Electrical Distribution System Analysis

Dr. Ganesh Kumbhar
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture - 22
Load Models in Distribution System Part I

Students, we have seen modeling of step voltage regulator in the last 2-3 lectures. And, today's lecture we will see Load Models in the Distribution System. So, before going to the load model let us see what we have studied in the step voltage regulator modeling section of this particular chapter.

So, we have seen why voltage regulation is required that is basically what are the effects of under voltage or over voltage on distribution system components. And, then we have seen various techniques which you can used to control the voltages over the distribution feeder. And those are basically you have seen OLTC voltage boosters, step voltage regulator capacitor banks, then we have seen different types of regulator those are type a and type b regulator both are almost same only mirror image of each other.

So, basically type b regulators are widely used. So, we have studied the type b regulator in detail and we have got the voltage and current relationship on primary and secondary side of the voltage regulator. Then, we have seen the function of what is called as a line drop compensator circuit. And, we have seen that is basically required to control the taps of your regulator based on voltage at the far end of your feeder. And, then see we have seen modeling of 3 phase regulators.

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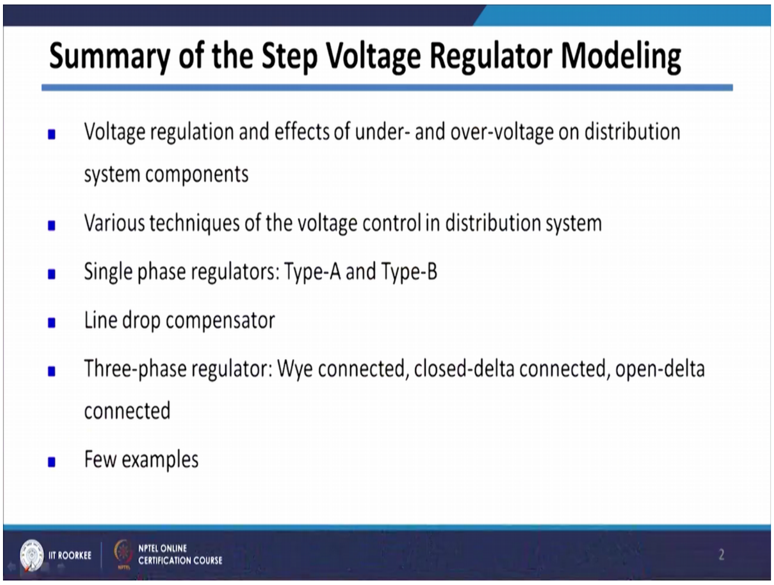

First we have seen: wye connected, then closed delta connected then open delta connected 3 phase regulators. And to understand it better we have seen few examples one example on single phased regulator, another example on 3 phase open delta kind of regulator.

Now, let us start what is called as load models, before going to the load models first we will see why load models are required. So, if you know while doing the load flow studies in your b tech class or under graduate course of power system analysis, you might have a used constant real and reactive power loads. Means in each iteration of your load flow studies your power remains constant means.

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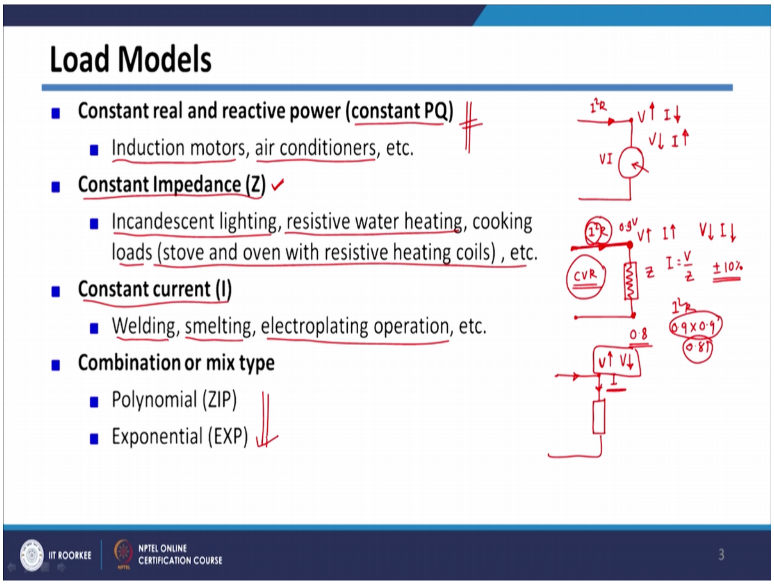

