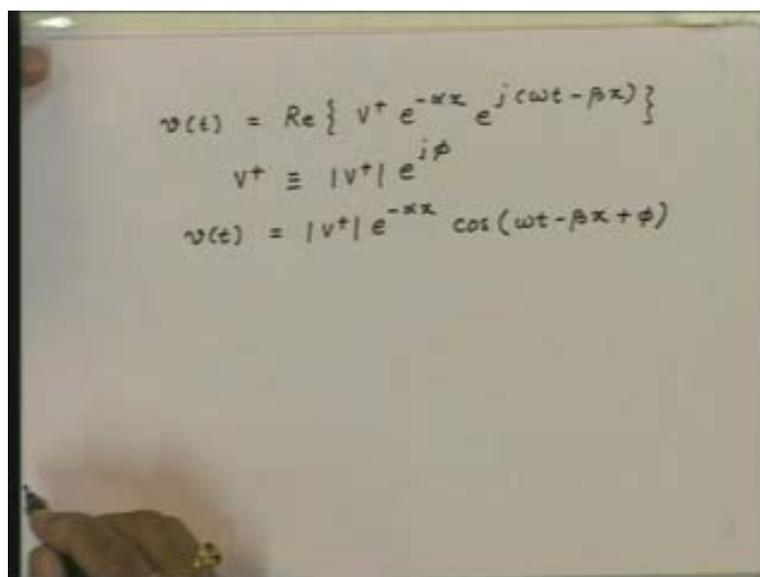
So if I write down the instantaneous voltage at any point on Transmission Line $v(t)$ which will be nothing but real part of $V^+ e^{-\alpha x} e^{j(\omega t - \beta x)}$.

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$$v(t) = \text{Re} \{ V^+ e^{-\alpha x} e^{j(\omega t - \beta x)} \}$$

Saying that this quantity V^+ in general is a complex quantity and if you write down V^+ as the magnitude of V^+ and some initial phase which is $j\phi$. Then this quantity $v(t)$ which is the real part of this expression can be written as $|V^+| e^{-\alpha x}$ and e to the power $e^{j\phi}$ will combine with this so real part of that will be nothing but $\cos(\omega t - \beta x + \phi)$.

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$$v(t) = \text{Re} \{ V^+ e^{-\alpha x} e^{j(\omega t - \beta x)} \}$$
$$V^+ \equiv |V^+| e^{j\phi}$$
$$v(t) = |V^+| e^{-\alpha x} \cos(\omega t - \beta x + \phi)$$

Now we are given that at $t = 0$ and at $x = 0$ this instantaneous voltage is 8.66 volts so I can substitute it into these equations I can say that this voltage is 8.66 volts that is equal to V^+ this voltage is given at $x = 0$ and $t = 0$ so this quantity becomes 1 at $x = 0$, this will be zero at $t = 0$ and this will be equal to zero at $x = 0$ and now the initial phase is required for calculation so it is not mentioned here but let us say I take the initial phase for the problem which is thirty degrees initial phase is 30° .

So this quantity ϕ which is the initial phase for the voltage wave that is equal to 30° so this quantity is zero, this quantity is zero so I get $\cos 30^\circ$. The $\cos 30^\circ$ is nothing but $\sqrt{3}/2$ which is 0.866 so from here we get $V^+ = 10$ Volts.

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The image shows a whiteboard with the following handwritten equations:

$$v(t) = \text{Re} \{ V^+ e^{-\alpha x} e^{j(\omega t - \beta x)} \}$$

$$V^+ \equiv |V^+| e^{j\phi}$$

$$v(t) = |V^+| e^{-\alpha x} \cos(\omega t - \beta x + \phi)$$

$$8.66 = |V^+| \cos 30^\circ$$

$$|V^+| = 10 \text{ Volts.}$$

Once I know this quantity V^+ then I can go back and substitute into this equation and I can find out the voltage at any instant of time at any given location on the Transmission Line. So now you have to find out this voltage $v(t)$ at $x = 1\text{m}$ and $t = 100$ nano second.

Now let us substitute these quantities into this expression So that I get a voltage here $v(t) = 10 e^{-\alpha x}$ where $x = 1$ and alpha in nepers per meter is 2.33 so you got $e^{-2.23}$ and $\cos \omega t$

which is 2π into 10^9 into t which is hundred nano second so this is hundred into 10^{-9} minus βx and as we have seen β for this Transmission Line is 28.2 radians per meter so I get 28.2 into 1 at $x = 1$ plus this quantity ϕ initial phase which is equal to 30° so that is $\pi/6$.

