

Semiconductor Devices and Circuits
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Module-05
Lecture - 23
Schottky Contact: IV Characteristics – Continued

(Refer Slide Time: 00:15)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

Diffusion:

- (i) Barrier height $\gg kT$
- (ii) Concentration at $x=0$ and $x=xd$ do not change from equilibrium values
- (iii) Non-degenerate semiconductor

$$J = q\mu_n n \xi + qD_n \frac{dn}{dx}$$

Using $\xi = -\frac{d\phi}{dx}$ and $D_n = \frac{kT\mu_n}{q} \Rightarrow \mu_n = \frac{qD_n}{kT}$

$$J = -q \frac{qD_n}{kT} n \frac{d\phi}{dx} + qD_n \frac{dn}{dx}$$

$$J e^{-q\phi/kT} = -q \frac{qD_n}{kT} n e^{-q\phi/kT} \frac{d\phi}{dx} + qD_n e^{-q\phi/kT} \frac{dn}{dx} = qD_n \underbrace{\frac{d(n e^{-q\phi/kT})}{dx}}_{\text{Diffusion}}$$

So, in order to you will first look at the diffusion current, so and we will see why it is called diffusion and where that comes about. So, we are going to make a few assumptions, you know without these assumptions the mathematics can get very difficult and therefore, we have to make assumptions in order to look at some answers, these assumptions might sometimes in general in semiconductor physics when you make assumptions you always do. So, in order to get a glance into what the non equilibrium condition is.

It is very hard to calculate the non equilibrium condition, while making 0 assumptions. And it is only to get a glance as to into the behaviour of the device in non equilibrium or may be very close to equilibrium that you make certain assumptions proceed with the analysis and develop an understanding on the behaviour.

So, here the assumptions that we are going to make are the barrier height is much greater than kT . So, first assumption that we are making is this barrier height $q\phi_B$ is much

greater than kT ok, the second assumption that we will make is that even though we are applying a voltage and taking the device out of equilibrium.

(Refer Slide Time: 01:40)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

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$$J e^{-q\phi/kT} = -q \frac{qD_n}{kT} n e^{-q\phi/kT} \frac{d\phi}{dx} + qD_n e^{-q\phi/kT} \frac{dn}{dx} = \underbrace{qD_n \frac{d(n e^{-q\phi/kT})}{dx}}_{\text{Diffusion}}$$

We are applying a voltage checking the device out of equilibrium, yet the carrier concentrations at these 2 nodes that is x equal to 0 and x equal to x_d have their equilibrium values that is the second assumption that we will make, they have not changed too much, they are just slightly out of equilibrium. We just going to take the system slightly out of equilibrium, we are going to apply a very small voltage v_a and you are going to assume that these concentrations do not change from that equilibrium value.

And the third assumption we make is that it is a non degenerate semiconductor; that means, that even though it is an n type material the doping is not so high that E_c minus E_f in the bulk is less than $3kT$, ok. So, it is most definitely the case than E_c minus E_f in the bulk is greater than $3kT$, all right. So, with these 3, let us write out a general expression for the current which is got the drift, it is called the diffusion term and that is my current density.

(Refer Slide Time: 03:05)

Metal-Semiconductor Junctions

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$$J = q\mu_n n \xi + qD_n \frac{dn}{dx}$$

$$-\frac{d\phi}{dx}$$

$$D_n = \frac{kT\mu_n}{q}$$

Using $\xi = -\frac{d\phi}{dx}$ and $D_n = \frac{kT\mu_n}{q} \Rightarrow \mu_n = \frac{qD_n}{kT}$

$$\mu_n = \frac{qD_n}{kT}$$

$$J = -q \frac{qD_n}{kT} n \frac{d\phi}{dx} + qD_n \frac{dn}{dx}$$

$$J e^{-q\phi/kT} = -q \frac{qD_n}{kT} n e^{-q\phi/kT} \frac{d\phi}{dx} + qD_n e^{-q\phi/kT} \frac{dn}{dx} = qD_n \frac{d(n e^{-q\phi/kT})}{dx}$$

Diffusion

$$J = qD_n \frac{dn_1}{dx}$$

Now, you are going to make a few substitutions here. Now this electric field that you see here, you are going to say that the electric field is minus d phi by d x where phi is a potential and we substitute minus d phi by d x in terms of in, in the place of E, the second substitution is that if you think about the diffusion coefficient we had earlier seen that the diffusion coefficient when we looked at the derivation of the diffusion equation we found that the diffusion coefficient was related to the mobility of my Einstein's relation right $kT \mu_n$ by q , you are going to say that this relation holds and your μ_n is simply $q D_n$ by $k T$.

So, you are going to make this assumption and instead of μ_n and we are going to throw in this particular term, instead of electric field we are going to throw in this particular term and we will end up with an expression for the current density to be this. What we are going to do is, we are going to take this expression and they are going to multiply this little factor which is $e^{-q\phi/kT}$ on both sides and we are doing this for a certain reason and it is because you will find that the expression on the right hand side is equal to this term here.

So, if you make these changes, if you were to make all these substitutions and multiply this factor you will end up with seeing the diff, the current across the junction the current, the current density to be equal to this term here.

And now this is nothing, but your expression for a diffusion current. So, if you imagine that to be some carrier concentration n_1 , the current is nothing, but $q D_n \frac{dn_1}{dx}$

which is nothing but your diffusion term. So, that is why we are talking about the diffusion of carriers across the in a Schottky contact and phi is my potential. So, there is a potential dependence here.

(Refer Slide Time: 05:24)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

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$$J = -q \frac{qD_n}{kT} n \frac{d\phi}{dx} + qD_n \frac{dn}{dx}$$

$$J e^{-q\phi/kT} = -q \frac{qD_n}{kT} n e^{-q\phi/kT} \frac{d\phi}{dx} + qD_n e^{-q\phi/kT} \frac{dn}{dx} = qD_n \frac{d(n e^{-q\phi/kT})}{dx} = J e^{-q\phi/kT}$$

Diffusion

So, with this being the current across the junction we just need to solve this differential equation. So, you have you have your $J e$ to the power minus $q \phi$ by $k T$ to be equal to this term and we need to solve this differential equation in order to obtain the current density.

(Refer Slide Time: 05:38)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$$J \int_0^{x_d} e^{-q\phi/kT} dx = \int_0^{x_d} qD_n d(n e^{-q\phi/kT})$$

$$J = \frac{\int_0^{x_d} qD_n d(n e^{-q\phi/kT})}{\int_0^{x_d} e^{-q\phi/kT} dx} = \frac{qD_n [n e^{-q\phi/kT}]_0^{x_d}}{\int_0^{x_d} e^{-q\phi/kT} dx}$$

$$= \frac{qD_n (n(x_d) e^{-q\phi(x_d)/kT} - n(0) e^{-q\phi(0)/kT})}{\int_0^{x_d} e^{-q\phi/kT} dx} = \frac{qD_n (n(x_d) e^{-q\phi(x_d)/kT} - n(0) e^{-q\phi(0)/kT})}{\int_0^{x_d} e^{-q \left(\frac{qN_D(x_d x - x^2)}{\epsilon_s} \right) / kT} dx}$$

$\uparrow \phi_s$

So, the current density is simply, if you just integrate on both sides you take the dx from the denominator put it on the left side and integrate, and then rewrite the expression you know keep J out, you will find that the current density is simply given by this factor here. We had seen that $J e^{-\frac{q\phi}{kT}}$ was equal to $q D n \frac{d}{dx} e^{-\frac{q\phi}{kT}}$.

So, we are taking the dx to the other side. So, we are saying that $J e^{-\frac{q\phi}{kT}} dx$ is equal to $q D n \frac{d}{dx} e^{-\frac{q\phi}{kT}}$ and we now integrate both sides, the J is common so, the J comes out and this term here the remaining term that integral is shifted over down to the denominator, that is how you end up with this expression.