This first load which I have shown it here that is nothing but your constant real and reactive power that is called as constant PQ kind of load. And, in this particular load your P and Q values of the load remain constant throughout all iteration we are not going to chance. So, you might be remembering you are not changed the value. So, P and Q during the iteration, even the voltage changes P and Q will remain constant.

Basically, this happens in case of induction motor. So, approximately it will not be exactly constant one, but approximately induction motor load or air conditioning kind of load, you can model using constant real and reactive power model. So, in this case we can see that if there is says induction motor kind of load and which is connected at the end. And, then if you see the voltage at this end if this voltage increases what will happen this load will try to keep VI constant.

So, when voltage is increasing your current through the network will go down to keep the power of this load constant PQ of this power load constant. Similarly, when voltage decreases it will try to take more current from the system. So, when the voltage decreases for this particular kind of load what will happen is more current will flow through the network, and it will create more I square r loses and also inside the motor there will be higher I square R loses.

So, this is how your constant PQ load will be a another type of load is called as constant impedance load. Here, the impedance of the load remains constant. The examples of this load are incandescent lighting resistive water heating, cooking loads, which are basically based on your resistive heating coils. Those kinds of load which are basically based on resistive heating kind of coils, they those can be modeled as a constant impedance loads.

Now, if you see the behavior of this load it will be exactly opposite to your constant PQ type of loads. So, this is your constant impedance kind of load. So, this impedances of these device is say constant. So, in this case your voltage so, impedance is remaining constant. Then your current will be just voltage divided by your impedance, which is constant. So, when your voltage goes up your current also goes up into this model. And, when voltage goes down current also goes down which is exactly opposite of your constant PQ kind of load.

So, in this case when you are decreasing the voltage you can see that current is decreasing. So, whenever we keep lower voltage here, your current is decreasing and you are getting less or I square are loss into your distribution network or if this is feeder here. So, feeder less current less some out of current is flowing that is why less I square or loss.

So, these particular characteristics of this load are used in what is called as CVR. So, it is called as conservation voltage reduction and in conservation voltage reduction if your distribution system has majority of constant impedance kind of load, then what they do they will try to put the low voltage across the feeder as low as possible. However, we have to keep limit into the mind.

We know that there is only plus minus of 10 percent variation is allowed. So, we can (Refer Time: 06:29) this point at say 0.9 nominal voltage. And, in that case if the plus minus 10 percent is allowed, in that case your current will be also decreased by 0.9, and then your, I square loses into the system will become your you can say 0.8 per unit. Because, it will be actually I square r. So, I is 0.9 multiplied by 0.9. So, it will become 0.8 1.

So, your losses will decreased 20 percent. So, if your decreasing voltage by 10 percent your current is also decreased by 10 percent, and in that case your loses into the system will decrease by 20 percent around 20 percent. So, they this philosophy will be used in what is called as conservation voltage reduction, where the majorities of the load in your distribution if they are constant impedance load.

Then, third type of load is called as constant current load and constant in constant current load, whatever voltage across your device if this is your load, whatever voltage across this device, your current will remain constant current is not going to change. So, even you voltage at this load increased or voltage or this load decreased, your current is remaining constant. So, losses will not change in this type of load. The examples us example of this kind of load are welding, smelting, or electroplating operations basically they require constant current. So, even though voltages are lower higher at that particular and your, current is remaining constant.

