INTELLIGENT CONTROL OF ROBOTIC SYSTEMS

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Lecture 03: Back Stepping Control

Good morning, everyone. Today, we are going to see a lecture on Backstepping control of robotic manipulators. So today, we will be seeing the introduction of backstepping controller, and then we will use an example which is a single-link robot manipulator.

Then, we will be moving to a special case of backstepping, which is called integrator backstepping. And we will be seeing one example using the design by integrator backstepping. So, coming to the introduction part of backstepping, in the year 1990, backstepping was designed for the purpose of developing a stabilizing controller for a special case of nonlinear systems. The backstepping approach provides a recursive method for stabilizing the origin of a system in its strict feedback form. It provides a recursive method for stabilizing the origin of a system in its strict feedback form. That is, I here. Okay, so we will see the design of the backstepping approach now. So, now consider, let us consider a second-order nonlinear system in its strict feedback form given by

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)\,x_2$$

and

$$\dot{x}_2 = f_2(x_1, x_2) + u \longrightarrow (2)$$

Okay, so these are the two subsystems forming the strict feedback form of a second-order nonlinear system.

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)\,x_2$$
$$\dot{x}_2 = f_2(x_1, x_2) + u \longrightarrow (2)$$

So, in the backstepping approach, the variable x_2 in the subsystem 1 is considered virtual control and is computed to make this subsystem, which is given by equation 1, stable. So,

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)\, x_2$$ subsystem 1.

In which x_2 is the virtual control that will make this first subsystem stable. So let us have this control be x_{2d}. We termed it as x_{2d}. Okay. And the control action u is computed so that the second subsystem is stabilized, as well as the state x_2 follows x_{2d}.

x_2 is meant for stabilizing the system, which means that the state variable x_1 will follow and reach x_{1d}, the desired value for state x_1. x_{1d} is the desired value for state x_1. Similarly, x_{2d} is the desired value for for the x_2 state. So the first statement. It is possible by having a control. Input. Which is given by x_{2d}. Because we consider x_2. As the virtual control.

Then for the second state x_2. To reach x_{2d}. We have the. Control input u to be designed. Now, with this concept, let us understand more with the help of an example of a single link manipulator. Let us consider the single link manipulator dynamics given by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -10 \sin(x_1) + u$$

So, we have the first subsystem, second subsystem. So, the purpose of this is to design, the objective of this is to design a back-stepping controller to stabilize the system above, the robotic system which is highly non-linear. So, the solution is we consider the X_2 state as the virtual control for the subsystem 1.

Let The error variable for subsystem 1 is given by $e_1 = x_1 - x_{1d}$

where x_{1d} is the desired value or desired output of subsystem 1. And if we choose The virtual control input as $x_2 = -k_1 e_1 + \dot{x}_{1d}$ Then equation 1 becomes, that is, subsystem 1 becomes $\dot{e}_1 = -k_1 e_1$

What do you mean by this system? This means that it is a linear system and also it is stable, which implies that for k_1 positive, we have e_1 tends to 0. e_1 becomes 0. Similarly, we have to see now for the second subsystem. So, thus the desired output for subsystem is 2 becomes

$x_{2d} = -k_1 e_1 + \dot{x}_{1d}$ and if the control input u to the second subsystem given by equation 2, that is, $u = +10 \sin(x_1) - k_2 e_2 + \dot{x}_{2d}$

If we choose this as a control input for the second subsystem given by equation 2, then equation 2 becomes, then the second subsystem becomes $\dot{e}_2 = -k_2 e_2$

which again implies that e_2 tends to 0 for positive k_2, that is, k_2 greater than 0 values. So, in other words, we can say that x_2 tends to x_{2d}.

Therefore, we also say x_1 tends to x_{1d}. So, this is the thing, and now let us see how to generalize this concept or the design of the backstepping controller. So, we are going to see how to generalize the backstepping control algorithm. So, let us consider a non-linear system, a general non-linear system given by

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)\, x_2$$
$$\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)\, x_3$$
$$\dot{x}_3 = f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)\, x_4$$

It goes up to $\dot{x}_m = f_m(x_1, x_2 \ldots, x_m) + g_m(x_1, x_2, \ldots x_m)\, u$

So, here we can see what are the x_m, x_i, and u. How can we say that? So, x_i belongs to the n-dimensional Euclidean space and where i equals 1 till m. Likewise, where x_i represents the state of the system, likewise we have u belonging to the equilibrium space of dimension n, which is a vector of control inputs, and the functions. The non-linear functions f_i and g_i belonging to n * n Euclidean space, where i equals 1 till m. These are non-linear functions that contain both parametric and non-parametric uncertainties. And g_i are known and invertible. So, about x_i, we have stated what x_i is.

