

**Mathematical Methods and Techniques in Signal Processing - I**  
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**Lecture – 43**  
**Problem on designing synthesis bank filters**

So, let us have some interactive problem-solving sessions by my students who have taken this course. Ah so you will see some illustrations and examples into problem solving ah, which is useful to understand and digest the concepts learnt during the lectures.

So, hello everyone I am Priya Jain Canio PhD student at IISC. Ah today we will solve a problem which came in the exam 1 of the MMTSP course in 2015.

. So, basically we have a structure here which is basically a 3-channel synthesis bank with 3 filters.

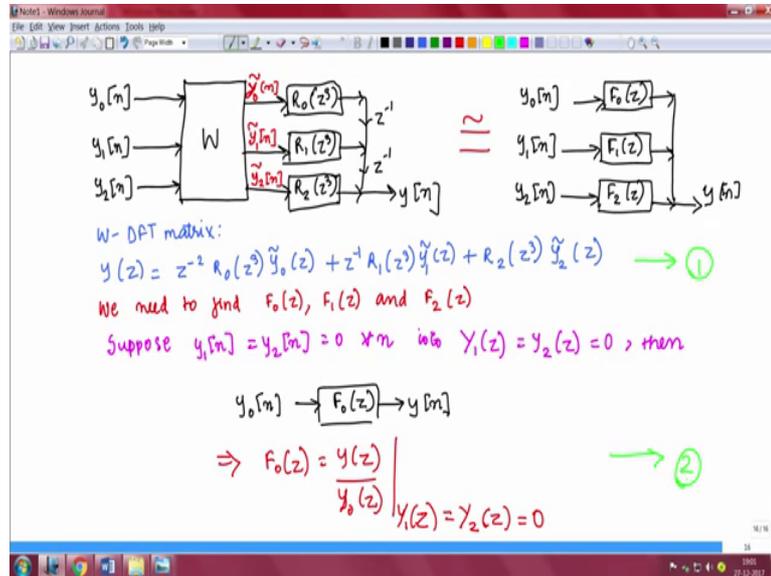
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**PROBLEM: (MMTSP 2015, Exam 1)**  
 Consider the structure shown in the figure below, where  $W$  is the  $3 \times 3$  DFT matrix. There is a three channel synthesis bank with three filters  $F_0(z)$ ,  $F_1(z)$  and  $F_2(z)$ . Assuming  $R_0(z) = 1 + z^{-1}$ ,  $R_1(z) = 1 - z^{-2}$ ,  $R_2(z) = 2 + 3z^{-1}$ . Find an expression for the three synthesis filters  $F_0(z)$ ,  $F_1(z)$  and  $F_2(z)$ .

So, which is given by  $F$  naught  $F_1$  and  $F_2$ , there is an equivalent structure of it given using a DFT matrix  $R$  naught of  $z$ ,  $R_1$  of  $z$  cube and  $R_2$  of  $z$  cube. And in the question it has been meant  $R$  naught of  $z$ , this is been given as  $1 + z^{-1}$   $R_1$  of  $z$  has been given as  $1 - z^{-2}$ ,  $R_2$  of  $z$  is  $2 + z^{-1}$  and what we

need to do is find the expressions for the 3 synthesis filters which is  $F_0(z)$ ,  $F_1(z)$  and  $F_2(z)$ .

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So, let us first understand the question. So, this is given to us and on the other hand the same thing can be represented as a synthesis bank with 3 filters  $F_0(z)$ ,  $F_1(z)$  and  $F_2(z)$ . And we need to get the representation of  $F_0(z)$ ,  $F_1(z)$  and  $F_2(z)$ . So, let us solve it. So, from this particular figure which has this DFT matrix  $W$  basically is the DFT matrix here. We obtain  $y(z)$  is equal to  $z^{-2} R_0(z^3) \tilde{y}_0(z) + z^{-1} R_1(z^3) \tilde{y}_1(z) + R_2(z^3) \tilde{y}_2(z)$ .