So, the upper integral is easy. So, this is integration from 0 to x_d sorry that is not very clear, but it is 0 to x_d . So, the numerator is quite straight forward because, there are no terms here that depend on this particular factor and therefore, the numerator is simply $q D n \int_0^{x_d} e^{-\frac{q\phi}{kT}}$ with the limits going from 0 to x_d . The denominator is something we will look at, little later. So, that is that expression.

Now, if you look at the numerator, we are going to apply these limits. So, all we are going to do is, you are going to take the n at x_d into $e^{-\frac{q\phi}{kT}}$ at x_d by kT minus n at x_0 into $e^{-\frac{q\phi}{kT}}$ at x_0 by kT , you could apply these limits, and the denominator has got this term ϕ , the ϕ is the potential and we know the potential varies, you know as the quadratically with x right. So, that is the potential. So, that is what you obtain by solving Poisson's equation, assuming the full depletion approximation.

(Refer Slide Time: 08:25)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

Handwritten notes on the diagram include: $\kappa \ll \kappa_d$, $J \cdot e^{-q\phi/kT} = qD_n d(n e^{-q\phi/kT})$, and $\int J \cdot e^{-q\phi/kT} dx = qD_n d \int n e^{-q\phi/kT} dx$.

$$J \int_0^{x_d} e^{-q\phi/kT} dx = \int_0^{x_d} qD_n d(n e^{-q\phi/kT})$$

$$J = \frac{\int_0^{x_d} qD_n d(n e^{-q\phi/kT})}{\int_0^{x_d} e^{-q\phi/kT} dx} = \frac{qD_n [n e^{-q\phi/kT}]_0^{x_d}}{\int_0^{x_d} e^{-q\phi/kT} dx}$$

$$= \frac{qD_n (n(x_d) e^{-q\phi(x_d)/kT} - n(0) e^{-q\phi(0)/kT})}{\int_0^{x_d} e^{-q\phi/kT} dx} = qD_n \frac{(n(x_d) e^{-q\phi(x_d)/kT} - n(0) e^{-q\phi(0)/kT})}{\int_0^{x_d} e^{-q\phi/kT} dx}$$

So, we are going to substitute that phi. So, because phi is a function of x and that is that function. So, we are going to make that substitution and rename our denominator to this term yeah. So, now, what we need to do is identify what is the carrier concentration at x d, the potential at x d the carrier concentration x equal to 0 and the potential at x equal to 0.

(Refer Slide Time: 08:45)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

Handwritten notes on the diagram include: $\phi = 0, \kappa = 0$, $\phi_{bi} - V_a$, and $\kappa \ll \kappa_d$.

$$\phi(x_d) = \phi_{bi} - V_a, \phi(0) = 0$$

$$n(x_d) = N_c e^{-q\phi_{bi}/kT} e^{qV_a/kT}, n(0) = N_c e^{-q\phi_{bi}/kT}$$

$$J \sim \frac{qD_n N_c e^{-q\phi_{bi}/kT} (e^{qV_a/kT} e^{-q(\phi_{bi}-V_a)/kT} - 1)}{\int_0^{x_d} e^{-q\phi} dx} = J \sim \frac{qD_n N_c e^{-q\phi_{bi}/kT} (e^{qV_a/kT} - 1)}{\int_0^{x_d} e^{-q\phi} dx}$$

$$= \frac{q^2 N_D (x_d)^2}{\epsilon_s kT} \left(\frac{q^2 N_D (x_d)^2}{\epsilon_s kT} \right)^{1/2} e^{-q\phi_{bi}/kT} (e^{qV_a/kT} - 1)$$

And if you identify these 4 terms, you have your numerator. So, the potential at x d so, this is the band bending, we had earlier referenced the potential at x equal to 0 to be 0.

So, that was our reference and at x equal to x d we had our built in potential minus v a, because we are now applied a bias voltage v a. So, it is phi bi minus V a.

So, the potential at x d is phi bi minus V a, the potential at 0 is 0 and the charge concentration at x d is the same as we had said its going to be the same as the equilibrium concentrations that is one of the assumptions we have made. So, the concentration at x d is N c e to the power minus q phi B by k T into e to the power q phi B by k T. So, how does that come about. So, would have been good to have a band bending diagram here, but we can just draw it.

(Refer Slide Time: 09:59)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$\varphi(x_d) = \varphi_{bi} - V_a, \varphi(0) = 0$
 $n(x_d) = N_c e^{-q\varphi_{bi}/kT} e^{q\varphi_{bi}/kT}, n(0) = N_c e^{-q\varphi_{bi}/kT}$

$J \sim \frac{qD_n N_c e^{-q\varphi_{bi}/kT} (e^{q\varphi_{bi}/kT} e^{-q(\varphi_{bi}-V_a)/kT} - 1)}{\int_0^{x_d} e^{-q\varphi} dx} = J \sim \frac{qD_n N_c e^{-q\varphi_{bi}/kT} (e^{qV_a/kT} - 1)}{\left(\frac{q^2 N_D(x_d)}{\epsilon_s kT} \right)}$

$J \sim \frac{q^2 D_n N_c}{kT} \left(\frac{2qN_D(\varphi_{bi} - V_a)}{\epsilon_s} \right)^{1/2} e^{-q\varphi_{bi}/kT} (e^{qV_a/kT} - 1)$

So, this is the case right. So, you have your conduction band that is the Fermi level and that is q times the barrier height and that is let us say x d and that is your built in potential minus V a and your E c minus E f. So, if you look at this term here E c minus E f, it is the barrier height minus this particular value.

So, it is q phi B minus q phi bi, if it was the same as equilibrium. So, if this is the equilibrium case q phi b minus q phi bi is your E c minus E f in the bulk. So, that is basically your E c minus e f at x equal to x d. So, N c into e to the power minus E c minus E f by k T at equilibrium is nothing, but N c e to the power minus q phi B by k T

into e to the power plus $q \phi_{bi}$ by $k T$. It is a basically this term minus this term and there is a negative sign then therefore, you end up with this expression.

(Refer Slide Time: 12:25)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$$\phi(x_d) = \phi_{bi} - V_a, \phi(0) = 0$$

$$n(x_d) = N_c e^{-q\phi_b/kT} e^{q\phi_n/kT}, n(0) = N_c e^{-q\phi_b/kT}$$

$$J \sim \frac{qD_n N_c e^{-q\phi_b/kT} (e^{q\phi_n/kT} e^{-q(\phi_{bi}-V_a)/kT} - 1)}{\left(1 - \frac{q^2 N_D(x_d^2)}{\epsilon_s kT}\right)} = J \sim \frac{qD_n N_c e^{-q\phi_b/kT} (e^{qV_a/kT} - 1)}{\left(\frac{q^2 N_D}{\epsilon_s kT}(x_d)\right)}$$

$$J \sim \frac{q^2 D_n N_c}{kT} \left(\frac{2q N_D (\phi_{bi} - V_a)}{\epsilon_s} \right)^{1/2} e^{-q\phi_b/kT} (e^{qV_a/kT} - 1)$$

So, that is the carrier concentration at x in the at x at equilibrium and we are going to say that in the non equilibrium case that carrier concentration has not changed much and at n equal to 0 once again the carrier concentration has not changed much from equilibrium case and your E_c minus E_f remains, remains to be your $N_c e$ to the power minus $q \phi_{bi}$ by $k T$. So, that is your E_c , that is your E_f and that is the difference which is nothing but your barrier height by $k T$.

So, we now have, a n at x_d we have n at 0, we have ϕ at x_d and ϕ at 0 and we now substitute these 4 parameters in your expression in your in the previous slide, if you look at the expression here, we have to calculate these 4 parameters, we now have defined all 4 of them we substitute them and we will end up with the numerator having a term like this.

