INTELLIGENT CONTROL OF ROBOTIC SYSTEMS

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Lecture 20: Path Planning of Robotic Needle

Good morning, everyone. Today, we are going to attend a lecture on path planning of robotic needles and the relationship between angular velocity and orientation angle rates. The outline of today's lecture will be as follows. First, we have the introduction on path planning for smart robotic needles used in percutaneous interventions.

Then, we will see RRT-based multiple target reaching. Next, we will examine the bevel-tip flexible needle kinematic model. The kinematics of a bevel-tip flexible needle. There are two methods to derive the kinematic model of a bevel-tip flexible needle. Method 1 is based on non-holonomic constraint equations.

Method 2 is based on transformation matrices. Finally, we will explore the relationship between the angular velocity of the tip of a robotic system and the rate of change of orientation angles, namely alpha dot, beta dot, and gamma dot, which are the angles around the x, y, and z axes, respectively. Then, the relationship between the angle rates alpha dot, beta dot, and gamma dot and the angular velocity omega. First, let us review the introduction to path planning techniques.

Towards a needle system: smart robotic needle for minimally invasive surgery. Path planning is a very important aspect in needle insertion procedures. With the help of proper path planning methods, we can reach our desired location through the needle inside the tissue because once the needle is inserted, the target will deform its location due to tissue deformation, and the target will move instantaneously. Path planning will help improve the efficiency of needle insertion procedures significantly. Due to their non-holonomic kinematics inside biological tissue, path planning for these needles is complicated and requires

a great deal of care. So, the objective of path planning techniques for the application of minimally invasive surgery through a robotic smart needle is to find an optimal path to reach the goal location by avoiding anatomical obstacles in a dynamic environment. There are various classifications among the methods for path planning. So, in path

planning, the first method is the inverse kinematics method, where the approaches involved include discretization techniques, real-time finite element models, and geometrical motion planning algorithms with constant time. Sampling methods are another general approach where In path planning,

a sampling-based approach where the approaches falling under this category are probabilistic roadmap.

RRT. RRT star. And BIT star. Now, the third. Category.

In path planning approaches is that. It is a graph-based method. Where the approaches are A star. Adaptive. Fractal tree-based approach, and the fourth approach in the path planning approach or method in needle interventions is the learning-based approach calculation method of arc radius universal distribution, that is, Q-learning algorithm-based approach.

Now, coming to the rapidly exploring random tree approach, which has been used in our method here for needle intervention to reach multiple targets. So, let us have a brush-up of the RRT approach. It is an incremental sampling and searching approach-based method, and here, no tuning parameters are needed. It incrementally constructs the search tree. And the exploration here is in the region of interest.

Where the tree densely covers the space. Hence, this method is suitable for high-dimensional space. Using this method. Motion planning for both algebraic constraints arising from the obstacles, which are dynamic in nature, as well as the differential constraints arising from non-holonomics and dynamics of the robotic needle. So, the definition of RRT is that a dense sequence of samples is used as a guide in the incremental construction of a tree, and if the sequence here is random, then

The resulting tree is called a rapidly exploring random tree (RRT). Here, the main idea of this approach is to bias the exploration toward unexplored portions of the space. Let us see the working steps associated with the rapidly exploring random tree. Step 1 is A tree in the search space from the starting node.

Starting node: these are the nearby nodes already connected to the starting node. So, this is the starting tree. Pick a random node in the search space. Search for the nearest node in the tree from this random node. This is near or this is near.

Obviously, this is near. And extend from the nearest node toward the random node in small steps. Add the node as a new edge and vertex to this. Now this becomes a new edge, and we draw a line to connect it as a branch of the tree. Repeat steps 2 to 5 to explore the search space by extending more nodes of the tree.

And then finally, step 7 is executing the path. Let us see here. This is a simulation. We have the start node and the goal node. We have the obstacle here.

You see how it is growing branches to reach the target by repeating steps 2 to 5 again and again. We can finally reach the goal because the goal condition is met. If the tree has a node that is the goal node or within the tolerance of the goal node, the search process stops. We are getting close to the goal, and now we track back. To reach from the goal back to the initial node.