However, when we are considering load for say load flow analysis or any type of analysis, what we do we take combination of load means we do not model exactly the each device or particular device instead of that we actually collectively of many devices we take together or many houses we take together, and we model it as a lump load at that particular point.

So, load will be always kind of next type, when there will be some compo equipment which will having constant PQ kind of characteristic, some equipment will having will be having constant impedance kind of characteristics and come some equipment will be having constant current type of characteristics. So, if you are modeling all the 3 types of load together you are having 2 models available; one is called as polynomial model we will see that and then another is exponential model, which consists of all the 3 types of load. Let us see one by one will start with constant real and reactive power load.

So, since we are considering distribution system we should always talk about 3 phase system.

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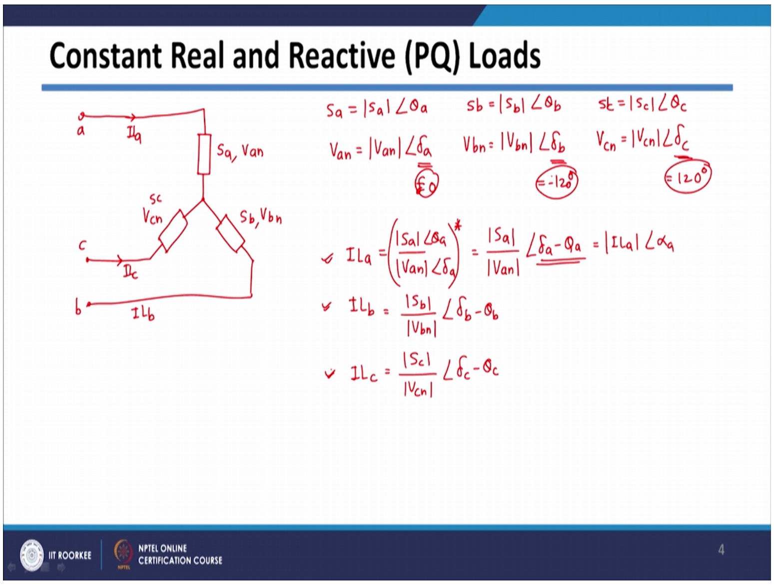

So, let us consider your 3 phase load. So, you are having this load here. So, one phase, this is another phase and this is 3 loads which are connected in say star delta also will consider after wards. And, this is say a terminal this is your c terminal, this is your b terminal, this is your current I a I c or I will say is sensitive load current so, I L a I L c and I L b.

Now, this load is having say power S a complex power voltage across this load is say Van, this is S b voltage across, this is S b n, this load is say S e and voltage across this is say V c n. Now, this since it is complex load S a will be equal to S a and it will be having some angle say theta a, then your S b will be equal to magnitude will be S b and say angle of it is see theta b if they may be actually unbalanced loads. So, all the thetas as well as magnitudes may different. Then S c will be equal to S c magnitude and angle will be theta c.

And, then voltages Van will be having magnitude of Van and angle say delta a voltage bn will be Vbn, and it is angle will be delta b. And, V cn it is magnitude is V cn and angle say delta c. If, these are balance to voltages the magnitudes of 3 voltages will be same and this delta a will be equal to 0, this delta will be will be say minus 120 degree shifted, 120 degree and then it will be plus 120 degree shifted.

However, as I told you all these loads as well as voltages may become unbalanced in case of distribution system. In that case these delta a delta b delta c they may not be even

20 degree phase shifted as well as your magnitudes also will not be same. So, you have to give that is why you have to represented by different angles here as well as magnitudes may be different. So, in that case we can easily calculate. So, in this case many loads full over studies, the loads will be modeled as equivalent current injection even generators will be modeled as equivalent current injection. So, whatever shunt component like capacitor, capacitor loads and generators, they may be modeled as equivalent current injection.

So, basically we need to calculate how much current is getting injected into the system. So, in that case I L a can be easily calculated from this data. So, it will be S a divided by Van and S a is actually S a angle theta a, Van is angle having angle delta a and you have to take star of it.