Now let us worry about the denominator. So, that is the numerator. So, that is done ok. So, now let us look at the denominator. So, this is definitely a typo here so, that is not complete. So, if you go to the previous slide, if you look at the denominator, you had a term which was e to the power minus q in q square N_D by ϵ_s into x_d^2 minus x_d^2 whole thing by $k T$, all right.

So, now we are going to say that, if x is really small because we are talking about regions that are here. So, we let us look at x being much less than x_d . In those regions, I can ignore the term x^2 because $x_d x$, $x_d x$ is much greater than x^2 .

So, I am going to simplify that expression to this term. So, you are going to write so the, this is a, there is a typo here. So, I am going to rewrite that expression, I am going to say it is 0 to x_d e to the power. So, let me just make sure I transfer it correctly minus $q^2 N_D$ by $\epsilon_s x$ by kT .

So, that should be the expression here, and then you perform the integral with respect to x . So, the integral of this term with the respect to x and after you apply the limits will give you this relation that you see here, all right and if you further, if you make 1 further assumption that is e to the power minus $q^2 N_D$ by $\epsilon_s x$ T^2 by kT is quite small as compared to 1 and you replace this term by 1, you will end up with the current density to be equal to this expression in a, we shown in the blue box here, wherein you have a, a big pre factor here, ok.

(Refer Slide Time: 15:25)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$$\phi(x_d) = \phi_{bi} - V_a, \phi(0) = 0$$

$$n(x_d) = N_c e^{-q\phi_b/kT} e^{q\phi_b/kT}, n(0) = N_c e^{-q\phi_b/kT}$$

$$J \sim \frac{qD_n N_c e^{-q\phi_b/kT} (e^{q\phi_b/kT} e^{-q(\phi_{bi}-V_a)/kT} - 1)}{1 - e^{-\left(\frac{q^2 N_D(x_d^2)}{\epsilon_s kT}\right)}} = J \sim \frac{qD_n N_c e^{-q\phi_b/kT} (e^{qV_a/kT} - 1)}{\left(\frac{q^2 N_D(x_d)}{\epsilon_s kT}\right)}$$

$$J \sim \frac{q^2 D_n N_c}{kT} \left(\frac{2qN_D(\phi_{bi} - V_a)}{\epsilon_s} \right)^{1/2} e^{-q\phi_b/kT} (e^{qV_a/kT} - 1)$$

$V_a > 0 \uparrow$

Now the key points of this pre factor term are since it is the diffusion current definitely there is a dependence on the diffusion coefficient. There is a significant dependence on the barrier height which basically means that if the barrier height is very large, the current is going to drop quite a lot most interestingly is dependence on V_a .

Now, when V_a increases what happens you will find that as V_a increases you have an exponential dependence on V_a so, V_a increases. So, if the bias, if the device goes to forward bias conditions let us say V_a is greater than 0 and it continues to increase, this term is going to shoot up quite significantly but then you also have a $\phi_{bi} - V_a$, but since it is a square root dependence on $\phi_{bi} - V_a$ and an exponential dependence on qV_a by kT , this term is going to dominate the current expression and the current will increase.

So, though it I have not shown it, it is worthwhile to imagine what the current voltage characteristics looks like. So, if you forgive this expression or you know if you sort of ignore this dependence on V_a and say that even a V_a changes a little this is not going to change significantly as compared to this particular term.

(Refer Slide Time: 17:09)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$$\phi(x_d) = \phi_{bi} - V_a, \phi(0) = 0$$

$$n(x_d) = N_c e^{-q\phi_b/kT} e^{q\phi_n/kT}, n(0) = N_c e^{-q\phi_b/kT}$$

$$J \sim qD_n N_c e^{-q\phi_b/kT} (e^{q\phi_n/kT} e^{-q(\phi_{bi}-V_a)/kT} - 1) = J \sim qD_n N_c e^{-q\phi_b/kT} (e^{qV_a/kT} - 1)$$

$$J \sim \frac{q^2 D_n N_c}{kT} \left(\frac{2qN_D(\phi_{bi} - V_a)}{\epsilon_s} \right)^{1/2} e^{-q\phi_b/kT} (e^{qV_a/kT} - 1)$$

$J \sim J_0 e^{qV_a/kT}$
 "Diode" → Schottky Diode
 Rectifying

If you go about with that understanding, you can say that J is as though you have a J naught of course, it is a light dependence on V_a but you know as we discussed you can ignore that J naught e to the power qV_a by kT minus 1.

So, what is this characteristics characters look like. So, we have to draw the current voltage characteristics here, you will find that this is the current, that is the voltage and at V_a equal to 0, the current is 0 which is, which makes sense because you do not want to have a current at V_a equal to 0 because that would mean it is a current at thermal equilibrium. So, that is V_a equal to 0.

So, that is the thermal equilibrium point and then as V_a increases, your current goes up quite a bit, it increases exponentially and when V_a is less than 0, the current decays, but not it is not that.

So, you have a curve that looks like this and therefore, it is a device that allows current to flow in 1 direction as compared to another. So, if I were to apply a plus 1 volt and minus 1 volt, I would have a lot more current flowing through the device at plus 1 volt as compared to the current flowing through the device at minus 1 volt. So, you could substitute the values here and you can calculate that. Therefore, it is a device that allows current in 1 direction and therefore, it behaves like a Diode.

So, this is a technical term we will, I think some of you might be familiar with p n Junction Diodes but these devices are something called as the Schottky Diode and they have a rectifying nature and that is why, it is called as a rectifying contact and what do you mean by rectifying, Now, that the term rectifying comes from the use of Diodes in a processing sinusoidal or ac waveforms, but essentially it means that it allows current in one direction it aids current in 1 direction and does not permit current in the other direction.

So, let us just leave it at that. So, it is a Diode it is called a rectifying nature therefore, if you think of if you somebody says a metal semiconductor contact with a rectifying property then they talking about the Schottky contact and it is a Schottky Diode. So, just like you can buy p n junction diodes in a market, you can also buy Schottky Diodes and Schottky Diodes have got some very interesting features. So, the first point is since we talked about electron transport. So, you have a n type semiconductor when we are talking about the transport of the electrons across the junction.

So, we are talking about the majority carriers. So, these Diodes are using their majority carriers for charge transport. So, this is a point remember and I mean discuss p n junction diodes will come back to this point.

(Refer Slide Time: 20:35)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

Sch. Diodes → Majority Carriers
 ϕ_{bi}

$$\phi(x_d) = \phi_{bi} - V_a, \phi(0) = 0$$

$$n(x_d) = N_c e^{-q\phi_{bi}/kT} e^{q\phi(x_d)/kT}, n(0) = N_c e^{-q\phi_{bi}/kT}$$

$$J \sim qD_n N_c e^{-q\phi_{bi}/kT} (e^{q\phi_{bi}/kT} e^{-q(\phi_{bi}-V_a)/kT} - 1) = J \sim qD_n N_c e^{-q\phi_{bi}/kT} (e^{qV_a/kT} - 1)$$

$\int_0^{x_d} e^{-q\phi_{bi}/kT} \frac{dx}{\epsilon_s} \cdot \int_0^{x_d} e^{-q\phi_{bi}/kT} dx$

$$\left(\frac{1 - e^{-\left(\frac{q^2 N_D(x_d^2)}{\epsilon_s kT}\right)}}{\left(\frac{q^2 N_D(x_d)}{\epsilon_s kT}\right)} \right)$$

$$J \sim \frac{q^2 D_n N_c}{kT} \left(\frac{2qN_D(\phi_{bi} - V_a)}{\epsilon_s} \right)^{1/2} e^{-q\phi_{bi}/kT} (e^{qV_a/kT} - 1)$$

*$J \sim J_0 e^{qV_a/kT}$
"Diode" → Schottky Diode Rectifying*

So, Schottky Diodes have a current voltage that is dependent on the majority carriers carrying the current, say it is a Schottky Diode based with a based on a metal p semiconductor then it is going to be the holes that are going to be responsible for the current. So, majority carriers are responsible for the current. So, that is a key point to remember, there are other features that we that will become more clear when we discuss p n junction Diodes but this is a key point to remember.