So, this is basically obtained by considering these signals here and then multiplying them by the respective filters impulse response, and then considering these delays. So now, we need to find out  $F_0(z)$ ,  $F_1(z)$  and  $F_2(z)$ . So, we can see that so let us first write down what if  $F_1(z)$  and  $F_2(z)$ .

Now, suppose  $y_1[n] = y_2[n] = 0$  for all  $n$ . That that is  $Y_1(z) = Y_2(z) = 0$ . Then the system reduces to this  $y[n]$ ,  $F_0(z)$  and  $y_0[n]$ . So, therefore, we have  $F_0(z)$  in this case can be written as  $y(z)$  by  $y_0(z)$ , given  $Y_1(z) = Y_2(z) = 0$ . So, given that  $Y_1$  and  $Y_2$  are 0 we can write  $F_0(z)$  in this format.

So, let us mark this as let us mark this equation as number 1 and this is number 2.

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On similar terms,

$$F_1(z) = \frac{y(z)}{y_1(z)} \Big|_{y_2(z)=y_0(z)=0} \rightarrow \textcircled{3}$$

$$F_2(z) = \frac{y(z)}{y_2(z)} \Big|_{y_0(z)=y_1(z)=0} \rightarrow \textcircled{4}$$

From the DFT circuit, we have

$$\begin{bmatrix} \tilde{y}_0(z) \\ \tilde{y}_1(z) \\ \tilde{y}_2(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix} \begin{bmatrix} y_0(z) \\ y_1(z) \\ y_2(z) \end{bmatrix} \rightarrow \textcircled{5}$$

3 pt DFT

$\omega = e^{-j2\pi/3}$   
 $\Rightarrow \omega^3 = 1 \Rightarrow \omega^4 = \omega$

So, on similar terms we write  $F_1(z)$  is equal to  $y(z)$  by  $y_1(z)$ , when  $y_2(z)$  equals  $y_0(z)$  equals to 0. And  $F_2(z)$  equals to  $y(z)$  by  $y_2(z)$  given  $y_0(z)$  equals  $y_1(z)$  equals to 0.

So, let us label this 3 and 4. So now, going back from this particular part let us obtain the equation. So, from the DFT circuit we have  $\tilde{y}_0(z)$ ,  $\tilde{y}_1(z)$  and  $\tilde{y}_2(z)$  which is basically the output from the DFT block is equal to  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}$  and then put  $y_0(z)$ ,  $y_1(z)$ ,  $y_2(z)$ .

So, basically this is 3-point DFT. So, let us label this as 5, one more thing to mention is that  $\omega$  here is  $e^{-j2\pi/3}$ . So, this basically implies  $\omega^3 = 1$  and  $\omega^4$  is nothing but  $\omega$ .

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$$F_0(z) : \text{Set } y_1(z) = y_2(z) = 0 \text{ in equation (5).}$$

$$\Rightarrow \tilde{y}_0(z) = \tilde{y}_1(z) = \tilde{y}_2(z) = 1$$
 Using (1) & (2) we get
 
$$F_0(z) = z^{-2}R_0(z^3) + z^{-1}R_1(z^3) + R_2(z^3)$$

$$= z^{-2}(1+z^3) + z^{-1}(1-z^{-4}) + 2+3z^{-1}$$

$$= 2 + z^{-1} + z^{-2} + 3z^{-3} + z^{-5} - z^{-7}$$

$$F_1(z) : \text{Set } y_0(z) = y_2(z) = 0 \text{ \& we obtain from eq. 5}$$

$$\tilde{y}_0(z) = 1, \tilde{y}_1(z) = \omega, \tilde{y}_2(z) = \omega^2$$
 Using (1) & (3), we get
 
$$F_1(z) = z^{-2}R_0(z^3) + z^{-1}R_1(z^3)\omega + R_2(z^3)\omega^2$$

$$= z^{-2}(1+z^3) + z^{-1}\omega(1-z^{-4}) + \omega^2(2+3z^{-3})$$

$$= 2\omega^2 + \omega z^{-1} + z^{-2} + 3\omega^2 z^{-3} + z^{-5} - \omega z^{-7}$$

So, from all these equations let us compute  $F_0(z)$ ,  $F_1(z)$  and  $F_2(z)$ . So, first let us compute  $F_0(z)$ . So, we set  $y_1(z)$  equals  $y_2(z)$  to be equal to 0 in equation 5. So, this basically gives us  $\tilde{y}_0(z)$  equals  $\tilde{y}_1(z)$  equals  $\tilde{y}_2(z)$  is equal to 1.

