

Electronics Enclosures Thermal Issues
Prof. N. V. Chalapathi Rao
Department of Electronic Systems Engineering
Indian Institute of Science, Bangalore

Lecture – 09
Published correlations 2

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boundary conditions. (c) Adiabatic surface at midplane. Only surface 2 benefits from convection cooling.

The rate of heat transfer through the sphere is then

$$q = -4\pi r^2 \frac{\partial T}{\partial r} = \frac{T_o - T_i}{\frac{r_o - r_i}{4\pi k r_i}}$$

and the thermal resistance is found by

$$\theta = \frac{r_o - r_i}{4\pi k r_o r_i}$$

2.3.1.3 Plane Wall with Heat Generation²

In the plane wall studied previously we neglected heat generation, q_G , within the wall. If we now calculate for heat generation (see [Figure 2.9](#)) and constant thermal

The subsequent things are a matter of more for the.

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conductivity, k , the equation becomes

$$k \frac{d^2 T(x)}{dx^2} = -q_G$$

We find the temperature distribution, $T(x)$, by

$$T(x) = -\frac{q_G}{2k} x^2 + \frac{T_2 - T_1}{L} x + \frac{q_G L}{2k} x + T_1$$

If the two surface temperatures are equal, $T_1 = T_2$, the temperature distribution simplifies to a parabolic distribution about the centerline of the plane wall, described as

What you call understanding the record is what I was talking to about if two surface temperatures are equal temperature distribution, simplifies to parabolic distribution.

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$$T(x) = \frac{\dot{q}_G L^2}{2k} \left[\frac{x}{L} - \left(\frac{x}{L} \right)^2 \right] + T_1$$

Since the centerline, which is $x = L/2$, has the maximum temperature, we can find the temperature rise by calculating

$$\Delta T = \frac{\dot{q}_G L^2}{8k}$$

[Table 2.1](#) shows the solutions to a variety of conductive plate and wall problems.

2.3.1.4 Cylinders and Spheres with Heat Generation

In this section we will examine heat transfer in a radial system such as a cylinder or sphere with internal heat generation. Such cases occur in current-carrying bus bars, wires, resistors, and a flex circuit rolled into a cylindrical shell. The following equations apply to both cylinders and spheres (see [Figure 2.10](#)). The temperature distribution in a cylinder is found by

So we have all these you know expressions, which can be easily be evaluated provided you know things like L, you know the things like you know various are that you know the amount of area and so on.

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[Table 2.1](#) shows the solutions to a variety of conductive plate and wall problems.

2.3.1.4 Cylinders and Spheres with Heat Generation

In this section we will examine heat transfer in a radial system such as a cylinder or sphere with internal heat generation. Such cases occur in current-carrying bus bars, wires, resistors, and a flex circuit rolled into a cylindrical shell. The following equations apply to both cylinders and spheres (see [Figure 2.10](#)). The temperature distribution in a cylinder is found by

$$T(r) = \frac{\dot{q}_G r_o^2}{4k} \left[1 - \left(\frac{r}{r_o} \right)^2 \right] + T_s$$

The maximum temperature is at the centerline of the cylinder, $r = 0$; therefore,

$$T_{max} = T_o + \frac{\dot{q}_G r_o^2}{rk}$$

Now, from here onward cylinders and spheres with heat generation are the ones that manufacturers we have evaluated based on chip geometry that is how they are able to

make high power per electronic chips, otherwise you cannot imagine something which dissipates tens of kilowatts, I mean it generates tens of kilowatts by understanding the geometry they are able to evaluate cylinders and spheres in a radial system as a with internal heat generation occur in current carrying bus bars wires resistors flex circuit rolled into a cylindrical shell.

It is very much possible he has given an example here which is generic saying you have wires and resistors. So, you two would have noticed if you have a copper conducting wire which is put a concealed wiring in your building inside you have copper on top of it you have, usually a simple p v c at the moment you bring it out the same thing if you keep it outside it can carry a higher current two things, first of all you have convection which takes place all the time.

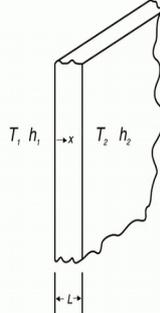
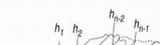
Secondly it is not looped and confined like this things, change a lot when you want to make underground cable you have usually two more sheets around it, do you have the basic insulator then there is a on top of it one more anchoring what you call metallic or plastic things are on top of it usually one or two other waterproofing devices are the these things the heat carrying changes, but then on the outer surface we have earth.

So, things are there and then depending on the conditions they would have done a test saying in the case of loose soil or in the case of its and concrete or in the case if it is under a pavement meaning a road. What is the heat carrying capacity in the case of a length of it, these things are evaluated using the thing here if you have a length of resistance of the what you call material.

So, let say you have a underground cable in the case of in India, where we live we have a 230 volts usually 15 amps peak rated two wire system to our homes. So, we have 15 amps peak in the movement is a 15 amps peak usually a factor of safety of two is there. So, the most of the things are designed for our 30 amps with 230 volts input.

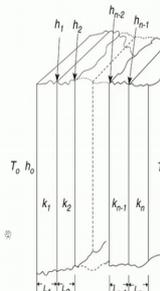
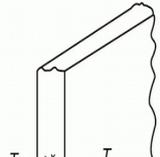
Does it matter that 230 volts yes or no because right now if we are considering heating it is only the length and the heat dissipation when 30 amps passes through as you increase the cross section; obviously, the resistance comes down and total amount of internal heat generation is less you have a very thin section or if there is a kink or some dense formed you see that the whole thing will come down. So, we have this relations here how these things work.

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TABLE 2.1 Conduction in Plates and Walls ²	
Description	Equations
Convectively heated and cooled plate 	Convectively heated and cooled plate $q'' = \frac{h_1 \Delta T_{T_1 - T_2}}{Bi_1 + 1 + \frac{h_1}{h_2}}$ $\frac{\Delta T_{T_2 - T_1}}{\Delta T_{T_2 - T_1}} = \frac{\frac{h_1 x}{k} + 1}{\frac{h_1 L}{k} + 1 + \frac{h_1}{h_2}}$
Composite plate 	Composite plate $q'' = \frac{\Delta T_{T_0 - T_n}}{\dots}$

So, I will just move it saying, this is more in a general things. So, you have here conduction in plates and walls, I suggest since these are expressions which depend a lot on this heat transfer coefficients and the directions and all that.

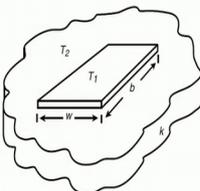
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Composite plate 	Composite plate $q'' = \frac{\Delta T_{T_0 - T_n}}{\sum_{i=1}^n \left(\frac{L_i}{k_i} + \frac{1}{h_i} \right) + \frac{1}{h_0}}$ for $J > 1$ $\frac{\Delta T_{T_1 - T_0}}{\Delta T_{T_n - T_0}} = \frac{\sum_{i=1}^{J-1} \left(\frac{L_i}{k_i} + \frac{1}{h_i} \right) + \frac{x_j}{k_j} + \frac{1}{h_0}}{\sum_{i=1}^n \left(\frac{L_i}{k_i} + \frac{1}{h_i} \right) + \frac{1}{h_0}}$
Plate with temperature-dependent thermal conductivity 	Plate with temperature-dependent thermal conductivity ³ for $k = k_1 + \beta \Delta T_{T-T_1}$ $q'' = k_n \frac{\Delta T_{T_1 - T_2}}{L}$ $\Delta T_{T_2 - T_1} = \frac{1}{\beta} \left(\frac{\sqrt{k_1^2 + 2\beta k_m \Delta T_{T_2 - T_1} x}}{L - k_1} \right)$ where

All you need to do is like shown there before you have several contact surfaces and then a composite plate with temperature dependent thermal conductivity. So, a very important thing thermal conductivity itself is sometimes dependent on the temperature.

I have shown you a plate, which is used in a mosquito what you call repellent coil that heaters there is made with a ceramic which has a two or three slow temperature coefficient. So, when the temperature changes the heat transfer coefficient also changes in it. So, they are all generic cases meant for thoroughness of what you call of analysis you have.

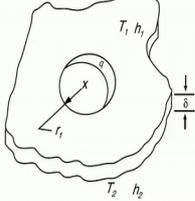
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TABLE 2.1 (continued) Conduction in Plates and Walls ²	
Description	Equations
Thin rectangular plate on the surface of a semi-infinite solid 	Thin rectangular plate on the surface of a semi-infinite solid ⁴ $q = \frac{k w \pi \Delta T_{T_1 - T_2}}{\ln\left(\frac{4w}{\delta}\right)}$
Infinite thin plate with heated circular hole 	Infinite thin plate with heated circular hole for $T = T_2$ at $r = r_1$ and $r > r_1$ $\frac{\Delta T_{T_1 - T_2}}{\Delta T_{T_2 - T_1}} = \frac{K_0 \left(\frac{B r_1}{\delta}\right)}{\nu' / B r_1}$

A large number of these things and here is where known you can use this as a handbook, so that you can get back. Especially you are one of those people who are trying to write your own code and understand for every what do you call a typical case of a heat, what you call thermal condition they have given.

The operational equations that are there thin rectangular plate on the surface of a semi infinite solid, seen there is no that is in relation to the total mass of the heat generating device is outside thing, maybe 100 or a 1000 times no one not to order speaker. So, it is practically semi-infinite solid, so they have given examples of how the heat conducts here.

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$$\frac{k\delta\Delta T_{T_1-T_2}}{q} = \frac{K_0\left(\frac{Br}{\delta}\right)}{2\pi\left(\frac{Br_1}{\delta}\right)K_1\left(\frac{Br_1}{\delta}\right)}$$

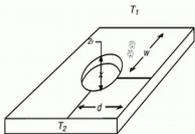
where:

$$B = \sqrt{Bi_1 + Bi_2}$$

$$T_\infty = \frac{T_1 + HT_2}{1 + H}$$

$$H = \frac{Bi_1}{Bi_2}$$

Finite plate with centered hole



Finite plate with centered hole³

$$q' = \frac{k\Delta T_{T_1-T_2}\left[\frac{\pi d}{2w} + \ln\left(\frac{w}{\pi r}\right)\right]}{2\pi}$$

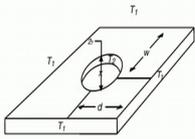
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Infinite thin plate with heated circular hole is what I was telling you if you have a straight mounted transistor. So, you have a inside usually they are made of copper, so you clamp it and then you can treat it like, if you are to have a large plate and if you are having several of these things how these things behave.

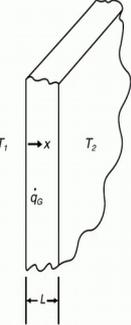
So, we have all these you know relationship saying as heat is transferred then with the finite plate how these things will do right now there is shown only steady state nerd rate of heating eventually even the rate of heating comes into place you need next higher level of differential equations.

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TABLE 2.1 (continued) Conduction in Plates and Walls ²																			
Description	Equations																		
Tube centered in a finite plate 	Tube centered in a finite plate ⁴ for $r < \frac{d}{10}$ $q' = \frac{2\pi k \Delta T_{T_1 - T_2}}{\ln\left(\frac{4d}{\pi r}\right) - c}$ <table border="1"> <thead> <tr> <th>w/d</th> <th>c</th> </tr> </thead> <tbody> <tr><td>1.00</td><td>0.1658</td></tr> <tr><td>1.25</td><td>0.0793</td></tr> <tr><td>1.50</td><td>0.0356</td></tr> <tr><td>2.00</td><td>0.0075</td></tr> <tr><td>2.50</td><td>0.0016</td></tr> <tr><td>3.00</td><td>∞</td></tr> <tr><td>4.00</td><td>1.4×10^{-5}</td></tr> <tr><td>∞</td><td>0.0</td></tr> </tbody> </table>	w/d	c	1.00	0.1658	1.25	0.0793	1.50	0.0356	2.00	0.0075	2.50	0.0016	3.00	∞	4.00	1.4×10^{-5}	∞	0.0
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Infinite plate with internal heat generation	Infinite plate with internal heat generation ² $T = T_1, \quad x = 0$																		

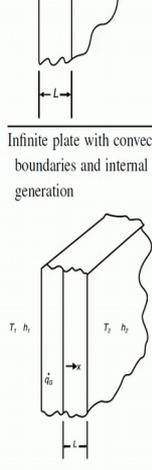
Tube centered in a finite plate, so you have this you know width divided by seeing all sorts of conditions.