So, the built in potential so, we will see that when it comes to p n junction diodes the built in potential in a Schottky Diode can be made much lower as compared to the built in potential in a p n Diode but we will save that point full lead. So, this is the, this is the derivation of the diffusion current through the Schottky contact. So, just on the few features. So, just to think of it very intuitively.

So, why is it that you have this exponential dependence on the voltage. If you think about it in a very intuitive manner, if you just forget the mathematics and you say that you know I just want to birds eye view or but it is a very simple understanding of what the diffusion current is.

But the electrons moving from the metal to the semiconductor, so no influence on their barrier height, it is still ϕ_B and why is it still ϕ_B , because it is a constant right this is nothing but your $q\phi_m - q\phi_s$, right it is a constant. So, you can even though we have applied a voltage V at the metal to the semiconductor current it has not changed because the electrons moving from the metal to the semiconductor remain the same.

But on the other hand the electrons coming in from the semiconductor to the metal suddenly see a mass, a much lower barrier and therefore, their flux increases dramatically. So, this is the reason that applying a forward bias suddenly allows a lot of electrons to migrate from the semiconductor to the metal while not having any influence on the flux of electrons from the metal to the semiconductor. So, if a lot of electrons flow from the semiconductor to the metal, it implies that we have established a large current from the metal to the semiconductor,

So, we want to think of this p-n junction, now sorry if you want to think of this Schottky junction. So, you have a metal semiconductor contact, that is your positive, that is your negative, it is like as though you may have a Diode. So, that is a symbol for a Diode and it is got this rectifying or you know a Diode like nature. It allows current in this direction while not permitting too much of current in the opposite direction.

So, that is, that is basically a good overview of the current due to diffusion in the junction Diode in the metal semiconductor Schottky junction Diode and these are called as Schottky Diodes.

(Refer Slide Time: 25:49)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

Thermionic Emission

Thermionic Emission

All carriers greater than the energy barrier will transfer across the barrier

$$J_{S \rightarrow M} = \int_{E=E_C}^{E_F} qv_x dn = \int_{E=E_C, \text{bulk} + q\phi_b}^{E_F} qv_x \frac{dn}{dE} dE = J_{M \rightarrow S} @ \text{equilibrium}$$

$$\frac{dn}{dE} = g(E)f(E) = \frac{4\pi(2m_e^*)^{3/2}(E-E_C)^{1/2}}{h^3} \underbrace{e^{-(E-E_C)/kT}}_{e^{-(E-E_C)/kT}} \underbrace{e^{-(E_C-E_F)/kT}}_{e^{-(E_C-E_F)/kT}}$$

$E - E_C = m_e^* v^2 / 2$

v is defined in spherical - contains x,y,z components of velocity.

$$dE = m_e^* v dv$$

$$\frac{dn}{dE} dE = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_C-E_F)/kT} e^{-m_e^* v^2 / 2kT} (4\pi v^2) dv$$

So, we will now look at another mechanism and this mechanism is something called as Thermionic Emission. This too, this mechanism too gives you; you know shows you gives you insights into the rectifying nature of this edge it shows you that the Schottky contact behaves like a Diode.

So, we will see that towards the end of this discussion this mechanism also supports the diffusion current expression, but it is just that the terms are a little different and therefore these 2 mechanisms are sometimes treated together and under something called the thermionic diffusion theory, you can think of both these mechanisms the same, you can model them in a more in a better manner but we will just talk about the thermionic emission separately.

(Refer Slide Time: 26:52)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

Thermionic Emission

All carriers greater than the energy barrier will transfer across the barrier $\rightarrow J_{ms}$

$$J_{S \rightarrow M} = \int_{E=E_C}^{E_F} qv_x dn = \int_{E=E_C, \text{bulk} + \phi_{B0}}^{E_F} qv_x \frac{dn}{dE} dE = J_{M \rightarrow S} @ \text{equilibrium}$$

$$\frac{dn}{dE} = g(E)f(E) = \frac{4\pi(2m_e^*)^{3/2}(E-E_C)^{1/2}}{h^3} \underbrace{e^{-(E-E_C)/kT}}_{e^{-(E-E_F)/kT}} e^{-(E-E_F)/kT}$$

$$E - E_C = m_e^* v^2 / 2$$

v is defined in spherical - contains x,y,z components of velocity.

$$dE = m_e^* v dv$$

$$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_C-E_F)/kT} e^{-m_e^* v^2 / 2kT} (4\pi v^2) dv$$

So, what was the Thermionic Emission you know, as we discussed Thermionic Emission takes into account all the carriers that I have got a high enough velocity or you know high enough energy. So, you have an electron there and that is got a high enough energy that it really does not care about the nature of the barrier. So, it is simply this electron moving from the semiconductor to the metal that is going to create a current from the metal to the semiconductor.

So, let us see in a let us take a count on the number of electrons and see what their velocity is. So, that should be the approach to calculating the thermionic emission or the thermionic current. So, it is called thermionic because its temperature dependent because the velocity of the carrier is dependent you know it is it comes from the temperature.

(Refer Slide Time: 27:27).

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

Thermionic Emission

All carriers greater than the energy barrier will transfer across the barrier

nq (velocity)
dn/dE

$$J_{S \rightarrow M} = \int_{E=E_c}^{E_{\infty}} q v_x dn = \int_{E=E_c, \text{bulk} + \phi_{B0}}^{E_{\infty}} q v_x \frac{dn}{dE} dE = J_{M \rightarrow S} @ \text{equilibrium}$$

$$\frac{dn}{dE} = g(E) f(E) = \frac{4\pi(2m_e^*)^{3/2} (E - E_c)^{1/2}}{h^3} e^{-\frac{(E - E_c)kT}{kT}} e^{-\frac{(E_c - E_f)kT}{kT}}$$

1 → e^{-(E-E_f)/kT}

$E - E_c = m_e^* v^2 / 2$
v is defined in spherical - contains x, y, z components of velocity.

$dE = m_e^* v dv$

$$\frac{dn}{dE} dE = \frac{(2m_e^*)^{3/2}}{h^3} e^{-\frac{(E_c - E_f)kT}{kT}} e^{-\frac{m_e^* v^2}{2kT}} (4\pi v^2) dv$$

So, let us now define 3 directions. So, we need to first define an axis before we talking about there is all this mathematics. So, let us define this to be the x axis, you know as we have always been using, we will define the y axis to be say out of the plane ok. So, which I will just draw in this manner and the z axis is you know going upward there the y and z are interchangeable because you will see that it is really irrelevant as to what you pick as your y and what you pick as your z axis.

So, the current we are interested in is basically the flux of the carriers right it is essentially nq into the velocity. So, that is the current, the only thing is this velocity is not because of an applied electric field if it was due to the applied electric field we would call this a drift current and then it would have been new into the electric field, but that is not the case. So, this velocity is simply because of the energy possessed by the carriers, you know due to which they are sitting well above the conduction bandage ok.

So, we will just and we are interested only in the velocity in the x direction, we really do not care if a carrier sitting on the semiconductor side is moving in the y direction or the z direction because that is not contributing to the current because the semiconductor has to cross the junction, I am sorry the electron has to cross the junction in order to, contribute to the current and therefore, we are only interested in the velocity in the x direction.

So, n into q into the velocity in the x direction gives me the current but I need to sum up all the electrons or the electron count all the free electron free electrons sitting from E_c

to infinite. So, therefore, I need to perform an integral form here today and at equilibrium this current is equal to the current from the metal to the semiconductor side.