If I simplify this then I will get the voltage at $x = 1\text{m}$, at time $t = 100$ nano seconds which will be equal to -0.88 Volts.

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The image shows a whiteboard with the following handwritten equations and steps:

$$v(t) = \text{Re} \{ V^+ e^{-\alpha x} e^{j(\omega t - \beta x)} \}$$

$$V^+ \equiv |V^+| e^{j\phi}$$

$$v(t) = |V^+| e^{-\alpha x} \cos(\omega t - \beta x + \phi)$$

$$8.66 = |V^+| \cos 30^\circ$$

$$|V^+| = 10 \text{ Volts.}$$

$v(t)$? at $x = 1\text{m}$ and $t = 100 \text{ nsec}$

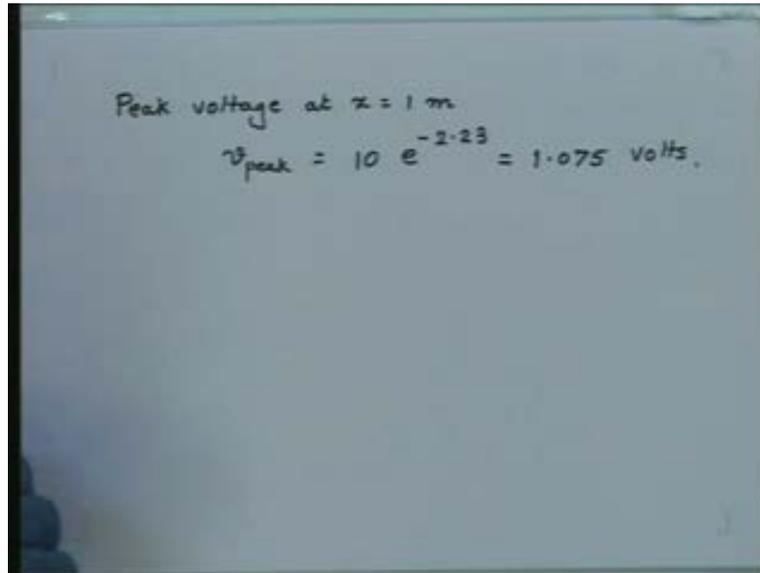
$$v(t) = 10 e^{-2.23} \cos(2\pi \times 10^9 \times 100 \times 10^{-9} - 28.2 \times 1 + \pi/6)$$

$$= -0.88 \text{ Volts.}$$

It is also asked to find out the peak voltage at this location that is $x = 1\text{m}$ so if I go in to this expression the cosine term essentially tells you how the signal is varying as a function of time at given location $x = 1\text{m}$. This quantity is nothing but the amplitude of the voltage at that location $x = 1\text{m}$.

So the peak voltage v_{peak} at $x = 1\text{m}$ will be equal to $10 e^{-2.23}$ which is nothing but one 1.075 volts.

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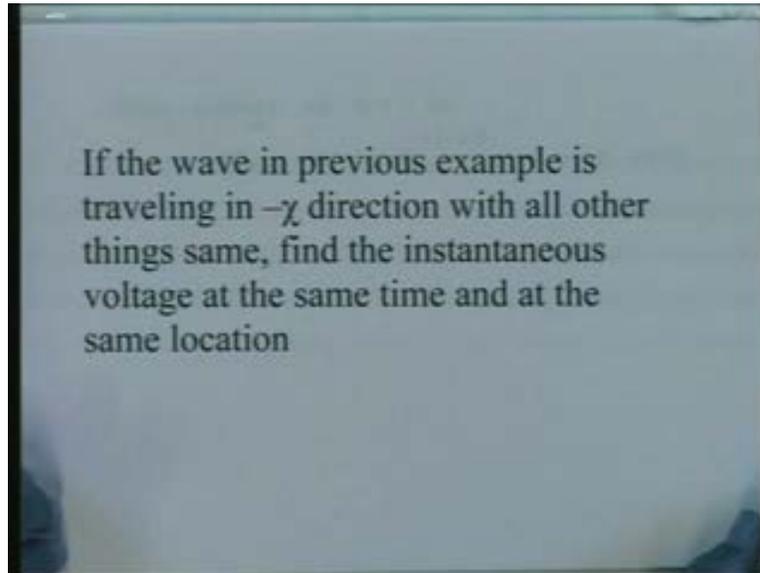
Peak voltage at $x = 1 \text{ m}$
 $V_{\text{peak}} = 10 e^{-2.23} = 1.075 \text{ volts.}$

So this is one of simplest problem we can think on Transmission Line where the Transmission Line primary constants are given, the voltage at some instant of time and some location of Transmission Line is given and we are asked to find out the voltage at some other location and some other instant of time on the Transmission Line.

