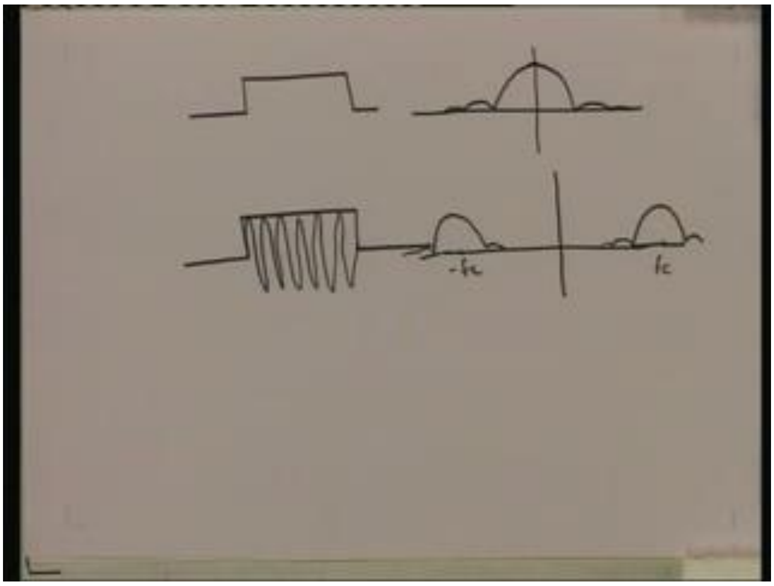
Digital Communication
Prof. Bikash Kumar Dey
Electrical Engineering Department
Indian Institute of Technology, Bombay

Lecture - 13
Digital Modulation Techniques (Part - 2)

Welcome to the class. We have been discussing digital modulation techniques for few classes now. We actually started digital modulation in the last class. Previous to that, we discussed about baseband representation or bandpass signals in the first two classes in this module. And, in that context, we have discussed lowpass equivalent signal of a bandpass signal. And, we have seen how to… We have seen that, we can convert a bandpass signal into lowpass signal. That signal we call lowpass equivalent signal. And, it does not lose any information. And, we can convert the lowpass equivalent signal back into passband signal. So, this conversion from bandpass to lowpass is called the down conversion, because the frequency comes down and the conversion from the lowpass equivalent signal to the equivalent bandpass signal is called the up conversion.

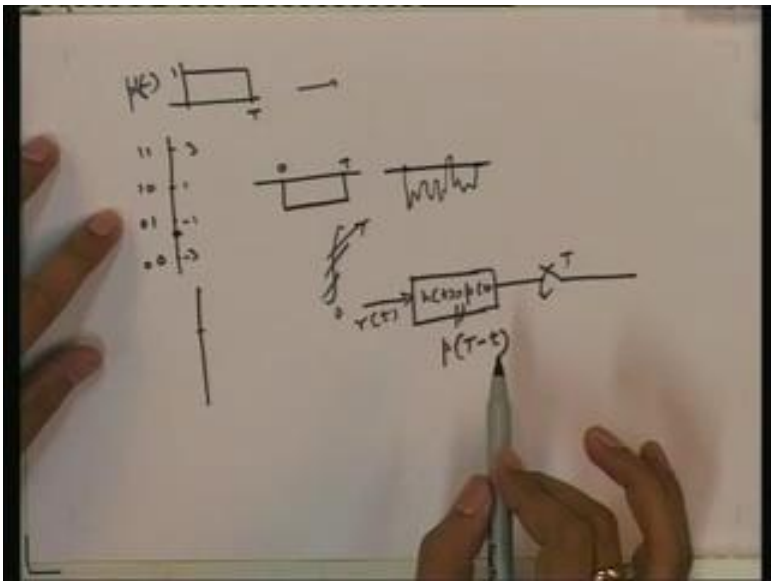
And, we have seen that, this conversion have certain properties – nice properties; for example, if we take also the lowpass equivalent system of a bandpass system and that system fully characterizes the original system. So… And then, we have discussed one modulation technique called pulse amplitude modulation, that is, one digital modulation technique we have discussed so far. And, we have seen that, pulse amplitude modulation is basically just modulating one single pulse depending on the symbol that is to be transmitted. So, if it is binary pulse amplitude modulation, that is, 2 PAM, then we have a pulse g t or p t; then, we scale, that is, to change the amplitude depending on what you want to transmit. If you want to transmit 1, we will transmit 1 amplitude; if you want to transmit 0, we will transmit another amplitude of the same pulse. So, now, this pulse p t may be a baseband pulse just like a rectangular pulse in which case the spectrum of the resulting transmitted signal will be a lowpass signal or it may be a passband pulse; meaning by it may be a shape that it is modulated by sinusoid. Then, the spectrum of that pulse itself is not lowpass; it is bandpass. It will have center frequency that FC, which is the frequency of that sinusoid. So, we have seen that if we…

(Refer Slide Time: 03:50)



For example, if we take this pulse for baseband model or PAM, we get one spectrum like this. But, if we take a bandpass pulse, it is not this, but it is modulated by this. So, like this. Then, the spectrum will be centered at FC. So, it will be like this. So, if you want to do passband modulation, we just need to modulate the pulse by a sinusoid. So, if we use such a modulated pulse as the carrier for our pulse amplitude modulation, then we will have a bandpass spectrum of the resulting transmitted signal. So, that is called passband PAM. Now, instead of binary, we have seen that, we can have different M eddy-modulation scheme, where M equal to 2 power B. So, if you want to transmit of B number of bits per symbol, then those 2 power B possible values will be mapped first to some real numbers – some amplitude levels like 1, 2, 3; then, minus 1, minus 2, minus 3 – several amplitudes we can map. And then, take those amplitudes of the PAM pulse. So, that will be multiplied to the pulse p t and then that will be transmitted. So, that will be done in every symbol. So, we have seen such examples. And so, we have also seen in the last class, how to demodulate a simple binary PAM signal or binary or in fact, any PAM signal when the pulse is rectangular – baseband rectangular pulse.

