**Digital Communication**
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**Lecture - 31**
**Fundamentals of OFDM**

Hello every, one and welcome to the class. Today we will discuss very important communication technology that is coming in every application today and that is OFDM. The title of today's lecture is DFT and its application OFDM because we will specially see how, a such a simple transform technique as DFT plays how important role in the OFDM technology.

Before going into the OFDM technology we will first discuss the basics of the DFT transform. And even before that we will discuss some basic results in signals and systems because, those basic results are very fundamental in understanding the OFDM technology based on DFT. So, we will consider a discrete time version of the channel, the transmitter signal and the receipt signal. The whole system will be in discrete time and we will denote the discrete time signals as a sequence like this.

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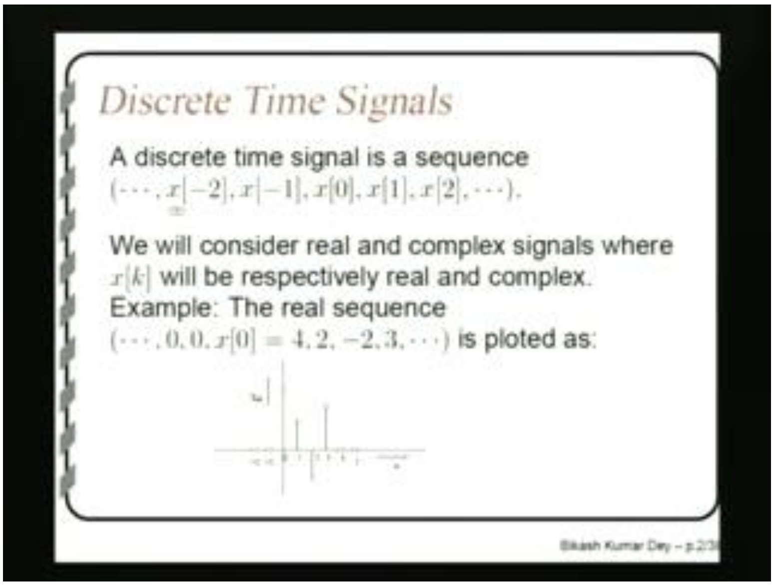


Where this is the zeroth sample this is the minus 1 sample this is sample number minus 2 and on this side sample number 1 2 and so on. We will consider real signal as well as complex signal. So, sometimes the signal may be real or complex; if the signal is real all these sample values are real, if it is complex all these samples are complex numbers. Example of a real sequence is this where this starts from 0 the zeroth sample onwards before that all the samples are 0 in this particular example.

And the plot of this signal is like this; such signal will be plotted on the real axis at integer values on x axis. So, this is the zeroth sample the value is 4 and this first sample is of magnitude 2 and so on.

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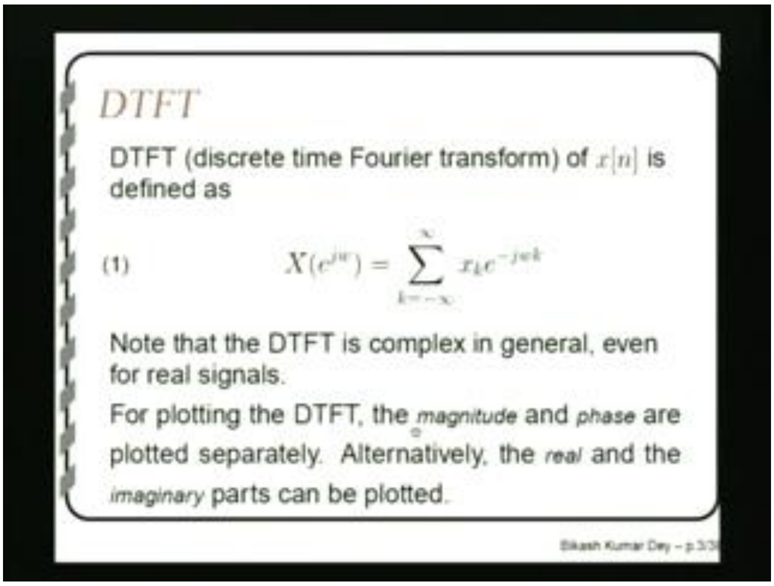


And for any given discrete time signal the discrete time Fourier transform is defined as this, this is the function of omega the angular frequency. And on this side there is a summation over all time instances that is integer valued time instances because, this is a discrete time signal. And 1 can note that even if xk this xn is also sometimes denoted by x subscript n. So, here that is the notation that is used we will use both the notations interchangeably to mean the same thing.

So, this is the k-th sample of this discrete time signal and this is the DTFT of the signal. And we see that even if xk is real the signal is real its DTFT may be complex because, this quantity is complex. So, the DTFT of a signal is complex in general even if that signal is real value. And for plotting the DTFT since it is complex we can plot it in either of the 2 ways: either to plot the magnitude and phase separately or to plot the real and imaginary parts separately.

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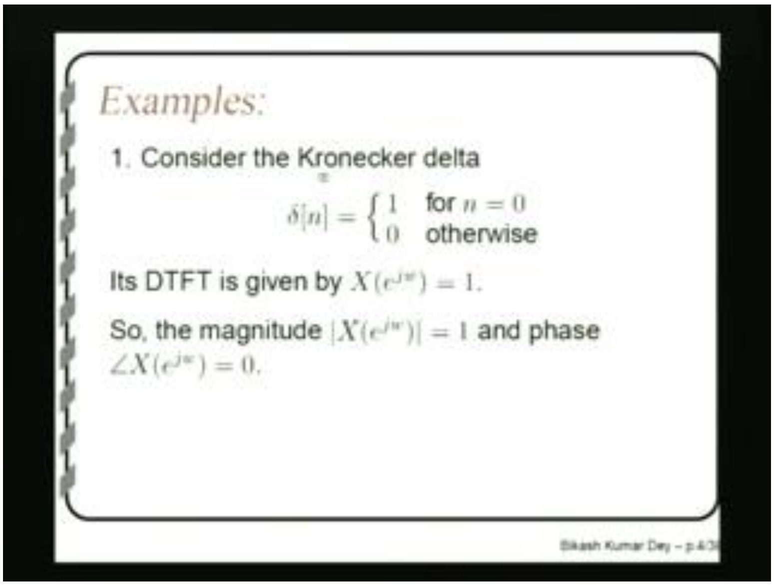


For example, consider the Kronecker delta function. This signal delta n is 1 at n equal to 0 and 0 in all the other times. So, there is only 1 time since n equal to 0 for that the signal has nonzero value in all other n the value is 0. DTFT of this signal is; obviously, 1 because for k not equal to 0 all these xk's are 0 and only for k equal to 0 we have x 0 then e to the power 0 which is 1. So, and x 0 is 1 so we have 1. So, for any omega this side is 1.

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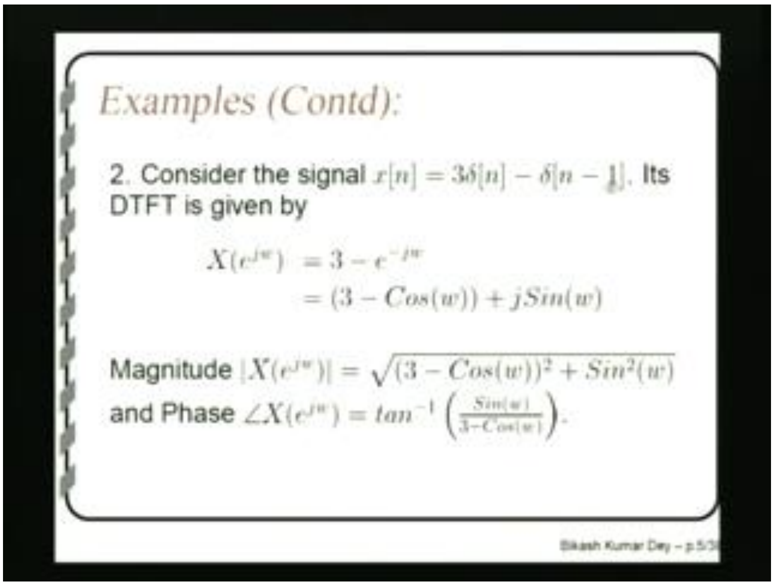

So, DTFT of the Kronecker delta signal is 1. So, the magnitude is 1 and phase is 0 for all omega. Now, let us take another example 3 times delta n minus delta n minus 1. So, which basically means that the magnitude of the signal at n equal to 0 is 3 and at n equal to 1 the magnitude is minus 1 and at all other n the magnitude is 0.

So, DTFT of this signal is; obviously, the DTFT of this minus DTFT of this because DTFT is linear. So, DTFT of this is 3 times 1 that is 3 minus the DTFT of this is e to the power minus j omega because, this is the shifted delta function shifted by 1. So, it will be the DTFT of delta n time’s e to the power minus j omega. So, we can you can manipulate this and see that this is nothing, but 3 minus cos omega plus j times sin omega this is the real part and this is the imaginary part.

And we can compute the magnitude of this DTFT as the square of this plus square of sin omega and then root over the whole thing. This is the magnitude of the DTFT that is this. And the phase is the tan inverse imaginary part by the real part. So, we can compute magnitude and the phase of the DTFT this way.

