Neural Networks for Signal Processing-I

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Lecture – 30

Kernel Functions

Kernel functions can take on a variety of forms. As I mentioned earlier, two key properties are essential: symmetry and normalization. Often, these functions have a mean of zero and are centered around zero. They are typically real-valued functions.

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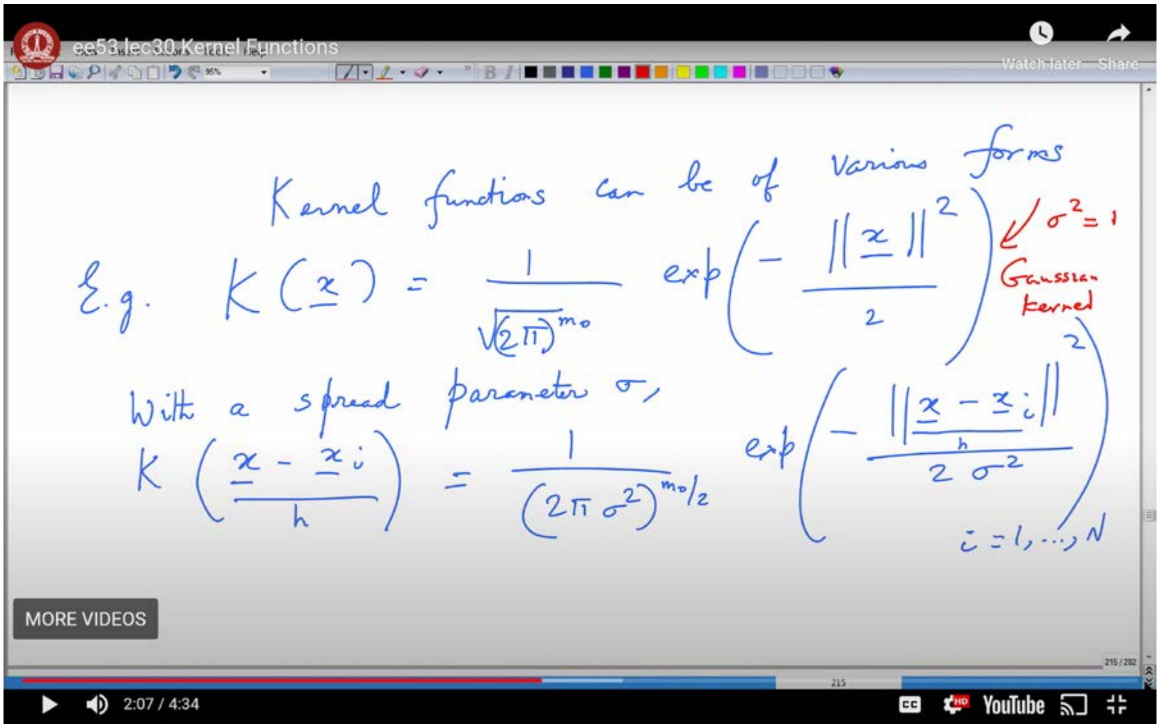

What makes a good kernel function? Examples include a triangular function centered at the origin, a cosine-like function such as a raised cosine centered at the origin, and a bell-shaped Gaussian kernel. Each of these functions has distinct characteristics, but they all serve the purpose of providing a smooth, continuous measure of similarity.

Let's consider one specific example. The Gaussian kernel function k(x) can be defined as:

$$k(x) = \frac{1}{\sqrt{(2\pi)^{m_0}}} \exp\left(-\frac{x^2}{2}\right)$$

where $\sigma^2 = 1$. This is a standard Gaussian kernel. If we introduce a spread parameter, the kernel function becomes:

$$k\left(\frac{x - x_i}{h}\right)$$

where h is the bandwidth parameter. This modification adjusts the spread of the Gaussian function. For every data point x_i, we have a kernel defined in this manner.