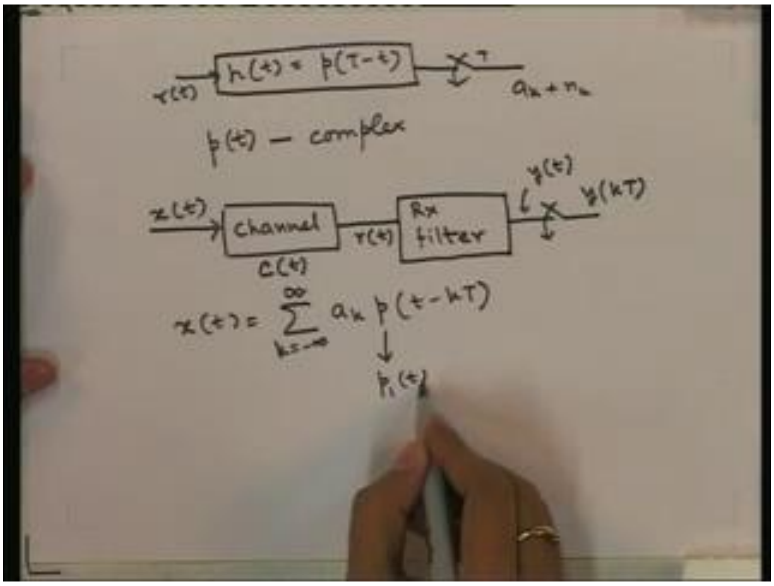
(Refer Slide Time: 06:02)



So, if we have this p t rectangle T-1; then, we have seen in the last class; then, the best things to do to… No, not the best thing, but we have not seen that; but, we have seen that, one way to demodulate this is to integrate the received signal. So, received signal if we transmit one; so, we will have two levels for 1 0. Or, if we have M eddy PAM; then, we will have many more levels – M number of levels. So, for four PAM, for example, we can have minus 1, minus 3, minus 1, 1, 3. So, we can have the different levels minus 1, 3, 1, 3. We can have these four levels. And then, we can multiply those numbers to this and transmit depending on what you want to transmit. If we want to transmit 0 0, we will probably take this 0 1 to this, 1 0 to this, and 1 1 to this. So, if we want to transmit 0 1, we will transmit minus p t. So, now, whatever we transmit, it will be ultimately corrupted by noise. So, what we will receive if we… For example, transmit minus 1 0 1, we will have transmitted this in 0 to T; but, we would receive something like this – some corrupted by noise.

So, now the way to one had to demodulate is integrate this pulse in this interval. So, take 0 to T and then integrate this value and then if that is… So, that value we can compare with… Suppose if it is minus 1, it would give some value; if it is give minus 3, it will give some value; if it 1, it will give some value and so on for 3. So, we will integrate and then see to which of those values it is nearest, so that we will pick. So, if we integrate, we will see that, it is nearest to it; it will probably… It will be some here probably. Then, we will pick minus 1, because it is nearest to minus 1. So, we have also seen that, doing this operation, this operation can be done in terms of a filter and sampler. So, we can have a filter with impulse response p t, which is same as p t minus capital T minus t in this case. So, we will take r t – will be passed through this and then sampled at T. Then, we will get the same output as the integration of this here. We can write here p t, but… – impulse response as p t. But, we will see that, for other pulse shapes, it is not actually p t in this case; we should have p capital T minus t. So, for other pulses, it will matter; for this particular pulse shape – rectangular pulse shape, the both p t and p capital T minus t are same. So, it does not matter what you call it. But, for other pulse shapes, it will matter. That we will see later. So, we will now investigate; a generalization of this we will see.

(Refer Slide Time: 09:09)

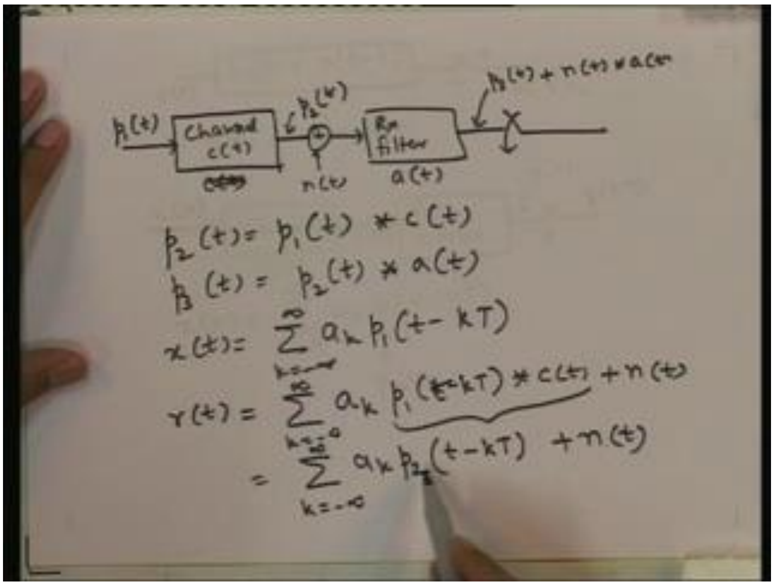


So, in this case we have seen that, we should take h t at… Unless this is one choice h t equal to p capital T minus t; pass it through this filter and then sample it. So, first pass is through this filter and then sample at T. This is one choice. Now, we will get an estimate of a k. If there is no noise, we will get exactly a k; if there is some noise, it will be added by some noise component. Now, we will consider a generalized situation. We will assume that, we are using some arbitrary pulse p t and investigate how to demodulate the modulated signal. So, in this analysis, we will generalize the notations we have been using. We are assuming in this… When we were discussing PAM, all the examples we took have real valued pulse. But, it need not be.

