

Course Name: Design of Electric Motors

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Title: Stator and Rotor Leakage Inductances of Induction Machine

Greetings to all of you. In the last lecture, we have discussed the equations to find the magnetizing inductance. In this lecture, we will discuss the leakage inductances, how to calculate the leakage inductances and other inductance values. If we will see the torque production in an electrical machines involves the interaction of two fluxes or two currents. This is the one we have discussed in the initial lectures. The torque is equals to reluctance terms plus some mutual term.

This is the mutual term reluctance terms. So, the interaction of these two fluxes will result the torque. So, the stator currents and rotor currents depends upon the leakage reactances or leakage inductances. These leakage inductances will be caused by the leakage fluxes. The leakage fluxes are the prime source for the leakage inductances.

In particular, the key parameters such as breakdown torque and then starting torque, inrush currents are nearly independent of magnetizing current or slightly dependent, but the leakage inductance will be dominant factor to find the breakdown torque and starting torques and inrush currents. In order to find the leakage fluxes and leakage inductance, what kind of procedure or what kind of equations we have to consider, some equations are we can derive, some equations are empirical thing. So, we have to utilize the both things to calculate the leakage inductances. Now, we will see, let us take the three phase machine.

It is three phase distributed winding and balance and it is forming a four pole flux lines. We can see in this figure, these are the four pole flux lines with respect to the three phase symmetrical supply as well as three phase symmetrical winding excitation. So, the leakage flux components, we can classify into five types. One will be useful flux, other will be leakage flux. The useful flux, the flux which is linking with the rotor, we can see here, whatever the flux coming from the stator is linking and flowing through the, contacting with respect to the rotor, linking with the rotor windings or rotor conductors.

The flux which is not linking with the rotor or which is not flowing through the air gap is nothing, but the leakage flux. This leakage flux, we can classify into five types slot leakage, end winding leakage and harmonic or belt leakage flux, zigzag leakage flux and skew leakage flux. These are the five different components of the leakage flux. After knowing all these leakage fluxes, we will calculate the effective inductance, that is, effective leakage inductance. First we will discuss the slot leakage flux.

If we will see here, right side images, we are placing one conductor here, depends upon the cross or dot current direction. We are seeing some flux line. Some flux will flow through the teeth and air gap and then links with the rotor and here this is LG and this is the teeth. This is the useful flux, the red color one, but the pink color one is the leakage flux with respect to the slot. This flux only link with the stator windings only and it is flowing through the stator back iron only.

We can see in this region only, it is flowing stator back iron from the image 1 and image 2. We can observe that the flux lines are flowing through the back iron of stator portion. In this region, the flux lines are flowing. This slot leakage flux mainly depends upon the slot shape. If it is a closed slot, then the slot leakage will be very high.

If it is semi-open type of slot, slot leakage will be slightly less as compared to the closed slot. If it is open slot, the slot leakage will be very less. The flux only links with the stator or rotor windings only. It will not flow through the air gap and it will not link with the rotor windings. It is observed that the flux path of the stator leakage flux is perpendicular to the main flux, which is passing radially downwards towards the teeth.

We can observe here red color 1 is flux flowing through the teeth and this is the flux flowing through the slot leakage. Both are perpendicular to each other or we can see here. It is in this direction horizontal, the flux flowing through the teeth will be vertical direction. If it is noticed that small amount of leakage flux links with the bottom side of the slots, where some flux is linking with the own conductors in a same particular slot. The slot leakage flux is nothing but the flux, which is linking or which is flowing through the stator core or near to the stator slots only.

It is not linking with the rotor core or it is not flowing through the air gap. The next thing will be end winding leakage flux. If we will see here, this is the machine. This is the stator and this is the rotor. Here, the winding portion whatever it may be the winding portion coming out of the core, this one this portion is nothing but end winding or overhang.