So this is the path obtained. Next we see how we have developed an approach to reach multiple targets in the search space for the percutaneous needle interventions towards cancerous treatment. So the point here is you can see that initially the needle is associated with the position and orientation. In order to find the closest neighbor, then needle is in the beginning, this is the needle tip.

Then in order to find the closest path point, Or the closest neighbor based on RRT. We need to provide. These three information. Which is.

Initialize the state of the needle. With the position. And orientation of the needle. Being the initial states of the needle. Then finding.

Obstacle-free position. If this is the case. Nearby point. In order to have the Nearby node.

We also need to see whether it is an obstacle-free location in space. And the generation of the reachable space. That means whether it can be reached or not. You found a node close by after a random node. Then we find which is closest, and that node is selected.

Now, this node and the initial starting node should not have any obstacle between them. So that test we are also doing here, and then we need to generate the path to reach the node that is a random node, and this generated path is feasible for the needle to Reach there in a curved manner. Because the needle is a flexible needle. At the same time.

It does not matter. Or it. It is not. A compulsion that. Given any curved trajectory.

The needle will pass through. Because of the geometry. And the mechanism of the needle. It may not be able to. Pass through this curved trajectory.

Path that means the path is this curvature may not be reachable by the flexible needle, whereas the path this can be reachable. So that test is done here, then finding the closest neighbor after that estimate the parameters associated with the needle. Okay then. The control law to find the new state. That means we need to provide the motion planning of the needle. That means we can have estimated the parameters associated with the needle path, then we need to have the needle move on or pass through to steer through this path. That can be done by providing the

insertion velocity and spinning velocity. This is the spinning velocity. The needle is rotated around its axis and the needle is inserted with a velocity. These two velocities are given as input for the motion planning. Then check the collision free solution.

While moving there is no collision with the No if it is collided then go back find the free obstacle area and then provide the trajectory generation path. If yes it is not colliding it is a solution collision free solution. So in that case you can add the vertex and edge of the tree. So to add this vertex and edge of the tree we require the optimal condition to get the optimal path.

That means so the shorter path is considered as the optimal path. It is almost an offline approach. That means after finding out there is no obstacle between the node, random node and the initial node. Then the needle can travel a curvilinear path which is feasible for it as per its geometry and mechanism.

Then this procedure is continued for several iterations in order to bring the optimal path with minimum path length, the minimum path length associated. After the vertex is added, we check the target-reaching condition, which means verifying whether the goal has been reached inside the tree. We have several nodes; is any of the nodes the goal node or not? That way we can check, and then we proceed to other targets—multiple targets. If this is reached, then proceed to the next target, repeating the process from the starting node to the goal node, which is given by the multiple targets, such as target 1, target 2, target 3, and so on. For each target, we start from the start node and repeat this process.

Then, we check if all targets are reached or not. If yes, the formation of desired paths for all targets will be shown through the simulation results we have obtained. If no, then find

the neighbor and proceed with the same procedures, the same steps. So, here the input for this process is the starting node, the goal node, and the obstacle-avoided region, the obstacle positions.

This allows us to avoid them while traversing through a curvilinear path to reach the target node. Then the output is the optimal path connecting the start node to the goal node, and the input for the motion planning, which is the insertion velocity and the spinning velocity for the needle to move or steer inside the tissue region. Now, let us discuss how this motion planning will be done, obviously through the modeling of the system. The needle that we consider here is the bevel-tip needle, the bevel-tip needle.

This is the bevel angle beta could be beta is 30 degree 45 degree, 60 degree, 75 degree like that. Depending on that the face will have certain inclinations. And being this flexible needle being this needle being bevel tipped and flexible. So, based on the tissue reaction force when it is inserted in the tissue region.

So, the needle will Okay, so we will take this shape. Okay, so accordingly it takes this shape. That means once it is inserted and based on the face of the bevel tip, it gets a reaction force from the tip from the tissue and it gets bent. Because the body is flexible.

Okay. So, let me erase this and let us start with the kinematic model approaches. There are two approaches as I mentioned earlier in the beginning of the class. So, what are the two methods? Method 1 is based on non-holonomic constraints equations. So, the non-holonomic constraints equation for the unicycle model is given by

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & \lambda \end{bmatrix} \times \begin{bmatrix} v \\ 0 \\ \kappa_0 v \end{bmatrix}$$

Where z dot y dot is the planar surface. This is z, this is y. And the needle is going to bend like this. Okay. Alright.