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	<table border="1"> <tbody> <tr><td>1.50</td><td>0.0356</td></tr> <tr><td>2.00</td><td>0.0075</td></tr> <tr><td>2.50</td><td>0.0016</td></tr> <tr><td>3.00</td><td>0.0003</td></tr> <tr><td>4.00</td><td>1.4×10^{-5}</td></tr> <tr><td>∞</td><td>0.0</td></tr> </tbody> </table>	1.50	0.0356	2.00	0.0075	2.50	0.0016	3.00	0.0003	4.00	1.4×10^{-5}	∞	0.0
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Infinite plate with internal heat generation 	Infinite plate with internal heat generation ² $T = T_1, \quad x = 0$ $T = T_2, \quad x = L$ $\frac{\Delta T_{T_2 - T_1}}{\Delta T_{T_2 - T_1}} = X + \frac{\text{Po}X(1 - X)}{2}$ where $X = \frac{x}{L}$ ∞												
Infinite plate with convection	Infinite plate with convection boundaries and internal												

A large number of such things are covered here as I said this is more.

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Infinite plate with convection boundaries and internal heat generation

Infinite plate with convection boundaries and internal heat generation³

$$\frac{\Delta T_T - \tau_2}{\Delta T_{T_1 - T_2}} = \frac{1 - \text{Po} \left(\frac{1}{\text{Bi}_2} + 1 \right)}{1 + \text{Bi}_2 + H} + \frac{\text{Po}}{\text{Bi}_2} + \frac{\text{Po}}{2} (1 - X^2)$$

$$+ \frac{\text{Bi}_1 \left[1 + \text{Po} \left(\frac{1}{\text{Bi}_2} + 0.5 \right) \right] (1 - X)}{1 + \text{Bi}_1 + H}$$

where $H = \frac{h_2}{h_1}$

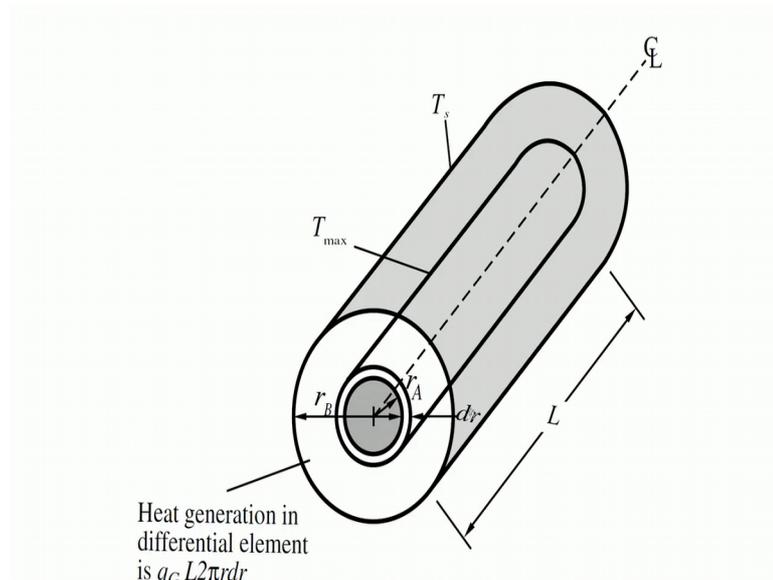
To be treated as a handbook.

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TABLE 2.1 (continued) Conduction in Plates and Walls ²	
<i>Notes:</i>	
Bi = Biot Number, hL/k	q'' = rate of heat flux, W/m^2
c = value for w/d	q' = linear heat flux, W/m
d = diameter, m	\dot{q}_G = volumetric heat flux, W/m^3
h = heat transfer coefficient, $\text{W/m}^2 \text{K}$	w = width
k = thermal conductivity, W/m K	x, y, z = Cartesian coordinates
L = length, m	X = length ratio (x/L)
$\text{Po} = \frac{\dot{q}_G L^2}{k}$	β = coefficient of thermal expansion ($^{\circ}\text{C}^{-1}$)
q = rate of heat flow, W	δ = thickness

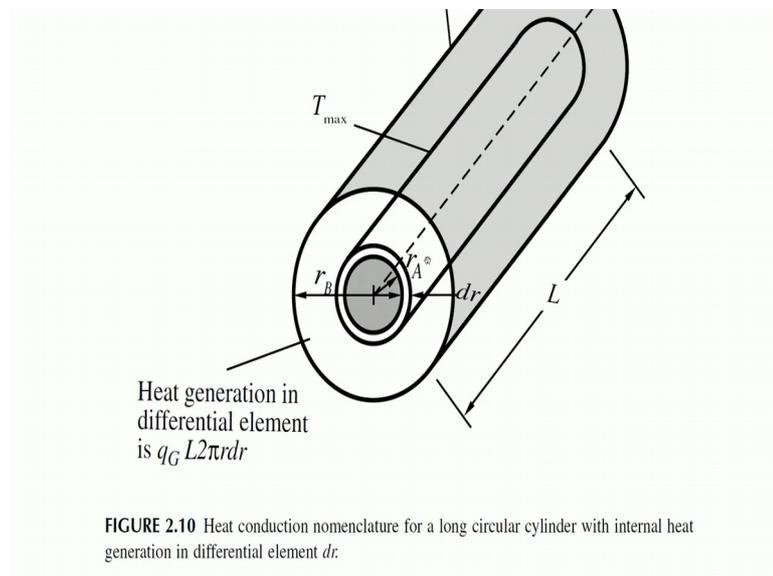
Handbook information, so where all these numbers you know are all mentioned here as you go down.

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See you have more and more.

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Heat conduction nomenclature for long cylindrical with internal heat generation in a differential element dr as the small this thing, what you call if you take a thin layer as it moves how the heat generation takes place, so the subsequent things talk to you about.

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If we evaluate the temperature distribution at the centerline of the cylinder, we find the nondimensional temperature distribution

$$\frac{T(r) - T_s}{T_{max} - T_s} = 1 - \left(\frac{r}{r_B}\right)^2$$

To find the surface temperature of a tube, T_s , having a flowing cold fluid at T_∞ , we evaluate with a simplified energy balance equation which yields

$$T_s = T_\infty + \frac{\dot{q}_G r}{2h_c}$$

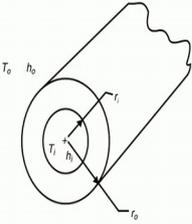
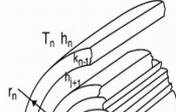
The effective heat transfer coefficient for the tube is then

$$h_c = \frac{\dot{q}_G(r_o^2 - r_i^2)}{2r_i(T_i - T_\infty)}$$

Tables 2.2 and 2.3 show the solutions to a variety of conductive cylinder and sphere problems.

Various things of.

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TABLE 2.2 Rods, Tubes, Cylinders, Disks, Pipes, and Wires ²	
Description	Equations
Infinite hollow cylinder 	Infinite hollow cylinder $q' = \frac{2\pi k \Delta T_{T_o - T_i}}{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_i} + \frac{1}{Bi_o}}$ $\frac{\Delta T_{T_i - T_o}}{\Delta T_{T_i - T_o}} = \frac{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_o}}{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_i} + \frac{1}{Bi_o}}$ where $Bi_o = \frac{h_o r_o}{k}$
Composite cylinder 	Composite cylinder $q' = \frac{2\pi \Delta T_{T_n - T_1}}{\sum_{i=1}^{n-1} \left(\frac{1}{k_i} \ln\left(\frac{r_{i+1}}{r_i}\right)\right) + \sum_{i=1}^n \frac{1}{r_i h_i}}$ for $j > 1$

The tubes and so on, so you have rods tubes cylinders disks pipes and wires, so a huge number of the tables explain to you how an infinite hollow cylinder works, so in this case you have.

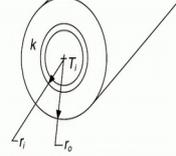
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	$q' = \frac{\Delta T_{T_n - T_1}}{\sum_{i=1}^{n-1} \left[\frac{1}{k_i} \ln \left(\frac{r_{i+1}}{r_i} \right) + \frac{1}{r_i h_i} \right] + \frac{1}{r_n h_n}}$ <p>for $j > 1$</p> $\frac{\Delta T_{T_j - T_1}}{\Delta T_{T_n - T_1}} = \frac{\sum_{i=1}^{j-1} \left[\frac{1}{k_i} \ln \left(\frac{r_{i+1}}{r_i} \right) + \frac{1}{r_i h_i} \right] + \frac{1}{k_j} \ln \left(\frac{r_j}{r_j} \right) + \frac{1}{r_j h_j}}{\sum_{i=1}^{n-1} \left[\frac{1}{k_i} \ln \left(\frac{r_{i+1}}{r_i} \right) + \frac{1}{r_i h_i} \right] + \frac{1}{r_n h_n}}$ <p>where T_j is the temperature in the jth layer</p>
<p>Insulated tube</p>	<p>Insulated tube</p>
	$q' = \frac{2 \pi k \Delta T_{T_i - T_o}}{\ln \left(\frac{r_o}{r_i} \right) + \frac{1}{\text{Bi}_o}}$ <p>where</p> $k \ll k_{tube} \quad \text{and} \quad \text{Bi}_o = \frac{h r_o}{k}$ <p>Maximum heat loss occurs when $r_o = \frac{k}{h}$</p>
<p>Infinite cylinder with temperature-dependent thermal conductivity</p>	<p>Infinite cylinder with temperature-dependent thermal conductivity³ with:</p>

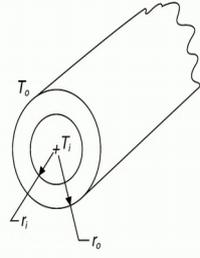
Composite cylinder which is what I was talking to about the underground cable or a wiring cable where are these important, these are important because in the case why it looks easy, eventually your conductors and all and get bunched in an electronic equipment especially power electronic.

So, what looks very easy the moment you make it a bunch and it becomes anything above 10 or 15 m m, you notice that all these effects will come into place and. So, manufacturers take a various things and then second thing related here is the insulated tube do we have insulated yes and you are seen here.

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Infinite cylinder with temperature-dependent thermal conductivity



where

$$k \ll k_{\text{tube}} \quad \text{and} \quad \text{Bi}_o = \frac{hr_o}{k}$$

Maximum heat loss occurs when $r_o = \frac{k}{h}$

Infinite cylinder with temperature-dependent thermal conductivity³ with:

$$k = k_o + \beta \Delta T_{T-\tau_o}$$

$$k = k_o \text{ at } r_o$$

$$k = k_i \text{ at } r_i$$

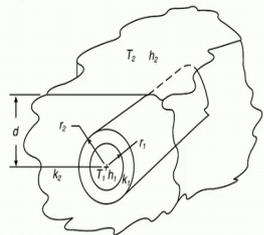
$$q' = \frac{2\pi k_o \Delta T_{T_i-\tau_o}}{\ln\left(\frac{r_o}{r_i}\right)} \left[1 + \frac{2\beta k_o \ln\left(\frac{r_o}{r_i}\right)}{k_o^2 \ln\left(\frac{r_o}{r_i}\right)} \Delta T_{T_i-\tau_o} \right]^{0.5} - 1$$

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Usually you know huge amount of calculations they are not important, if you know little about all the surface conditions and then. If you also know the where the k changes to the temperature it is possible for you to evaluate the expression.

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TABLE 2.2 (continued) Rods, Tubes, Cylinders, Disks, Pipes, and Wires ²	
Description	Equations
Pipe in semi-infinite solid 	Pipe in semi-infinite solid ⁴ $q' = \frac{2\pi k_1 \Delta T_{T_2-\tau_1}}{\frac{1}{\text{Bi}_1} + \ln\left(\frac{r_2}{r_1}\right) + \left(\frac{1}{k}\right) \ln\left(2D \frac{2KD}{\text{Bi}_2}\right)}$ where: $\text{Bi}_1 = \frac{h_1 r_1}{k_1} \quad \text{Bi}_2 = \frac{h_2 d}{k_2}$ $K = \frac{k_2}{k_1} \quad D = \frac{d}{r_2}$

So, we have.