Because of the current flowing through this overhang, there will be a some flux. This flux is flowing through the air gap and it is not linking with the iron part either stator or rotor. Because of this reason, this flux with respect to the end winding, we will call it as a

end winding leakage flux. We can see here the overhangs are at this point. The flux is represented with blue color line.

So, this side also overhangs, we can see the blue color lines are the leakage flux with respect to the end winding. This end winding leakage flux will be dominant with respect to the number of poles. How it will be? We will see now. We can see here there will be a 4 poles are there. This is one winding, second, third, fourth.

If 2 pole 2 pole pairs are there or 2 poles are there only that means, only one coil will be here and one coil will be here that means, coil length is increasing. That coil length is nothing but πD by P arc length with respect to one coil. End winding length if we will calculate it arc length that is πD by P . If the number of poles were decreasing, then the total coil length at the end winding side will increase that we can see as per the literature. For a 2 pole machine 55 to 75 percent of the total leakage inductance will be with respect to the end winding leakage and 49 to 68 percent of the total leakage inductance share will be end winding only.

Similarly, 43 to 60 percent for a 6 pole, 39 to 52 percent for 8 poles. So, if you are increasing the number of poles, then the length of the end winding also will come down and the leakage flux with respect to the end winding also will come down. The complexity of the flux paths associated with this practical type of windings overhangs and end windings, there will be a complex mathematical models. So, to make the simplification, we will go ahead with some empirical factors to find the accurate end winding leakage flux. Next one is harmonic or belt leakage flux.

Harmonic or belt leakage flux is nothing but in a symmetrical machine, the MMF we can see here in a symmetrical machine, the MMF waveform or MMF distribution with respect to the stator and with respect to the rotor, both will be identical or both having the same harmonic profile. If this kind of MMF is same, there is no harmonic or belt leakage flux. If the MMF distributions are not same, then the harmonics fields at the stator side or rotor side will not link with the stator or rotor. So, that flux because of the harmonics with respect to the MMF is nothing but harmonic or belt leakage flux. The next thing will be zigzag leakage flux.

We can see here the image at the right side. The zigzag leakage flux is vary with respect to the zigzag wave where it is shown as a pink color one. It will flow through the tooth tips of the stator as well as rotor side. So, at the bottom side of the stator tooth and top side of the rotor tooth, it will flow in a zigzag manner. The magnitude of this flux depends upon the length of air gap and the relative instantaneous positions of the tooth tips.

It depends upon the slot shape also because of its nature. It cannot be clearly assigned to either stator side or rotor side because it is linking with respect to the stator tooth, this

portion as well as this portion. So, we cannot say that this leakage flux exactly related to stator or rotor. So, empirically we can take that the zigzag leakage flux will be half of will be at the stator side, half of the zigzag leakage flux will be at the rotor side. The next thing will be skew leakage flux.

In the earlier lectures, we have discussed the skewing type, right, skewing type of rotor where the number of bars are winding at the rotor side is skewed. Because of this skewing, the fundamental flux which is linking with the rotor also coming down. So, that decrement or that factor how much we are losing with respect to the fundamental thing is nothing but leakage flux. We can see that thing the same effect in the increase in stator leakage flux because of the skewing flux. Let us say with skewing, we are getting fundamental will be 0.98, then remaining 0.02 will be the leakage flux. We have discussed the skewing factors for various harmonics, right. K_{sq} for the fundamental, we got 0.98. So, the remaining 0.02 will be the leakage flux component. So, in order to find the leakage inductance with respect to the stator, we have to calculate the leakage inductance with respect to the slot, leakage inductance with the respect to the end winding, leakage inductance with respect to the belt, leakage inductance with respect to the zigzag flux and leakage inductance with respect to the skeillo analysis or ske Moun type. These are the 5 components of the leakage inductances based upon the 5 leakage types of the fluxes. So, by utilizing equation number 55, we will calculate the leakage inductance with respect to the stator. First, we will calculate the slot leakage inductance.