So, is a needle tape and this is the bevel angle beta, okay. So, z dot equal to cos theta into v where v is the insertion velocity and minus sin theta into 0, okay. So, z dot equal to v into cos theta and y dot equal to v sin theta And theta dot is given by lambda into K naught V, where K naught is the constant here, precisely the curvature constant.

Now, once the lambda is positive 1 plus 1, we can have the bevel angle facing up. Accordingly, we can have the rotation while the insertion of the needle will have this type of bending upwards. Similarly, when the lambda is minus 1, we can have the needle

steered inside the tissue with this bending downwards. So, now we can see how the forward steering and inverse steering can be done using this model kinematic model acting through the non-holonomic constraint equations. So, for the forward steering the input here is insertion and orientation speed insertion velocity and spinning velocity is required.

You provide that to the needle and it goes accordingly wherever it wants. We do not know the destination is because we just provide the input. And the output here is the needle tip trajectory. Somewhere it goes because you have provided the two inputs to the needle. That is one is insertion velocity another one is the spinning velocity.

Let us see how it is getting simulated. This is how. By giving this insertion velocity and the spinning velocity we can have this type of needle tip trajectory. And wherever we stop here that we can say that is a final node. In the beginning we do not know the final node.

Only the starting node we know. And with the input of insertion velocity and the spinning velocity we could have the needle to have this actual path. Now what is inverse steering? Here the input is end position of the needle. Target node and the output will be the tip trajectory.

The needle tip trajectory between the start node and the target node. Now, let us see how it is done. So, this is the target node fixed now. We already have the target node on our side. That is the trajectory output from the inverse tiering.

So here, for the inverse tiering. What do we need to go waypoint-wise? So, given the target position x-coordinate and y-coordinate, we can get the radius of curvature

$$r_i = \frac{y_i^2 + z_i^2}{|y_i|}$$

$$l_i = r_i \, atan2(z_i, r_i - |y_i|)$$

$$v_i = \frac{l_i}{\Delta}$$

where delta t is the sampling time and l_i is the length of the curved path or the length of the curvature. Okay, so this is it.

And now, we can see the second method representing the kinematics of the bevel-tip needle. The second method is based on the transformation matrix, which is a 4x4 homogeneous transformation matrix that will definitely provide the information of successive frames. Say you have frame 0. You have frame 1.

Or frame I and frame I plus 1. Between these two. What is the position and orientation of frame I plus 1

with respect to frame I or frame I with respect to frame I minus 1. The successive frame location in terms of position and orientation will be given by this transformation matrix. So that is obtained by rotation around Z by alpha I and then translation along the X-axis by the length OA, where this is the schematic that we are going to take now to explain. And this is the bevel-tip robotic needle rotation.

$$g_i = Rot(z, \alpha_i) * Trans(x, OA) * Trans(z, AB) * Rot(y, \phi_i)$$

$$g_i = \begin{bmatrix} cos(\alpha_i)\cos(\phi_i) & -sin(\alpha_i) & cos(\alpha_i)\sin(\phi_i) & x_n \\ sin(\alpha_i)\cos(\phi_i) & cos(\alpha_i) & sin(\alpha_i)\sin(\phi_i) & y_n \\ -\sin(\phi_i) & 0 & cos(\phi_i) & z_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where we have the tip frame and the world frame or the reference frame. So, this is your O, this is point A, this is point B. Now, let us talk about this. Rotation around the Z-axis, this is the Z-axis, here is X pointing out of this plane, and in the plane, we have the Y-axis and Z-axis.

First is rotation around Z axis by an amount alpha I. That is what it is. So your X axis becomes here. Then the translation along X axis by a length OA. This is O. This is A. So you bring it here. And then we have the translation along Z axis by a length AB.

So, your needle tip was like this, then it has been brought here, then it has been lifted up, then it is going to have the rotation around this y axis, this is the y axis by like that. So, it is going to have the rotation around the y-axis. This is y-axis by an amount phi i. That leads to this entire transformation matrix where the position of the needle tip now

$$x_n = r cos(\alpha_i)(1 - \cos(\phi_i))$$
$$y_n = r sin(\alpha_i)(1 - \cos(\phi_i))$$
$$z_n = r sin(\phi_i)$$

That is how we get the homogeneous transformation matrix. Based on this, we can have the kinematic model representing the tip of the needle with respect to the base frame.