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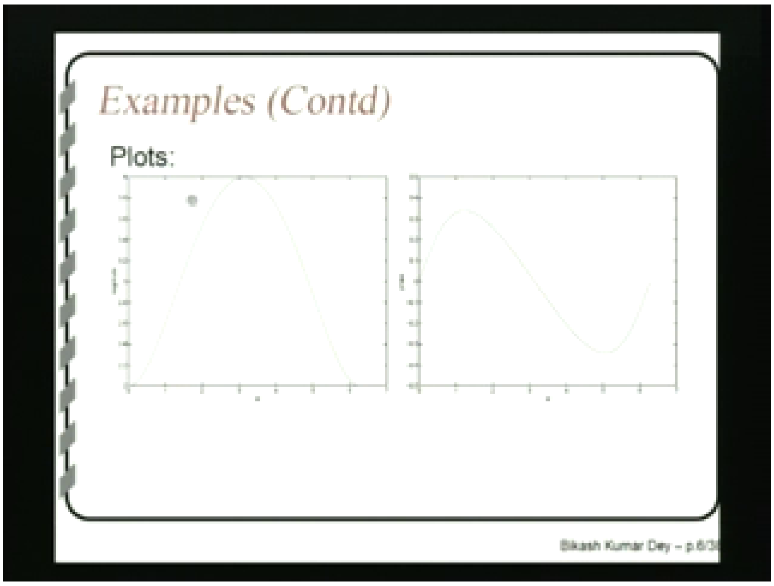


And we can plot the magnitude and the phase from those expressions. So, the for that example this is the these are the plots.

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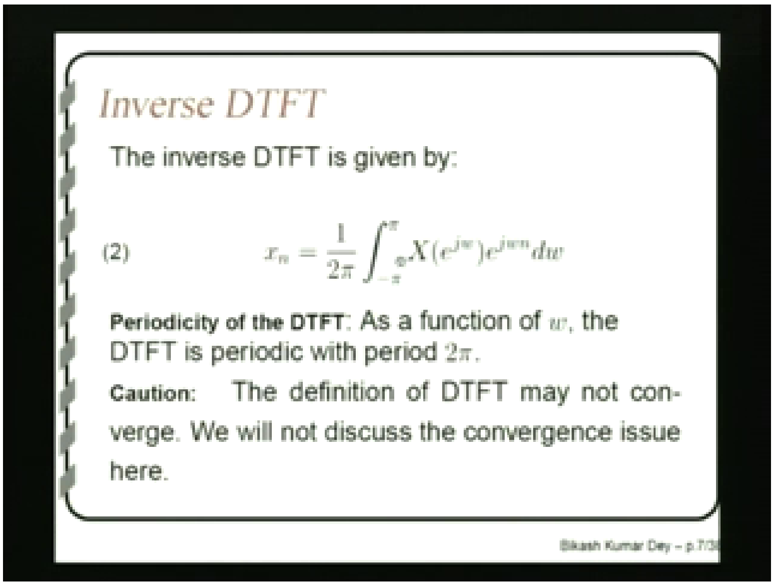


Now, if the DFT of the signal is given we can also compute the signal back from the DTFT. If the DTFT is given we can compute the signal back from the DTFT. So, the formula for that is this. The DTFT is X e to power j omega and the signal at n can be obtained from the DTFT from this integral. And 1 property of DTFT is that it is periodic with period 2 pi.

So, the integration you can see is from minus pi to pi because outside that it repeats this quantity repeats. So, there is no extra information there. And. So, the DTFT is periodic with period 2 pi. Of course, we have to keep in mind the definition of DTFT may not converge and we will not discuss those convergence issues in detail here. We will assume that for all practical signals are all practical signals that we will consider are nice enough and DTFT exists.

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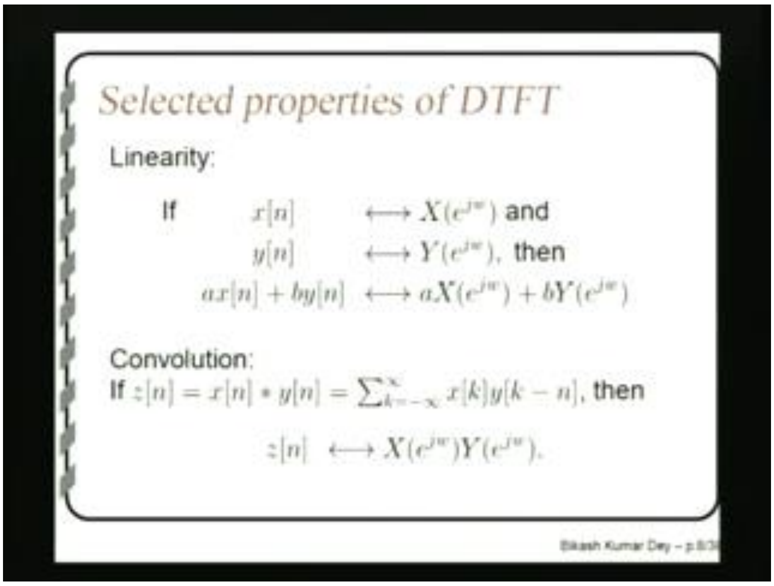


Linearity as we mentioned DTFT is linear; that means, if there are 2 signals xn and yn and their DTFT's are X e to the power j omega and Y e to the power j omega respectively. Then, for any 2 complex numbers a and ax n plus by n is another signal and DTFT of that signal will be a times X e to the power j omega plus b times Y e to the power j omega.

So, the same linear combination of the DTFT will be the DTFT of the linear combination. So, this is the linearity. And then another important property is convolution when you convolve 2 signals in the time domain, the DTFT is multiplied in the frequency domain. So, if you convert if you do convolution in the time domain the frequency domain the DTFT's are multiplied.

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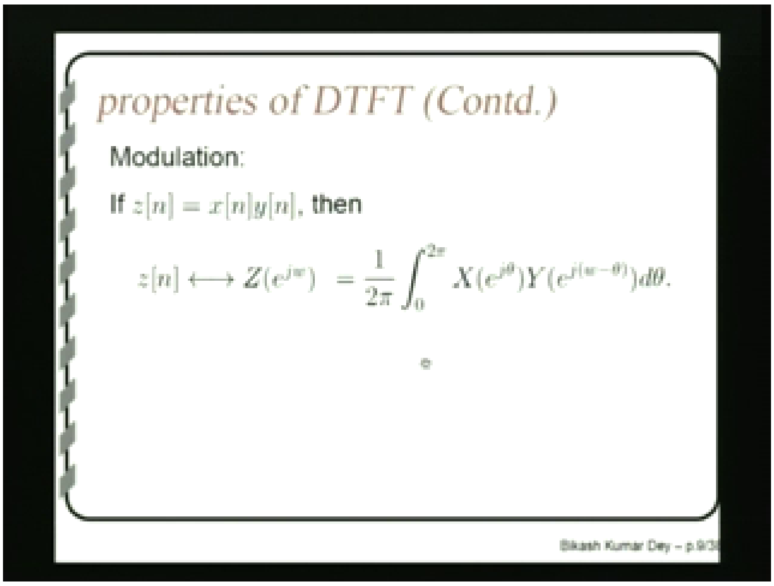


So, if Zn is Xn convolution Yn then the DTFT of Zn will be X e to the power j omega times Y e to the power j omega. And just the converse of that is that if you instead take product of the 2 signals in time domain that is sample wise product. That is, z 0 is x 0 times y 0 z 1 is x 1 times y 1 and so on. That is zn is xn times yn for any n then in the frequency domain the signals are convoluted. And one can see from the expression that this is really the convolution of these 2. Basically, this is the cyclic convolution of these 2 because this x and y are periodic.

We will not elaborate and prove these results here we will just discuss them in brief so as to, remind you i am sure all of you know it. But, in case you have forgotten please go back and look at any signal processing book or signals and systems book this will be there.

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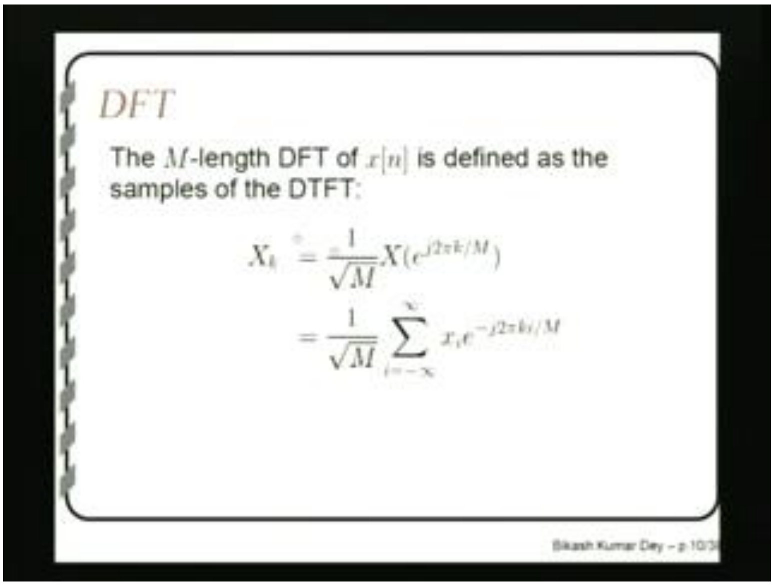


Now, let us consider DFT. DFT is basically a sampled version of the DTFT in the frequency domain. We have the DTFT which is periodic in 0 to 2 phi it is periodic with period 2 phi. We sample the DTFT we take M number of samples equally spaced in frequency then that becomes our M point DFT. The DFT is discrete in frequency domain also. The DTFT was taken of a discrete time signal, but the DTFT itself was in continuous frequency.

In the time domain, the signal was in discrete time, but in frequency domain the transform was in continuous frequency. Whereas, the DFT is taken of a discrete time signal and the DFT itself is in discrete frequency. The transform itself has finite number of components M number of components. The formula for DFT is the Xk see that there is no omega now because; the frequency domain is also discretized. So, the kth component of the DFT is obtained by taking the DTFT of the signal at this frequency 2 pi k by M.