If you are doing complex baseband case – equivalent complex baseband model and you are first generating the complex baseband signal; and then, from that, we will do up conversion to transmit. So, to do that, we can have a complex pulse. So, we will assume that, the pulse may be complex. We will assume that it is complex. If it is real, it is a special case. And then, we will just draw some… We will use some notations, which I will introduce here. This is a block diagram of what is happening. We are transmitting here. This is x t. Then, what we receive; this is the impulse response of the channel will be assumed to be c t; and, this received signal will be assumed to be r t. And then, we will pass through a filter.

Here we have seen that, we can pass through this filter and then sample at t. So, we will take care one filter and sample. And, we will see whether for arbitrary pulse shape, this can be done. And, if it can be done, what kind of filter we should take. So, this is received filter; this filter is called receiver filter or received filter. And, this signal what comes out of the received filter is denoted by y t. And then, we sample at every t seconds. So, the samples will be y k times capital T. So, at k equal to 1, it will be sampled at t; it will be sampled at 2T for k equal to 2; and, so on. So, x t we know is the modulated signal. So, x t looks like summation. So, this channel also has actually noise. This is the only impulse response, but it has also noise. So, here k equal to minus infinity to infinity a k and then p t minus kT. So, now, this will go through channel. So, here what we will receive is c t convolution this plus noise. So, that convolution can be taken inside the summation. And then, what we will get is p – this pulse convolution c t inside. So, for convenience, now on, we will use the notation that, p t will be denoted by p 1 t. We will see why, because there will be several pulses we will consider. So, this is p 1 t.

(Refer Slide Time: 13:28)



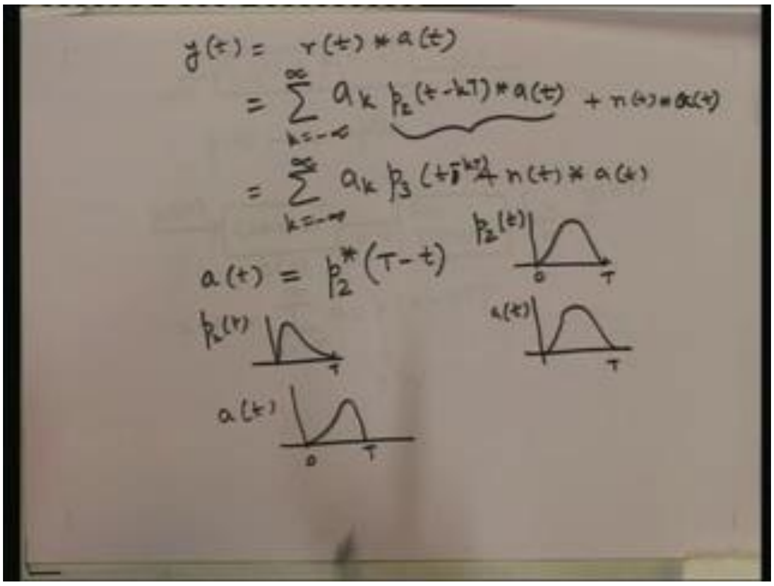
And, we have the channel. If we break this summation, you see that, there are so many terms – individuals pulses are transmitted after delaying and scaling. So, we will see what is happening to these individual pulses when it goes through all these systems. So, you have impulse response c t; there is actually noise here. So, we will assume that, this is the filter only and we will add the noise outside. Then, this goes to R x filter has impulse response a t suppose. We will see what a t should be. This is…

Now, suppose I give only p 1 t as input to this. Then, what will come out here is also a kind of pulse; it is p 1 t convolution c t. So, that is, that we will denote by p 2 t. And then, here again it is p 2 t convolution a t plus the noise convolution a t. So, this pulse we will assume that, it is p 3 t plus some noise n t convolution a t. So, if we denote it this way; so, remember these things – this is the output only when you give this as input; only one pulse you give as input. So, p 2 t is nothing but p 1 t convolution c t. And, p 3 t is nothing but p 2 t convolution a t. Then, we are sampling at every t seconds. So, which means transmit simply p 1 t; we will sample at capital T and see what we get.

Now, x t in terms of p 1 t; x t we have written as x t is summation k equal to minus infinity to infinity a k p 1 t minus kT. Now, what we receive here r t is x t convolution ct plus n t. So, what we receive when you give x t as input to the channel. We receive r t k equal to minus infinity to infinity – this whole thing convolution c t; it is going through channel plus noise. So, the convolution can be taken inside summation. So, you can write a k; then, p 1 convolution c t plus n t. Now, what is this term? We see that p 1 t convolution c t is assumed to be p 2 t. So, if we shift this pulse by kT, the whole convolution result will be shifted. So, it will be p 2 shifted by kT. So, we can have write here k equal to minus infinity to infinity a k; then, p 2 shifted by kT plus n t as it is. So, what have we done here? We assume that, p 2 t is convolution of p 1 t and c t; that is, p 2 t is the output when you give p 1 t as input to the channel. And similarly, p 3 t is the output when you give p 2 t as input to the received filter a t. So, that is p 3 t is convolution of p 2 t and a t.

Now, the actual transmitted signal is this – summation of so many pulses – shifted pulses after scaling. So, when you do this convolution, what we receive here is this noise plus what we transmit here convolution c t. So, this whole thing convolution c t. But, that c t convolution can be taken inside summation and then that convolution – individual convolution can be written as p 2 shifted by kT. So, we have here the noise plus this pulse p 2 t differentiated version. So, it is like what we transmitted. What has happened; what we transmitted is this summation; what we have received is this summation. So, what is the difference? Difference is that, there is noise here; there was no noise here. So, channel has added noise and then the pulse was actually p 1 t that was transmitted. But, here what we have received is p 2 t. So, it is like it is as if we have just transmitted using p 2 t; as if we did pulse amplitude modulation using a different pulse p 2 t. So, we can assume that and see how to do demodulation. There is no need to consider channel, because ultimately what has come out is just like a PAM signal with a different pulse p 2 t and some noise is added. So, now, this signal goes to the received filter; this signal goes to the received filter; r t goes to the received filter. So, what happens then we will see.