So, further using the equations 1 and 2 we get  $F_0(z)$  to be  $z^{-2}R_0(z^3) + z^{-1}R_1(z^3) + R_2(z^3)$ . So, this is by replacing all these parameters into equation 1.

So, we know the values of  $R_0(z)$ ,  $R_1(z)$  and  $R_2(z)$  let us substitute that. So, by substituting that we get  $z^{-2}(1+z^3) + z^{-1}\omega(1-z^{-4}) + \omega^2(2+3z^{-3})$ .

So, on simplification we get this to be  $2 + z^{-1} + z^{-2} + 3z^{-3} + z^{-5} - z^{-7}$ . So, similarly let us compute  $F_1(z)$ . So, again here we set  $y_0(z)$  and  $y_2(z)$  to be equal to 0 and we obtain so basically from equation 5.  $\tilde{y}_0(z)$  is one  $\tilde{y}_1(z)$  is  $\omega$   $\tilde{y}_2(z)$  is  $\omega^2$ .

So, again using 1 and 3 we get,  $F_1(z)$  is equal to  $z^{-2}R_0(z^3) + z^{-1}R_1(z^3)\omega + R_2(z^3)\omega^2$ .

So, substituting again we get  $z^{-2} + z^{-3} + z^{-1} + \omega^{-1} z^{-6} + \omega^2 z^{-3} + 3z^{-3}$ , and simplifying that we get  $2\omega^2 z^{-3} + \omega z^{-1} + z^{-2} + 3\omega^2 z^{-3} + z^{-5} - \omega^2 z^{-7}$ .

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The image shows a handwritten derivation in a software window. The text is as follows:

$$F_2(z): y_0(z) = y_2(z) = 0$$

From eq (5),  $\tilde{y}_0(z) = 1, \tilde{y}_1(z) = \omega^2, \tilde{y}_2(z) = \omega$

using (1) & (4),

$$F_2(z) = z^{-2} R_0(z^3) + z^{-1} \omega^2 R_1(z^3) + \omega R_2(z^3)$$

$$= z^{-2} (1 + z^3) + z^{-1} \omega^2 (1 - z^6) + \omega (z + z^3)$$

$$= 2\omega + \omega^2 z^{-1} + z^{-2} + 3\omega z^{-3} + z^{-5} - \omega^2 z^{-7}$$

One observation:

$$F_1(z) = \omega^2 F_0(\omega z)$$

$$F_2(z) = \omega F_0(\omega^2 z)$$

So, similarly we can do it for  $F_2$  of  $z$ , where we put  $y_0$  of  $z$  to be equal to 0 and  $y_2$  of  $z$  to be equal to 0.

So, again so again from equation 5 we get  $\tilde{y}_0$  of  $z$  to be equal to 1,  $\tilde{y}_1$  of  $z$  to be equal to  $\omega^2$ ,  $\tilde{y}_2$  of  $z$  is equal to  $\omega$  and using 1 and 4 we obtain  $F_2$  of  $z$  is  $z^{-2} R_0$  of  $z^3$  plus  $z^{-1} \omega^2 R_1$  of  $z^3$  plus  $\omega R_2$  of  $z^3$ .

So, again substituting it back so on simplification we get  $2\omega + \omega^2 z^{-1} + z^{-2} + 3\omega z^{-3} + z^{-5} - \omega^2 z^{-7}$ . So, one observation is that  $F_1$  of  $z$  is equal to  $\omega^2 F_0$  of  $\omega z$  and  $F_2$  of  $z$  is equal to  $\omega F_0$  of  $\omega^2 z$ . So, therefore, by using all this we have obtained  $F_0, F_1$  and  $F_2$ .

Thank you.