It is basically the state of a system, with u_i being the control vector, and g_i and f_i are the non-linear functions that have both parametric and non-parametric uncertainties, where g_i is specifically known and invertible. So, the backstepping algorithm can be applied to a special case of non-linear systems as long as the internal dynamics of these systems are stabilized. Okay, in this method of control strategy, we need to first select a desirable value of x_2, possibly a function of x_1. So, in this method, first, we need to select a desirable value for x_2.

Obviously, the desirable value will be represented by x_{2d}, which is a function of x1, such that in the ideal system $\dot{x}_1 = f_1(x_1, x_{2d})$, we can have stable tracking of $x_1(t)$ towards x_{1d}.

By choosing a desired value for the x_2 state, we can ensure that state $x_1(t)$ can track the desired value x_{1d} in a stable manner. Next, Then, in the next step, choose x_{3d} for the state x_3 such that state variable x_2 tracks x_{2d}, and this process is repeated. That is why it is a recursive process.

Eventually, we need to select the control action u of t such that the last state x_m tracks x_{md}, the desired value for x_{md}. That is all. Now, let us talk about integrator backstepping. Integrator backstepping allows us to have Lyapunov function candidates in order to provide stable control of special cases of non-linear systems.

Integrated backstepping is a special case of backstepping controller design. The control input is connected to the system via an integrator or through a chain of integrators. Again, I repeat, integrated backstepping is a special case of backstepping controller design in which the control input is connected to the system via an integrator or a chain of integrators. Let us consider a system. A nonlinear system is given by $\dot{x}_1 = f(x) + g(x)\lambda$

where f (0) = 0 and $\dot{\lambda} = u$ where lambda is known to be a virtual control. Suppose that a control law given by u equals alpha of x stabilizes the system x dot equal to f of x plus g of x into u. We can define in continuation to this statement, suppose A control law given by u equals alpha of x stabilizes the system $\dot{x} = f(x) + g(x)u$

At the same time, we define an error variable z, which is the difference between the virtual control and the desired value of the control. The virtual control and the desired value of the control. Thus, the new variable in this case z, is given by $z = \lambda - \alpha(x)$

Let the Lyapunov function candidate for the system 1 be given by V(x). The Lyapunov function candidate for the system 1, which is represented by the equation $\dot{x} = f(x) + g(x)u$

In that case, let us consider the Lyapunov function for that subsystem being V(x). Let us also consider an augmented Lyapunov function candidate to be written as

$$V_a(x, z) = V(x) + \frac{1}{2}z^2$$

Now, taking the time derivative of this augmented Lyapunov function candidate will give

$$\dot{V}_a = \dot{V}(x) + z\dot{z}$$

which is again further expanded in the next line as. $= \frac{\partial V}{\partial x}(f + g\lambda) + z\left(u - \frac{\partial \alpha}{\partial x}(f + g\lambda)\right)$

We know what lambda is. $\lambda = z + \alpha(x)$ which can be taken from this equation. Next, substituting $\lambda = z + \alpha(x)$ in the above equation, we get

$$\dot{V}_a(x,z) = \frac{\partial V}{\partial x}\left(f + g\alpha + gz\right) + z\left(u - \frac{\partial \alpha}{\partial x}\left(f + g\alpha + gz\right)\right)$$

which can be given by $\frac{\partial V}{\partial x}\left(f + g\alpha\right) + z\left(u - \frac{\partial \alpha}{\partial x}\left(f + g\alpha + gz\right) + \frac{\partial V}{\partial x} g\right)$

I close it. So, this can lead to this inequality. $\le -W(x) + z\left(u - \frac{\partial \alpha}{\partial x}\left(f + g\alpha + gz\right) + \frac{\partial V}{\partial x} g\right).$

where thus V a dot of x comma Z is given by this expression, where the control signal u can be chosen such that. The derivative V A dot can become negative definite, be a negative definite. So, we have to choose a proper U in this expression so that V A dot becomes negative definite. Let us see an example to understand this concept of an integrator controller, an integrator backstepping controller, in a better way.