So, I just want to have a clear schematic to you to make sure how it is getting moved. First, it is oriented around Z by an amount alpha i. So, this is there. This is first rotated and then it is after rotation it is here and then it is translated and then finally it has a rotation which is this around the y axis. So, this is the form. Now, this is the trajectory that is going to happen.

And the wave point trajectory it is going to happen is to divide this phi i with this small increment. That means, if the phi i is 45 degree, you start from 1 degree, 1 degree, you divide that into 45 times say 1 degree, 1 degree. For each degree, we will find out R i and L i. Based on that, you will have this motion obtained for this needle, so that the needle gets travelled. We are talking only of the needle tip with the assumption that the entire, this is the needle tip, this is the

flexible body. So, it is with the assumption that the entire body of the flexible needle follows the needle tip because entirely they are flexible in nature. And based on the transformation method which is method 2, we have obtained the kinematic model for which the forward steering is obtained like this. starting node where input is orientation angle alpha i steering angle phi i and the radius of curvature r i and output from forward steering is this trajectory because we gave the input accordingly we get the output this is it similarly the inverse steering is obtained like this approach which means We have the end position of the needle tip which is called target node and then we get the needle tip trajectory that is between the start node and the target node what is the trajectory.

And for this we obtain this wave point trajectory between the start node and the target node is by having the Curvature constant

$$k = \sqrt{x_n^2 + y_n^2} \ .$$

$$r_i = k/2 + z_n^2/2k$$

$$\phi_i = \pi - 2\arctan(z_n/k)$$

$$l_i = r_i \times \phi_i$$

Now let us see more about the path planning for multiple targets using greedy heuristic reachability guided RRT. Greedy method is once you reach the random node from the start node you try to reach the target and the reachability guided approach which means that after reaching the first random node from the start node immediately we check for the

Feasibility to reach the target node is what is meant by the greedy heuristic approach and reachability guidance, which means that whether from the first random node to the target node can be reachable or not will be feasible for this flexible needle to traverse that curved path—yes or no. That is what is now coming to the objective here: to perform. Greedy heuristic reachability-guided RRT-based path planning for steering a flexible bevel-tip needle inside biological tissue to reach multiple targets in a dynamic environment with dynamic obstacles. Here, the assumption is that the tissue deformation

during the movement of the needle inside the soft tissue has not been considered. There are two main approaches. One is the greedy heuristic strategy, which means the greediness to build a tree from the starting node and also for the goal node to achieve the path. The reachability-guided strategy is to speed up the path-finding approach, and the following condition should be checked.

What is that? That means the length is greater than or equal to the minimum. The obtained length is greater than, say, this is the R minimum. The obtained length is greater than this R minimum. The R curvature, you can say, is greater than the minimum curvature. $r_n \geq r_{min}$

Which is nothing but from the tip position; we can say that the P_Z coordinate is greater than at each instant, which is

$$P_z \geq \sqrt{2r_{min}\sqrt{P_x{}^2 + P_y{}^2} - (P_x{}^2 + P_y{}^2)}$$

where P_x, P_y, P_z are nothing but the tip of the needle. Now, coming to the working steps associated with the greedy heuristic reachability-guided RRT approach, we have categorized them into two steps. The first part is the generation of candidate solutions. First, we start with a random node—the generation of a standard random node.

Search the nearest node of the tree from this random node. Extension of the tree in a curvilinear path by avoiding the obstacles because we need to connect the tree to the random tree random node with the nearest node of the tree and that connection is not by straight line rather it is by a curved path. By checking whether there is no obstacle between these two nodes and also it can be reached by the needle or not by reachability guided strategy. Then we see after reaching we see through greedy heuristic strategy that from the first random node can it be reached to the goal node that is what it is through greedy heuristic If yes go to the second part that is this finding the optimal path.

If no try extend the other tree. Now what is second part of this working step is to find the optimal path. Here we are finding the cost values of all candidate solutions and optimal path from the start node to the target. So the objective of the optimization is to minimize the path length which is an offline optimization technique. We reach the target through a curvilinear path and we do this several times between these two nodes.