Now what we will do is we will take this dn and rewrite it as dn by dE into dE because we know what dn by dE is dn by dE is a number of electrons per unit volume per unit energy right and that is nothing but the density of states into the Fermi function. So, we will just write dn by dE is equal to the density of states into the Fermi function and that is the density of states there and this is the Fermi function here which is approximated by the Boltzmann distribution here, ok.

So, I, we have rewritten 1 by $1 + e^{(E - E_f)/kT}$ to the power $E - E_f$ by kT as simply $e^{-(E - E_f)/kT}$ because we have assumed that this term is greater than 1 that is the Boltzmann approximation for f of e . And furthermore, we have split you know for the sake of clarity in the analysis we have split $E - E_f$.

(Refer Slide Time: 30:53)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

Thermionic Emission
All carriers greater than the energy barrier will transfer across the barrier

Handwritten: $n, q(\text{velocity})$

Handwritten: $\frac{dn}{dE} \cdot dE$

$$J_{S \rightarrow M} = \int_{E=E_C}^{E_{F_M}} qv_x dn = \int_{E=E_C}^{E_{F_M}} qv_x \frac{dn}{dE} dE = J_{M \rightarrow S} @ \text{equilibrium}$$

$$\frac{dn}{dE} = g(E)f(E) = \frac{4\pi(2m_e^*)^{3/2}(E - E_C)^{1/2}}{h^3} e^{-(E - E_C)/kT} e^{-(E_C - E_f)/kT}$$

Handwritten: $E - E_C = m_e^* v^2 / 2$

Handwritten: v is defined in spherical - contains x,y,z components of velocity.

Handwritten: $dE = m_e^* v dv$

$$\frac{dn}{dE} dE = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} e^{-m_e^* v^2 / 2kT} (4\pi v^2) dv$$

You have rewritten that term as $E - E_C + E_C - E_f$ and I have retained this term here and that term there.

(Refer Slide Time: 31:13)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

Thermionic Emission

All carriers greater than the energy barrier will transfer across the barrier

Handwritten: n_x (velocity)

$$J_{S \rightarrow M} = \int_{E=E_c}^{E_f} qv_x dn = \int_{E=E_c, \text{bulk} + q\phi_b}^{E_f} qv_x \frac{dn}{dE} dE = J_{M \rightarrow S} @ \text{equilibrium}$$

Handwritten: $\frac{dn}{dE}$

$$\frac{dn}{dE} = g(E)f(E) = \frac{4\pi(2m_e^*)^{3/2}(E-E_c)^{1/2}}{h^3} \frac{e^{-(E-E_c)/kT}}{e^{-(E-E_c)/kT} + e^{-(E_c-E_f)/kT}}$$

Handwritten: $E - E_c = m_e^* v^2 / 2$

Handwritten: $E - E_f = E - E_c + E_c - E_f$

v is defined in spherical - contains x,y,z components of velocity.

$$dE = m_e^* v dv$$

$$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_c-E_f)/kT} e^{-m_e^* v^2 / 2kT} (4\pi v^2) dv$$

So, why are we making the splits? So, this is the conduction band edge, we are interested in looking at the velocity in the x direction of all these carriers and these carriers have got some energy E has got, have got some energy E and this energy here is E_c .

So, the carrier sitting at E_c has got 0 velocity and $E - E_c$, this gap is specially important for me because that is the energy, that is given to the carrier in the form of kinetic energy. So, that is why we are trying to rewrite $E - E_f$ as $E - E_c + E_c - E_f$, we are trying to get a gauge on what that energy is plus $E_c - E_f$ or $E_c - E_f$ is our 2 well known quantities and therefore, that is a constant, ok.

Now, $E - E_c$ is essentially the kinetic energy of the carriers. So, here we are making an assumption that all the energy of the carriers above the conduction band is the kinetic energy and we are doing so in order to get a measure of this velocity ok. So, $E - E_c$ is mv^2 by 2. So, m_e is the effective mass of the electrons and here the v is not $v_x v_y v_z$ it is the V vector ok.

(Refer Slide Time: 32:40)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

Thermionic Emission

All carriers greater than the energy barrier will transfer across the barrier

$J_{S \rightarrow M} = \int_{E=E_c}^{E_{\infty}} q v_x dn = \int_{E=E_c, \text{bulk} + q\phi_b}^{E_{\infty}} q v_x \frac{dn}{dE} dE = J_{M \rightarrow S} @ \text{equilibrium}$

$\frac{dn}{dE} = g(E) f(E) = \frac{4\pi(2m_e^*)^{3/2} (E - E_c)^{1/2}}{h^3} \frac{e^{-(E-E_c)/kT} e^{-(E_c-E_f)/kT}}{e^{-(E-E_f)/kT}}$

$E - E_c = m_e^* v^2 / 2$ $dE = m_e^* v dv$

v is defined in spherical - contains x, y, z components of velocity.

$dE = m_e^* v dv$

$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_c-E_f)/kT} e^{-m_e^* v^2 / 2kT} (4\pi v^2) dv$

So, it is, if that is your x y and z you can think of v as in all 3, it is got all 3 components in it. So, essentially your V square is, V is V x square plus v y square plus v z square because that is the way the energy is going to distribute itself. So, E minus E c is m v square by 2 and if you take the derivative, if you take what is de by dv de by d v is nothing but m e v into 2 by 2. So, it is m e into v ok. So, a m e star into v and therefore, my d is nothing but m e v into d v and what we are going to do is now we are going to substitute these 2 terms and we are going to try to estimate what that quantity is. So, I am trying to estimate calculate that integral and that in and I know my d n by d e is nothing but g e into f of e so.

Now, I have d n by d e into de which is basically my d n has nothing but this term here into e to the power E c minus E f into all these terms put together. So, you can see that the whole expression is now written out in terms of the velocity of the carriers which was our goal we are trying to get an estimate on the velocity.

So, now all this is in you do not see the v x v y v z components because we need to convert this to the Cartesian system, you need to split out the V in into these 3 components.

(Refer Slide Time: 34:25)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

In cartesian,

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$(4\pi v^2)dv = dv_x dv_y dv_z$

$$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} e^{-m_e^* v_x^2 / 2kT} e^{-m_e^* v_y^2 / 2kT} e^{-m_e^* v_z^2 / 2kT} dv_x dv_y dv_z$$

$$J_{S \rightarrow M} = \int_{E = E_{C, bulk} + q\phi_B}^{E_{F0}} qv_x \frac{dn}{dE} dE$$

$$= \frac{q(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} \int_{-\infty}^{\infty} (e^{-m_e^* v_y^2 / 2kT} dv_y) \int_{-\infty}^{\infty} (e^{-m_e^* v_z^2 / 2kT} dv_z) \int_{v_{min}}^{\infty} (v_x e^{-m_e^* v_x^2 / 2kT} dv_x)$$

$$m_e^* v_{min} / 2 = q(\phi_{bi} - V_a) = q\phi_B - (E_C - E_f)_{bulk}$$

Note: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

Now, v square is v x square plus v y square plus v z square and this volume element is simply this volume element here. So, we now rewrite this expression d n by d e into d e in terms of v x v y and v z because now we need to, we have a measure of the velocity, but in particular we want to get a measure of the velocity in the x direction because that is what is going to cross, that is what those are the carriers that are going to cross the junction and that those are the carriers that are going to contribute to the current.

So, that turns out to be, I will not read out this expression but it turns out to be this big term here, ok. So, here you can see or the summation of the 3 components. So, that would have been v square by 2 k T. So, that is now split into these 3 exponential components.

So, now, we can write out the current expression right. So, the current was nothing, but this term here. So, that is why we started and we now have estimated this component in terms of the velocities. So, that is what all this you know all these equations were about.