Let us see an example here. So, consider the following system: $\dot{x}_1 = -x_1^3 + x_2 \rightarrow (1)$

$\dot{x}_2 = x_2^2 + u \rightarrow (2)$

Now, design a suitable controller that can stabilize this system. So, we need to proceed by integrated backstepping controller to stabilize the system.

So, as per the solutions, we have the following steps. The solution contains a few steps, say step 1. So, in this problem, x_2 can be treated as the control input to subsystem 1. Now, define a virtual control alpha for system 1 and an error variable z such $z = x_2 - \alpha_2$

The solution starts with selecting x_2 as the control input to equation 1 or subsystem 1. Now, define a virtual control, which is given by α_2 for subsystem 1, and also define, in parallel, an error variable z such that z is the difference between the control input and the virtual controller. So, equation 1 becomes $\dot{x}_1 = -x_1^3 + z + \alpha_2$

Because I am representing equation 1 in terms of z, since $z = x_2 - \alpha_2$

Now, moving to step 2. Let us proceed to step 2. Define a Lyapunov function candidate for subsystem 1 such that $V(x_1) = \frac{1}{2}x_1^2$

This implies that
$$\begin{aligned}\dot{V}(x_1) &= x_1\dot{x}_1\\ &= x_1\left(-x_1^3 + z + \alpha\right)\end{aligned}$$

This can be equation number 4. Now, step 3 is to select a virtual control that is alpha, which is $\alpha = -k_1 x_1$ where $k_1 \geqq 0$

The step 3 is to select a virtual control such $\alpha = -k_1 x_1$ that where $k_1 \geqq 0$

So, this implies

$$\dot{V}(x_1) = -x_1^4 + x_1 z - k_1 x_1^2$$
$$= -x_1^4 - k_1 x_1^2 + x_1 z$$

I just exchanged the terms 2 and 3. Let us have this equation as equation number 5. And

$$\dot{\alpha} = -k_1(-x_1^3 + x_2)$$

That is the sixth equation.

Let us move on to step number four. So, here the virtual state equation, that is in terms of

$$\dot{z} = \dot{x}_2 - \dot{\alpha}$$
$$\Rightarrow x_2^2 + u - \dot{\alpha}$$

when we substitute, we have

$$= x_2^2 + u + k_1(-x_1^3 + x_2)$$

This is equation number 7.

Finally, we have step 5. The augmented, we have seen about the Lyapunov candidate for system 1, subsystem 1, and we also stated about the virtual equation with the alpha value. Now, we can see step 5, where we can have the augmented Lyapunov function candidate. Augmented Lyapunov Function candidate being chosen as

$$V_a(x, z) = V + \frac{1}{2} z^2$$

so, $\dot{V}_a = \dot{V} + z\dot{z}$

Upon substituting what is v dot z dot, we will be having

$$\dot{V}_a = -x_1^4 - k_1 x_1^2 + x_1 z + z\left(x_2^2 + u + k_1(-x_1^3 + x_2)\right)$$

So, this term is V dot. Likewise, this term, the second term Is coming out to be the z dot term.

$$\dot{V}_a = \left(-x_1^4 - k_1 x_1^2 + x_1 z\right) + z\left\{\left(x_2^2 + u + k_1(-x_1^3 + x_2)\right)\right\}$$

Another term, which can be further written as

$$= -x_1^4 - k_1 x_1^2 + z\left(x_1 + x_2^2 + u + k_1(-x_1^3 + x_2)\right)$$

Thus, step 6 will be targeting to select a proper u. So, u is to be chosen such that VA dot is negative definite, which will make the second order system stabilizable. Stabilizable. So. I will also give the simplest form of the control action u for our problem.

Example 1 is. Say $u = -k_2 z - (x_1 + x_2^2 + k_1(-x_1^3 + x_2))$

That's all. In this lecture. Now, come to the conclusion part. So, we have seen a simple design of the backstepping algorithm, and we have also seen in this lecture a special case of backstepping called Integrator Backstepping Controller, where we have used individual Lyapunov candidate functions for each subsystem in order to stabilize the entire non-linear system. Thank you very much for your patience. With this, I stop here.