(Refer Slide Time: 19:19)



So, what we have here now, is that, y t, which is the output of the received filter if r t is given as input is now… Now, convolution of r t with a t. Now, r t is now this whole thing. So, r t will be convoluted to this summation and n t. So, n t we can write; n t convolution r t and then here this summation convolution r t. But, this convolution with r t can be taken inside the summation again. r t is this; so, convolution with a t. So, convolution with a t with noise and then convolution with a t of this summation. So, that convolution with a t can be taken inside. And then, we have a k as it is. Then, p 2 t minus kT convolution a t; that can be written as another pulse; and, what is that? We have here p t minus kT convolution a t.

Now, this whole thing – what is this? p 2 shifted convolution a t. Now, p t without shifting – p t convolution at is assumed to be p 3 t – another pulse. So, this will be… because this we have shifted p 2 by kT. This whole – this result itself will be shifted kT. That is the result of convolution. If we shift one component by sometime – tau, then the result also will shift by tau. So, here we consider this is now k equal to minus infinity to infinity a k p 3 t plus n t convolution a t. So, what do we have? We have here… We have assumed the response of the channel to p 1 t to be p 2 t and then c t is the channel response; received filter response impulse response is a t. Then, p 2 t can be written as c t convolution p 1 t. p 3 t can be written as p 2 t convolution a t. And then, x t is we know that, it is PAM signal. So, it is of this type. And then, we have seen what we receive here; what we receive here – r t. r t is this convolution and this we have brought to this form. This is p 2 shifted. So, it is like a PAM signal with this pulse plus some noise.

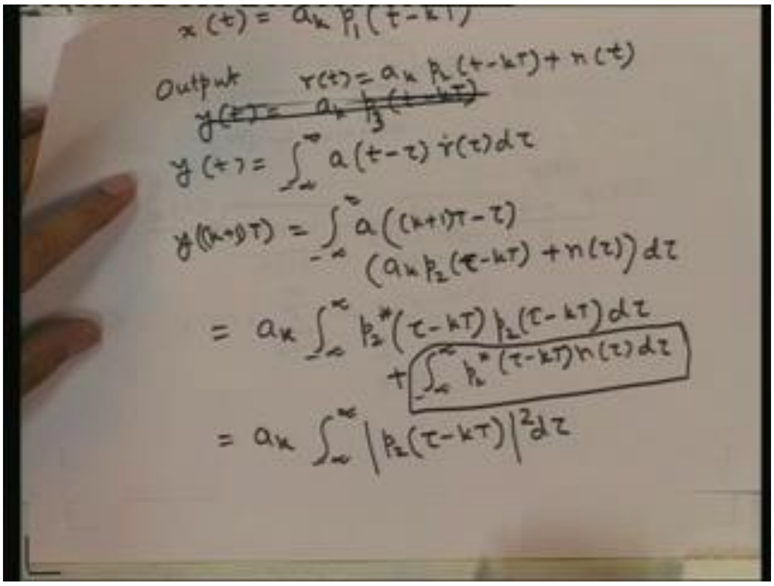
And then, after received filter, what we have got is again like a PAM signal. This is p 3 t minus kT. So, this is also like a PAM signal with again a different pulse – p 3 t; then, plus the noise, which is not quite noise now. So, now, what we are doing is we are sampling at multiples of time t. So, we will sample at t, we will sample at 2t and so on. So, we will now see what we get if we sample. Now, this whole thing we have assumed that, it is… There is no assumption on this that, p 1 t is real or anything; p 1 t can be complex; c t can be complex; a t can be complex; noise can be complex; everything is complex here. If it is real, it is a special case of this.

So, now, we want to see what a t should be; how should we actually design the received filter; what is the criteria. So, to do that, we will go into that; but, before that, we will actually assume something; we will assume that, the received filter is of something and then analyze the case and then prove that, that is the best thing to do. So, we will do what is called… We will take what is called the match filter. So, a t is matched to this pulse. So, a t is being convolved with this pulse if you see here. So, we will take a t such that, it is matched to p 2 in some sense. So, in this sense, we will take p 2 star capital T minus t. So, this is… So, here this is… It is matched in some sense you can see.

Now, what is this? If we… We will take an example. Suppose p 2 t is like this; here it is T; it is 0. If it is p 2 t, then what is this p 2 star t minus... Star does not mean anything here because we have taken an example of real pulse. So, p 2 capital T minus t; star means nothing. So, p 2 capital T minus t is just split version of this. So, it is the same thing even in this case, because this signal is… This pulse is symmetric. On the other hand, if we take this kind of pulse – p 2 t, this is a t. If we take for example, this kind of pulse; then a t will be… It will be flipped; it will be flipped. So, we see what it means to do this. Capital T minus t means basically flipping; 0 comes to T; T comes to 0. So, this is called the matched filter; a filter with impulse response this, is called the matched filter to this pulse; a filter matched to the pulse p 2 t; that is the matched filter.

Now, we will take this particular filter as the received filter and see what we get. Now, remember that, for rectangular pulse – rectangular baseband pulse, we actually took this filter. When we took example of a demodulation scheme, we actually took this filter and then sampled at the output. So, in this case also, for general other pulse also, we are taking the same thing and then we will see that, we will able to demodulate using this filter. So, with this, we use this filter; and then at the output of this filter, we sample. So, to analyze, what will happen; we will not assume that, we are transmitting so many symbol after symbol; we will assume what happens to one symbol. Will you be able to demodulate if we transmit just one symbol. Then, we will see that, if we can demodulate one symbol, we can demodulate the next symbol also. So, we will take just one symbol and then see what happens.