So, in this case your Val will be magnitude of S a divided by magnitude of Van, and this angle will be delta a minus theta a which is basically I can say IL a magnitude and it is will angle, which is basically this angle I can say just alpha a. Similarly, I can get IL b current, which will be equal to S b magnitude of this divided by Vbn magnitude and it is angle will be delta b minus theta b. And IL c load will be equal to S c magnitude divided by V cn magnitude, and it is angle will be delta c minus theta c.

So, these are nothing but equivalent current which are coming out or current injected will be opposite of that. So, basically you have got the current, which are flowing to the load which we can be calculated like this. Now, let us consider delta connected load. So, in case of delta connected load it will be modeled like this.

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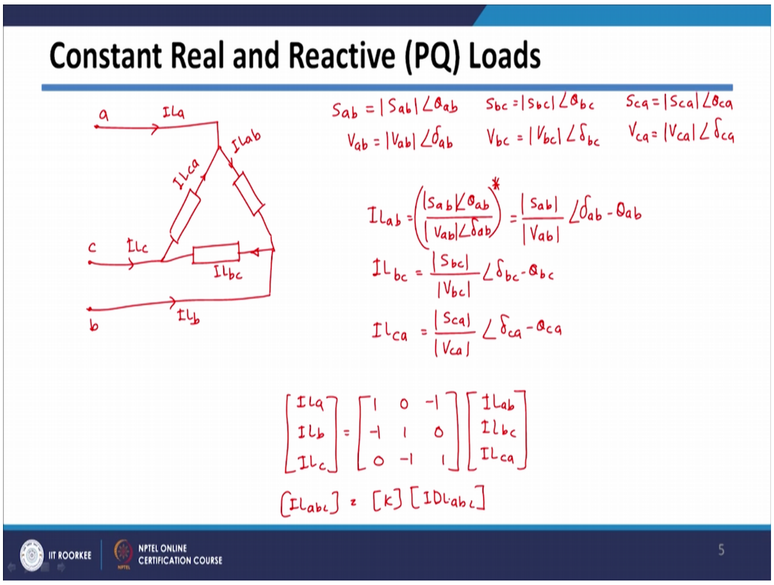

So, these are 3 delta connected loads. So, this we can connect to terminal a, this we can connect to terminal b, and this we can connect to terminal c. So, this is your I L a, this will be I L b, and this will be I L c. Now, this current is I L a b, this current will be I L b c, and this current will be I L c a, which will flowing basically like this. So, I L b c will be like this.

So, in this case similar to our star connection we can again write the equation. So, in this case S ab, which is basic basically load between a and b phases will be having S ab magnitude. And, it is angle is say theta will be similarly S bc will be S bc and it is angle will be theta bc, and S ca will be having S ca magnitude and it is angle will be theta ca. And, voltages will be V ab will be V ab magnitude and it is angle will be delta ab, V bc will be having V bc magnitude and it is angle will be delta bc, and V ca will be having V ca magnitude and it is angle will be delta ca.

And, in this case also we can easily write equation similar to what we have seen earlier, your current I L a b will be just depend upon S ab divided by it say ab angle theta ab, divided by V ab angle these are magnitudes, angle delta ab, and you have to take complex conjugate of it. Which will be equal to S ab magnitude, divided by V ab magnitude, and s angle will be delta ab minus theta ab.

Similarly, I can write for other 2 currents I L b c will be equal to S bc magnitude divided by V bc magnitude and it is angle will be delta bc minus theta bc. And, I L c a will be equal to S ca magnitude, divided by V ca magnitude and it is angle will be delta ca minus theta ca.

Now, you have got your delta currents; however, we have seen that for all these study we need line currents. So, that they can be injected during the power flow so, we are not keeping anything, which is basically delta connected while doing load flow storage or short circuit connect. So, you have to get the equivalent line current. So, we have seen that how to convert your delta phase current to the line currents.