So that is 2 pi is divided in to M parts by taking 2 phi by M and then multiplied by k. So, for k equal to 0 1 2 till M minus 1 we get some samples of the frequency in 0 to 2 phi. And when you write it down in terms of the time domain signal from the formula of DTFT we get this. And this is the well known DFT formula.

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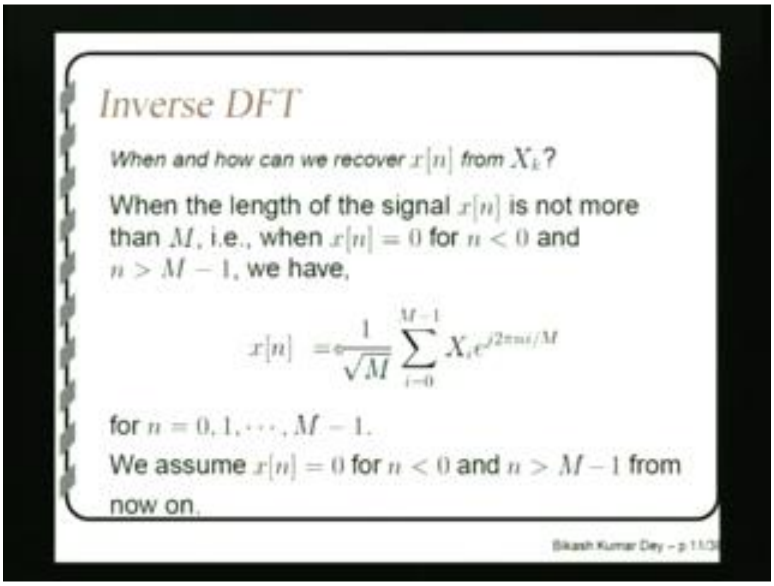


And the inverse DFT can be written in this form and the inverse will be same as the time domain signals that we started with provided the original single itself; had utmost M number of samples, M number of non 0 samples. If the original signal had more than M samples and we took DTFT of that signal and took M samples of the DTFT. Then we cannot get the original signal back from those M samples. Because the time domain signal had more than M components and we have taken only M number of components in the frequency domain.

We cannot get those more than M samples in the time domain back from the M samples in the frequency domain that, we have taken. So, for being able to recover the time domain signal from the frequency domain samples the original signal itself should have less than equal to M number of samples. So, DFT is actually defined for M length sequences whose number of nonzero components or consecutive components are is M.

So, we have an M length sequence xn we will assume that the length is M and that is xn is 0 from for n less than 0 and n greater than M minus 1 that is for n equal to 0 2 M minus 1 only the value can be nonzero. Then we have this inverse DFT that we can get xn back from X i using this formula. So, from now on we will assume that xn is 0 for n less than 0 and n greater than M minus 1.

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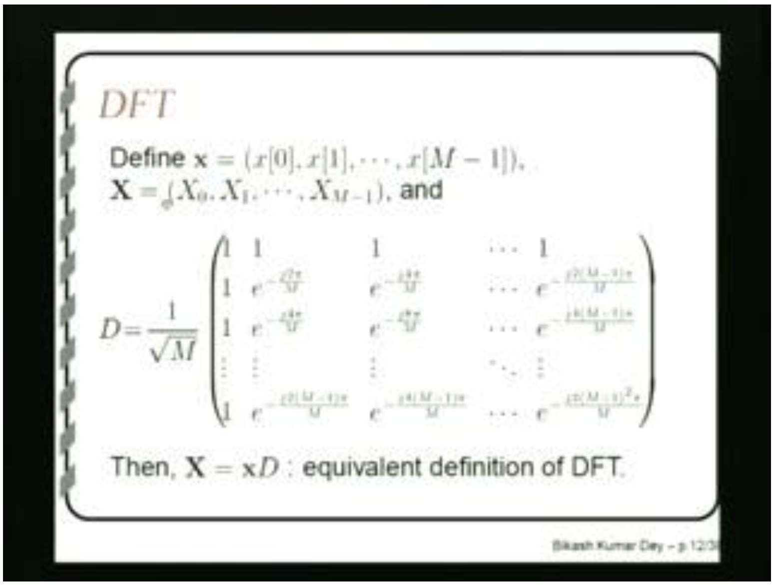


So, now from the formula we can also write DFT in the following way. If you define a matrix D as this that is 1 by root over M times the first row is 1 1 1 all ones. Second row is 1 e to the power minus j 2 phi by M e to the power minus j 4 phi by M and so on. That is, 2 phi by M times k where k is varying from 0 1 2 and so on. Then 2 phi times 2 times k where k is varying from 0 1 2 and so on.

So, all these are multiple of 4 pi you can see above in the numerator. And then the next will be all will multiple of 6 pi and so on. Here, all are multiple of M minus 1 times 2 M minus 1 times pi. So, once we have this matrix and we denote the time domain time domain sequence as a vector because, it now has only M number of non 0 components. We can write only those non 0 components and we do not need to write the zeros outside that range.

So, we have a vector representation of the sequence x 0 to x M minus 1. And the DFT has also M components written as X 0 to X M minus 1. Then this vector can be expressed in terms of this vector as X equal to this x multiplied by this matrix D. So, DFT of the time domain vector x is obtained simply by taking by multiplying this matrix to the time domain vector.

This is the time domain vector and this is the DFT vector we can also call it the frequency domain vector. So, the DFT is obtained by simply multiplying by this matrix

D. And so that this matrix D is called the DFT matrix and this can be taken as an equivalent definition of DFT.

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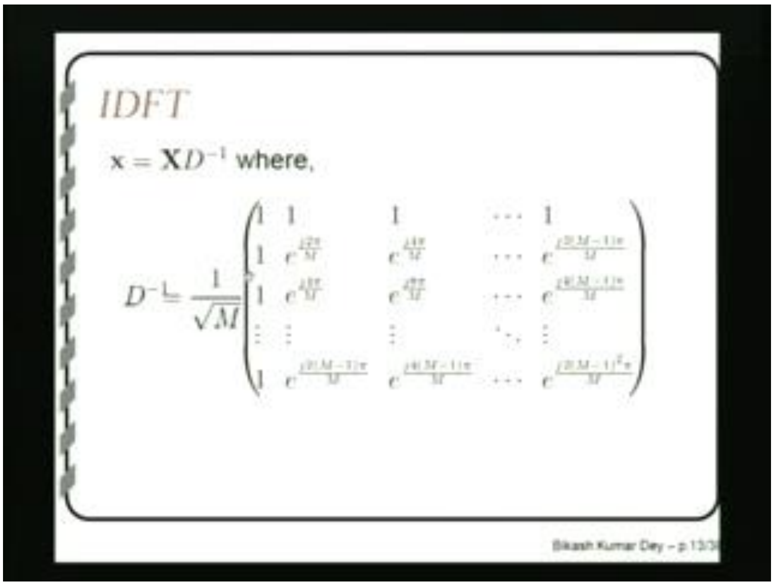


And then, IDFT is; obviously, written in terms of D in a very simple manner as x equal to X time's D inverse. How do we get this vector back from D back from this X DFT? We simply multiply by D inverse from the right and then we get on the right hand side simply small x and on the left hand side we get X times D inverse.

So, small x is equal to X time's D inverse that is the inverse DFT formula. Previously, we had a determinant in terms of summation like this and this now expressed as this. And the matrix D inverse is also very similar to matrix D. Instead of this matrix we now have the same matrix except for these minus signs. These are minus signs here whereas, this has no minus sign on these exponentials; otherwise the matrix is same.

So, this is quite clear also from the corresponding definitions in terms of summations. In the definition of DFT we have minus here and the definition of inverse DFT we do not have this minus, otherwise the formulas are same.

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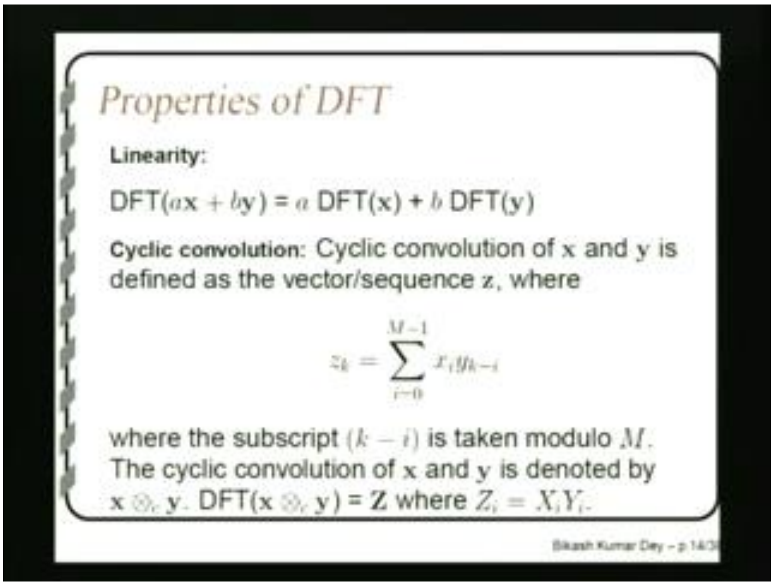


So, we have defined DFT and IDFT. The properties of DFT and IDFT are also very similar to the properties of the DTFT because, they are the sampled version of the DTFT. It also has satisfies linearity it satisfies a similar property like convolution, similar property like modulation and let us just go through them 1 by 1. Linearity If we take 2 vectors we take a linear combination of them a and b are 2 scalars.