(Refer Slide Time: 26:41)

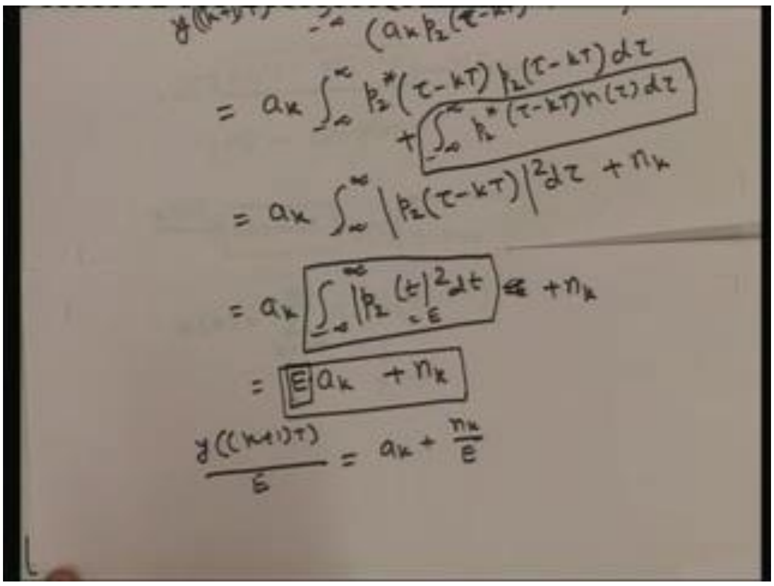


So, suppose that we transmit just one symbol, so that x t is just one symbol k-th symbol. Suppose we transmit only the k symbol. So, it is transmitted between kT and k plus 1 T. So, p 1 t minus k capital T. This is what we have transmitted. Then, what will be the output? y t – output where output here; output here – this y t. So, output y t is… So, actually, we do not… Output y t is nothing but a k p 1 t minus kT – p 3 because p 1 will be convoluted with c t and then with a t. So, that will give us p 3 – this. So, what is p 3 we will see. So, basically this is… So, we will write in a different way. So, y t is minus infinity to infinity; we are writing the convolution. So, add the receiver. Before receiving filter, we will receive r t equal to a k p 2 t minus kT plus n t.

Then, we pass it through the received filter a t. So, we will get a convolution output as t minus – say t minus tau; then, r of tau d tau. This is the output. But, we will take… We are sampling. So, for demodulating the first symbol, that is, the zero-th symbol, which does not have any shift – that we will sample at t and demodulate. And then, see what is the best symbol, which symbol was transmitted. But, for the k-th symbol, we will sample at k plus 1 times t. So, the output we have is k plus 1 times T; that is, from here we can see now that, it is minus infinity to infinity; then, a k plus 1 T minus tau; then, a k p 2 t minus kT; that is, I am now writing r… So, this is tau minus kT. We are evaluating at tau. So, t will be replaced by tau; tau minus kT plus n tau d tau. This is the sampled version; y t is this, but we are sampling at k plus 1 T. So, this is now a k minus infinity to infinity. Now, a is…

Now, you can see that, we are taking the matched filter. So, for matched filter, we have this; a t is p 2 star capital T minus t. So, we will replace a t by that. So, a of this whole thing will be put… This whole thing will be put in place of t. So, here… So, we have p 2 star; then, capital T minus this whole thing; capital T minus this whole thing will be tau minus k capital T. And then, we take this part – p 2 tau minus kT. This is a k; there is another summation. This summation minus infinity to infinity p 2 star tau minus kT n tau d tau. So, this part one can see is a noise – is the noise convoluted with this. And, this thing – what is this thing? This is… You can see that, this is the conjugate of this term. So, it is a real number, which is the magnitude of this square. So, this is a k p 2 tau minus kT square d tau. Now, what is this? There is no… Actually if you do a change of variable; in place of tau, if we put t plus kT, then this will be… This can be written as…

(Refer Slide Time: 32:20)

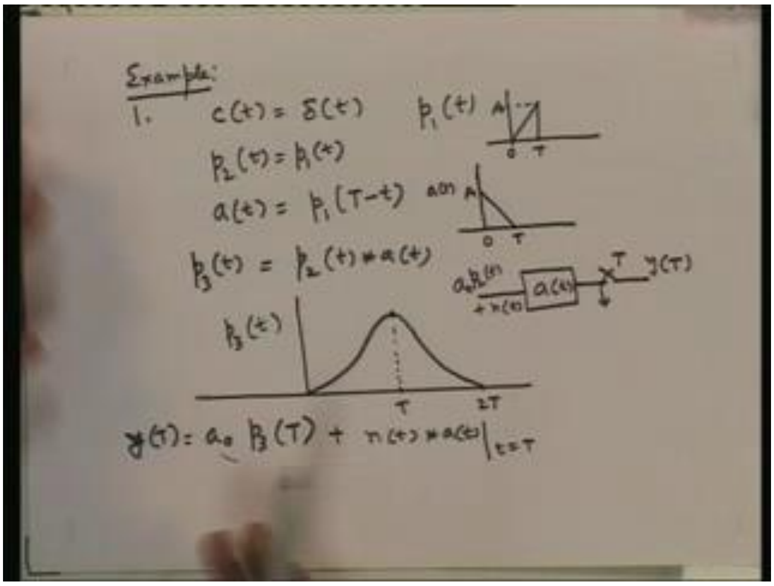


This can be written as a k minus infinity to infinity p 2 t square dt. So, this is basically the energy of p 2 t. So, this is E – the energy. So, this part is energy E of p 2 times a k. Now, we have not written another term, that is, noise; here we have noise. So, this plus we call it n k. So, this plus n k; this plus n k. So, what have we done here? We have taken a… Particularly, a matched filter. We have taken this particular case. And, the receiver filter is the matched filter. And then, with that we have computed the output first; output of the receiver filter, that is, y t; that is, convolution of the matched filter plus impulse response with the received signal. And, that signal we have sampled at k plus 1 times t time. So, that sample value we have computed to be this.