Generally, the inductance is nothing but N^2 by reluctance. If you will write the inductance in terms of permeance, that is N^2 into permeance. Here, permeance is nothing but $\frac{1}{\mu}$, I represented here and number of turns will be n_s . If you have to calculate the slot inductance, slot leakage inductance, so in single slot, how many number of conductors we are keeping? Based upon that thing, we have to calculate the slot leakage inductance. So, here the number of turns will be n_s^2 with respect to a single slot, where the number of turns per slot will be n_s and permeance will be P_s and the empirical gain factor, we are adding l_e also.

n_s^2 into l_e into permeance will give the slot inductance. l_e factor, we are multiplying with respect to the n_s^2 into permeance as a empirical gain or assumption. If the machine is three phase balanced system and winding is designed for Q_s number of slots, then the leakage inductance associated with one stator phase belt of a three phase machine is this one. L phase belt is nothing but slots per pole per phase into single slot inductance with respect to single slot. So, inductance per phase belt is equal to number of slots per pole per phase into L slot is nothing but $n_s^2 l_e$ into permeance.

So, slots per pole per phase is nothing but Q_s by m into number of poles. So, if we will substitute this equation in this thing, then we will get the equation number 56 that is leakage inductance for phase belt. So, in order to calculate the inductance per n number

of circuits with respect to the P number of poles, then we have to multiply the poles per circuit with respect to the phase belt that will give the inductance per circuit. Number of poles into inductance per phase belt is nothing but leakage inductance per circuit. C is nothing but parallel circuits with respect to the winding.

Generally, the C equals to 1 for a symmetrical distributed winding. For higher current thing, we will make the parallel circuits. Let us say 10 ampere we want to make. Each winding is carrying 5 ampere means we have the 2 coils in parallel where C is equals to 2. So, if we will represent the slot leakage inductance in terms of total number of phases, total number of turns per phase, so the total number of turns with respect to all 3 phases will be n_s equals to 3 into 2 into n phase.

And n_s in terms of slots number of conductors per slot is equals to total number of slots into conductors per each slot that is equation this one. So, if we will substitute this n_s value in terms of number of turns per phase, the inductance per phase or leakage inductance with respect to the slot is equals to 36 into n phase square divided by number of phases into slots Q_s into l_e into permeance that is equation number 58.

Based on this slot leakage inductance thing, if the slot shape is varying, how to calculate the permeance? We can see here some equations, some empirical equations we can see and some proof is given in this reference with respect to this permeance. Permeance with respect to the circular slot will be this one μ_0 into 0.60623 plus d_0 is nothing but height of the slot opening this one and b_0 is nothing but slot opening.

So, depends upon the b_0 and d_0 value, we can calculate the permeance of a circular slot. Next if we will consider the semi-open type of slot, this kind of slots generally we will utilize it for most of the electrical machines. So, how to calculate the permeance with respect to this type of slots and this type of winding arrangement with respect to the semi-open type of slots. Let us consider the slot opening width will be b_0 and d_1 is the height of the slant portion of the slot that is this one and d_0 is nothing but slot opening height and the depth of the slot will be d_4 minus d_2 or d_2 will be the distance between the coil to the top portion of the slot that is this one. From this point to this point is nothing but d_2 up to this level, we have placed the coil and this point to this point is nothing but d_2 and d_4 is the space between the coil to slot.

This one is nothing but d_4 and b_s is nothing but slot width and d_3 is nothing but slot depth or winding height from this point to this point is nothing but d_3 . d_3 we can write it as d_4 total slot depth minus d_4 minus d_2 minus d_1 . If the total slot depth is d , then minus d_2 minus d_1 based on this thing we can calculate the d_3 value. So, the permeance for this kind of slots in terms of different widths of the slots will be like this. So, μ_0 into d_3 is nothing but winding coil height in a slot, b_s is the slot opening width of the slot and d_2 is nothing but this one, the distance or space between the coil top side of the coil

to the slot opening type from this point to this point is nothing but d_2 and then 1 by π minus 2α , α is nothing but inclination angle with respect to the slot opening and the point where the square type of portion is starting.