Then the DFT of that vector is a times DFT of x plus b times DFT of y. So, the DFT is linear. Then cyclic convolution; in the case of DTFT we had linear convolution in the time domain then that meant point wise multiplication in the frequency domain. Whereas, here the time domain we need to take cyclic convolution. Cyclic convolution is defined in this way Z is the cyclic convolution of the x and y then zk is defined as this. For k equal to 0 to M minus 1 and this z is still an M length vector unlike the linear convolution.

So, that is because this yk minus 1 this k minus 1 is also taken module of M. So, if it is greater than M only the residue after dividing by M is taken here. So, if we take the cyclic convolution this way then the DFT of this vector the cyclic convolution will be simply point wise product of the DFT of x and y. So, that is the DFT of this cyclic convolution of x and y. If we call that Z then Zi will be Xi times Yi the point wise product of the DFT components of X and Y.

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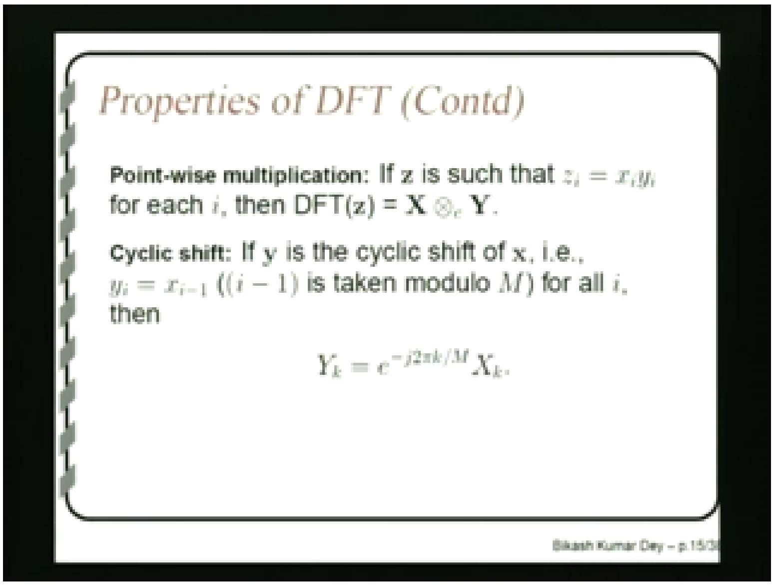


Then in the time domain if you take point wise product that is if we take a vector z such that the Zi the ith component is; the product of the i-th components of x and y. Then the DFT of z will be the cyclic convolution of the corresponding of DFT's of X and Y. So, cyclic convolution in the time domain will mean point wise product in the frequency domain.

Whereas, the point wise product in the time domain will mean cyclic convolution in the frequency domain. And then cyclic shift this is actually a special case of the convolution. If we have a sequence x a vector x and we cyclically shift this vector then. So, yi is xi minus 1 that is this x vector is cyclically shifted towards left. So, y 1 is equal to x 2 y 2 is x 3 and so on. So, x vector is cyclically shifted towards left to get a y vector y vector is nothing, but the left wise cyclically shift of x vector.

So, if we do that then the DFT of y has a relation with the DFT of x and that relation is also very simple. It is simply that that kth DFT component is the kth DFT component of x times this factor.

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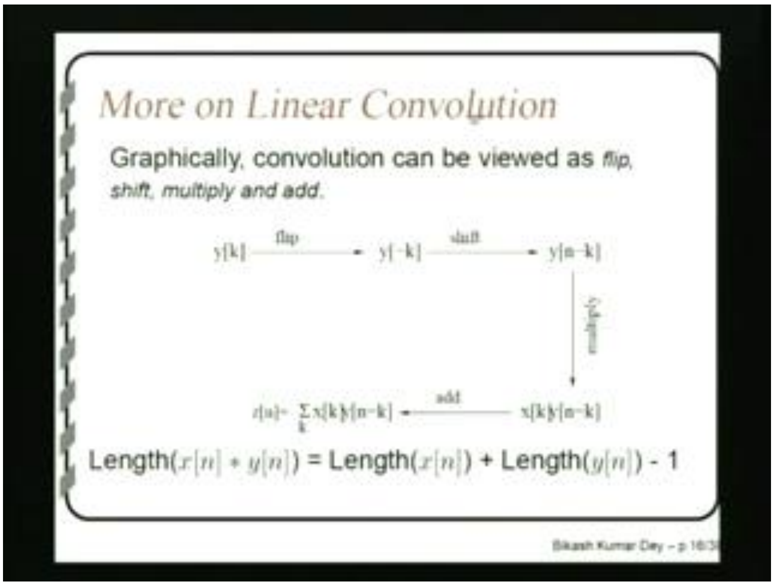


Now, let us just have revise the graphical representation of the linear convolution and cyclic convolution. Graphically the convolution is simply the 4 operations together. That is first we flip 1 of the vectors then shift that flipped vector. Then multiply that vector with the other vector with which we are taking the convolution then add all the components of the resulting vector. That will give us 1 component of the convolution.

So, if you want to take linear convolution of x and y sequences xk is 1 sequence and yk are y another sequence. Then let us say we choose yk and flip that sequence yk. We get y minus k sequence that is the flipped version of yk sequence. Then we shift this sequence by n. So, if you want to compute the linear convolution at time n then we shift the flipped sequence y by n. So, we get y n minus k then we multiply this by the x sequence xk times this.

So, k is the running variable here and n is fixed we want to compute the convolution at n. So, n is fixed and k is running. So, xk times y n minus k is what we obtain after multiplying by the shifted and flipped yk with the x sequence. So, after that we we add all the components of this product sequence. So, for different k we have different values different samples of this sequence we add all the components and we get summation over k of this.

And that is our zl the convolution of x and y. And we know that the length of the convolution of 2 sequences, linear convolution of 2 sequences is the length sum of the lengths of the individual sequences minus 1. So, length of the convolution of x and y is length of x plus length of y minus 1.

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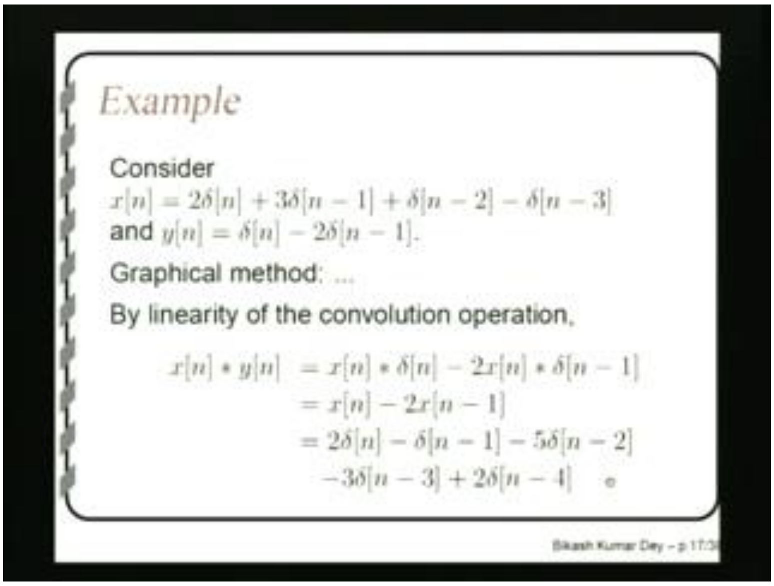


Let's see an example. This is the sequence x it has 4 non 0 samples 0 to 3, so the length is four. Another sequence is yn has length 2 0 to 1 and we want to take convolution of these 2 sequences. We can do it by graphical method, but of course, for this case it is simpler do it by using linearity of convolution. So, this is this plus this and we take convolution of xn with delta n and convolution of xn and delta n minus 1 and take the linear convolution given by this.

So, it is xn convolution delta n minus 2 times xn convolution delta n minus 1. Now, what is xn convolution delta n? It is xn itself and what is xn convolution delta n minus 1 it is x n minus 1 it will be shifted by 1. So, it is x n minus 2 times x n minus 1 and that can be simply computed as this. And the length becomes 5 0 to 4 5 0 1 2 3 4. And length should be 5 because the length of this is 4 and length of this is 2.

So, 2 plus 4 minus 1 6 minus 1 that is 5. So, the this is how we can compute the convolution here. So, graphical methods also let us just see the graphical method how to do it by the graphical method because, that will give us some insight into the convolution into convolution.

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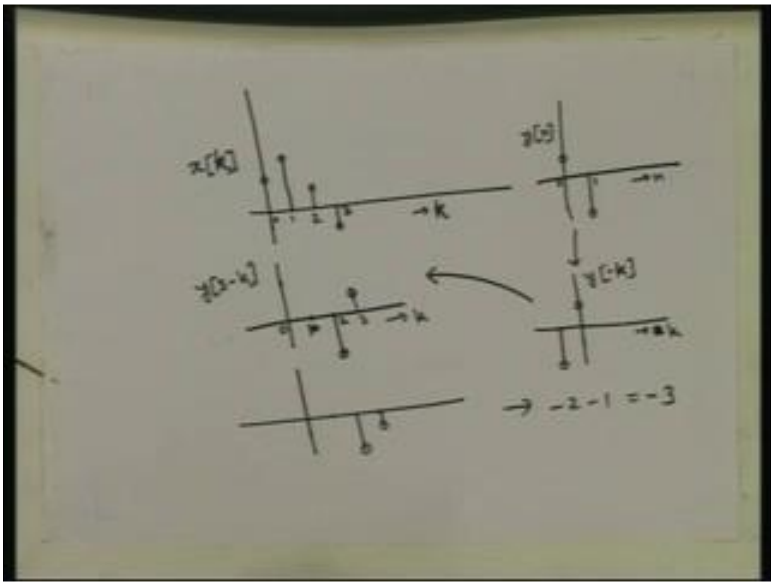


So, we have xn we plot xn we have 2 at 0 at 1 we have the value 3 at 1 we have value 3 at 2 we have value 1 and at at 3 the value is minus 1. So, this is m this is 0 1 2 3. So, this is xn and yn is at n equal to 0 it is 1 and n equal to 1 it is minus 2. So, from the graphical method how should we compute the linear convolution? Let us take yn let us suppose we want to compute the convolution at n equal to 3.