Now, what is this? What is this? This has two parts: 1 is E times a k; a k is the transmitted symbol value; a k is the transmitted symbol value – the level; the amplitude we have transmitted; we have multiplied to p 1 t and then transmitted. So, that a k is the crucial part; if you know a k, we will know what is the bit; what is the bit stream we had transmitted in that symbol. So, if we are using 8 PAM, then we will we would have transmitted 3 bits in that symbol. Then, from a k, we will come to know which 3 bits we have transmitted; whether it is 0 1 0 or it is 1 1 0 we will know if you know a k. So, now, E is fixed; we can compute energy of p 2 t beforehand, because we will know the channel impulse response c t; we will know p 1 t what we are using for PAM. We will take the convolution and then that is p 2 t; we can compute the energy of the p 2 t. So, E we know.

So, now, we can divide this. So, y k plus 1 T; we divide by E; that we will get as a k plus n k by E, because E we know. So, we sample and then divide by E. Then, we get a k – plus some noise. So, if the noise is small enough; if the noise is small enough, we can estimate a k. This is an estimate of a k – a k plus some noise; this is an estimate of a k. And, how good the estimate is depends on the noise value; if the noise is small, then we will be able to estimate a k fairly well. So, from a k, we will be able to recover the bits that we transmitted. So, this is the idea. So, we have seen in these few slides that, by using a matched filter matched to the p 2 t, that is, the output pulse of the channel when you give p 1 t as the input. So, the output pulse is p 2 t. So, if we take the receiver filter as the matched filter of that pulse, that is, matched to that pulse; then, the output if we sample at k plus 1 T; then, we get… And then, we can divide and then we get an estimate of a k; that is the level of the pulse that we transmitted. From that level of course, we can again get the bits back. So, we have seen now this matched filter can be used to demodulate. Now, we will see whether matched filter is really the optimum. Can we use some other filter. So, we will see that. But, before doing that, let us see some example of matched filter and what we receive at the output, etcetera.

(Refer Slide Time: 36:42)



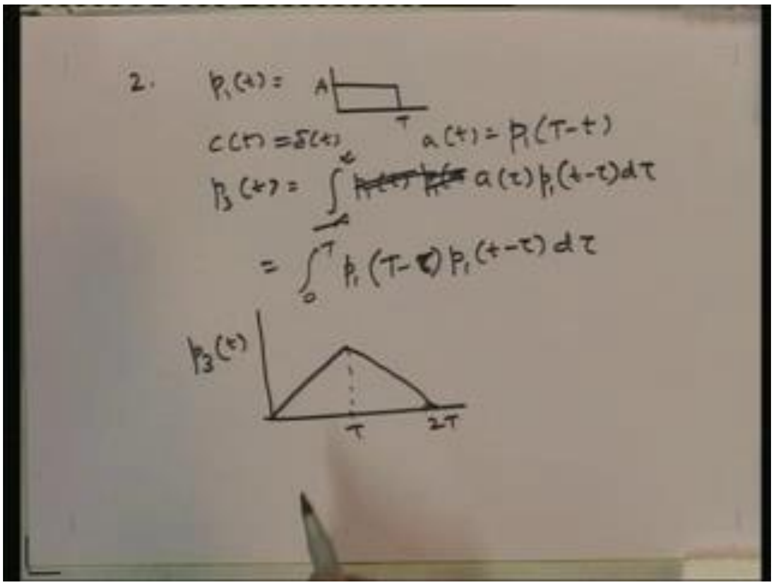
So, example – example 1; suppose the channel is ideal – just delta t. The filter – there is no channel filter; just noise is being added at the channel. So, we will take that, this is a simple case and we will take that case to see what is happening. So… And, we take let us say p 1 t of this shape – some level A. You take this pulse; then, what is p 2 t? p 2 t if you remember; p is p 1 t convolution c t. Now, c t is delta t as assumed. So, this is nothing but p 1 t. Then, what is the matched filter to p t? It is a t equal to p 1 capital T minus t; where, p 1 is same as p 2. So, what is… Can we plot a t? If we plot, it will be reversed. So, T will come to 0; 0 will go to T. So, this is T 0 A; this is a t.

Now, we actually pass a k times this as we assumed in the last analysis that, we transmit a k times p 1 this. So, let us say we are transmitting at the zero-th symbol itself, not the k-th symbol; k equal to 0 we are assuming. Then, what are we transmitting? a k times p 1 t. So, what will be the output of the received filter? If we forget the noise for time being, what will be the output? p 3 t is the output; a k times p 3 t. But, it is sufficient to analyze what is p 3 t. What is p 3 t? It is p 2 t convolution a t. So, if you do this convolution, we will see that, p 3 t is of this shape. T is here; 2T is here; it will be this shape. At T, it is maximum. So, what we will receive at the output of the matched filter; this is matched filter a t. And, we will sample here at T, because we are transmitting from 0 to T; the zero-th symbol; k equal to 0. So, here we have p 2 t plus noise; which we neglected, but we are transmitting; we are actually receiving this at the receiver and that we are passing through the matched filter.