(Refer Slide Time: 35:47)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

In cartesian,

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$(4\pi v^2) dv = dv_x dv_y dv_z$$

$$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} e^{-m_e^* v_x^2 / 2kT} e^{-m_e^* v_y^2 / 2kT} e^{-m_e^* v_z^2 / 2kT} dv_x dv_y dv_z$$

$$J_{S \rightarrow M} = \int_{E = E_{C, \text{bulk}} + q\phi_B}^{E = \infty} q v_x \frac{dn}{dE} dE \leftarrow \int f(v_x, v_y, v_z)$$

$$= \frac{q(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} \int_{-\infty}^{\infty} (e^{-m_e^* v_x^2 / 2kT} dv_x) \int_{-\infty}^{\infty} (e^{-m_e^* v_y^2 / 2kT} dv_y) \int_{v_{\text{min}}}^{\infty} (v_x e^{-m_e^* v_z^2 / 2kT} dv_z)$$

$$m_e^* v_{\text{min}} / 2 = q(\phi_{bi} - V_a) = q\phi_B - (E_C - E_f)_{\text{bulk}}$$

Note: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$



We wanted to get this entire expression as a function of v_x and we have done that we have got a function of v_x , v_y and v_z .

So, now we substitute this expression for this quantity here and we rewrite the integral in this form in the y direction we really do not care about what the velocity is. So, we consider all velocities from minus infinite ok. So, that is the velocities in either direction z direction. We again do not care about what the velocities are, but in the x direction we are only interested in those carriers.

(Refer Slide Time: 36:24)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

In cartesian,

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$(4\pi v^2) dv = dv_x dv_y dv_z$$

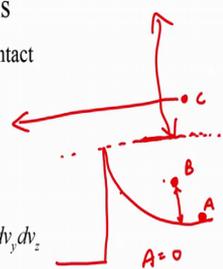
$$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} e^{-m_e^* v_x^2 / 2kT} e^{-m_e^* v_y^2 / 2kT} e^{-m_e^* v_z^2 / 2kT} dv_x dv_y dv_z$$

$$J_{S \rightarrow M} = \int_{E = E_{C, \text{bulk}} + q\phi_B}^{E = \infty} q v_x \frac{dn}{dE} dE$$

$$= \frac{q(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} \int_{-\infty}^{\infty} (e^{-m_e^* v_x^2 / 2kT} dv_x) \int_{-\infty}^{\infty} (e^{-m_e^* v_y^2 / 2kT} dv_y) \int_{v_{\text{min}}}^{\infty} (v_x e^{-m_e^* v_z^2 / 2kT} dv_z)$$

$$m_e^* v_{\text{min}} / 2 = q(\phi_{bi} - V_a) = q\phi_B - (E_C - E_f)_{\text{bulk}}$$

Note: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$



So, let us say I have a carrier sitting here at the conduction band edge, let us take another example of a carrier sitting here and let us take a carrier sitting here. So, let us say we have these 3 carriers which we call as a electron A, electron B and electron C.

Now, the velocity of the carrier here, this a is 0, 0 velocity. So, that is not going to contribute to any thermionic currents, the velocity of carrier b is not 0 but it is still not high enough, it is not going to, it is not something that is going to walk pass this barrier without you know really caring about the barrier. The electron B really depends upon the nature of the barrier and you know how all these changes and you know the width of the barrier here etcetera in order to contribute to any current.

So, we are not interested in carrier b as well, but carrier c has got enough velocity ok. So, essentially it is above this dotted line and it is got enough velocity to just walk fast, this barrier without really worrying about the nature of the barrier and that is the essence of the thermionic current. So, it is carrier c that we are interested in.

So, it is only those carriers with a velocity corresponding to this energy level, this dotted line and above that are going to contribute to any thermionic emission. So, we are only interested in those velocities.

So, if you think about this entire integral from minus infinity to infinity with everything in there the everything that is below v_{min} does not contribute any current. So, that entire current is essentially 0 and it is only that which is which is got a minimum velocity of v_{min} and above that is going to contribute to any current.

(Refer Slide Time: 38:37)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

In cartesian,

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$(4\pi v^2) dv = dv_x dv_y dv_z$$

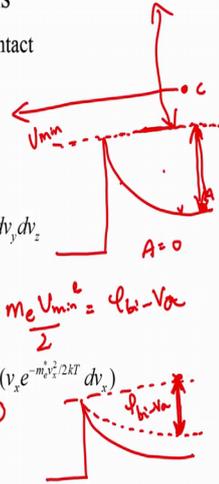
$$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} e^{-m_e^* v_x^2 / 2kT} e^{-m_e^* v_y^2 / 2kT} e^{-m_e^* v_z^2 / 2kT} dv_x dv_y dv_z$$

$$J_{S \rightarrow M} = \int_{E = E_C, \text{bulk} + q\phi_B}^{E = \infty} q v_x \frac{dn}{dE} dE$$

$$= \frac{q(2m_e^*)^{3/2}}{h^3} e^{-(E_C - E_f)/kT} \int_{-\infty}^{\infty} (e^{-m_e^* v_x^2 / 2kT} dv_x) \int_{-\infty}^{\infty} (e^{-m_e^* v_y^2 / 2kT} dv_y) \int_{v_{\min}}^{\infty} (v_x e^{-m_e^* v_z^2 / 2kT} dv_z)$$

$$m_e^* v_{\min} / 2 = q(\phi_{bi} - V_a) = q\phi_B - (E_C - E_f)_{\text{bulk}}$$

Note: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$



So, it is these carriers at this dotted line are said to have a velocity of v minimum. That is the minimum velocity of the carrier for it to contribute to a thermionic current and we only consider the integral from that minimum velocity to above and what is that minimum velocity so, what, what velocity are we talking about, if A the carrier at A has got 0 velocity, it is this energy which corresponds to v minimum.

So, v minimum, so if you think about the kinetic energy of the carrier here $m_e v_{\min}^2 / 2$ is essentially your $\phi_{bi} - V_a$ it is this. Now, here for the first time you see the impact of V_a all right. So, far we never saw V_a appear anywhere but V_a determines what the minimum velocity is.

So, if my V_a is. So, let us say, that is my equilibrium condition and if my V_a is greater than 0, the requirement for my v_{\min} has changed because the carrier here now has got 0 velocity and I only need a very small amount of energy because I only need to cross $\phi_{bi} - V_a$ in order to achieve the V_{\min} required to get past the barrier to contribute to thermionic currents.

So, v_{\min} is dependent on $\phi_{bi} - V_a$ and we rewrite $\phi_{bi} - V_a$ in terms of all these quantities then ok. We can we can rewrite $\phi_{bi} - V_a$ in terms of those quantities.

(Refer Slide Time: 40:34)

Metal-Semiconductor Junctions

Current Voltage Characteristics: Schottky Contact

In cartesian,

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$(4\pi v^2) dv = dv_x dv_y dv_z$$

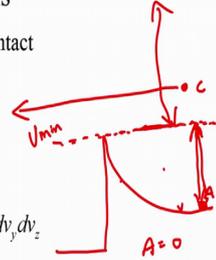
$$\frac{dn}{dE} = \frac{(2m_e^*)^{3/2}}{h^3} e^{-(E_c - E_f)/kT} e^{-m_e^* v_x^2 / 2kT} e^{-m_e^* v_y^2 / 2kT} e^{-m_e^* v_z^2 / 2kT} dv_x dv_y dv_z$$

$$J_{S \rightarrow M} = \int_{E=E_c, \text{bulk} + q\phi_B}^{E=\infty} q v_x \frac{dn}{dE} dE$$

$$= \frac{q(2m_e^*)^{3/2}}{h^3} e^{-(E_c - E_f)/kT} \int_{-\infty}^{\infty} (e^{-m_e^* v_x^2 / 2kT} dv_y) \int_{-\infty}^{\infty} (e^{-m_e^* v_z^2 / 2kT} dv_z) \int_{V_{\text{min}}}^{\infty} (v_x e^{-m_e^* v_x^2 / 2kT} dv_x)$$

$$m_e^* v_{\text{min}} / 2 = q(\phi_{\text{bi}} - V_a) = q\phi_B - (E_c - E_f)_{\text{bulk}}$$

Note: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$



$$m_e v_{\text{min}} / 2 = \phi_{\text{bi}} - V_a$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

So, now how do we perform this integral. So, look at these 2 terms ok, look at these 2 terms here. They are an integral of say some e to the power minus x square dx and I will not say, I will not use the symbol x . So, let us say something else. So, let us say α square $d\alpha$ ok. So, that is, that is the integral we are talking about.