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	<p>where:</p> $\frac{1}{Bi_1} + \ln\left(\frac{2}{r_1}\right) + \left(\frac{1}{k}\right) \ln\left(2D \frac{h_2}{Bi_2}\right)$ $Bi_1 = \frac{h_1 r_1}{k_1} \quad Bi_2 = \frac{h_2 d}{k_2}$ $K = \frac{k_2}{k_1} \quad D = \frac{d}{r_2}$
<p>Row of rods in semi-infinite solid</p>	<p>Row of rods in semi-infinite solid³</p>
	<p>For one rod</p> $q' = \frac{2\pi k \Delta T T_2 - T_1}{\frac{1}{Bi_1} + \ln\left[\left(\frac{d}{\pi D r_1}\right) \sinh\left[\pi 2\left(D + \frac{D}{Bi_2}\right)\right]\right]}$ <p>where:</p> $Bi_1 = \frac{h_1 r_1}{k}, Bi_2 = \frac{h_2 d}{k}, D = \frac{d}{s}$
<p>Row of rods in wall</p>	<p>Row of rods in wall³</p>
	<p>For each rod</p> $4\pi k \Delta T T_2 - T_1$

This whole thing ha we are coming to more and more practical example. So, when these are embedded into another, what you call another medium it could be a combined thing or it could be a pin fin heat sink you have a base in, which you know you have these things and sometimes they may be different materials it is very much possible for us to evaluate.

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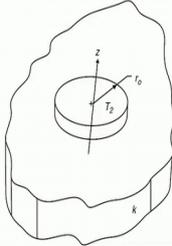
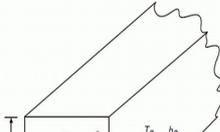
	$q' = \frac{2\pi k \Delta T T_2 - T_1}{\frac{1}{Bi_1} + \ln\left[\left(\frac{d}{\pi D r_1}\right) \sinh\left[\pi 2\left(D + \frac{D}{Bi_2}\right)\right]\right]}$ <p>where:</p> $Bi_1 = \frac{h_1 r_1}{k}, Bi_2 = \frac{h_2 d}{k}, D = \frac{d}{s}$
<p>Row of rods in wall</p>	<p>Row of rods in wall³</p>
	<p>For each rod</p> $q' = \frac{4\pi k \Delta T T_2 - T_1}{\frac{1}{Bi_1} + \ln\left[\left(\frac{d}{\pi D r_1}\right) \sinh\left[\pi 2\left(D + \frac{D}{Bi_2}\right)\right]\right]}$ <p>where:</p> $Bi_1 = \frac{h_1 r_1}{k}, Bi_2 = \frac{h_2 d}{k}, D = \frac{d}{s}$
<p>Circular disk on the surface of a semi-infinite solid</p>	<p>Circular disk on the surface of a semi-infinite solid⁵</p>
	<p>For</p>

Things here, so especially these things make sense, I do not know if I have if you recollect. So, the next thing if possible I will get back to the video which I have shown

you. We have usually internally some structures inside which could be embedded materials.

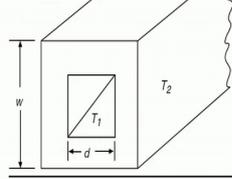
If you recollect the phase change graphic, which I have shown you there you have copper which is sitting in an aluminum and some of the copper tube inside the copper tube we have the working fluid, but outside the of the copper which is again embedded into a heat sink, conditions like this you know happened there especially on the surface of that, so as you go down we have all this.

(Refer Slide Time: 12:04)

TABLE 2.2 (continued) Rods, Tubes, Cylinders, Disks, Pipes, and Wires ²	
Description	Equations
Circular disk in an infinite solid 	Circular disk in an infinite solid ³ $q = 8r_0 k \Delta T_{T_2 - T_1}$
Infinite hollow square rod 	Infinite hollow square rod ³ $q' = \frac{2\pi k \Delta T_{T_2 - T_1}}{\frac{k}{h_1 \tau_2} + \ln\left(\frac{1.08w}{2r_o}\right) + \frac{\pi k}{2h_2 w}}$

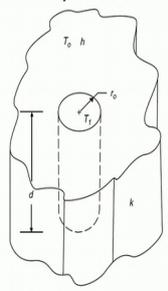
More and more.

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Vertical cylinder in a semi-infinite solid

$$q' = \begin{cases} 0.785 \ln\left(\frac{w}{d}\right) \\ \frac{2\pi k \Delta T_{T_1 - T_2}}{0.93 \ln\left(\frac{w}{d}\right) - 0.0502} \end{cases} \quad \frac{w}{d} > 1.4$$



Vertical cylinder in a semi-infinite solid³

$$q = \left\{ \frac{2D}{\ln\left[2D\left(1 + \frac{1}{Bi_d}\right)\right]} + \frac{Bi_d}{D} \right\} \pi r_o k \Delta T_{T_1 - T_o}$$

where $Bi_d = \frac{hd}{k}$, $D = \frac{d}{r_o}$

(Continued)

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Large number of expressions, so I suggest you go back to the these things.

(Refer Slide Time: 12:12)



$$\frac{2k_2}{\pi(k_1 + k_2)} \sin^{-1} \left\{ \frac{2}{\left[(R-1)^2 + Z^2 \right]^{0.5} + \left[(R+1)^2 + Z^2 \right]^{0.5}} \right\} Z < 0$$

$$\frac{\Delta T_{T - T_o}}{\Delta T_{T_1 - T_o}} =$$

$$1 - \frac{2k_1}{\pi(k_1 + k_2)} \sin^{-1} \left\{ \frac{2}{\left[(R-1)^2 + Z^2 \right]^{0.5} + \left[(R+1)^2 + Z^2 \right]^{0.5}} \right\} Z > 0$$

$$q = \left(\frac{4r_o k_1 k_2}{k_1 + k_2} \right) \Delta T_{T_o - T_1}$$

where $Z = \frac{z}{r_o}$, $R = \frac{r}{r_o}$

Heat flow between two rods in an insulated infinite plate

Heat flow between two rods in an insulated infinite plate⁵

And then try to know how these things vary a lot.

(Refer Slide Time: 12:23)

$$r_o = r_{crit} = \left(\frac{\alpha}{(1-n)k} \right)^{\frac{1}{n-1}}$$

where:

α = thermal diffusivity of the convective media, $k/\rho c_p$

n = 0.5 for laminar forced convection or 0.25 for natural convection

k = thermal conductivity of insulation, W/m K

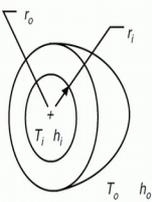
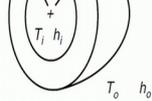
2.3.2 CONDUCTION IN COMPLEX GEOMETRIES

In the previous section we studied one-dimensional heat flow. In this section we will examine heat transfer in multidimensional systems. Multidimensional heat transfer occurs when we transfer the heat from different locations and the temperature may vary in more than one dimension. One example is an active component in a potting compound, an irregularly shaped object, or a corner where we join three chassis walls. [Figure 2.11](#) shows two-dimensional conduction.

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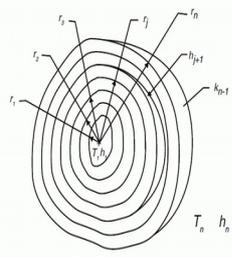
Large number of things comes here well above. So, far one dimensional heat flow multidimensional systems also.

(Refer Slide Time: 12:36)

TABLE 2.3 Conduction in Spheres ²	
Description	Equations
<p>Spherical shell</p> 	<p>Spherical shell</p> $q = \frac{4\pi r_o k \Delta T_{T_i - T_o}}{\frac{r_o}{r_i} - 1 + \left(\frac{r_o}{r_i}\right) \frac{k}{h_i r_i} + \frac{k}{h_o r_o}}$
<p>Composite sphere</p> 	<p>Composite sphere</p> $\frac{\Delta T_{T_i - T_o}}{\Delta T_{T_i - T_o}} = \frac{\frac{r_o}{r_i} - 1 + \frac{k}{r_o h_o}}{\frac{r_o}{r_i} - 1 + \left(\frac{r_o}{r_i}\right) \frac{k}{r_i h_i} + \frac{k}{r_o h_o}}$

In this case of multidimensional you have this fear you have seen that you know and. So, in just in one either the radial or in the axial direction you have things which will move in all the directions.

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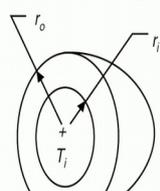


$$q = \frac{4\pi\Delta T_{T_i - T_n}}{\sum_{i=1}^{n-1} \frac{1}{k_i} \left(\frac{1}{r_i} - \frac{1}{r_{i+1}} \right) + \sum_{i=1}^n \frac{1}{r_i^2 h_i}}$$

$$\frac{\Delta T_{T_j - T_i}}{\Delta T_{T_n - T_i}} = \frac{\sum_{i=1}^{j-1} \left[\frac{1}{k_i} \left(\frac{1}{r_i} - \frac{1}{r_{i+1}} \right) + \frac{1}{r_i^2 h_i} \right] + \frac{1}{r_j^2 h_j}}{\sum_{i=1}^{n-1} \frac{1}{k_i} \left(\frac{1}{r_i} - \frac{1}{r_{i+1}} \right) + \sum_{i=1}^n \frac{1}{r_i^2 h_i}}$$

where T_j is the local temperature in the j th layer

Sphere with temperature-dependent thermal conductivity



Sphere with temperature-dependent thermal conductivity?

with

$$T = T_i \quad r = r_i$$

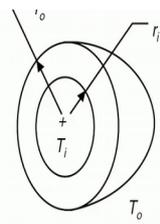
$$T = T_n \quad r = r_o$$

$$k = k_o + \rho$$

$$q = \dots$$

So, you have several gradients and so on, these are all the more important when we have transient heat phenomena as in if you have a switching device and then it is already sitting inside a finite, what do you call material all this much of analysis is required for us to make this summation. But then kind you remember everywhere you know they are able to reduce everything to a network of resistances.

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$$T = T_i \quad r = r_i$$

$$T = T_n \quad r = r_o$$

$$k = k_o + \beta\Delta T_{T - T_o}$$

$$q = \frac{4\pi r_o k_m \Delta T_{T_i - T_o}}{\frac{r_o}{r_i} - 1}$$

where $k_m = \frac{k_o + k_i}{2}$

with

$$k = k_o \quad T = T_o$$

$$k = k_i \quad T = T_i$$

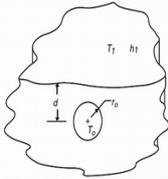
$$\frac{\Delta T_{T - T_o}}{\frac{k_o}{\beta}} = \sqrt[4]{1 + 2\beta\Delta T_{T_i - T_o} \frac{k_m}{k_o^2} \frac{r_o - r_i}{r_i} - 1}$$

(Continued)

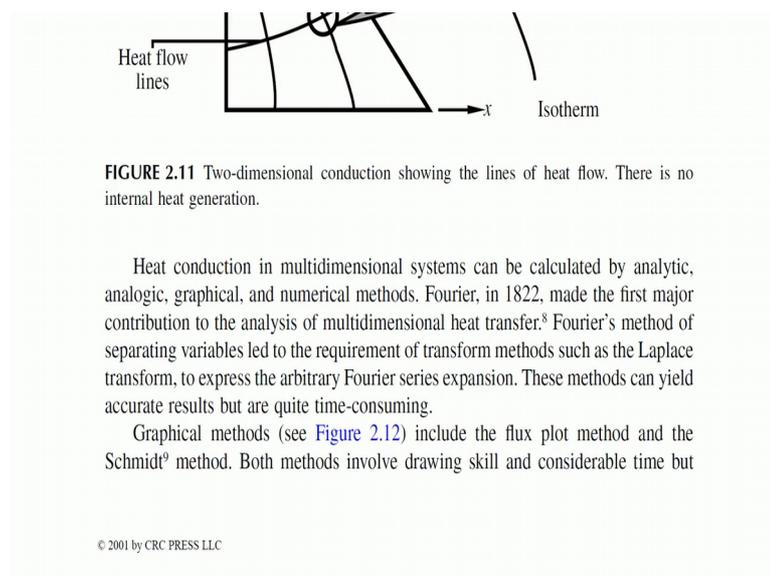
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It is very much possible for us and then you see here conduction in spheres.