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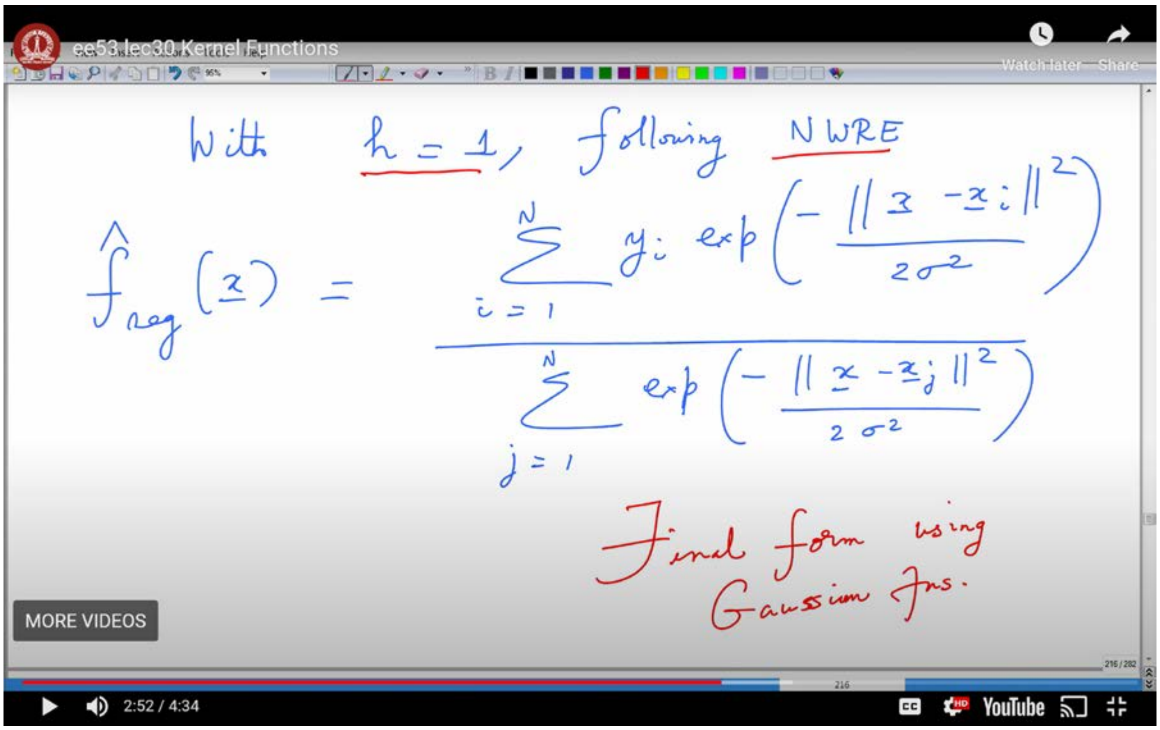

With h = 1, following the Nadaraya-Watson regression estimation, the regression function $f_{\text{reg}}(x)$ is given by:

$$f_{\text{reg}}(x) = \frac{\sum_{i=1}^{N} y_i \cdot k\left(\frac{x - x_i}{h}\right)}{\sum_{i=1}^{N} k\left(\frac{x - x_i}{h}\right)}$$

Here, y_i and x_i are the observations and data points, respectively. We compute the weighted sum in the numerator using the Gaussian function, and normalize it by the sum of the basis functions evaluated at x_i. This provides the final form of the regression function $f_{\text{reg}}(x)$.

Radial Basis Functions (RBFs) play a crucial role in regression estimation. Recall that in earlier modules, we covered linear and logistic regression models. After exploring Multilayer Perceptrons (MLPs) and discussing RBFs, we understand how radial basis functions are used in learning problems. Their application in regression is a direct extension of their use in learning networks.

In summary, RBFs have two primary applications: solving regression problems and serving as the foundation for learning networks. In our upcoming discussions, we will delve into regularization theory, support vector machines (SVMs), and related topics. This will further our understanding of how these concepts fit into the broader context of machine learning.