So, at n equal to 3 the value should be minus 3 we want to get this value how do we do it graphically. First flip this sequence yn we will we shift this we flip the sequence yn we get from here at 0 we have value 1 at minus 1 we have value minus 2, at all other values it is zero. So, this is our y minus n instead of n we will denote it by k as we have denoted in the slide. So, let us call the running variable k.

So, this is minus 2 this is 1 then we shift it by 3, because we want compute the convolution at n equal to 3. So, we shift it towards right by 3 so we shift it by 3 to get so from here we get, so this minus 1 will go to 2 so 1 2. So, here the value will be minus 2 and this zeroth sample will go to 3; so this will be one. The next operation is take product of these 2 sequences xk. So, this is this is also this can be taken as k. And this is our y 3 minus k this is k this 2 this is 3.

Now, let us take the product the product is 0 this is also 0 this is 1 times minus 2 that is minus 2 and this is minus 1 times 1 that is minus 1. Then we need to take the sum of these 2 samples. So, we get minus 2 plus minus 1 is equal to minus 3 and that is what we should get as indicated by this. This component is also minus three. So, we have got minus 3 by graphical method also.

So, what have we done? We have taken 1 sequence we have flipped it then we have shifted by 3. And then we have multiplied these 2 sequences to get this and then added all the components of this sequence and that is the value of the convolution at 3. Then the cyclic convolution is just the same thing except that the flipping operation and the shifting operation both are done cyclically. That is if any component goes outside M it is brought back to inside cyclically.

So, if the if anything goes to crosses M minus 1 and goes to M it is brought back to 0, if it goes to M plus 1, it is brought back to 1 and so on. So, everything is brought back to inside the 0 to M minus 1 range. So, then the other things are exactly same.

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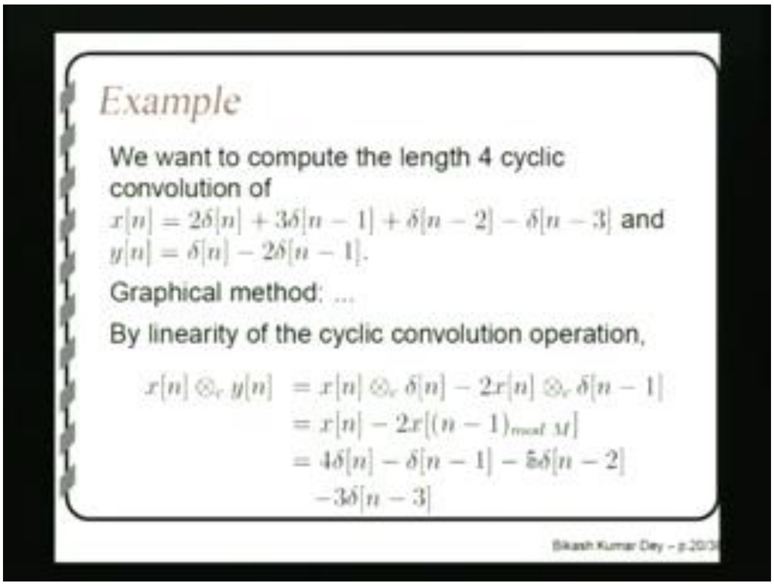


This shifting this flipping and this shifting both are taken module of M. Mathematically cyclic taking cyclic operation means taking module of M. Divide by M and then take the take the element. The cyclic convolution of those 2 sequences again can be computed either by using linearity just like before for linear convolution or by graphical method in a similar manner. We will not go into the details now.

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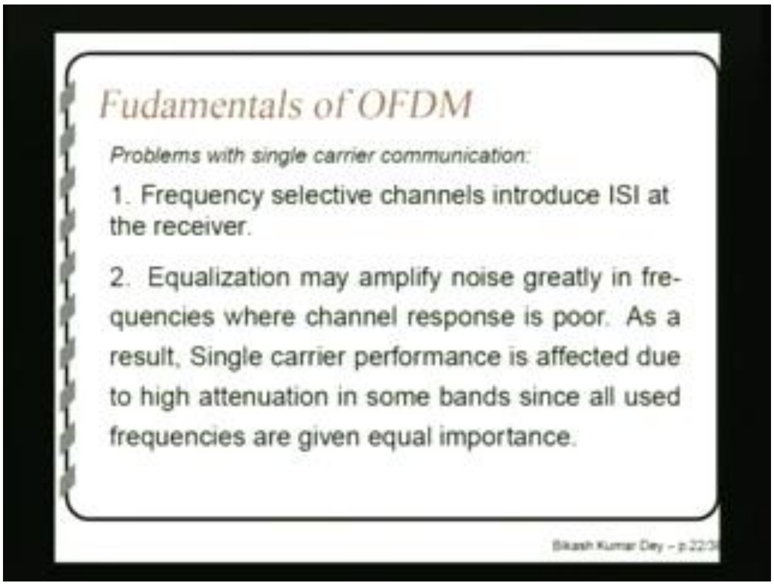


Now we will start the OFDM. First of all let us see why we need OFDM. OFDM is basically a multi carrier modulation meaning by we have a certain bandwidth to be used; we want to use the certain bandwidth for our communication purpose. But we do not want to take a single carrier and then modulate that carrier in some way and transmit. We want to take multiple carriers we want to divide the band into many small bands and then take many carriers in 1 carrier each in each band. And then divide the data stream also into many parallel data streams of lower rate and then modulate individual carriers by individual lower rate data streams.

And then add all the signals and transmit together. So, that we will be using the whole bandwidth, but we will be modulating individual sub carriers we will call those as sub carriers. And we modulate those sub carriers individually by lower rate data streams and then add those signals and transmit. So, the advantage here is many fold we will discuss that in a moment.

So, the problem with single carrier modulation is that the frequency selective channels introduce ISI at the receiver. Because, the channel may not have flat response in the frequency domain and as a result there will be inter symbol interference in the receiver. We have discussed this when we discussed equalizers in this course. So, the main problem is that the channels are frequency selective.

They have different attenuation and different frequency. So, that causes inter symbol interference at the receiver if you use, single carrier modulation. Also, even if we do equalization at the receiver that does not solve the problem. Because by equalization we may actually make the channel flat, we may amplify the regions where regions in the frequency domain where the attenuation was too much and so on, so that the overall response is flat.

But then, the noise in those bands will be also amplified to a high extent and that will cause an overall amplification of the noise. Overall noise variance will increase and that is not desired that will actually create high error probability.

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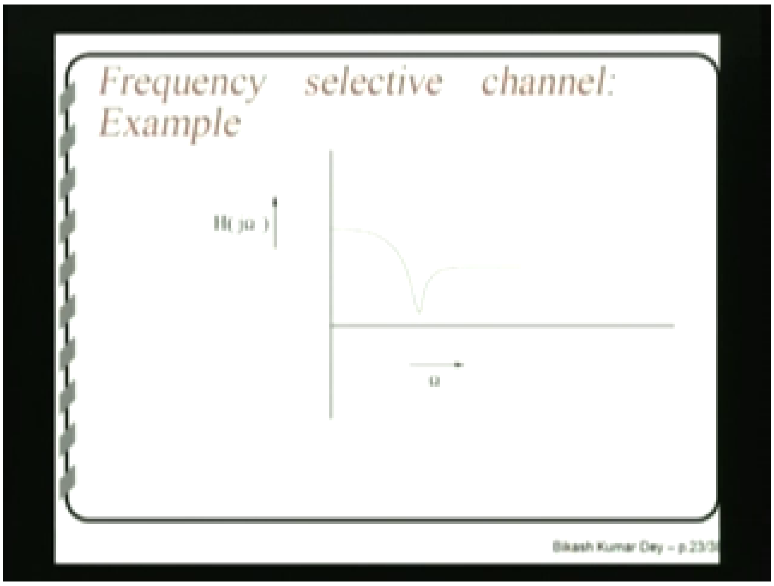


So, instead in OFDM what we do is that if there is such a channel like this where the high frequency selectivity this part of the channel is very good this part on the other hand is very bad because of its very high attenuation. Then this is the frequency response of the channel. So, in this frequency there is very high attenuation. So, if we, instead of using this channel to transmit single carrier modulated signal in which case this needs to be amplified at the receiver in equalization and that cause's high amplification of the noise in this band.

Instead of doing that if we instead break this frequency into many bands and then transmit very little information in this band and at higher rate in these bands. Then this part of the band frequency this band will not affect our communication much because we will not be using this band so much. We for example, may opt to opt not to communicate not to use that band at all. So, in that case there is no application of this noise and that does not affect others at least the other bands it has an advantage.

And also in each sub band if we take the sub bands narrow enough then the equalization becomes very simple in each sub band because each sub band is narrow and as a result it is almost flat. So, there is no frequency selectivity in each sub band.