So, we know that the conversion metrics is basically your k metrics which you have seen. So, that k metrics is generally your I L a, I L b and I L c, which are basically line currents, which will be equal to you are seen that it is 1 0 minus 1 minus 1 1 0 0 minus 1 1, into your delta currents I L a b I L b c and your I L c a basically this is nothing but your, I L a b c metrics, which is k metrics, which you have seen earlier and it is I L I d IDL abc.

So, we are seen how to get the injected current in case of star connected load and delta connected load. So, every time I will not go for delta connected load you can easily derive them if you know how to do it in star connection. So, for other loads, I will just go for only star connection. Now, let consider your constant impedance kind of load. So, we have seen the behavior of constant impedance kind of load where the impedance of the load is remaining constant.

So, what ever to do is first you have to get the value of impedance and then whenever your voltage across that impedance is changing, injected current is calculated based on that constant impedance. So, let see how we can do it?

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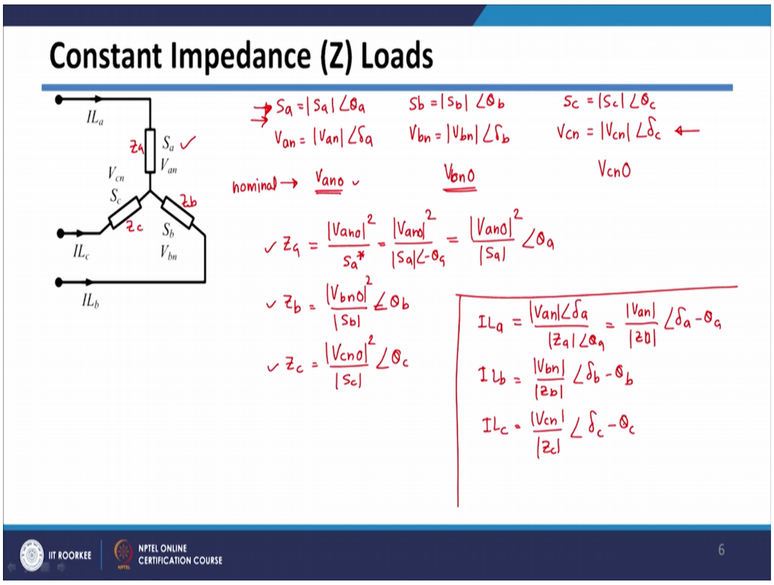

So, in this case again I will take a I will be taking your S a, which will be equal to S a magnitude and it is angle will be theta a S b will be equal to S b magnitude and angle will be theta b, and S c will be equal to S c magnitude and angle will be theta c.

And, voltages we are already seen they are Van V an and is angle is delta a V bn, and it is angle is delta b and V cn is actually equal to V cn magnitude and it is angle is delta c.

Now, these are say actual voltages here ; however, there will be always the voltages you can say nominal voltages will be defined for this particular load. So, say nominal voltages. So, I am saying nominal or rated voltages, nominal or rated voltages magnitude for this particular load is say Van 0, which is I am saying nominal voltage of this particular load. Nominal voltage for this load is say Vbn 0 and this say V cn 0.

So, these are basically nominal voltage related voltages. So, these are related loads and these are related voltages; so from this rated contiduce first gate; impedances of the loads. So, impedance of the load first load is say this load will be calculated say this is impedance say Za, this impedance is Zb and this impedance is Zc. And, as I told you throughout the calculation impedance will remain constant here because these are constant impedance kind of load.

So, Za will be calculated like this it will be Van 0 square divided by your S a star or we can say. So, this will basically equal to Van Van 0 square divided by S a angle minus theta I, because we are taking complex conjugate of it and it is magnitude here. So, this will be equal to V an 0 it is square divided by S a, which will be magnitude of this impedance and impedance angle will be theta a. Similarly, Zb will be V bn 0 divided by S b magnitude of this square and angle of it will be theta b. Similarly Zc will be equal to V cn 0, it is square divided by your S c and it is angle will be theta c.