(Refer Slide Time: 13:31)

TABLE 2.3 (continued) Conduction in Spheres ²	
Description	Equations
Sphere in a semi-infinite solid 	Sphere in a semi-infinite solid ⁴ $q = \frac{4\pi r_0 k \Delta T_{T_1 - T_2}}{1 + 0.5 \left(\frac{d}{r_0} + \frac{1}{\text{Bi}} \right)}$ where $\text{Bi} = \frac{hr_0}{k}$
Sphere in an infinite medium 	Sphere in an infinite medium ⁵ with $T = T_2$ at $r \rightarrow \infty$ $q = 4\pi r_0 k \Delta T_{T_1 - T_2}$

(Refer Slide Time: 13:36)

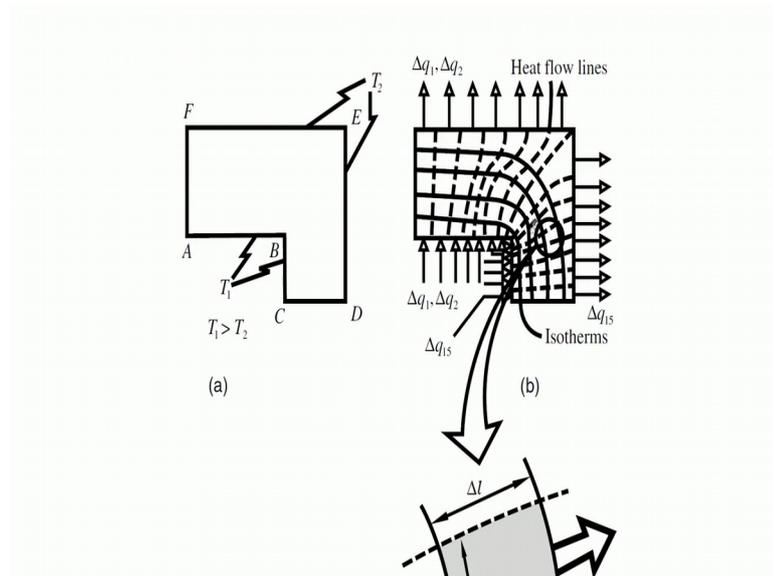


I will just keep all these things and read this is somebody says I am a new person who has started working and discovered all these things, if he is holding an examination you believe him and repeat it and you will be surprised to see that it is about nearly quite a long time ago 200 years back, for here already made a analysis of multidimensional heat transfer.

Later on we had this Laplace transform, arbitrary Fourier series and accurate results, but important thing is time consuming you understand you know nearly 200 years back. In

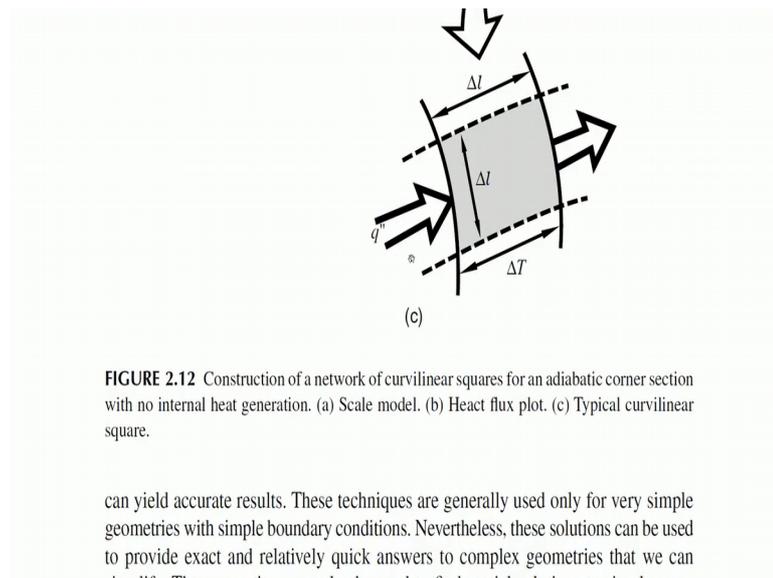
800, in this case you know they have mentioned 20 that is just short of 200 years back. Fourier made this observation and also some analytical methods saying using this it is very much possible for us to.

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We have this beautiful graphical methods also now, which appear very intuitive you understand, know.

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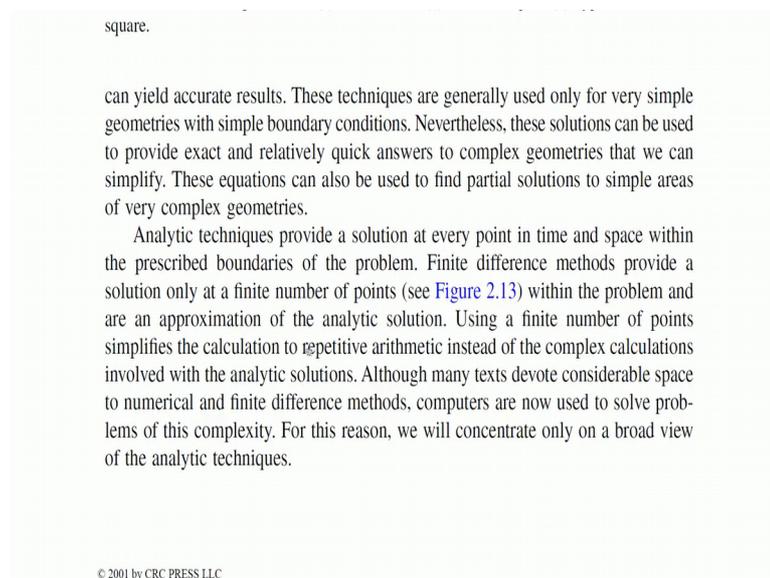


These things look a little like things which are used in the other fields. So, anywhere you have things of equal potential it has possible for us to draw a lines with the equal

potential. So, we have here various types of you know what you call heat flow geometries and so on.

So, you will see that whenever there is a you know little bit of crowding, you have these what you call beautiful lines which are reasonably easy to plot this lecture is not about plotting this thing and which are all available I am sure if you take a proper heat transfer, heat and mass transfer, course you may will be able to solve these things. My interest has been how to take commercial available devices and then try to work with simplified, equation this word even schramberg starts in the beginning saying while it is possible for us to calculate all these.

(Refer Slide Time: 16:38)



So in reality, you have seen this read it equations can be used to find partial solutions to simple areas of very complex geometries, analytical techniques provide a solution within the prescribed boundaries finite differences methods provide a solution only at a finite number of points, within the problem and are an approximation of the analytical solution and so on and so on.

The using a finite number of point and simplify the calculation to repetitive arithmetic instead of the complex involved with analytical solutions, many texts devote considerable space to numerical and finite different methods, computers are now used to solve this I will considered only in broad delivery of the analytical techniques you have

seen that the man who sort of you know wrote the book he himself admits that computers can be used safely.

So, you have here things like flow them and several other computational things, and if you are a student or a fresh to the field engineer you can probably download beta versions or trial versions which work with a few number of points and then try to show you how to work with it. And then if you are part of a larger this thing if you are part of an educational institution, if you get into proper what you call m o y with them they will allow you to park those programs here.

Luckily for where I am working where I have you worked in the where this is being recorded we had access to this for a long time, and only thing is even what is used for the educational purposes it will allow view it will allow me to you know teaches the students and how to use them, adventually when they go to the corporate sector or when they go to the defense electronics sector they buy the original full package.

And why do they give it to us is now we have firsthand experience in it I have no experience, but my students know how to use it the moment they step out and go to either a r and d lab or the other places they buy the original. And as you say numerical and how to write the program is not given here computers are used to solve the problems of the complexity.

(Refer Slide Time: 19:23)

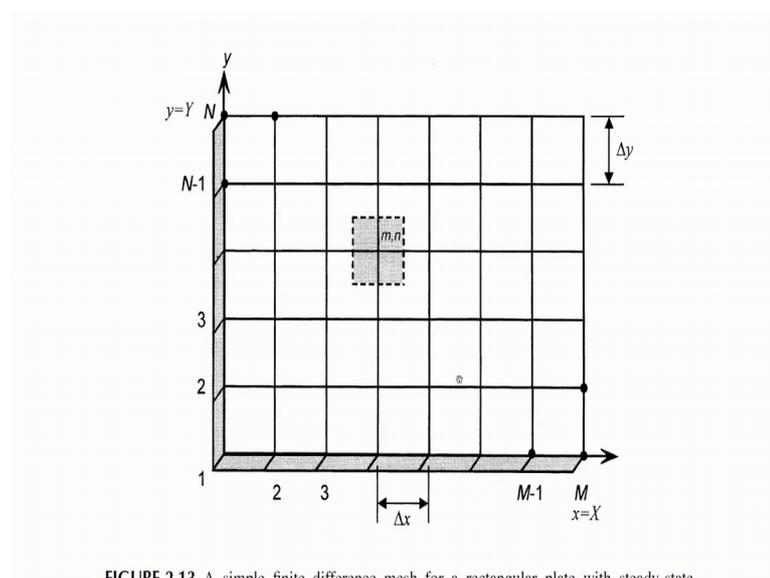


FIGURE 2.13 A simple finite difference mesh for a rectangular plate with steady-state

So succeeding lectures.

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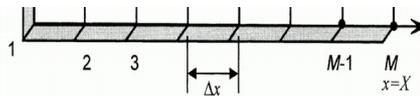


FIGURE 2.13 A simple finite difference mesh for a rectangular plate with steady-state conduction.

2.3.2.1 Multidimensional Analytic Method

In a two-dimensional system without internal heat generation and with uniform thermal conductivity, the general conduction equation has been found as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

The total rate of heat transfer is a vector. The vector is dependent upon the rate of heat flow in x , which is q_x , and the rate of heat flow in y , which is q_y . The total rate of heat transfer is then perpendicular to an isotherm within the boundaries of the geometry. Therefore, if we solve for the temperature distribution, the heat flow can be found easily. Examine a rectangular plate that is insulated at two opposite

Tries to explain how these things come including wind up with partial differential equations yes I admit I am no good at it, but it doesn't mean I have a problem we have help on hand we live close to the. In fact, my neighbor is one of the top mathematics professors and he puts his students on the job and then beautifully.

(Refer Slide Time: 20:03)

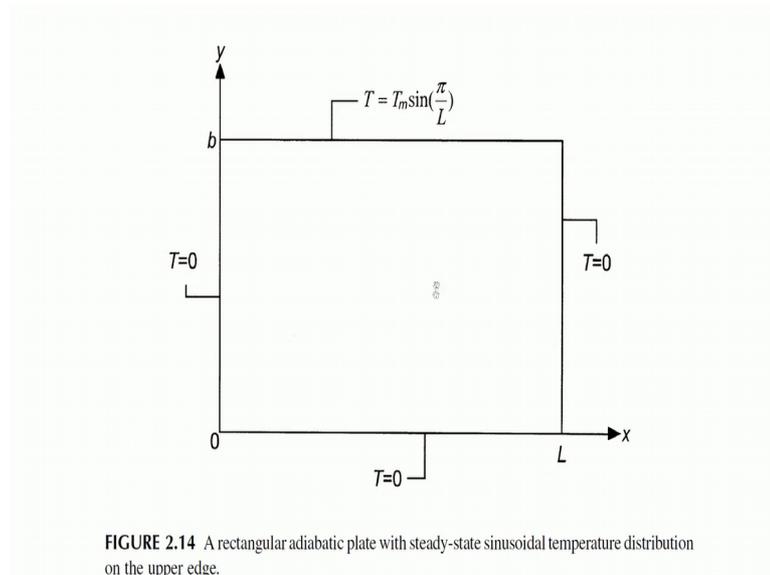
The total rate of heat transfer is a vector. The vector is dependent upon the rate of heat flow in x , which is q_x , and the rate of heat flow in y , which is q_y . The total rate of heat transfer is then perpendicular to an isotherm within the boundaries of the geometry. Therefore, if we solve for the temperature distribution, the heat flow can be found easily. Examine a rectangular plate that is insulated at two opposite sides (see Figure 2.14). Since the problem is linear, $T = XY$, $X = X(x)$, and $Y = Y(y)$. The solution to the temperature distribution is

$$T(x, y) = T_m \frac{\sinh\left(\frac{\pi y}{L}\right)}{\sinh\left(\frac{\pi b}{L}\right)} \sin \frac{\pi x}{L}$$

The solution to the temperature distribution is shown graphically in Figure 2.15. When we specify more complex boundary conditions, the series can become infinite.