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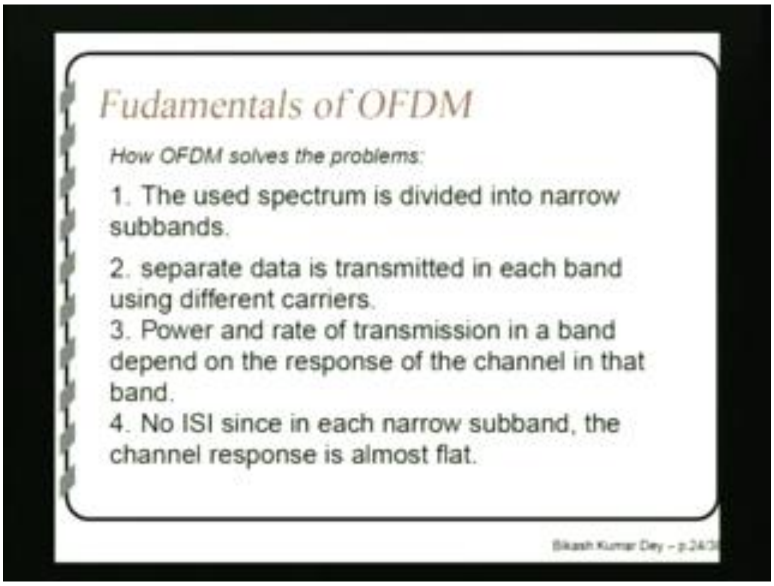


So, as a result the equalization becomes very simple. So, that is another advantage. So, in OFDM the spectrum is divided into narrow sub bands separate data is transmitted in each band using different carriers. And power and rate of transmission in different bands may be different. Different in different bands we may transmit at different power and at different rates depending on how that channel is whether it is good or bad.

So, and no ISI since the since each sub band is narrow and as a result it is almost flat so, each sub band is narrow and it is almost flat that. So, there will not be any ISI and also there will be only an amplification or attenuation and we can take care of that by simply amplifying or attenuating at the receiver.

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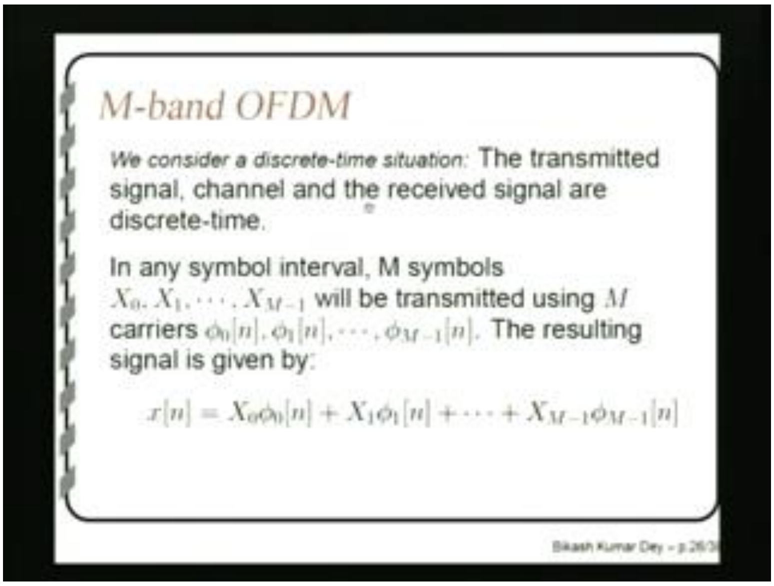


So, we divide the spectrum in this kind of bands sub bands. Let us say that we divide the spectrum in M number of band. We consider a discrete time situation as we said. So, we consider M symbols that will be transmitted using M number of carriers phi 0 n to phi n minus 1 n are M number of carriers in the time domain. So that, we will be multiplying X naught with this carrier X 1, with this carrier and so on.

So, that we get n separate signals and we will add them and transmit through the channel. So, here is the here the sub bands coming we have M different carriers. Now, the interesting thing is that these carriers will be at different frequency.

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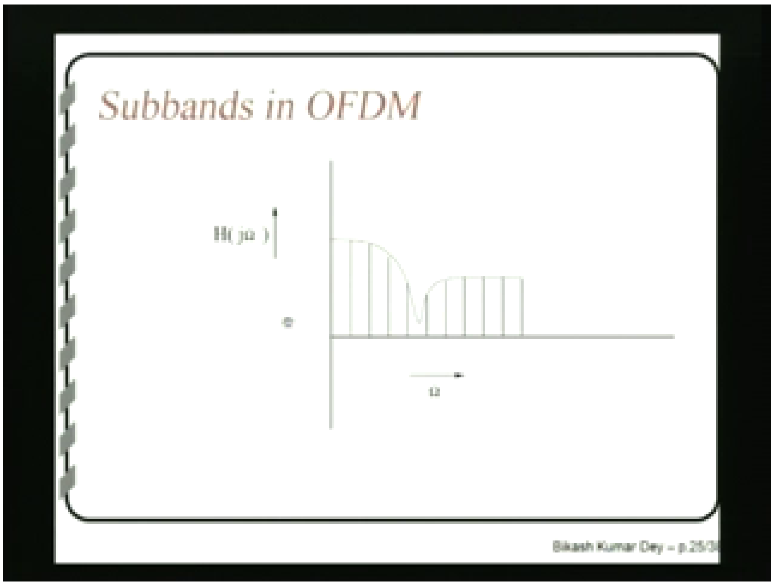


So, that for example, the first carrier phi naught may be somewhere here, it may be its in the frequency domain the DTFT of that may look like the Fourier transform may look like this. Somewhere here and the phi 1 may look like will lie here phi 2 lie will here and so on. So, as a result in this overall signal this component will lie in the first band this component of the signal will lie on the second band and so on. So, all these components are actually separated in the frequency domain. So, we are transmitting X naught in the first band X 1 in the second band and so on.

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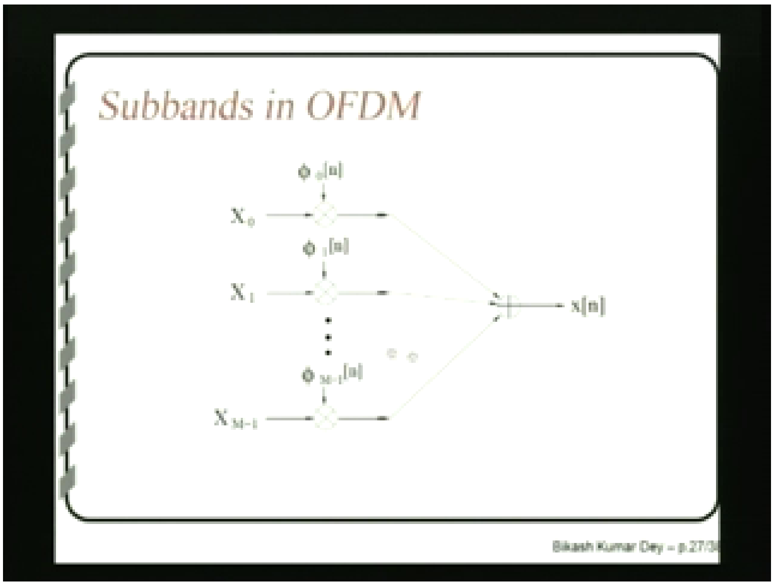

So, the block diagram looks like this X naught is multiplied by this carrier and then and so on. And all these components are added and the transmitted signal is obtained this way.

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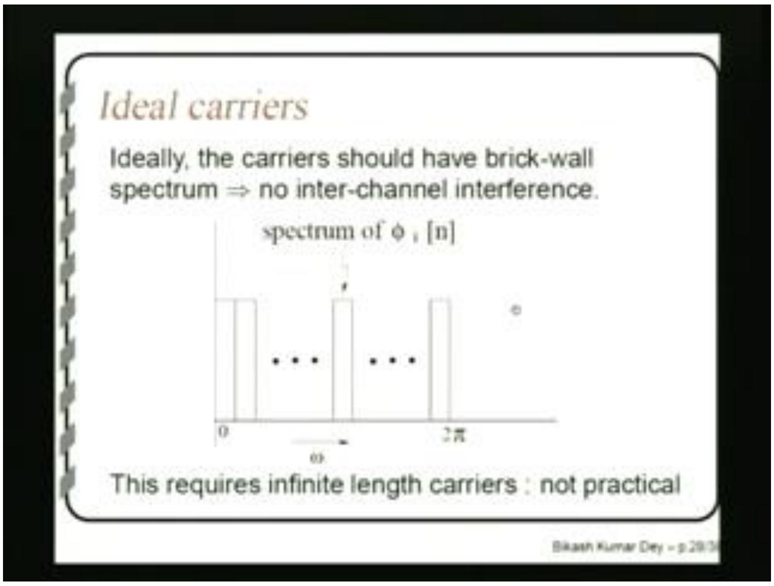


Now, ideally the carriers should have this kind of spectrum. So, that there is no interference between them there is no inter carrier interference. So, but this is not very practical because to have this kind of spectrum the phi i n should be infinite in time it should have infinite length. Because, the inverse Fourier transform of this will be sync and that has infinite length.

So, this is not practical this is what is desired in the frequency domain, but this is not practical because the carriers are infinite length. So, instead of that we want to choose different kind of spectrum, but even then we do not want any inter carrier interference. So, how do we get that, we can still get that provided we ensure that this different phi i's are orthogonal. Even though they overlap in frequency domain they are still orthogonal they are not frequency orthogonal, but they are still orthogonal. So, one very popular and probably the only used set of carriers is the DFT carriers. We have discussed the DFT matrix.