And, as I told you throughout the operation these impedances will remain constant and these impedances, we have got from then name plate ratings of the load. So, name plate ratings gives you the load values as well as your rated voltages rated voltage magnitude will only we need here. So, using the magnitude we have calculated impedances.

Now, when you want calculate injected current for different voltage level, which are different than your nominal voltages, then those can be calculated. So, your I a I L a will be equal to Van, it must be different voltage than your it may be I am saying. It would be different voltage than your nominal voltage and angle is delta angle may also different, and then divided by your Za magnitude angle theta a, this is your magnitude.

So, in this case I can easily get it will be Van magnitude divided by Z Za magnitude and it is angle will be delta a minus theta a. Similarly, I can write for I L b, which will be equal to Vbn divided by your Zb, a angle will be delta b minus your theta b and your I L c in this case will be V cn magnitude divided by Zc, magnitude and it is angle will be delta c minus theta c.

So, processor in this case as I told you from the name platating first you have to get the impedances of the load. And, once you get the impedance you have to use these impedances to get the injected current even network that is voltage divided by impedances will give you injected currents. So, we are not going for delta connection, because it will be easier like we have already seen it for constant PQ kind of load.

Now, let say how constant current loads behave? So, in this case you have to first calculate that values of constant currents. Now, again I am taking this S a.

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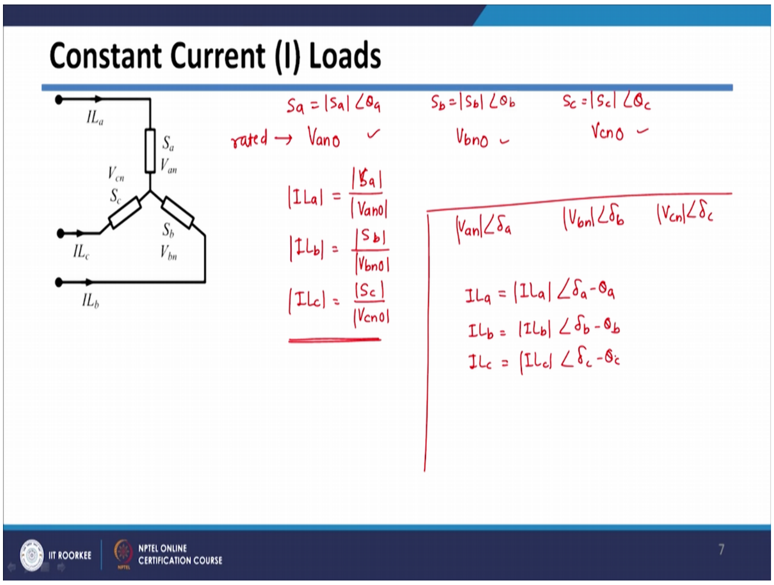

Which will be equal to S a magnitude and angle theta a S b will be S b magnitude angle theta b, and S c will be equal to S c magnitude angle theta c and as I told you nominal voltage help us tried. So, Van 0, which is nominal voltage or rated voltage of the loads of the second load Vbn 0, and third load V cn 0, only magnitudes we need it here. Now, in constant current load current magnitude only remains constant angle may change. So, in that case current magnitude can be easily calculated; so current magnitude of load L a. So, I L a magnitude can be calculated from S a S a magnitude divided by Van 0 and it is magnitude only Vbn.

Similarly, magnitude of b will be calculated S b magnitude divided by your Vbn 0 magnitude. And, then I L c will be equal to S c divided by S c magnitude divided by V cn 0 magnitude. So, this will give you the currents which are basically remain constant, throughout the operation or throughout the load flow solutions. So, in that case if there are different voltages than these nominal voltages some time as I told you, there will be different voltages. So, say the voltages now are say Van angle delta a, Vbn angle delta b, and V cn angle delta c.

Now, for this these voltages even though voltage will change you need to keep the current magnitude constants. So, current injected current in this case it will be I L a will be equal to I L a magnitude always remain constant even the voltages change. However, angle in this case will be calculated delta a minus theta a of the load.