What will you get here? We will… So, it is actually a 0 times p 2 t; the zero-th symbol; some scaling is also there. So, what we will receive here is p 2 t convolution a t evaluated at capital T times a naught. So, we will receive a naught p 3 capital T; p 3 is p 2 T convolution a t plus n t convolution a t; that we have to evaluate at t equal to capital T. So, this is what we will receive. This is y T – y capital T here. Now, what is this? Here we see that, a 2 – this is fine; this a t depends on this. But, why are you sampling at capital T? That we want to see now, see if we use this filter; why we are using this kind of matched filter we will see later. But, suppose we use this filter; then, we want to see why we are sampling at capital T. So, if we use this filter, this noise is – it capital T or it whatever; it will have the same variance. So…

But, here p 3 t is this. So, this p 3 t is maximum at capital T; you can see here. So, we want to amplify this a naught maximum. That is why we are sampling at capital T, because when you sample at capital T, p 3 t gives the maximum value. And, as a result, this is… This a naught value is scaled up to the maximum level; that is possible. And, this noise then becomes negligible compared to this. So, it is… This is sample at T – capital T or multiples of capital T, because we want to amplify the signal part of the received signal – signal part as much as possible. So, we are multiplying a naught by the maximum value by sampling at capital T, because this variance does not vary with sampling time. So, we have seen in this example, why we sample at capital T when we use the matched filter as the receiver filter. We have seen that. Sampling at capital T gives you the maximum amplification of the noise part. And, as a result, it gives you the maximum SNR. If you sample at other place, the SNR will reduce. So, now, we will also see in the next slides; how, why matched filter itself is the best? Sampling at capital T is fine. But, what should we use as the filter? That we will see later.

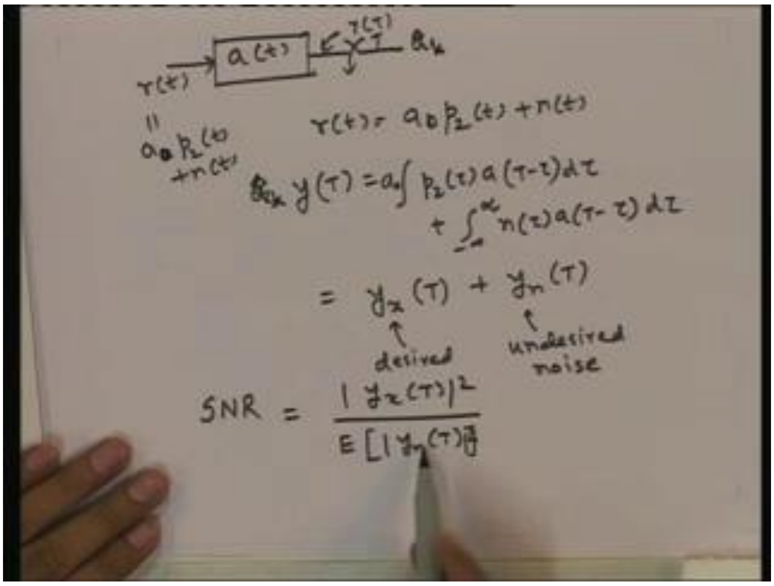
(Refer Slide Time: 42:32)



But, take… Let us take another example of the matched filter. Suppose we have p 1 t is as this capital T; this some level A. Now, again c t is taken as delta t. Then, p 3 t is the convolution of p 1 t and p 2 t is p 1 t itself. So, p 1 capital T; at is p 1 capital T minus t. We want convolution of this with p 1 t. So, let us write it in a different way. So, a t; then, p 1 a tau; then, p 1 t minus tau d tau. This the convolution formula. This integration can be taken from appropriate 0 to T itself, because at becomes 0 otherwise. So, this will become 0. So, we can take 0 to T itself. And then, a t is p 1. So, p 1 T and p 1 small t minus tau d tau. So, a tau… t will be replaced by tau.

Now, this if we plot, we will see that this becomes again T, 2T; it takes maximum value at T. This is linear 2T; T; it gives us maximum value at capital T. So, in both the examples, we see that, it is best to sample at capital T, because that gives us the maximum SNR by providing the maximum scaling to the signal part of the output signal. We take the output signal to the matched filter. If we sample it at capital T, we get the maximum scaling. And, that gives us maximum SNR on the sampled value. So, now, we will see why matched filter gives us the best SNR. Sampled at T, matched filter gives the best SNR. So, now, let us take an arbitrary filter as the received filter with impulse response a t.

(Refer Slide Time: 45:17)

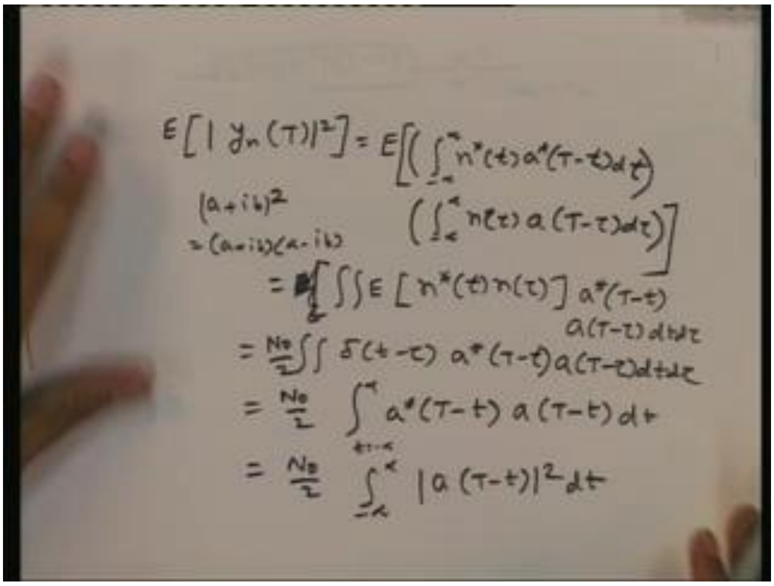


So, we have this r t. We will assume that, we are transmitting again only one symbol p 2 t plus n t. So, because we are transmitting only… So, this is not… So, we will assume that, we are using a 0. We are just transmitting one symbol – zero-th symbol plus a 2 n t. And then, we are receiving that passing through the matched filter and then sampling at capital T. So, we will see what is the best filter to take to maximize the SNR here; signal part here – the energy of signal part by the energy of noise part should be maximum. So, what should be a t; that we will see. So, r t is a naught p 2 t plus n t.