Now, what is integral e to the power minus α square $d\alpha$? It is nothing but the Gaussian integral which is square root of π . So, I have put a note here and in fact, this note is a bit confusing because I am using x for something else. So, I would not want this need to be confusing. So, this x is not related to anything else it is just to indicate think of it as a purely mathematical note here, it is just indicate that integral of minus infinity to infinity of e to the power minus x square dx is square root of π this is something called as a Gaussian integral and we are going to use that relation in that you know and therefore, we can easily identify these derivatives these integrals.

So, these 2 are easily calculated they are constants they are error they are independent of v_y , they are just constants, but this integral is special because this integral is not the Gaussian integral, it is got a v_x into e to the power minus $m_e v_x^2$ by $2kT$ dv_x .

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Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

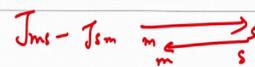


$$J_{m \rightarrow s} = e \cdot \left[2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q(\phi_B - V_a)/kT} \right]$$

$$J_{s \rightarrow m} = e \cdot \left[2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q\phi_B/kT} \right]$$

Thermionic

$$J_{eff} = J_{ms} - J_{sm} = \frac{4\pi m^{*2} k^2 T^2}{h^3} e^{-q\phi_B/kT} (e^{qV_a/kT} - 1)$$



So, this integral is going to give you a different answer. So, you solve for these 3 integrals and you will end up with an expression for the current from the semiconductor to the metal is going to be this term here and we will find that the current from the semiconductor to the metal is dependent on V_a because the barrier height now requirement is dependent on V_a and therefore, this is the current from d. So, I should say that, this is the electron flux. So, the electron flux from the semiconductor to the metal depends upon V_a .

And the electron flux from the metal to the semiconductor does not depend on V_a because of this barrier height and therefore, the effective current. So, electron flux of the semiconductor to the metal is the same as the metal to semiconductor current and this is nothing but the semiconductor to metal current. So, the effective current across the barrier is $J_{ms} - J_{sm}$.

So, it is this current going from the metal to the semiconductor minus the current coming from the semiconductor to their metal and that is given by this expression here. So, this is the final expression for the thermionic emission current.

(Refer Slide Time: 43:43)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact



$$J_{m \rightarrow s} = e \sigma_{s \rightarrow m} = 2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q(\phi_B - V_a)/kT}$$

$$J_{s \rightarrow m} = e \sigma_{m \rightarrow s} = 2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q\phi_B/kT}$$

Thermions

$$J_{\text{eff}} = J_{ms} - J_{sm} = \left(\frac{4\pi m^{*2} k^2 T^2}{h^3} \right) e^{-q\phi_B/kT} \left(e^{qV_a/kT} - 1 \right)$$

$J_1 (e^{qV_a/kT} - 1)$



So, once again if you think of all these terms as a constant ok. So, this entire thing if you say its J_1 ok, the current is $J_1 e$ to the power $q v a$ by $k T$ minus 1 which is again very similar to your Diode like behaviour which we saw on the diff in the case of a diffusion current, the current is going to have a current voltage characteristic level, that has not changed.

But what is different is to look at is the pre factor term. So, here once again another aspect that is common with the diffusion current expression is that the exponential dependence on the barrier height seen from the metal to semiconductor.

(Refer Slide Time: 44:17)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact



$$J_{m \rightarrow s} = e \sigma_{s \rightarrow m} = 2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q(\phi_B - V_a)/kT}$$

$$J_{s \rightarrow m} = e \sigma_{m \rightarrow s} = 2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q\phi_B/kT}$$

Thermions

$$J_{\text{eff}} = J_{ms} - J_{sm} = \left(\frac{4\pi m^{*2} k^2 T^2}{h^3} \right) e^{-q\phi_B/kT} \left(e^{qV_a/kT} - 1 \right)$$

$(e^{qV_a/kT} - 1)$



So, the larger the barrier, larger the cliff the smaller the current, so that is also the same, but if you look at the remaining terms you find that there is a dip strong dependence on the temperature. So, as the temperature increases this current goes up dramatically. So, therefore, there is a very strong dependence in the temperature and there is a strong dependence in the barrier height and an exponential dependence on the applied voltage and this pre factor here is something called as a Richardson's coefficient.

(Refer Slide Time: 44:47)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact



$$J_{m \rightarrow s} = e \cdot \frac{2}{h} \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q(\phi_b - V)/kT}$$

$$J_{s \rightarrow m} = e \cdot \frac{2}{h} \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q\phi_b/kT}$$

Thermionic

$$J_{\text{eff}} = J_{ms} - J_{sm} = \left(\frac{4\pi m^2 k^2 T^2}{h^3} e^{-q\phi_b/kT} \right) (e^{qV/kT} - 1)$$

Richardson's coeff $(e^{qV/kT} - 1)$

So, that is about Thermionic Emission. So, once again it, it, it agrees with the fact that the current has got an exponential dependence on the voltage.

(Refer Slide Time: 45:03)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

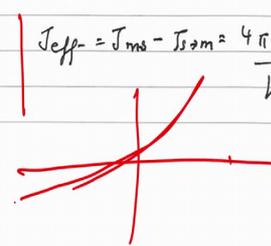


$$J_{m \rightarrow s} = e \cdot n_{s \rightarrow m} = 2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q(\phi_B - V)/kT}$$

$$J_{s \rightarrow m} = e \cdot n_{m \rightarrow s} = 2 \left(\frac{m^*}{h} \right)^3 \left(\frac{2\pi kT}{m^*} \right) \left(\frac{kT}{m^*} \right) e^{-q\phi_B/kT}$$

Thermionic

$$J_{eff} = J_{ms} - J_{sm} = \frac{4\pi m^{*2} k^2 T^2}{h^3} e^{-q\phi_B/kT} (e^{qV/kT} - 1)$$



$(e^{qV/kT} - 1)$

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Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

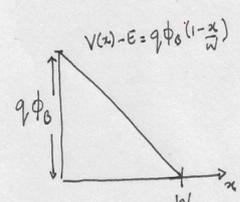


$$\psi(w) = \psi(0) e^{-\int_0^w \frac{\sqrt{2m^*(V(x)-E)}}{\hbar} dx}$$

Since $\psi(x+dx) = \psi(x) e^{-\frac{\sqrt{2m^*(V(x)-E)} dx}{\hbar}}$

$\alpha = \frac{\text{probability of finding } e^- \text{ at } w}{\text{probability of finding } e^- \text{ at } 0}$

$$= \left| \frac{\psi(w)}{\psi(0)} \right|^2$$



And that the metal semiconductor contact behaves like a rectifying contact. So, now let us come to the last bit which is the current voltage characteristics with the Schottky contact but why are the process of tunnelling. So, what do we mean ok so the as I described very briefly. So, far we have looked at tunnelling through a rectangular barrier. So, all right so, it is always been tunnelling through this barrier the carrier even though does not have enough energy.

So, let us say this is the potential energy that is the total energy is e and despite e being less than v the carrier is able to get past this barrier and have a non 0 probability on this side.

So, that is tunnelling and this essentially is a transport mechanism, the carrier has gone from this side to the other side with some probability. So, what we need to find out is where is the tunnelling. So, if you have your band diagram, let us say, that is the conduction band all right. So, the electrons from the semiconductor to the metal will tunnel through this barrier here and that is the tunnelling that we want to, what do you say capture that is, that is the current that we want to capture.