Now one can ask that if I have same problem if the wave was moving in the negative x direction instead of wave moving in the positive x direction and if the initial conditions were same that means at $t = 0$ and $x = 0$ the voltage was 8.66 volts then how the things would change. so we are still want to find out the voltage at this location $x = 1\text{m}$ this is time $t = 100$ nano second only thing now we say is that the direction of the wave is reversed that means now the wave travels in the negative x direction.

so what we are saying is if the wave in the previous example the example which we saw just now is traveling in negative x direction with all other things same find the instantaneous voltage at the same time and at the same location that means find the voltage at $x = 1\text{m}$ and $t = 100$ nano second.

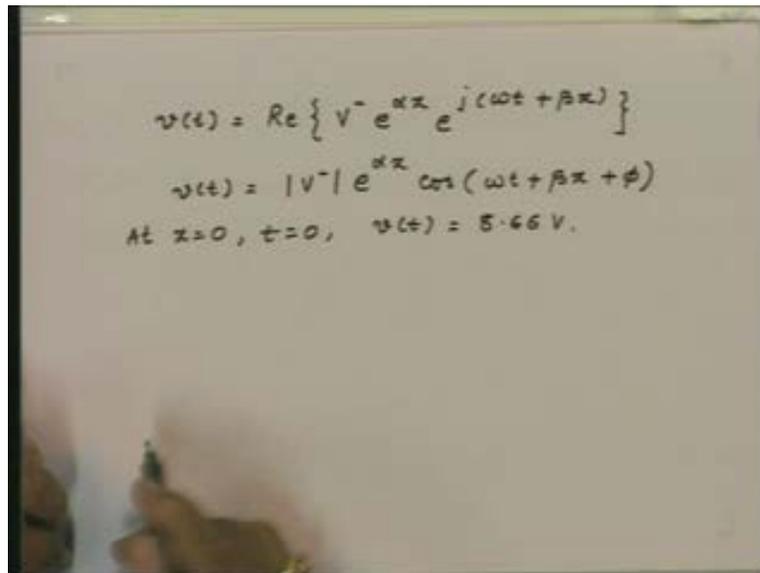
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So first thing you will notice is since the wave is traveling in the negative direction the phase variation of the wave as a function of space will be $e^{+j\beta x}$. Now I have the voltage which is traveling in the negative direction I can write down the instantaneous voltage $v(t)$ which will be nothing but the real part of the amplitude of the voltage wave with an amplitude V attenuation constant $e^{\alpha x}$ where earlier this quantity used to be $e^{-\alpha x}$ because wave was traveling in the positive x direction now the wave travels in the negative x direction so you have a variation of amplitude which is $e^{\alpha x}$ and phase term is $e^{j(\omega t + \beta x)}$ where as in the previous case this quantity is to be $-\beta x$.

Once we get this expression then the other procedure is exactly same as what we had done earlier we can substitute the values for different parameters and again we say that the voltage is 8.66 volts at $x = 0, t = 0$. So in this case also the $v(t)$ will be equal to the $|V| e^{\alpha x} \cos(\omega t + \beta x + \phi)$. And the $v(t) = 8.66$ volts at $x = 0, t = 0$.