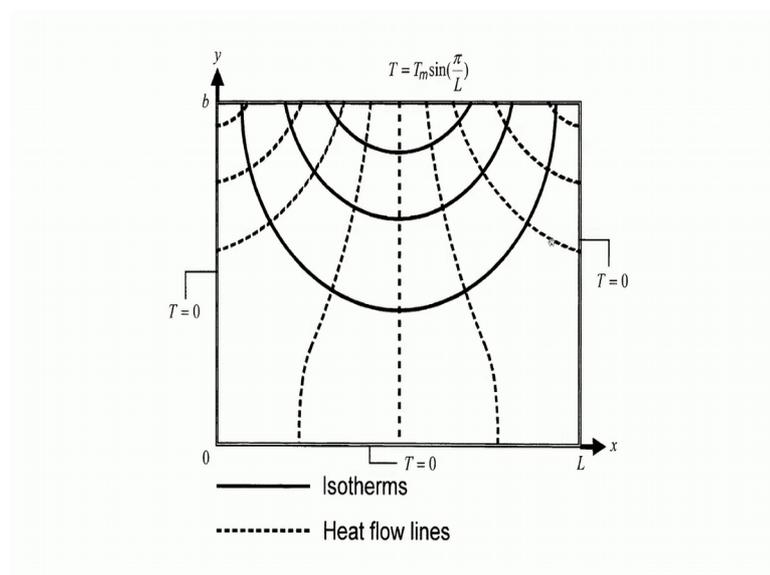
We have all these.

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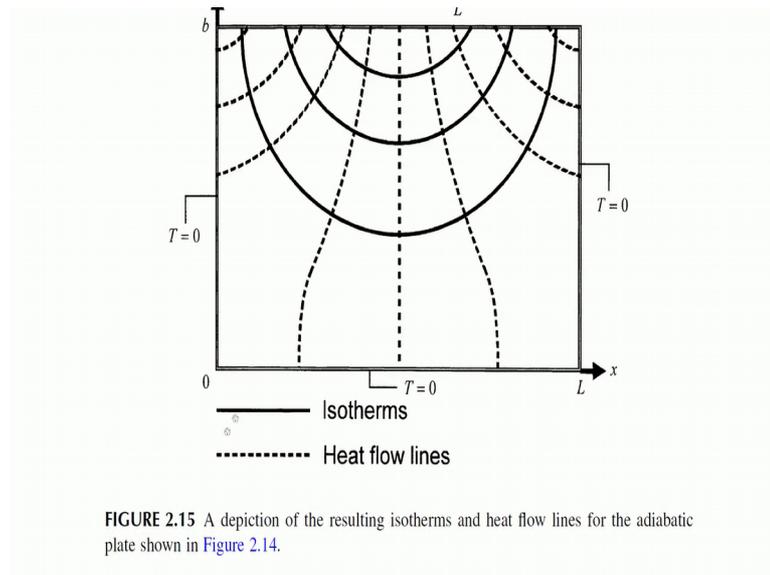
Expressions that can be easily evaluated you are seen here. So, I am just too where the purposes of completion of reading these chapters.

(Refer Slide Time: 20:12)



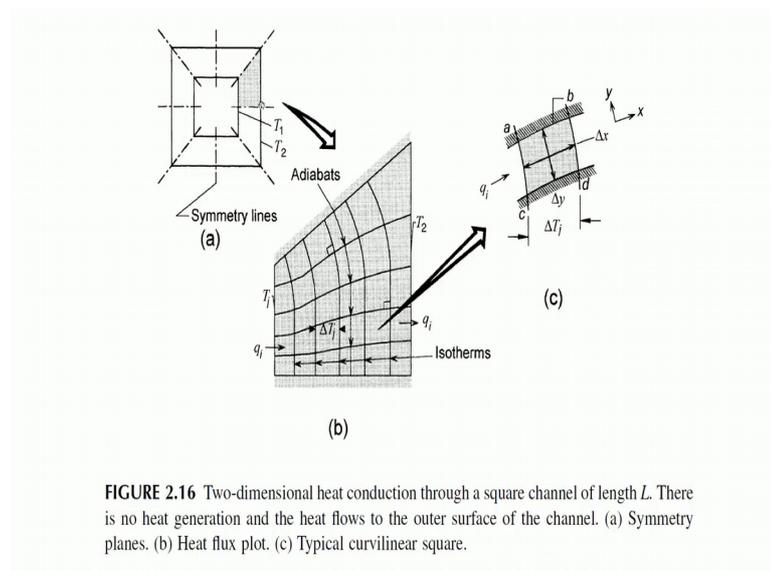
I will show you here saying even in the case when we have this and in the case of this how these what you call isothermal points can be plotted.

(Refer Slide Time: 20:25)



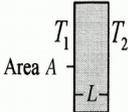
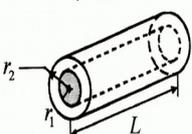
And how actually, heat flow lines can be shown, you would have seen these cross sections in fluid mechanics that there show at you and beautiful flow cross sections are shown saying in the case of a what do you call turbulent flow, how these things are and in the case of a laminar flow how these are, this thing also seem a little like this depending on the difference of temperature and the this thing these isothermals we have a little like that.

(Refer Slide Time: 21:04)



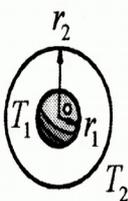
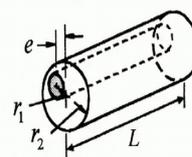
So I will just keep going down you have seen this whenever there is a cross section like this what actually reality what happens. So, we have huge number of these expressions.

(Refer Slide Time: 21:18)

Configuration	Restrictions	Conduction Shape Factor
Plane wall 	$A = \text{area}$ $L = \text{wall thickness}$	$\frac{A}{L}$
Concentric cylinders 	$L \gg r_2$ $r_1 = \text{inner cylinder radius}$ $r_2 = \text{outer cylinder radius}$	$\frac{2\pi L}{\ln\left(\frac{r_2}{r_1}\right)}$
Concentric spheres	$r_1 = \text{inner sphere radius}$	$\frac{4\pi}{\frac{1}{r_1} - \frac{1}{r_2}}$

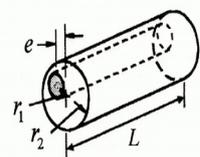
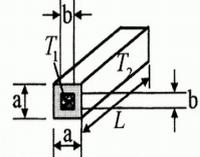
And finally, we come into more and more practical things saying shape factors if you already know a shape, meaning of its a square and then we have an area and then we have wall thickness and so on, whether this can be directly used here, so in the case of plane wall then concentric.

(Refer Slide Time: 21:47)

Concentric spheres 	$r_1 = \text{inner sphere radius}$ $r_2 = \text{outer sphere radius}$ if $r_2 \rightarrow \infty$	$\frac{4\pi}{\frac{1}{r_1} - \frac{1}{r_2}}$ $4\pi r_1$
Eccentric cylinders 	$L \gg r_2$ $e = \text{axial centerline offset}$ $r_2 = \text{radius of outer cylinder}$ $r_1 = \text{radius of small cylinder}$	$\frac{2\pi L}{\cosh\left(\frac{r_2^2 + r_1^2 - e^2}{2r_2 r_1}\right)}$
Concentric square cylinders 	$L \gg a$ if $a/b > 1.4$	$\frac{2\pi L}{0.93 \ln\left(\frac{a}{b}\right) - 0.0502}$

Cylinders concentric spheres.

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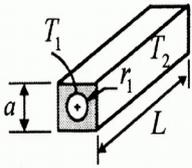
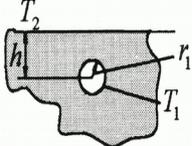
		
Eccentric cylinders 	$L \gg r_2$ e = axial centerline offset r_2 = radius of outer cylinder r_1 = radius of small cylinder	$\frac{2\pi L}{\cosh^{-1}\left(\frac{r_2^2 + r_1^2 - e^2}{2r_2 r_1}\right)}$
Concentric square cylinders 		
	$L \gg a$ if $a/b > 1.4$ if $a/b < 1.4$, where a = side of large square, b = side of small square	$\frac{2\pi L}{0.93 \ln\left(\frac{a}{b}\right) - 0.0502}$ $\frac{2\pi L}{0.785 \ln\left(\frac{a}{b}\right)}$

(Continued)

A large number of you see here this may be a reality in the case of a eccentric cylinder does it occur here's, look at your soldering iron or right do not now we use the word rod some people it is called a soldering iron how there is a heating element inside, and then there is a sleeve outside and we expect that the sleeve is in perfect contact may or may not be contact and it may be off center heating element may be off center.

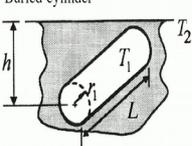
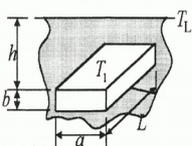
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TABLE 2.4 (continued)
Shape Factors for Steady-State Conduction

Configuration	Restrictions	Conduction Shape Factor
Circular cylinder in a square cylinder, concentric 	$a > 2r$ a = side of square	$\frac{2\pi L}{\ln(0.54 \frac{a}{r})}$
Buried sphere 	h = distance below surface $h > r_1$ if $h \rightarrow \infty$	$\frac{4\pi r_1}{1 - \frac{r_1}{2h}}$ $4\pi r_1$

So, most known items are all shown here.

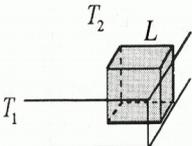
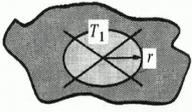
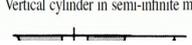
(Refer Slide Time: 22:30)

	$L \gg r_1$ $h = \text{distance below surface}$ if $h > 3r_1$ if $\frac{h}{r_1} \rightarrow \infty, s \rightarrow 0$	$\frac{2\pi L}{\cosh^{-1}\left(\frac{2h}{r_1}\right)}$ $\frac{2\pi L}{\ln\left(\frac{2h}{r_1}\right)}$
	$L \gg h, a, b$ $h = \text{distance below surface}$ $a = \text{box width}$ $b = \text{box height}$	$2.756L \left[\ln\left(1 + \frac{h}{a}\right) \right]^{-0.59} \left(\frac{h}{b}\right)^{-0.078}$
	$L = \text{wall thickness}$ $W = \text{length of attachment}$ $W > L/5$	$0.54W$

In this in the case of a barrier rectangular box and so on.

(Refer Slide Time: 22:35)

TABLE 2.4 (continued)
Shape Factors for Steady-State Conduction

Configuration	Restrictions	Conduction Shape Factor
	$W > L/5$	$0.15L$
	$r = \text{radius of disk}$	$4r$
	$L \gg D$	$\frac{2\pi L}{\ln(4L)}$

Large number of for in the case of heats as this thing huge amount of this calculations and all are given here I will skip all these things.

(Refer Slide Time: 22:48)

2.4 CONDUCTION—TRANSIENT

The solution for some heat conduction problems varies with time. Even problems involving steady-state conditions require time for the system to reach a steady state. We call such problems unsteady, or transient, conduction. Such cases may occur when the boundary conditions change, as when an object is immersed in a different temperature bath or with variable internal heat generation. There are two basic types of transient conduction problems, those that have boundary conditions that change once—balanced systems—and those that have continually changing boundary conditions—unbalanced systems. An example of the first type is a resistor in a system that is powered up. The temperature of the resistor will increase until there is an energy balance between the internal heat generation and the cooling effects of convection, radiation, and conduction. An example of an unbalanced system is the same resistor with a sine wave input voltage. The temperature of the device will continue to change with the changing input voltage conditions.

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And come back to important thing we end up with transient conduction, time for the system to reach a steady state saying non steady or transient conduction, such cases may occur when the boundary condition change object is immersed in a different temperature the variable internal heat generation which is the reality in the case of electronics.

So, inside the heat is not uniform it is not coming out at the same thing it may be simple example it may be a sine wave other example it may be square wave and variance of all these things, saying them if you have a pulse width modulated how these things will work a temperature of the resistor will increase until there is an energy balance. So, example is resistor unbalanced system is a resistor with the sine wave input voltage temperature the device will continue to change with the changing input voltage conditions .

You have seen you have both if it is a sine wave we can do something and if it is a square where we can do something as with $p_w m$ if you know the overall, duty factor it is very much possible for us to evaluate this condition, so this solid shows.