So, this angle is nothing but α and then d_{naught} by b_{naught} , d_{naught} is the slot opening height and b_{naught} is the slot opening. This is another equation to find the permeance with respect to the single layer winding.

Now, if we will take the double layer winding, one coil at the top side, other coil at the bottom side and different dimensions with respect to the double layer winding, if we will consider in this manner, the formula to find the permeance for a double layer winding will be like this. Permeance is equals to one-fourth of permeance with respect to the top side coil that is P_{et} and permeance with respect to the bottom side coil and permeance with respect to the mutual flux link or mutual permeance with respect to the top coil and bottom coil is nothing but P_{tb} and C pitch is nothing but whether it is coil or winding is short pitched or full pitch. So, C pitch is nothing but slots per coil pitch divided by slots per pole pitch that is pitch factor I can consider pitch ratio.

So, the permeance at the top side of the coil will be this one and permeance at the bottom side of the coil will be this one. The mutual permeance P_{TB} or P_{TB} is nothing but this one. Here, all these equations based upon some assumptions and some proof with respect to this reference. So, we have to consider these values like d_3 , d_2 , d_1 and other values b_{naught} and d_{naught} depends upon the slot shape. For a given slot shape like this, the permeance value with respect to the double layer winding will be this one at the top side, bottom side and mutual permeance.

Based upon this permeance values, we have to substitute here the resultant permeance value is equals to the combination of permeance at the top side, permeance at the bottom side and permeance with respect to the mutual coupling. All these three values we have to calculate based upon the equation number 62 and then substitute in the equation number 59 to calculate the slot leakage inductance.

Similarly, end winding leakage inductance in order to find the end winding leakage inductance, here we can see the different dimensions. le_2 is nothing but the winding length which is coming outside the core. We can see here this is le_2 and this is le_2 , the winding portion which is coming out of the stator core and the remaining portion from this point to this point and then this one is nothing but end winding length.

le_1 and le_3 and T_c is nothing but space between the two coils and b_c is nothing but breadth of one coil like how much width of each coil is nothing but b_c and distance or space between the two coils is nothing but t_c . So, based upon these two values b_c and t_c , we can calculate the end winding leakage inductance. That formula for end winding leakage inductance will be this one. $L_{end\ winding}$ is equals to 12 into μ_{naught}

number of tons square divided by total number of slots into pitch factor square, distribution factor square into 2.4 some gain into $l_e 2$ plus $l_e 1$ by 2. $l_e 2$ is nothing but the length of the coil stride portion which is coming out of the slot and $l_e 1$ is the length with respect to the end winding. So, $l_e 1$ we can calculate based upon the slot pitch and pole pitches and here pole pitch is nothing but π into d by p based on that equation we can calculate $\tau_p 1$ is equals to π by P into D I s plus d s and $\tau_s 1$ is nothing but P by Q_s into $\tau_p 1$ and coil pitch sorry not coil pitch it is the pitch ratio with respect to the slots per pole slots per coil pitch to slots per pole pitch. How many slots we are short pitching with respect to the winding based upon that thing we have to calculate the $l_e 1$ value. So, this is the end winding leakage inductance.

Harmonic or belt leakage inductance for a symmetrical machine where the MMF at the stator side and rotor side distributions are same in symmetrical machine where this harmonic leakage flux will be negligible.

So, the belt leakage flux we can consider it as a 0 especially with respect to the squirrel cage rotor and zigzag leakage inductance formula will be. Let us consider the tooth width at the stator side will be $t_t s$ and tooth width at the rotor side will be $t_t r$ then the permeance with respect to the zigzag flux zigzag flux or zigzag inductance will be this one and zigzag leakage inductance equation is equals to 3 into number of tons square by 2 by Q_s into length of a core into 21 times c pitch that is the ratio with respect to the slot. How many slots we are short pitching in the winding minus 5 divided by 4 into permeance with respect to the slot zigzag leakage inductance this is the permeance. So, that permeance value we can calculate based upon the equation number 66.