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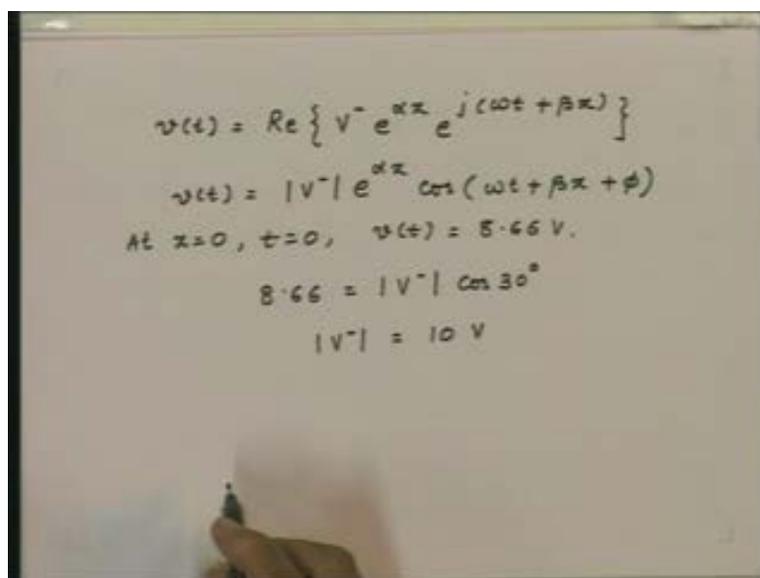
Handwritten equations on a whiteboard:

$$v(t) = \text{Re} \{ V^- e^{\alpha x} e^{j(\omega t + \beta x)} \}$$
$$v(t) = |V^-| e^{\alpha x} \cos(\omega t + \beta x + \phi)$$

At $x=0, t=0, v(t) = 8.66 \text{ V}.$

So if I substitute into this we get 8.66 will be equal to $|V^-| \cos 30^\circ$. So again you get this parameter $|V^-|$ which will be equal to 10 volts.

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Handwritten equations on a whiteboard:

$$v(t) = \text{Re} \{ V^- e^{\alpha x} e^{j(\omega t + \beta x)} \}$$
$$v(t) = |V^-| e^{\alpha x} \cos(\omega t + \beta x + \phi)$$

At $x=0, t=0, v(t) = 8.66 \text{ V}.$

$$8.66 = |V^-| \cos 30^\circ$$
$$|V^-| = 10 \text{ V}$$

Now if I substitute this quantity $|V^-| = 10$ volts in this expression then I get the voltage at $t = 100$ nano second and $x = 1$ m will be $v(t) = 10 e^{2.23}$ cos of the whole thing the ωt which we have already calculated earlier plus βx plus the angle ϕ which is equal to 30° which is $\pi/6$. If I simplify this then we get the voltage which is minus -83.77 volts.

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$$v(t) = \text{Re} \{ V^- e^{-\alpha x} e^{j(\omega t + \beta x)} \}$$

$$v(t) = |V^-| e^{-\alpha x} \cos(\omega t + \beta x + \phi)$$

At $x=0, t=0, v(t) = 8.66 \text{ V.}$

$$8.66 = |V^-| \cos 30^\circ$$

$$|V^-| = 10 \text{ V}$$

voltage at $t = 100 \text{ nsec}$ and $x = 1 \text{ m}$

$$v(t) = 10 e^{2.23} \cos(\omega t + \beta x + \pi/6)$$

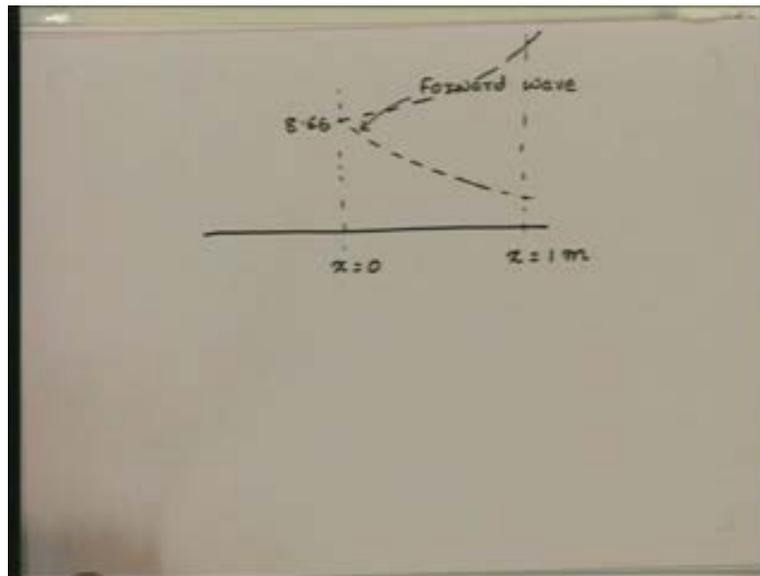
$$= -83.77 \text{ volts.}$$

So what you notice here now is that if I look at the situation that this is my Transmission Line and this was the location which is let us say $x = 0$, this is the location which is $x = 1$ m. If the wave travels in the forward direction then the wave will attenuate in the direction of propagation so initially you had certain value of the voltage which is 8.66. But since the wave is attenuating in the forward direction you have the amplitude variation of the wave which looks like this so 8.66 becomes something which we calculated earlier 0.88 volts so this quantity was 8.66 volts and this at 0.88 volts. So this would be the variation for the forward wave.