(Refer Slide Time: 24:20)

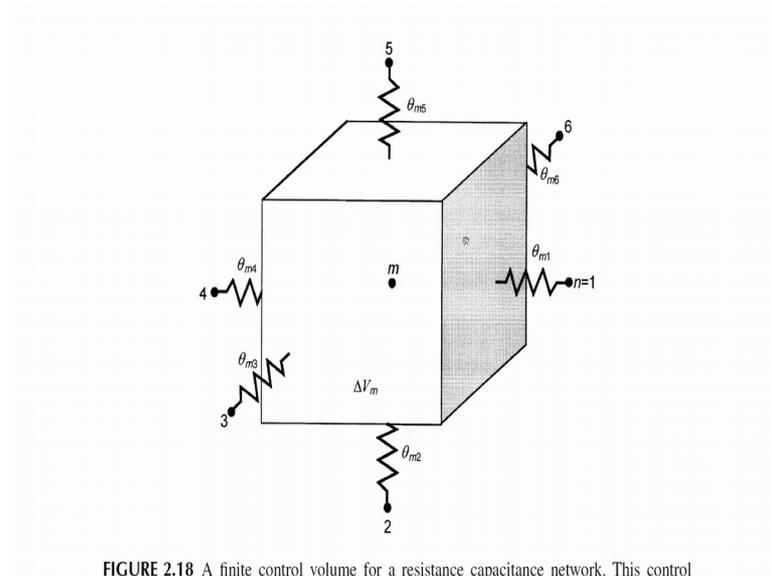


FIGURE 2.18 A finite control volume for a resistance capacitance network. This control

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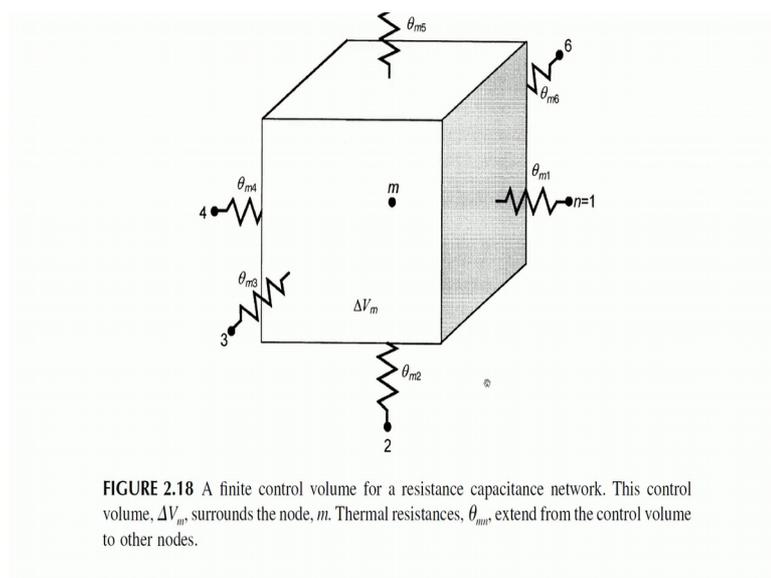


FIGURE 2.18 A finite control volume for a resistance capacitance network. This control volume, ΔV_m , surrounds the node, m . Thermal resistances, θ_{mn} , extend from the control volume to other nodes.

Resistance capacity or network, so we have a huge amount of what do you call.

(Refer Slide Time: 24:34)

TABLE 2.6
Internal Nodal Resistance Equations for Cartesian, Cylindrical, and Spherical Coordinate Systems

	Coordinate System		
	Cartesian	Cylindrical	Spherical
Coordinates used	x, y, z	r, ϕ, z	r, ϕ, ψ
Volume element	$\Delta x \Delta y \Delta z$	$r_m \Delta r \Delta \phi \Delta z$	$r_m^2 \sin \psi \Delta r \Delta \phi \Delta \psi$
θ_{m+}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta r}{\left(r_m + \frac{\Delta r}{2}\right) \Delta \phi \Delta z k}$	$\frac{\Delta r}{\left(r_m + \frac{\Delta r}{2}\right)^2 \sin \psi \Delta \phi \Delta \psi k}$
θ_{m-}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta r}{\left(r_m - \frac{\Delta r}{2}\right) \Delta \phi \Delta z k}$	$\frac{\Delta r}{\left(r_m - \frac{\Delta r}{2}\right)^2 \sin \psi \Delta \phi \Delta \psi k}$
θ_{e+}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{r_m \Delta \phi}{\Delta r \Delta z k}$	$\frac{\Delta \phi \sin \psi}{\Delta r \Delta \psi k}$
θ_{e-}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{r_m \Delta \phi}{\Delta r \Delta z k}$	$\frac{\Delta \phi \sin \psi}{\Delta r \Delta \psi k}$
θ_{t+}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta z}{r_m \Delta \phi \Delta r k}$	$\frac{\Delta \psi}{\sin\left(\psi + \frac{\Delta \psi}{2}\right) \Delta r \Delta \phi k}$
θ_{t-}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta z}{r_m \Delta \phi \Delta r k}$	$\frac{\Delta \psi}{\sin\left(\psi - \frac{\Delta \psi}{2}\right) \Delta r \Delta \phi k}$

Calculations that are possible nodal resistance equation cylindrical and spherical coordinate systems.

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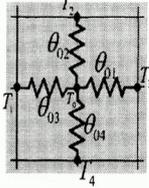
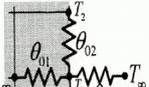
Coordinates used	x, y, z	r, ϕ, z	r, ϕ, ψ
Volume element	$\Delta x \Delta y \Delta z$	$r_m \Delta r \Delta \phi \Delta z$	$r_m^2 \sin \psi \Delta r \Delta \phi \Delta \psi$
θ_{m+}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta r}{\left(r_m + \frac{\Delta r}{2}\right) \Delta \phi \Delta z k}$	$\frac{\Delta r}{\left(r_m + \frac{\Delta r}{2}\right)^2 \sin \psi \Delta \phi \Delta \psi k}$
θ_{m-}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta r}{\left(r_m - \frac{\Delta r}{2}\right) \Delta \phi \Delta z k}$	$\frac{\Delta r}{\left(r_m - \frac{\Delta r}{2}\right)^2 \sin \psi \Delta \phi \Delta \psi k}$
θ_{e+}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{r_m \Delta \phi}{\Delta r \Delta z k}$	$\frac{\Delta \phi \sin \psi}{\Delta r \Delta \psi k}$
θ_{e-}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{r_m \Delta \phi}{\Delta r \Delta z k}$	$\frac{\Delta \phi \sin \psi}{\Delta r \Delta \psi k}$
θ_{t+}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta z}{r_m \Delta \phi \Delta r k}$	$\frac{\Delta \psi}{\sin\left(\psi + \frac{\Delta \psi}{2}\right) \Delta r \Delta \phi k}$
θ_{t-}	$\frac{\Delta x}{\Delta y \Delta z k}$	$\frac{\Delta z}{r_m \Delta \phi \Delta r k}$	$\frac{\Delta \psi}{\sin\left(\psi - \frac{\Delta \psi}{2}\right) \Delta r \Delta \phi k}$

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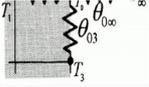
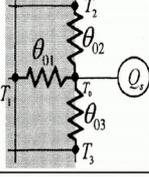
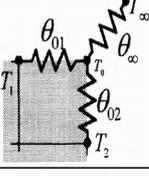
TABLE 2.7

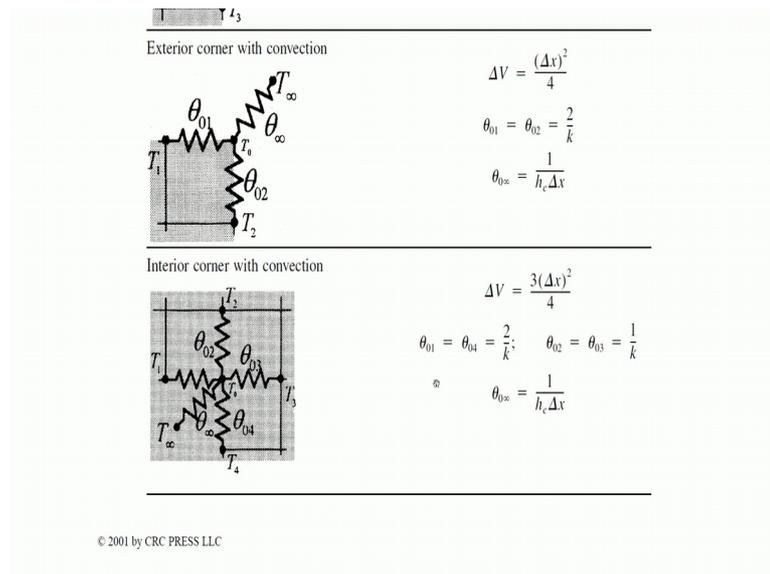
Control Volumes and Resistances for Various Two-Dimensional Boundary Conditions

Configuration	Control Volume and Resistances
<p>Interior node</p> 	$\Delta V = (\Delta x)^2$ $\theta_{01} = \theta_{02} = \theta_{03} = \theta_{04} = \frac{1}{k}$
<p>Plane surface with convection</p> 	$\Delta V = \frac{(\Delta x)^2}{2}$ $\theta_{01} = \frac{1}{k}; \quad \theta_{02} = \theta_{03} = \frac{2}{k}$

It is very much possible for.

(Refer Slide Time: 24:44)

	$\theta_{0\infty} = \frac{1}{h_c \Delta x}$
<p>Plane surface with unknown surface heat flux</p> 	$\Delta V = \frac{(\Delta x)^2}{2}$ $\theta_{01} = \frac{1}{k}; \quad \theta_{02} = \theta_{03} = \frac{2}{k}$ $\dot{Q}_s = q_s \Delta x$
<p>Exterior corner with convection</p> 	$\Delta V = \frac{(\Delta x)^2}{4}$ $\theta_{01} = \theta_{02} = \frac{2}{k}$ $\theta_{0\infty} = \frac{1}{h_c \Delta x}$
<p>Interior corner with convection</p> 	$\Delta V = \frac{3(\Delta x)^2}{4}$



To evaluate this thing saying in case of you know you have interior corners.

(Refer Slide Time: 24:50)

2.4.1 LUMPED CAPACITANCE METHOD

Several methods are commonly used to find the solution to transient conduction problems. One approach used quite often with good results is the lumped capacitance method.

Consider a microprocessor in a powered computer system. The microprocessor operates at a steady-state temperature of T_i . We turn off the system and the microprocessor begins to cool and approach T_∞ . If we turn off the computer at $t = 0$, the temperature of the device will decrease until it reaches T_∞ , at $t > 0$. To use the lumped capacitance method we assume that the temperature of the device is uniform. Although this assumption is not always accurate, the level of accuracy can be determined.

Consider the thick plane wall shown in Figure 2.19. One surface is at $T_{s,1}$. The opposite surface is in a coolant media having a temperature of T_∞ . Therefore, the opposite surface is at some temperature between $T_{s,1}$ and T_∞ , which we will call $T_{s,2}$. We can describe the steady-state surface energy balance by the equation

$$\frac{kA_s}{L}(T_{s,1} - T_{s,2}) = \bar{h}A_s(T_{s,2} - T_\infty)$$

Lumped capacitance problem a method to considered a microprocessor in a powered computer system, operates in a steady state temperature we turn off the system when the microprocessor begins to cool we turn off the computer how the temperature decreases and so on.

(Refer Slide Time: 25:11)

opposite surface is at some temperature between $T_{s,1}$ and $T_{s,2}$, which we will call $T_{s,2}$. We can describe the steady-state surface energy balance by the equation

$$\frac{kA_s}{L}(T_{s,1} - T_{s,2}) = \bar{h}A_s(T_{s,2} - T_\infty)$$

where k is the thermal conductivity of the solid object. We then rearrange the equation to find the temperature characteristics:

$$\frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_\infty} = \frac{\left(\frac{L}{kA_s}\right)}{\left(\frac{1}{\bar{h}A_s}\right)}$$

We then see that the equation is equal to the ratio of the conductive thermal resistance θ_{cond} to the convective thermal resistance θ_{conv} , and

$$\frac{\theta_{cond}}{\theta_{conv}} = \frac{\bar{h}L_c}{k} = Bi$$

So, we have here very much possible for us to evaluate.