So that several curvilinear paths can be obtained depending on the number of total iterations and one can be selected based on the minimum path length where the objective function is the length of the path which is given by this expression in terms of the

coordinates and the curvature length or the angle of this arc of curvature or radius of curvature you can say precisely. Now here is this simulation result to reach the targets multiple targets. For each target we continue with the same step till we go for finding the optimal path.

Once the optimal path is reached we can find finally how many optimal paths depending on the number of targets. Each target will have one optimal path to be reached from the goal point start node and the target node. Target 2, target 4, target 5, target 6 like that. Next we will see the kinematic model of the bevel tip needle how we can obtain it which will be helpful for motion planning. The homogeneous transformation matrix between the universal frame and the tip frame that is here is a tip frame and here is the universal frame.

That is given by a general 4 cross 4 matrix which defines the 3 cross 3 rotation matrix for the tip position with respect to the universal frame and the position of the needle tip with respect to the universal frame and this 0 and 1 nothing but the scale factors associated with the homogeneous matrix which is of size 4 cross 4. So, you can see the needle is going to get traversed in the z y plane here as per schematics. We have two velocities as the inputs one is insertion velocity which is u_1 and the spinning velocity which is the velocity or the rotation around the needle axis u_2. Depending on u_2 being 0 degree or 180 degree you will have the

travel or the steering of the needle in the upward direction or the downward direction. So, the orientation is given by phi angle. We are continuing with the kinematic model. We know $v = J\dot{q}$ that is nothing but differential kinematics of a robotic system. Where V is the tip velocity and J is the Jacobian matrix that maps the joint angular velocity to the end effect velocity.

Here the velocity associated here is U1 and U2. And hence we can say that V can be given by U1 into mu1 plus mu2 into U2. The tip velocity is given by this. Where mu 1 is a vector 6 cross 1 vector which is given by 0 0 1 k 0 0 which means that it is responsible for the insertion velocity which is precisely the linear velocity around the z axis. And thereby you have a curvature around the X or around the X axis.

That is given by the constant amount K. Whereas U2 is associated with the, mu2 is associated with the spinning velocity U2. Where mu2 is given by 0, 0, 0 and 1. Which is the spinning around the axis X. of the robotic system which is z axis. So, here is the needle.

So, this needle axis is given by z axis. So, rotation around that is u 2. So, mu 2 is a vector 6 cross 1 vector associated with the spinning velocity of the needle. And with this mu 1 mu 2 and we know u 1 is the insertion velocity u 2 is the spinning velocity. We can have q dot is coming out to be that is why that is nothing but

$$\dot{q} = J^{-1}\mu_1 u_1 + J^{-1}\mu_2 u_2$$

where j is the Jacobian matrix which is

J a is the analytical Jacobian matrix which relates the end effector linear velocity and the angle rates alpha dot beta dot gamma dot to the joint angles velocity. So, here that is given by

$$J = \begin{bmatrix} (R_S^U)^T & 0_{3\times 3} \\ 0_{3\times 3} & j_A \end{bmatrix}$$

is nothing but the transformation matrix that relates the angular velocity and the joint angle rate that is what given by this. So, this is for one orientation type of the tip it can be Z Y Z Euler angle or x, y, z fixed axis angle based orientation. Depending on here in Euler angle there are 12 ways of orientation representation and in fixed angle there are again 12 ways.

So, out of this 12 plus 12, 24 ways of orientation representation we can have this transformation matrix coming out to be like this. And here finally, we will see how the modified kinematic model of a needle has the final expression coming out to be Q dot equal to this multiplied by U 1 U 2 which is J inverse into U 1 U 2. We segregated this term where Q is nothing but X Y Z theta phi and psi. Now, let us see the relationship between analytical Jacobian and

Geometric Jacobian. We know that geometric Jacobian. Will have. This form $J_a = \begin{vmatrix} J_p \\ J_0 \end{vmatrix} \rightarrow v \rightarrow \omega$

Of the Jacobian matrix. Which is responsible for the. Linear velocity. Of the end effector V. The J O is responsible for. The angular velocity of the end effector.