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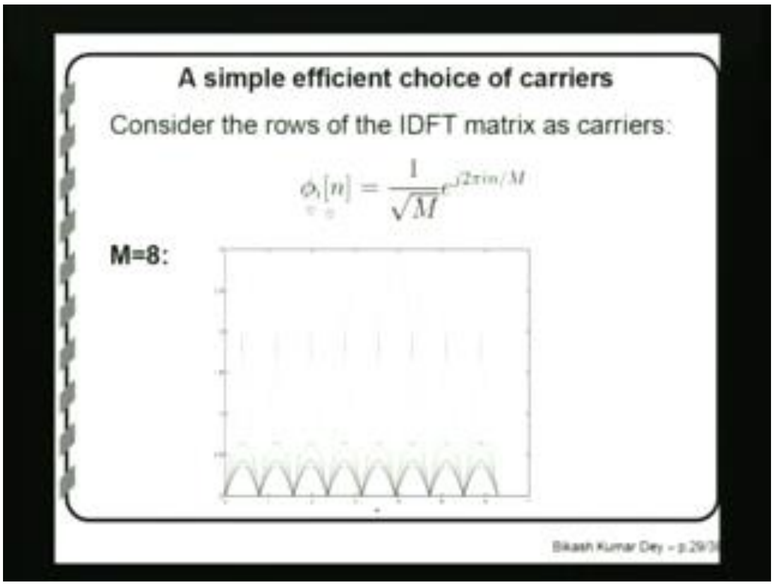


Now, we take the M different rows of the DFT matrix as M sub carriers. So, if we take those this is the ith carrier we take this to be the i-th row of the DFT matrix. This is the i-th row of the DFT matrix that we discussed instead of DFT we take IDFT because, we do not want to take the minus here. So, if we take the IDFT matrix and take the i-th row we get this basis of this carrier.

In the frequency domain this has a spectrum like this and then it has some ripples here. So, this shows M equal to 8 case where there are 8 carriers and their spectrums are drawn here separately. So, now let us see these carriers are known to be orthogonal because the DFT matrix is unitary matrix IDFT matrix is also unitary matrix.

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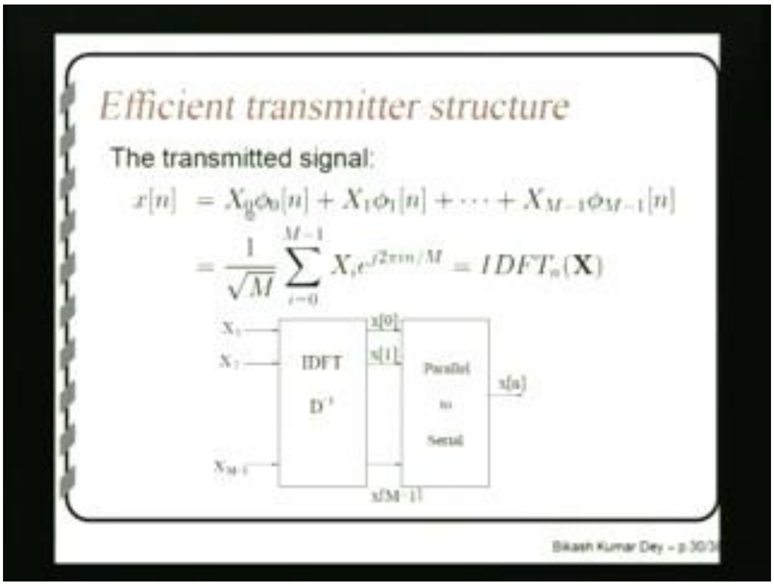


So, we can another important interesting thing here is that this transmitter as we illustrated here can be implemented. This part can be implemented in a very simple manner in an efficient manner using FFT. Here, this will turn out to be doing DFT and that can be implemented during effective. So, let us see that, so this is the signal that we want to transmit. But, if we replace the expression for phi i and that is this then 1 by root M times this then we get this and this is nothing, but the IDFT nth IDFT component of the vector X.

So, we can simply take the vector x x 1 to this should be x naught to x n minus one. Take the IDFT that is multiplied by D inverse matrix that is the IDFT matrix, we will get this signal that is x 0 to x M minus 1 we will get. And then we do parallel to serial conversion and transmit. So, the advantage of implementing it in this fashion is that this IDFT can be implemented using first Fourier transform algorithm this is very efficient. So, this is used in practice very much in all the OFDM technologies that are in used.

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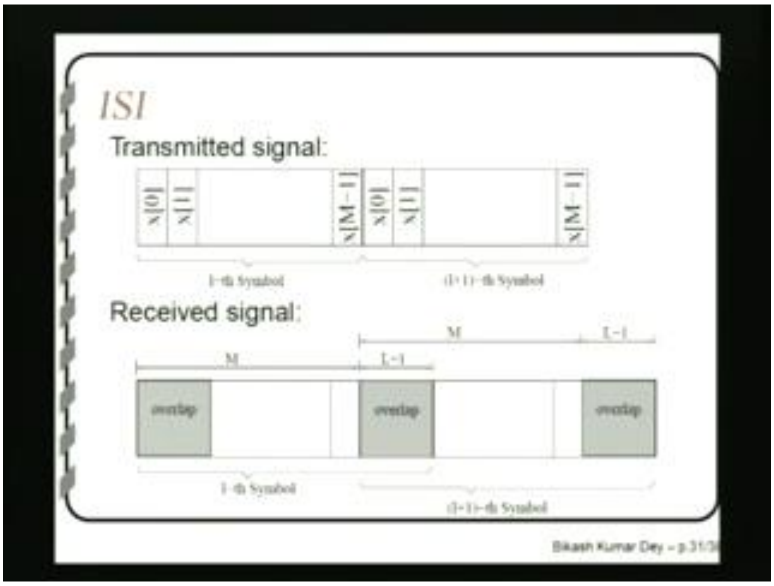


So, problem: problem is that if we now transmit these blocks. So, we have 1 set of symbols that we transmit using this and another set of symbols come you want to transmit that. Now, if we transmit those blocks 1 after another then what will happen is that. After the channel is channel impulse response is convolved with that there will be overlap between the blocks. So, that will cause inter block interference.

Because see this is the, suppose this is the l-th symbol l-th OFDM symbol x naught to x M minus 1 that is this vector the l-th vector. And again another vector comes and we get another IDFT vector that is this l plus one’th vector. Then if we transmit it this way what will happen is that the linear convolution of this vector with the channel impulse response will have length M, this is the M length M plus L minus 1. Where L is the impulse response length; channel impulse response length.

If L is the channel impulse length then the total length of the convoluted signal is M plus L minus 1. So, now those L minus 1 symbols or samples will come and interfere with the next block OFDM block. So, this will overlap with the next block and as a result this will spoil these first few samples of the next block. So, this is inter frame or inter block interference and we want to avoid that. How do we avoid that?

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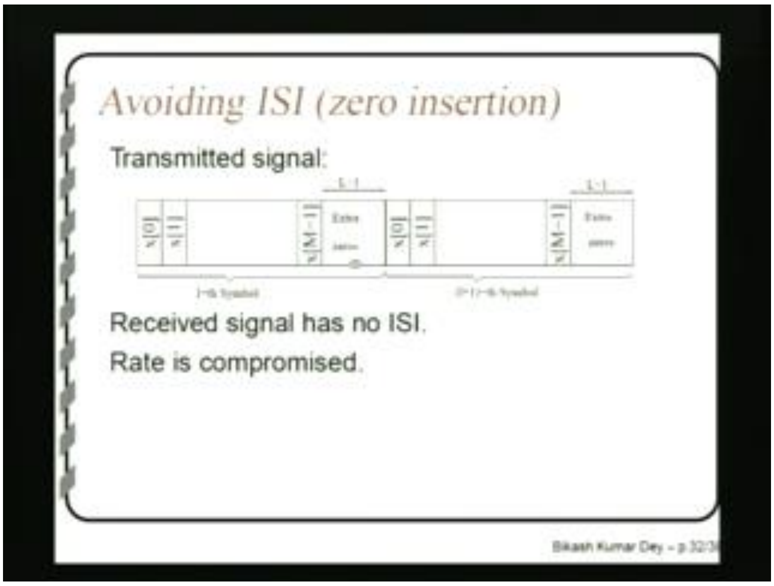


One way to avoid it is to put some extra zeros after every block. So, if you do that then even after convolution only these zeros will be affected. If you put L minus 1 or more zeros only these zeros will be affected and this block will be untouched even after, convolution of this block with the channel impulse response. So, the received signal will not have any ISI.

But the rate will of course, be compromised because we are not using some of the samples. We are transmitting 0 samples for L minus 1 samples, so we are compromising on it.

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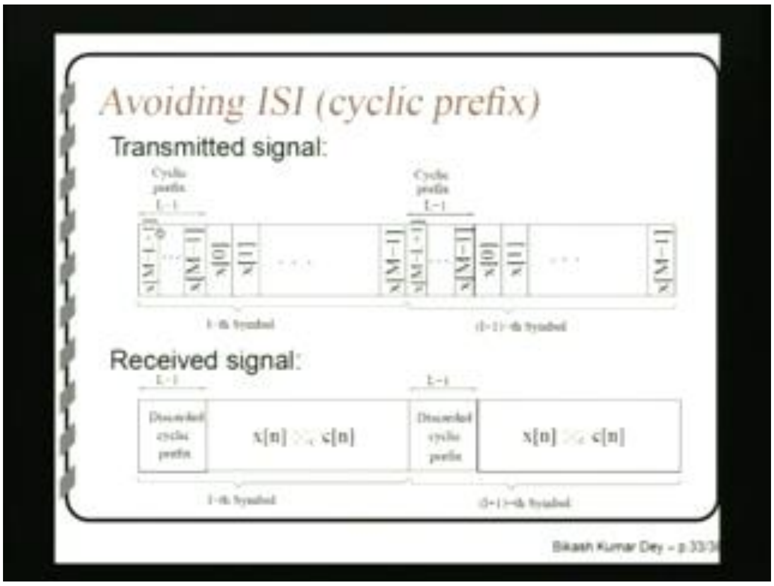


Another interesting way and that is what is used in practice is the cyclic prefix. Instead of padding some zeros at the end instead we take the last L minus 1 samples and put it copy it in the beginning of every block. So, this x naught to x M minus 1 out there, but we take x x M minus 1 x M minus 2 and so on till x M minus L plus 1 and put them here we repeat it them in the beginning. So, for the next block also we do the same thing we repeat the last L minus 1 samples in the beginning.