But to have the same amplitude here 8.66 and if the wave is traveling backwards then the attenuation is at the direction of wave propagation which is in this direction so for the same value here 8.66 volts now the amplitude variation would look something like that

and that was precisely is happening. Now the voltage amplitude which we get for this wave is which we saw is -83.77 volts so this problem clearly demonstrates that the direction of the wave propagation is very important for the same location on Transmission Line and same time the voltage values will be quite different depending upon whether the wave was traveling in one direction or other.

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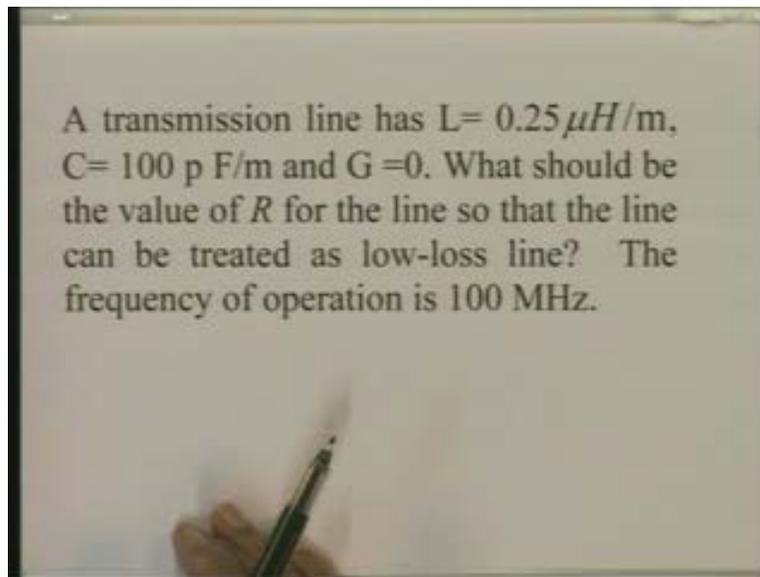


So this problem essentially gives a feel for how the voltages vary on a Transmission Line and how important is this aspect of the direction of wave propagation on the Transmission Line.

Now let us look at a problem of low-loss transmission line. In practice we always have a resistance in Transmission Line though you may always use a very high quality dielectric material so the conductance per unit length of Transmission Line could be very small but the resistance is not really very small for a practical Transmission Line so invariably a question is asked that what is the acceptable value of the resistance per unit length of Transmission Line so the line can be used or it can be treated as the low-loss transmission line.

So here is a question which says that A Transmission Line has $L = 0.25 \mu\text{H}/\text{m}$, the capacitance per unit length is $100 \text{ pF}/\text{m}$ and the conductance per unit length $G = 0$. What should be the value of R for the line that means the resistance per unit length of the line so that the line can be treated as low loss line? The frequency of operation is 100 MHz .

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If you recall the low loss transmission line is defined such that the attenuation constant for this line is much smaller compared to the phase constant. So we call a low loss line whenever we have the phase constant β which is much larger compared to the attenuation constant. So we say for a low loss line $\alpha \ll \beta$.

Now since the line is low loss we know that β is approximately equal to $\omega\sqrt{LC}$ and the

$$\text{attenuation constant } \alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}.$$

Since in our problem the conductance per unit length is zero this quantity is zero so the

attenuation constant α is $\frac{R}{2} \sqrt{\frac{C}{L}}$. Other quantities are given we are given value of L , we

are given value of C, also we know the frequency which is 100 MHz so we can substitute into this and we can find out the phase constant β which is equal to 2π into frequency which is 100 MHz so into 100 into 10^6 and \sqrt{LC} which is square root of 0.25×10^{-6} nothing but 0.25 micro Henry per meter multiplied by 100 pico Farad so $\sqrt{0.25 \times 10^{-6} \times 100 \times 10^{-12}}$.