Similarly, skew leakage inductance we can calculate based upon the skewing factor that is $\sin \pi P$ by $2 Q_s$ into Ssq in skewing angle divided by skew angle directly.

So, this is the empirical equation to find the skew inductance where L_{ms} is equals to magnetizing inductance. So, after knowing the all 5 inductance values with respect to the slot leakage, end winding leakage and belt leakage and zigzag leakage inductance and skew leakage inductance we can calculate the inductance with respect to the leakage inductance with respect to the stator.

Next we will calculate the leakage inductance term with respect to the rotor. The rotor leakage inductance per bar same equation inductance is equals to n square into permeance thing and permeance value with respect to the semi open type of slot will be this one. Here all these three terms we have to replace with $r d 1$ $r d 2$ $r d 4$ r like that based on that thing permeance equals to μ into $d 3 r$ by 3 into BSR plus $d 1 r$ divided by bsr into b naught bsr minus b naught r into log of bsr divided by b naught r plus d naught r divided by b naught r .

So, this d values and b values we have to calculate from the slot type or slot shape b naught r is nothing but slot opening and d naught r is nothing but height of the slot opening and $d_1 r$ and $d_2 r$ depends upon the slot type we can calculate it and $d_3 r$ is nothing but winding height.

Next rotor end winding leakage inductance per ring segment we can calculate it by utilizing this equation. These all equations are empirical based equations and some assumptions also will be there to derive this equations. If anyone is interested in deriving this equation they can go through this reference. Here I am giving the final equations with respect to the rotor end winding leakage inductance.

So, L_e is equals to the equation μ naught into D_b by $2Q_r$ into half of the $1 + \frac{d_e}{t_e}$ into t_e divided by D_b square. Here t_e is nothing but thickness of the end ring and d_e is nothing but width of the end ring. Based upon the width and thickness of the end rings we can calculate the end winding leakage inductance also. And rotor harmonic leakage inductance or belt leakage inductance we can calculate based upon this equation. This is another empirical formula to calculate the rotor bar harmonic leakage inductance.

It is related to the pole pitch and length of the core and effective length of the air gap that is l_{ge} and number of slots at the rotor side Q_r and poles P into summation of different harmonics. If we will see the harmonic leakage flux coefficient versus the number of rotor slots per pole will vary with like this manner. If the rotor slots per pole is increasing then the harmonic leakage flux coefficient that is nothing but this one. This coefficient value will come down if Q_r by P increases. Here this is Q_r by P and $\tau_p r$ is nothing but pole pitch at the rotor side that is πd naught r by P .

Then effective bar inductance in the last lecture we have realized the effective bar resistance right. Same way we can realize the effective bar inductance value that is effective bar inductance is equals to inductance of the bar plus inductance of the end winding divided by $2 \sin^2$ rotor slot angle. The effective leakage inductance per rotor mesh is nothing but L_{lr} effective leakage inductance is equals to 2 times the effective bar inductance plus 2 times the harmonic bar inductance. So, if we will substitute the L_{be} value and $L_{harmonic}$ value inductance of the bar with respect to the harmonics will be like this. These two equations we have to substitute and here L_b is equals to inductance of a bar based upon the permeance we can calculate it and inductance with respect to the end winding will be this equation.

All these three equations we have discussed just now substitute everything in the effective inductance per phase. Then effective leakage inductance per rotor mesh we can calculate based upon this equation number 73. So, with this I am concluding this lecture. In this lecture we have discussed the equations to find the leakage inductances even

though I have not derived the equations. These equations are realized based upon some empirical based approach and some assumptions.

So, these are the final equations to calculate the leakage inductances of a induction machine. Thank you.