And hence. V. And omega of the end effector equal to JG of Q into Q dot. So, this is the relationship between the end effector velocity and the joint angular velocity by geometric Jacobian. Now, we can say that if we have X, Y, and Z in

Euler angle orientation, what will happen to the relationship that is given by omega equal to T of phi into phi dot, where let us say phi is your alpha, beta, and gamma. Therefore, phi dot is alpha dot, beta dot, gamma dot. Now, let us have the relationship between the

angular velocity and the angle rate given by omega T of phi into phi dot. Let us have that we have X, Y, Z Euler angle rotation. So, what will be the relationship?

Let us see. What is the relationship between them? So, we can say that it is going to be For x, y, and z Euler angles, we will have x-axis by amount alpha, y-axis by amount beta, and z-axis by amount gamma. So, we start R with rotation matrix 1, 0, 0, 0, 1, 0, 0, 0, 1.

It is an identity matrix. The first one is around the x-axis of this frame. So, this frame is, say, x, y, and z. So, around x, we are going to have alpha, and then around y, it is going to be beta, and around z, it is going to be gamma. So, first, we are going to have around the x-axis by an amount alpha dot, which means omega here is alpha. 1, 0, 0 into alpha dot is the angular velocity for the first case.

Then, the rotation matrix is going to be now rotation around the x-axis by an amount alpha, which is 1, 0, 0, 0, cos alpha, -sin alpha, 0, sin alpha, cos alpha. So, omega here in the second case is around the y-axis by an amount beta dot. So, 0, cos alpha, sin alpha into beta dot is your omega for the second case. Now, we have to see for now the rotation matrix is R x comma alpha, then R y comma beta. So, that leads to

The matrix, which is cos beta, which is cos beta 0 sin beta and sin alpha sin beta cos alpha, then minus sin alpha cos beta, then minus sin beta cos alpha sin alpha cos alpha cos beta. Therefore, omega here is Z. X is multiplied by gamma dot, which is sin beta minus sin alpha cos beta cos alpha cos beta into gamma dot. And hence, we can say finally, this T, which is equal to 1 0 0. 0 cos alpha sin alpha and sin beta minus sin alpha cos beta and cos alpha cos beta. Thus, omega is equal to omega 1 plus omega 2 plus omega 3.

What we have obtained is that, okay. So, we will be having this T of phi multiplied by alpha dot beta dot gamma dot, where T of phi is given by this for this. X Y Z orientation by Euler angles. Next, having found this relationship, that is omega equal to omega x omega y omega z, which is equal to T of phi into alpha dot beta dot gamma dot, and T of phi depending on the orientation, whether it is Euler angles or fixed axis angle based. Among the 24 ways of orientation, we can get T of phi the way I have explained to you.

And then, once we get that, we can go for the relationship between analytical Jacobian and geometric Jacobian, as this J g, let us say. So, V omega is going to be T of phi into phi dot. That is equal to j g of q into q dot, so this can be written as i 0 and 0 t of phi into v and phi dot, that is equal to j g of q into q dot. Therefore, v phi dot is equal to i 0 0 t of

q. Whole inverse J G of Q into Q dot. That is nothing but B phi dot is equal to I 0 0 T of Q inverse into J G of Q into Q dot, which means that B phi dot is equal to J A of Q into Q dot, where J A of Q is equal to.

I 0 0. T of Q inverse. Into J G of Q. Thus, the relationship between. The analytical Jacobian. And the geometric Jacobian. And now, coming to the conclusion. My dear friends. In today's class. We have seen. The introduction to.

Path planning. And then we have. Seen the RRT-based path planning for smart robotic flexible needles for minimally invasive surgery, and then we have seen the relationship between JA and And JG, then omega and phi dot, which means now I can tell you very clearly that what we have seen today is the introduction to path planning towards percutaneous needling intervention for cancerous treatment through Rapidly exploring random tree-based path planning approach for reaching multiple targets.

Then, we have seen the kinematic model of the flexible bevel-tip needle using two methods: one is based on the non-holonomic constraint equations, and the other is based on the transformation homogeneous matrix. Then, finally, we have seen the relationship between the geometric Jacobian and the analytical Jacobian of a robotic system. Finally, we have also seen the relationship between angular velocity and the rate of change of the orientation angles alpha, beta, and gamma. With this, I conclude this session.

Thank you very much.