And what we want to do is we want to identify what is the probability of finding an electron at w ok. So, what is w ? So, the first thing we are going to do is this is a very difficult potential profile right.

(Refer Slide Time: 46:54)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$$\psi(x) = \psi(0) e^{-\int_0^x \frac{\sqrt{2m^*(V(x)-E)}}{\hbar} dx}$$
 Since $\psi(x+dx) = \psi(x) e^{-\frac{\sqrt{2m^*(V(x)-E)} dx}{\hbar}}$

$$\alpha = \frac{\text{probability of finding } e^- \text{ at } w}{\text{probability of finding } e^- \text{ at } 0}$$

$$= \left| \frac{\psi(w)}{\psi(0)} \right|^2$$

$$V(x) - E = q\phi_0 \left(1 - \frac{x}{w}\right)$$

$\propto x^2$

So, this is a quadratic dependence on distance. So, it depends on x square. So, we are going to approximate, this to our triangular barrier ok. So, this is a, this is the approximation you are going to make, we are going to say it is not a rectangular barrier but at the same time we do not want to use a quadratic barrier. So, you will give it some distance w and we will say that the tunnelling probability is like is equivalent to the probability of getting the carriers to tunnel across a barrier which is got this triangular shape.

(Refer Slide Time: 47:28)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$$\psi(x) = \psi(0) e^{-\int_0^x \frac{\sqrt{2m^*(V(x)-E)}}{\hbar} dx}$$

Since $\psi(x+dx) = \psi(x) e^{-\frac{\sqrt{2m^*(V(x)-E)} dx}{\hbar}}$

$$\alpha = \frac{\text{probability of finding } e^- \text{ at } x}{\text{probability of finding } e^- \text{ at } 0}$$

$$= \frac{|\psi(x)|^2}{|\psi(0)|^2}$$

Diagram labels: $V(x) - E = q\phi_B \left(1 - \frac{x}{w}\right)$, $q\phi_B$, dx , x , w , x .

So, this is my potential profile right. So, this is my potential well if you want to call it. So, that is potential energy and that is the barrier $q\phi_B$ and let us say at some distance w the potential profile hits 0. So, that is my potential profile, it goes like that it comes down like that and it is there. So, that is my potential profile. So, which means that V of x minus E . So, all this is offset you know. So, V of x minus E is this potential. So, it is $q\phi_B$ into 1 minus x by w . So, that is, that is defining this line here.

And therefore, if you remember the tunnelling through a rectangular barrier, you found that the probability had an exponential dependence on V x minus E on V minus E right. So, we are going to borrow that idea and say that you know if I know the Wave function at ψ of x , the wave function at ψ at x plus dx is simply going to be the Wave function at ψ of x into e to the power minus square root of $2m^*$ star to v of x minus e by \hbar bar b x ok.

So, that is essentially saying that this is, this can be imagined to be a rectangular barrier of width dx and I know the wave function, I know the magnitude of the wave function there and therefore, I can find the wave function here because it is got this dependence ok.

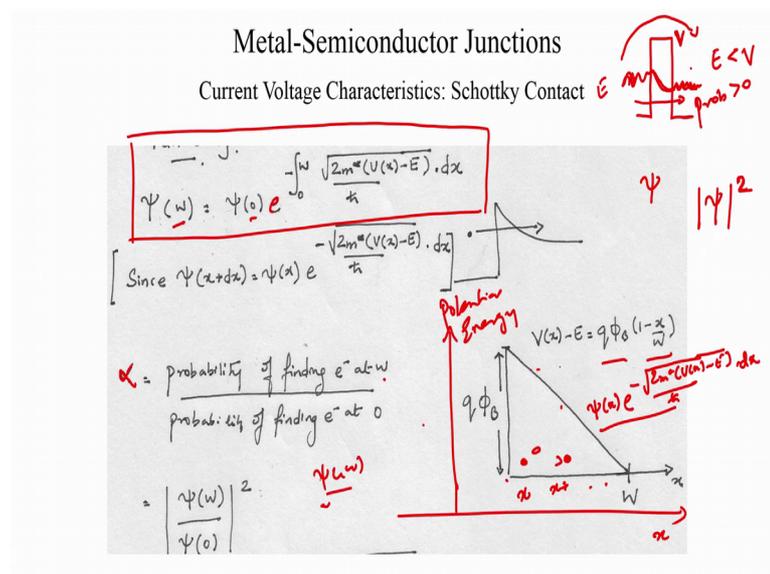
So, you are going to take an, we are going to take an infinitesimal width dx and say that that is a rectangular barrier and then proceed to that. Now, if you were to, now continue this you know you take ψ of x plus $2dx$ is equal to ψ of x plus dx into e to the power

so on so forth and go all the way till w you essentially end up with this relation the relation between the wave function at x equal to 0 and the wave function at x equal to w is simply e to the power the integral from 0 to w of this particular term that,

So, one can, if you are familiar with something called as a Beer Lamberts law when it comes to in optics when you throw a light let us say a light of certain intensity is thrown onto a material, let us say some liquid which is stored in a container in a transparent container you throw light on this material in you measure the intensity of light on the other side, you will find that the intensity decays, you know sort of exponentially with the distance depending upon the absorption coefficient.

So, this is a very similar analogy here. So, in some sense, the probability is absorbed ok, this is the absorption coefficient if you want to think about it ok. So, what is the probability of finding the electron on the other side?

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It is we know that, if the wave function is given to us the probability is simply psi square ok. So, it is the square of magnitude of the wave function. So, the probability the ratio or you know, if you say if we create a ratio which is the probability of finding an electron at w was is the probability of finding electron and 0, it is simply the ratio of psi of w by psi of 0 the whole square and we already know the relation between psi of w and psi of 0.

(Refer Slide Time: 51:22)

Metal-Semiconductor Junctions
Current Voltage Characteristics: Schottky Contact

$$\alpha = e \cdot 2 \int_0^w \frac{\sqrt{2m^*}}{\hbar} \sqrt{\phi_b \left(1 - \frac{x}{w}\right)} \cdot dx$$

$$\alpha \approx e \cdot \frac{4}{3} \frac{\sqrt{2q m^*}}{\hbar} \frac{\phi_b^{3/2}}{(\phi_b/w)} \leftarrow \epsilon_{\text{eff}}$$

$$J = q v n \alpha$$

$J \approx n q v \alpha$

And therefore, this probability is simply this term here. So, this is the probability. So, given n electrons at x equal to 0, if I put n electrons at x equal to 0, there is a probability that $n\alpha$ electrons. So, this is my term α right. So, $n\alpha$ electrons appear at x equal to w and that probability is given by this and we solve this integral and we end up with α being equal to this term here.

Now, this little coefficient here is, it is got the same dimensions as the electric field ok. So, this is not the electric field in the semiconductor it is not it is just ϕ_b by w and the problem is we have not defined our w very clearly. So, this is some in some sense a very, it is a parameter that needs to be modelled very carefully ϕ_b is a constant, but our w is you know arbitrarily take chosen we have chosen it in order to get this very nice triangular barrier that through which we have push the carriers through a lot of the carriers to tunnel through.

So, therefore, we retain all the other parameters but α could be modelled very carefully by modelling this term, this denominator here. So, typically the way see, if you think about it you know you could cancel these 2 terms and as ϕ_b power half, but then we do not know what w is and that cannot really it measured.

So, we leave it as this as some kind of an effective electric field which needs to be modelled better in order to get a good estimate for α . So, this is the probability of these carriers tunnelling through and we could very simply write an expression of say of

calling the current density as I am sure these things can be modelled much better but a simple expression is $n q$ into the velocity which normally be there which be the current normally for n carriers but since only α and carriers got across this would be the current through the metal semiconductor Schottky contact via the process of tunnelling all right.

So, with that we will complete our discussions on the current voltage characteristics of Schottky contacts.