So, this is cyclically repeated this is the prefix part of this block. So, what will happen? In the received symbol in the received symbol this will be linearly convoluted by the channel impulse response. So that, these will be affected. But then, this will not be affected because it will extend only till L minus 1 the effect of the last block will be till only L minus1. So, this will not be affected so we will still have these samples unaffected by the previous block.

So, if we remove this cyclic prefix of the receiver then, this does not have any contribution from the previous block only these samples had, but these samples are removed at the receiver. So, this cyclic prefix is discarded at the receiver then what is the remaining block. It is also not the linear convolution of the channel impulse response with this block because; linear convolution will also have some etcetera signal coming at the end which is removed.

But we can show that this will actually, will be cyclic convolution because these extra symbols are repeated cyclically from here. As a result what we get here will be the cyclic convolution of this block with the channel impulse response which is denoted by cn here. So, now at the receiver at the transmitter we did IDFT at the receiver we will do DFT of this received block.

When we do DFT of this received block we will get by property of convolution property of DFT we will get the DFT of this. Times the DFT of this convolution and the time domain becomes point wise multiplication in the frequency domain.

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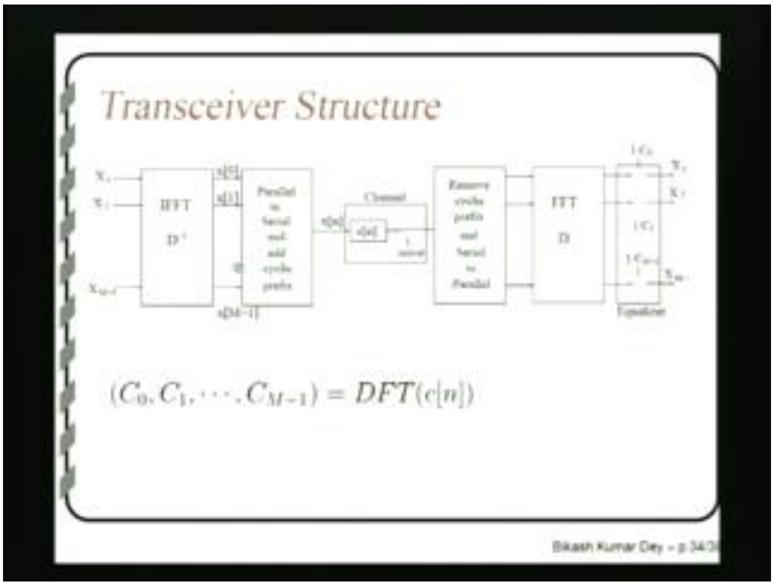


So, first at the receiver we remove the cyclic prefix at the transmitter after doing parallel to serially add cyclic prefix and transmit then the signal goes to convolution with cn. And then, here we removed that cyclic prefix part as we discussed and then make a serial to parallel. So, we have M length vector here after removal of cyclic prefix. Then we take FFT this is multiply by D. Then what we get is DFT of xn times DFT of cn.

Now, DFT of cn needs to be divided from that product to get DFT of xn. So, we divide by the DFT of the channel impulse response DFT of channel impulse response first component zeroth component is c naught, first component is c 1 and so on. So, that can be computed and then we divide by those. So, 1 by c naught is multiplied to this and so on. So, we will get the DFT of this vector the xn sequence which is this because, this is the IDFT of this. So, DFT of this is this. So, we will get this back if there is no noise of course, if there is noise there will be a noise component to this will be added.

So, we take DFT of the channel impulse response at the receiver and then divide by the DFT coefficients these components are divided by the DFT coefficients of the channel impulse response. So, this is the transceiver structure of the OFDM system. And here you see that how nicely the properties of DFT are used specially the cyclic convolution property. And 1 can see that if the channel is bad for example, if C 1 is very high; C 1 is very low then what will if the channel impulse response in the frequency domain is very low. That is C 1 is very low then this will be very high this quantity will be very high.

So, as a result this noise component in this branch will be multiplied by a high number. So, as a result the noise component added to this will be large and as a result this will have low SNR. So, some channels are good some channels are bad depending on the magnitude of these components. So, how do we how much do we use the different channels, to what extent do we use the different channels. The channel which is good we want to use that channel to a greater extent than the channels which are bad.

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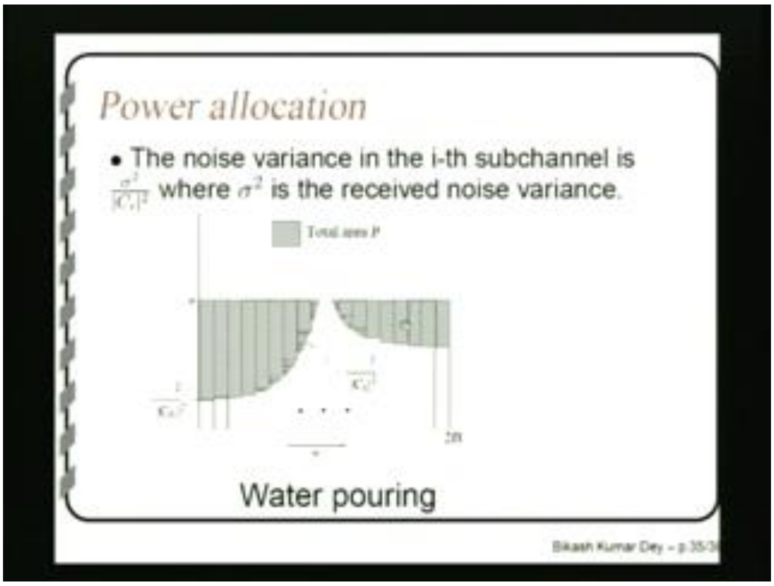


So, if you want to do it in optimum way this is the way we can derive that this is the way to go. If this is a band this is the if you plot this quantity the sigma square is the noise variance then, plot sigma square by Ci mod Ci square. So, in the i-th sub channel Ci square Ci is the DFT coefficient ith DFT coefficient of the channel impulse response. In sigma square by mod Ci square is basically proportional to the inverse of the impulse frequency response of the channel then it is like this.

We take the total power P that we have and we imagine a container with this shape of the bottom and we poured the power in the vessel of this shape. Then when the P the total amount of P which is behaving like water is finished we stop and we measure the height of the power level in each sub channel. And that is the power we transmit through that sub channel. So, this is called water pouring solution for power allocation.

So, we can compute the power that is to be transmitted through each sub channel in this fashion. And then to compute how many bits would be transmitted through each sub channel what we can do is.

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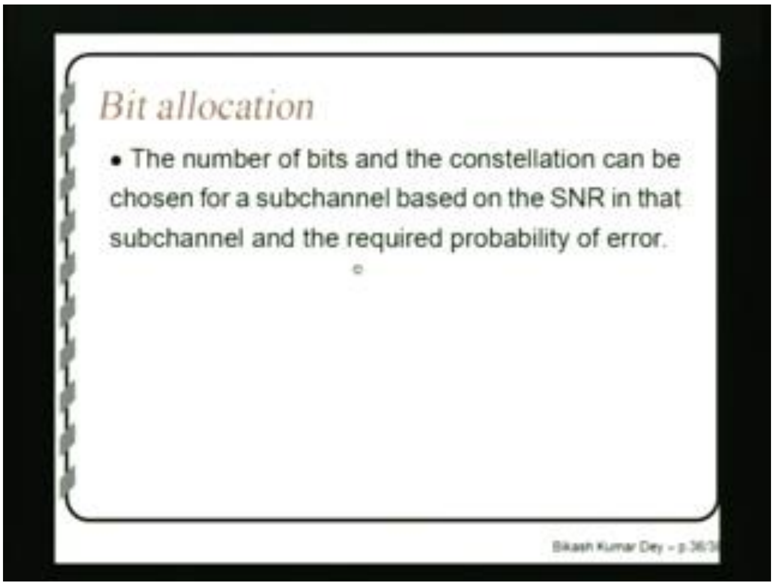


We can first decide what is the probability of error that we want? Then we have some SNR because the power that is to be transmitted is obtained from water pouring solution and the noise variance is also known. So, we can compute SNR. And we have some required probability of error then we can decide on a number of bits and the constellation that we want to use can be chosen.

So, in this class we have discussed OFDM technique we have first revised some basics of digital signals and systems and discussed DFT and it properties to some detail. And then we have seen how the properties of DFT's specially the convolution property is used very efficiently in an ODMF system. And that simplifies the OFDM implementation to a great extent. First of all the transmitter and the receiver is implemented using FFT which is very efficient algorithm.

And also the equalization becomes very simple by using cyclic prefix there is no interference between sub carriers. And the equalization is very simple just divide by the channel frequency response divide by Ci. So, that is how we see that way OFDM is simply implemented using DFT techniques.

Thank you.