Now, we will assume that, we are taking a k. This is the estimate of a k we want here. So, we want to see what is the estimate. So, what is a k? a k hat. So, whether it is a k hat or some scaling of a k hat; I mean we do not know. So, we will just sample at… This is y T; this is y T. And, we will evaluate y T. So, this is actually the convolution of r t with a t at capital T. So, a naught and convolution of p 2 t and a t. So, p 2 t and a t. So, this part is the signal part. We will call it y x of T. This is the signal part plus this is the noise part – y n of T. This is the signal part of this signal and this is the noise part of this signal.

We want to increase this. This is the desired part. We want to deject from here. This is of no use. This is only… It has noise; it does not have any continuation from a naught; whereas, a naught actually carries the information. So, this is undesired. This is desired. This is undesired – undesired noise. So, what is SNR? See from here if we have only this, that is best, because that from there we can estimate the value of a naught t, because p 2 t we know already; it does not depend on a naught t. And, a t also we know at the receiver. So, we can compute this convolution beforehand and keep it computed. That is a constant. And, just what we receive, we divide that by that constant. Then, we will a naught exactly. But, if there is some noise, that will corrupt the value and we will not be able to estimate a naught exactly. We will still divide the value by this part; but, that will be a naught plus some noise; this part by this integration. So, that will be some noise part. So, how much the noise is compared to the signal part? That is of interest. That tells us how well we can estimate the symbol a naught. So, let us compute the energy of this part and energy of this part. So, SNR will be actually y x T – the energy of y x T; and, the expected energy of y n T, because this term actually is a random variable, because n t is a random process; it is random. So, the expected energy of the noise and this is the signal part – energy of the signal part. So, this is SNR. Now, we just want to compute this and see what is the way to maximize this SNR.

(Refer Slide Time: 49:57)



So, what have we got? We want to compute first the expectation inside… The expectation of this noise part. This is… What is y n T? It is the convolution; it is the convolution; so, minus infinity to infinity n star… so, y n T. See mod y n T if we know, this a is a complex variable – a plus ib mod of this square is nothing but a plus ib a minus ib. So, this is the conjugate of this part. So, you multiply this by the conjugate of this. So, we are taking y n T conjugate and then y n T. We will evaluate two times; then multiply them. So, y n T conjugate is the conjugate of the convolution. So, conjugates will go inside. So, y n tau is star t minus tau d tau. This is the conjugate of y n T; y n star T is this. And then, y n T itself is n tau a T minus tau d tau. This is this expectation. So, we have taken the conjugate of this and then multiply it with this. So, this we want to evaluate. We take it as a double integral expectation.

Now, expectation of this integration will be whole thing; this n star T, a star n and a – all the things integral. But, these two are not random. So, these can come out of the expectation. So, we take expectation of only these two: n star tau n tau… We do not want to take the same running variable here. So, we take here T and here tau. So, here you will have T and then a star T minus t a T minus tau dt d tau. So, this expectation is not there anymore. Expectation is taken inside the integral. So, now, this we want to integrate. Now, if you see, this expectation noise is assumed to be quiet. So, there is no correlation between this difference. So, it is… There is correlation only when t is equal to tau. So, this is integral delta t minus tau a star T minus tau a T minus… This is a star T minus t a T minus tau dt d tau. This is equal to now… This is N naught by 2, because noise is assumed to have suppose N naught by 2 variance, that is, the power spectral density – both two sided power spectral density; then this expectation is delta t minus tau N naught by 2. At T equal to tau, it is N naught by 2 delta. And, at other places, it is 0. So, this… At t equal to… it is infinity; but, other places it is zero.

The variance here N naught by 2. So, this part is nonzero only at T equal to tau. So, only one integral will be there say T and delta t. So, this will give us… because delta t minus tau integration with this will give us a star evaluated at T minus t. Then a t minus tau remains; T minus integration with this is this; and then, this part remains. So, we actually have a T minus t again d t. This is now… This is the energy of this a. So, we have not finished this proving matched filter is optimum in this class. That we will continue in the next class. So, what we have done in that direction is that, we are assuming that, we are transmitting one only pulse – one symbol a times p 2 p 1 t and then we are receiving a times p 2 t plus n t. Then, we are passing that received signal through the filter and sampling at capital T. The output is sampled at capital T.

We have seen that, the output sampled has two parts: one is signal part; one is noise part. If the noise part is not there, we can exactly estimate a naught t. But, if the noise part is there, it will not give us exact estimate; it will have some noise. So, we want to minimize that. So, we want to maximize this scaling by this variance. So, SNR is this y x T. This is the energy of this – this by this one. We want to maximize the energy of this by the expected energy of this. So, towards that, we are computing now this denominator – expectation of the noise part. So, we are… We will continue doing it. We have computed this and then we will argue later that, matched filter actually gives us this best SNR here.

So, in this class, we have seen that, using a matched filter, we can demodulate a pulse amplitude modulator signal. We have then seen why we should sample whatever – if we use match filter, why should we sample at capital T. We have seen by seeing… We have seen two examples to really satisfy us that, we should actually sample at capital T to maximize the SNR. That will give us the maximum SNR. And then, we have started proving that matched filter is the optimum. We will continue and finish in the next class.

Thank you. We will see you again in the next class.