(Refer Slide Time: 25:16)

θ_{cond} to the convective thermal resistance θ_{conv} , and

$$\frac{\theta_{cond}}{\theta_{conv}} = \frac{\bar{h}L_c}{k} = Bi$$

where:

$$L_c = V/A_s, m$$

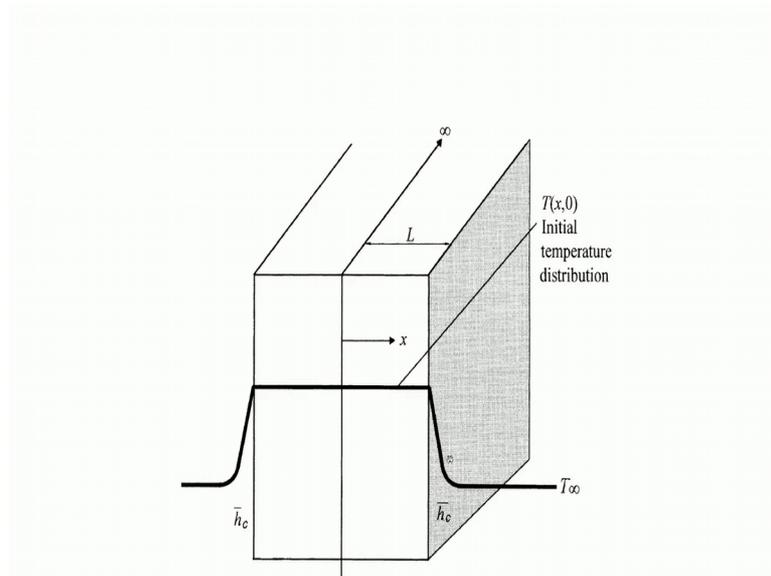
Bi = Biot number (dimensionless)

The Biot number is the basis for determining the validity of the lumped capacitance model of negligible internal temperature gradients. The Biot number is a measure of the temperature drop in the solid object relative to the temperature differential between the object's surface and the coolant media (see [Figure 2.20](#)). If $Bi \ll 1.0$, then the lumped capacitance model is valid, and we assume the internal temperature distribution within the solid object is uniform. For a simple object such

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And come about numbers like this you understand these have been derived from both the analytical thing and observed the biot number is the basis for determining the validity of the lumped capacitance model of negligible internal temperature is a measure of the temperature, drop in the relative to the temperature differential between the object surface if it is less than one lumped capacitance model valid and if we assume the internal temperature within the uniform.

(Refer Slide Time: 25:53)



So, they have given.

(Refer Slide Time: 25:55)

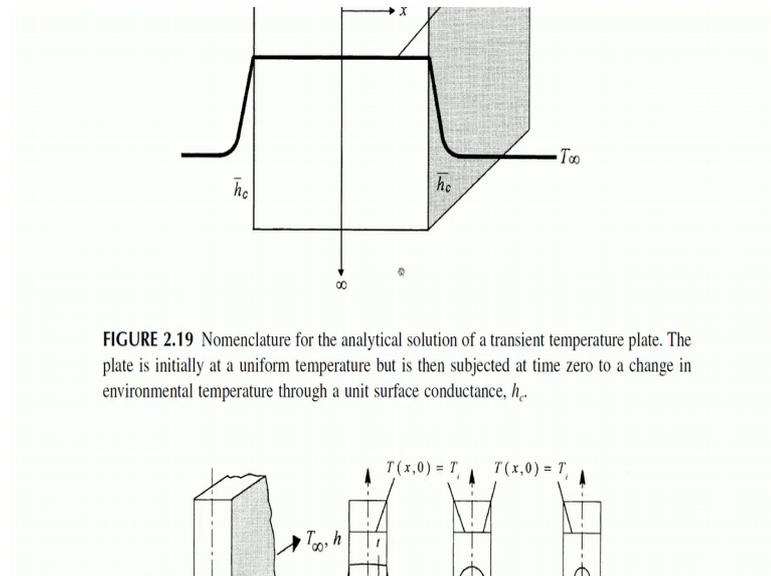
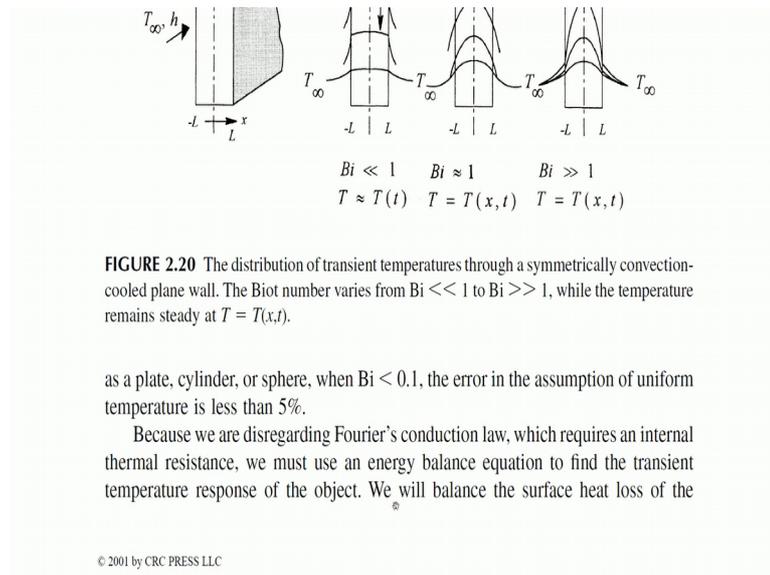


FIGURE 2.19 Nomenclature for the analytical solution of a transient temperature plate. The plate is initially at a uniform temperature but is then subjected at time zero to a change in environmental temperature through a unit surface conductance, \bar{h}_c .

Rather things are available for us.

(Refer Slide Time: 25:59)



For several of these things here.

(Refer Slide Time: 26:04)

object to the rate of change of the object's internal energy. This can be written

$$-\bar{h}A_s(T - T_\infty) = \rho Vc \frac{dT}{dt}$$

where:

- \bar{h} = average heat transfer coefficient, $W/m^2 K$
- A_s = surface area of object, m^2
- T = average temperature of object, K
- T_∞ = ambient temperature, K
- ρ = density of object, kg
- V = volume of object, m^3
- c = specific heat of object, $J/kg K$
- dT = temperature change, K , during time period dt , s

After separating the variables and integrating, we can find the temperature of an object at a point in time, t :

®

I will go through very fast we will get come back to it later on.

(Refer Slide Time: 26:12)

2.5 CONDUCTION IN EXTENDED SURFACES

One of the most widely used methods to solve problems in heat transfer is the extended surface. The extended fins of heat sinks are available in a variety of sizes, shapes, and forms. Usually constructed of aluminum, extended fin arrays are popular in low-cost extrusions and in higher-performance and more costly bonded configurations. It is important when evaluating fin designs to decide whether a simple fin shape will suffice or whether a more expensive fin cross section is needed. In a subsequent chapter we will explore convection, but in this chapter we will investigate the heat conductance through the most popular fin cross sections.

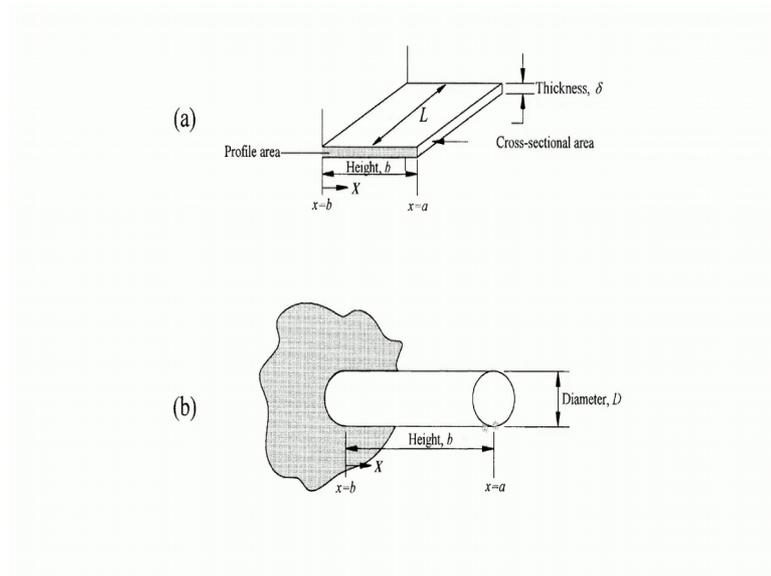
There are three basic fin shapes: longitudinal, spine, and annular. These three fin configurations are shown in Figure 2.21. The longitudinal fin, sometimes called a straight or plate fin, has one dimension, L , that is in the direction of flow and is usually greater than the other dimensions. The spine fin may be circular or any other shape. If the fin is circular, the thickness is the diameter, D . If the fin is another



The extended fin of heat sinks are available in a variety of sizes, these are from here onwards you know little more practical things, constructed of aluminum extended in eras are popular in low cost extrusion in higher performance, and more costly bonded configurations when evaluating fin designs to decide, whether simple fin shape will suffice or more expensive cross section is needed in a subsequent we will explore convection, but in this chapter we will investigate the heat conductance. So, the most popular fin cross sections.

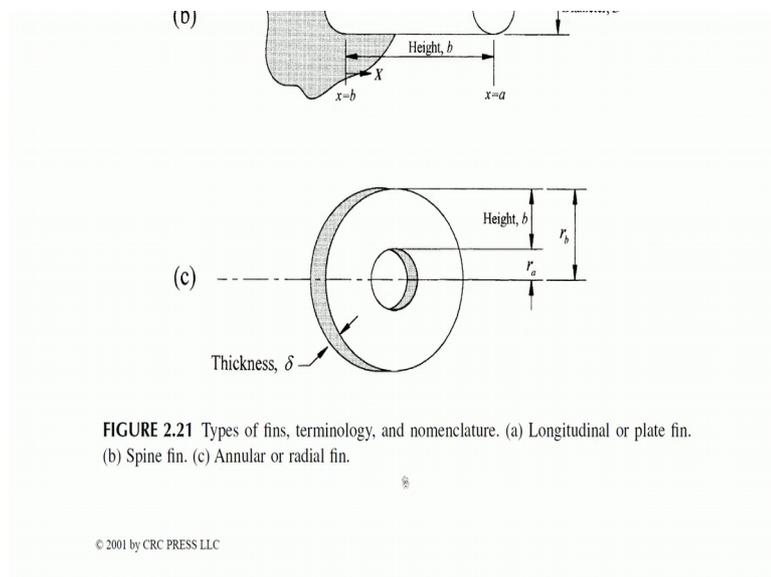
The very first lecture if you remember I have shown you yes I mean a long rod like thing. So, you have this long rod at one side there is something and then there is a heat generated and the other side it goes through the method we are thinking about as we go in.

(Refer Slide Time: 27:29)



You see we have this profile area the longitudinal fin called as straighter is has one dimension l that is in the direction of flow and is usually greater than the other dimension spine fin may be a circular.

(Refer Slide Time: 27:48)



So we have only is various types of fins that are there longitudinal spine fin annular a radiant fin.

(Refer Slide Time: 27:56)

shape, the length, L , and thickness, δ , are usually similar in value. The annular fin, also called the radial fin, follows a curved surface. The height, b , of this fin is the difference between the outer radius, r_o , and the inner radius, r_i .

In a fin of finite height, optimizing the fin geometry is necessary so that we conduct the maximum heat from the wall to the tip of the fin. Beyond this, we should maximize the conductance of the fin throughout the height of the fin. We can accomplish this using different cross sections. If the fin consists of a highly conductive material, we will enhance the temperature gradient along the fin and the heat characteristics of the wall. We accomplish maximum cooling of the wall if the entire fin is at the same temperature as the wall. We must weigh the shape, size, and material of a fin against the cost of the array to meet this goal.

2.5.1 FIN EFFICIENCY

We call the effectiveness of a fin the fin efficiency, η_f . We define fin efficiency as the ratio of the actual amount of heat transferred by the fin to the heat that we may have transferred if the entire fin was at the wall temperature. We can describe the fin efficiency for a circular spine fin, normally called a pin-fin, as:

The annular fin also called the radial fin follows a curved surface in a fin of finite height optimizing the fin geometry is necessary. So, that we conduct the maximum heat from the wall to the tip of the fin we should maximum the conductance of the fin throughout the height of the fin, we can accomplish by using different cross sections as we come down, you have see here we can accomplish maximum cooling of the wall if the entire fin as at the same temperature as the wall we must weigh the shape size and material of a fin against the cost of the array to meet the goal.

We invariably end up with fin efficiency it does not a say if we keep on randomly increasing the length of the fin or the depth of the cuts, though we have more area only the initial about maybe a one third of the fin will be as effective as we have thought about the remaining two thirds the heat doesn't reach that surface. So, there is no temperature differential from the rest of the heats in surface to the ambient especially if there is a force convection using a fan this is the one I have tried to show you earlier. So, if you go down here we will notice.

(Refer Slide Time: 29:32)

Most practical extruded rectangular fins are long and wide and have a thin cross section, where $\frac{P}{A_c} \cong \frac{2}{\delta}$. This simplification maintains the surface area from which the heat is lost. The fin efficiency for this simplification becomes:

$$\eta_f = \frac{\tanh \sqrt{\frac{2h_c b_c^2}{k\delta}}}{\sqrt{\frac{2h_c b_c^2}{k\delta}}}$$

where:

b = height of fin, m

$b_c = (b + \delta/2)$, m

h_c = average heat conductance, W/m² K

k = thermal conductivity of fin material, W/m K

D = fin diameter, m

δ = fin thickness, m

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In the given efficiency average heat conductance and so on, fin diameter and film thickness it can be easily evaluated because extruded rectangular fins are long and wide and I have a thin cross section, so you will notice.