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Low Loss line $\alpha \ll \beta$

$$\beta \approx \omega \sqrt{LC}$$

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}$$

$$= \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\beta = 2\pi \times 100 \times 10^6 \sqrt{0.25 \times 10^{-6} \times 100 \times 10^{-12}}$$

If I simplify this we get the value of β which is equal to π radians per meter. I can go now and substitute the value of L and C into this and I can find out the value of α so I get the value of α which is equal to $\frac{R}{2} \sqrt{\frac{C}{L}}$ so it is $\frac{R}{2} \sqrt{\frac{100 \times 10^{-12}}{0.25 \times 10^{-6}}}$ which is equal to 0.01 R nepers per meter.

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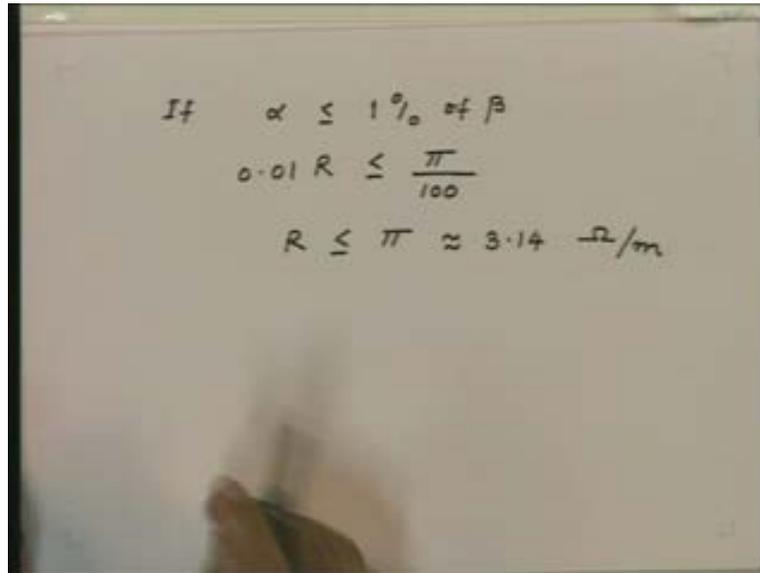
Low Loss line $\alpha \ll \beta$

$$\beta \approx \omega \sqrt{LC}$$
$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}$$
$$= \frac{R}{2} \sqrt{\frac{C}{L}}$$
$$\beta = 2\pi \times 100 \times 10^6 \sqrt{0.25 \times 10^{-6} \times 100 \times 10^{-12}}$$
$$= \pi \text{ rad/m}$$
$$\alpha = \frac{R}{2} \sqrt{\frac{100 \times 10^{-12}}{0.25 \times 10^{-6}}} = 0.01 R \text{ Np/m}$$

Now if the line is to be lossless this quantity α has to be much smaller compared to β . Now how much smaller it should be is a subjective matter but one can say that as a rule of thumb if this quantity is less than one percent of this value β then we can treat the line as a low loss line.

So I can say that if α is less than or equal to one percent of β then we can treat this line as the low loss transmission line. So here I can say that if $0.01 R$ is less than or equal to one percent of β which is $\frac{\pi}{100}$ we can get the value of R which is less than or equal to π so approximately $3.14 \Omega/\text{m}$.

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The image shows a whiteboard with handwritten mathematical equations. The first line reads "If $\alpha \leq 1\%$ of β ". The second line is $0.01 R \leq \frac{\pi}{100}$. The third line is $R \leq \pi \approx 3.14 \Omega/m$.

So for this Transmission Line if I take the resistance less than about $3 \Omega/m$ then this line can be treated as a low loss transmission line.

If the value of resistance exceeds this value then slowly the line will become a lossy transmission line and when the value of the resistance becomes comparable to β then we will get the variation or the performance of the line which will be extremely lossy transmission line.

So in this session essentially we first saw how to identify the load which is terminating the line so just qualitatively looking at the voltage standing wave patterns one can identify the type of load which is connected to the Transmission Line. Then we went to the laboratory and we saw the experimental setups which are used for measurement of VSWR or Voltage Standing Wave Patterns on Transmission Line. And then later we solved some very simple problems based on the voltage equations which we have derive for the Transmission Line.

In the next session we will go to the applications of the Transmission Lines that means having understood the behavior of voltage and current on Transmission Lines, how do we make use of sections of Transmission Lines in realizing various circuit elements in the high frequency circuits.

In the next session essentially we discuss the applications of Transmission Lines.

Thank you.