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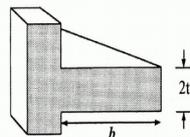
TABLE 2.8
Efficiency Factors for Fins of Various Shapes

Fin Configuration

η_f

Longitudinal fin of rectangular profile

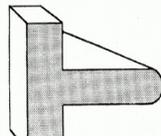
$$\eta_f = \frac{1}{mb} \tanh mb$$



$y = t$

Longitudinal fin of parabolic profile

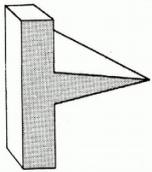
$$\eta_f = \frac{1}{mb} \frac{I_{2/3}(\frac{4}{3}mb)}{I_{-1/3}(\frac{4}{3}mb)}$$



Once again just like we have efficiency factors of fin shapes a lot depends on b and thickness.

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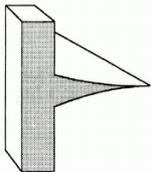
Longitudinal fin of triangular profile



$$y = t \left(\frac{1-x}{b} \right)$$

$$\eta_f = \frac{1}{mb} \frac{I_1(2mb)}{I_0(2mb)}$$

Longitudinal fin of parabolic profile



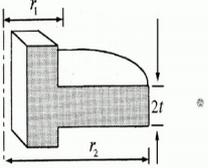
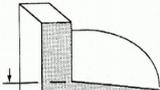
$$y = t \left(\frac{1-x}{b} \right)^2$$

$$\eta_f = \frac{2}{\left[\sqrt{4(mb)^2 + 1} \right] + 1}$$

So, of a parabolic profile, rectangular profile, a triangular profile though for simplification is put like that we expect that this whole thing is a parabola like this in the case of a triangular profile yes in this here, I am sorry its rectangular with a little bit of circular thing at the side and then this is the actual parabolic, which comes into a small thin shape how will these things can be calculated.

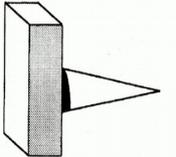
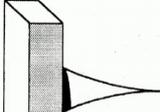
In reality you often find this and occasionally find this. So, instead of its being reduced directly into a sharp what do you call point usually you have a trapezoidal thing which I have tried to show you yesterday.

(Refer Slide Time: 31:16)

TABLE 2.8 (continued) Efficiency Factors for Fins of Various Shapes	
Fin Configuration	η_f
Annular fin of rectangular profile  $y = t$	$\eta_f = \frac{\frac{2r_1}{m} K_1(mr_1)I_1(mr_2) - I_1(mr_1)K_1(mr_2)}{r_2^2 - r_1^2 K_0(mr_1)I_1(mr_2) + I_0(mr_1)K_1(mr_2)}$
Annular fin of hyperbolic profile 	$\eta_f = -\frac{\frac{2r_1}{m} I_{2/3}(\frac{2}{3}mr_1)I_{-2/3}(\frac{2}{3}mr_2\sqrt{\frac{r_2}{r_1}}) - I_{2/3}(\frac{2}{3}mr_2\sqrt{\frac{r_2}{r_1}})I_{-2/3}(\frac{2}{3}mr_1)}{r_2 + r_1 I_{1/3}(\frac{2}{3}mr_1)I_{2/3}(\frac{2}{3}mr_2\sqrt{\frac{r_2}{r_1}}) - I_{-2/3}(\frac{2}{3}mr_2\sqrt{\frac{r_2}{r_1}})I_{-1/3}(\frac{2}{3}mr_1)}$

So, in the case of annular fin and so on similarly.

(Refer Slide Time: 31:26)

TABLE 2.8 (continued) Efficiency Factors for Fins of Various Shapes	
Fin Configuration	η_f
Spine fin of triangular profile  $y = t\left(\frac{1-x}{b}\right)$	$\eta_f = \frac{4 I_2[2(\sqrt{2})mb]}{2(\sqrt{2})mb I_1[2(\sqrt{2})mb]}$
Spine fin of parabolic profile 	$\eta_f = \frac{2}{\left[\frac{8}{3}(mb)^2 + 1\right] + 1}$

You have seen this this is a the reference surface a spine. So, if you are talking about pin fins you have spine fin or a triangular profile a.

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$y = t\left(\frac{1-x}{b}\right)$

Spine fin of parabolic profile

$y = t\left(\frac{1-x}{b}\right)^2$

$$\eta_f = \frac{2}{\left[\frac{8}{\sqrt{3}}(mb)^2 + 1\right] + 1}$$

Where : $m = \sqrt{\frac{2h_c}{k\delta}}$

Parabolic profile, I haven't seen these things it is rare one of the first reasons being these make sense only if you have the other side this is while this is conduction other side as a natural convective thing, when you have a what you call forced convection usually the dealing with it is easier and is very complicated to make which such things.

So, generally most of them they have probably an aluminum or copper block and then fins are milled off, and to make it easier it is extruded probably in one direction and then it is cut inside. So, arrays of rectangular or you know square fins are more common.

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extrusions, the amount of material a greater bearing on cost than fin cross-sectional complexity.

Usually, placing fins on the side of a heat exchanger where the heat transfer coefficient is lower is more desirable. Thin, closely spaced fins are generally more efficient than other patterns, and the fins should be constructed of a highly thermally conductive material. We prefer fins that are an integral part of the wall to fins that are bonded to a wall because of the penalty of contact resistance at the thermal interface.

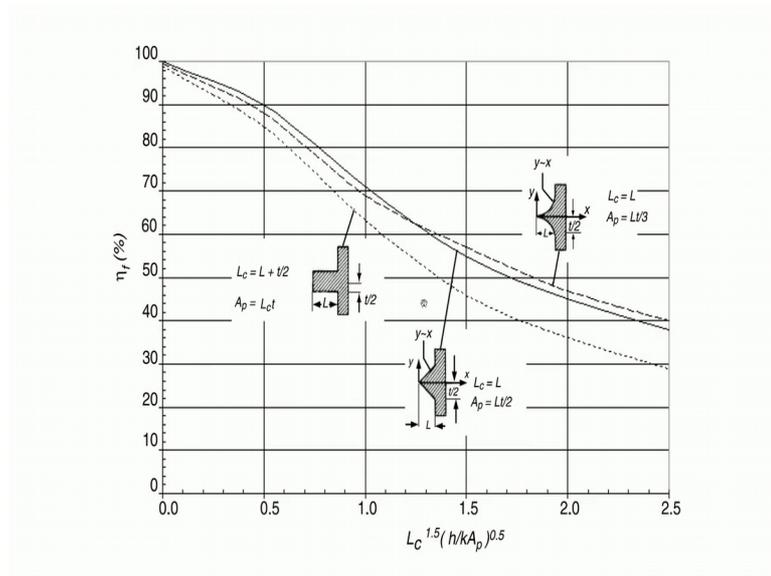
2.5.2 FIN OPTIMIZATION

For longitudinal fins of rectangular profile and for longitudinal fins of triangular profile, we can solve the efficiency equations to yield an optimum fin. There is a specific value of fin height, b , and fin thickness, δ , that results in maximum heat dissipation for a given area. For the longitudinal fin of rectangular profile, the heat flow through the base is found by

$$q_0 = k\delta L m \Delta T_0 \tanh mb$$

Longitudinal fins of rectangular profile and fins of triangular profile all the equations.

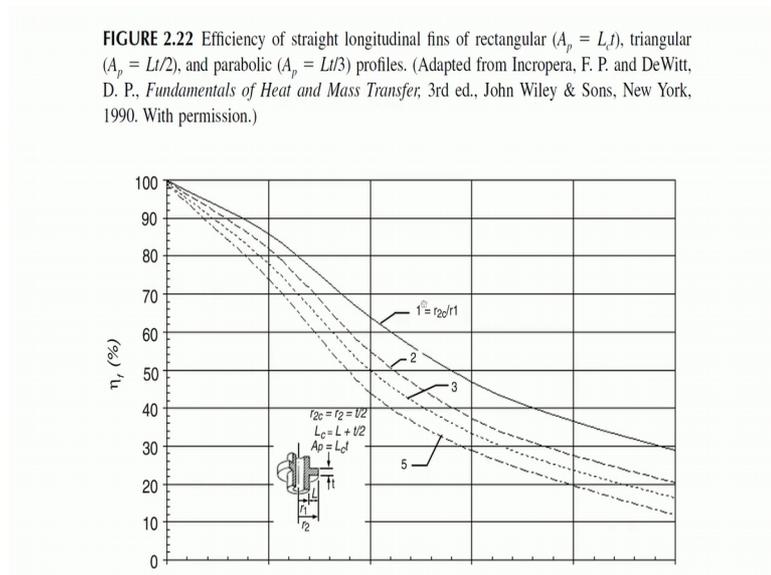
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You know these things have all been given here.

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FIGURE 2.22 Efficiency of straight longitudinal fins of rectangular ($A_p = Lt$), triangular ($A_p = Lt/2$), and parabolic ($A_p = Lt/3$) profiles. (Adapted from Incropera, F. P. and DeWitt, D. P., *Fundamentals of Heat and Mass Transfer*, 3rd ed., John Wiley & Sons, New York, 1990. With permission.)



So, I suggest you check see he himself has acknowledged that fundamentals of heat transfer, from f b what you call and these people they have given all these things nicely solved here it is for you to what you call see if it can use them.

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equal to a single fin eight times as large.

2.5.3 FIN SURFACE EFFICIENCY

Besides the fin efficiency, η , there is also a term for overall fin surface efficiency, η_0 . The fin efficiency characterizes the conductive efficiency of a single fin, while the overall fin surface efficiency characterizes an array of fins and the attachment surface. If the area of the fin array is separated into a finned area and a base area, we see that $A_{tot} = A_f + A_b$, and we can describe the overall surface efficiency as

$$\eta_0 = \frac{q_{tot}}{q_{max}} = \frac{q_{tot}}{h_c A_{tot} \Delta T_b}$$

where:

q_{tot} = total rate of heat transfer, W

q_{max} = maximum possible rate of heat transfer, $A_s = T_b$, W

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But in general these are not these are these are only used when you want to create a new configuration or make a optimum thing as in the case of probably spacecraft, because it is not easy for us to have a generic solution and each is unique each what you call spacecraft and condition is unique. So, it is worthwhile doing all these calculations for them.

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h_c = convective heat transfer coefficient, W/m² K

A_{tot} = total surface area of fins and base, m²

ΔT_b = temperature difference between fin base and ambient media, K

Using the separate values of the fin area, A_f , and the fin base area, A_b , the overall fin surface efficiency can be expressed by

$$\eta_0 = 1 - \frac{A_f}{A_{tot}}(1 - \eta_f)$$

2.6 THERMAL CONTACT RESISTANCE IN ELECTRONIC EQUIPMENT INTERFACES

Most electronic applications have at least one interface where heat flow must cross

So, we have the about the surface efficiency.

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2.6 THERMAL CONTACT RESISTANCE IN ELECTRONIC EQUIPMENT INTERFACES

Most electronic applications have at least one interface where heat flow must cross between two surfaces. At each interface there is a measurable temperature difference across the joint. This can occur dramatically within two rough surfaces under light joining pressure, or slightly in the contact between a device soldered to a heat sink, but it still exists. Each of these contacts contributes to the overall thermal contact resistance. Together, these additional resistances can cause excessive component temperatures. Altoz¹³ estimates that when we join seemingly identical cross-sectional-area components, only 5% of the apparent surface areas actually make intimate physical contact. We cannot ignore thermal contact resistances, but they are very difficult to quantify. No accurate models exist that are applicable across the range of electronic packaging applications.

2.6.1 SIMPLIFIED CONTACT RESISTANCE MODEL

Consider that heat flows from a microprocessor into a heat sink, both of which are in intimate contact. We assume that heat flow occurs in the axial direction, x , only.

And the contact resistance model, so I will stop my lecture at this point and we will meet again starting from electronic equipment interfaces. So, far things have been very general saying in everything including in the case of as I said no reactors, or in the case of automobile or in the case of other heat transfer all those are valid when we come to this electronic equipment things are slightly different in that the conditions are known if I said take a TO-220, you know typically how it would TO-220, behaves similarly if you take a diode package or if you take IGBT the internal constructions are known so.

Thank you.