Similarly, I L b will be equal to I L b main it will remain same and then it will be delta b minus your theta b. And, I L c will be equal to I L c magnitude and delta c minus your theta c. And, as I told you many times your loads are mix type of load. So, many times we need consider all the 3 types of loads together and in that case we have seen there are two types of model we use, one is called as polynomial load model.

And, in case of polynomial load model we represent load like this.

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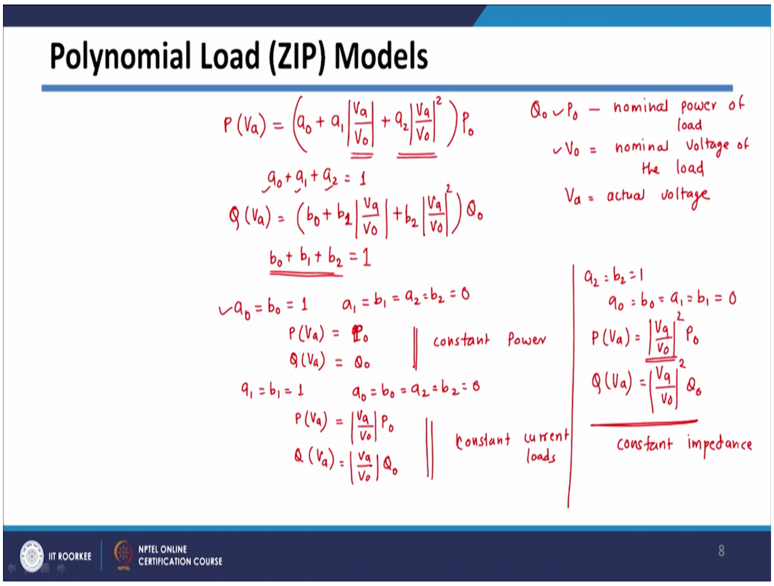

So, P which is actually basically voltage dependent so, I can say a Va which is basically actual voltage, which will be equal to some a 0 or we can say a 0 plus some a 1 into V actual divided by V nominal, plus a 2 into V actual divided by V nominal and it is square multiplied by nominal power of the load ok.

So, here P 0 is nominal power of the load and V 0 is nominal voltage of the load and Va is actual voltage at which we want to calculate the load. So, it is actual voltage during the load flow calculation as I told you voltages may change. So, at iteration Va will go change will go on changing. However, this V 0 which is nominal and P 0 which is nominal it will remain constant.

So, in this case, if you see your al already considering all the 3 types of load so, this a 0 multiplied by P 0 will represent basically constant power load P 0 multiplied by a 1 multiplied by this a Va divided by V 0 will basically represent constant current load. And, this P 0 multiplied by a 2 I am multiplied by Va divided by V 0 and whole square will represent your constant impedance kind of loads. So, there for since they are mixed type of load and in that case your, a 0 plus a 1 plus a 2 should be equal to 1. So, it should represent total 1 per unit load. And, this will be basically represent your percentage of 3 different kinds of loads. Exactly, similar way I can write Q we see also depends upon your Va and in this case your constants may be different b 1 plus b 2 Va divided by V 0 plus in this case b 0 I will say, this is b 1 and this is b 2 again it is Va divided by V 0 whole square into Q 0. I can Q 0 is nominal the active power of the load.

So, in this case also total proportion of the all the different types of loads will be equal to 1. So, in this case these coefficients are different, because we know that real power and reactive power they will behave differently with respect to voltage change. So, in this case if you observe your when your, a 1 we can say a 0 equal to b 0 equal to say 1 and all other quantities are 0 that is a 1 is equal to b 1 is equal to a 2 is equal to b 2 is equal to 0. That case I can write PVa is actually equal to your a a 0 is 1. So, 1 multiplied by P 0 will be actually just P 0 here.

Similarly, Q multiplied by sorry Q, which is function of Va will be equal to Q 0. So, we can see that they are these loads now when a 0 and b 0, they are one it is behaving as a constant power loads. So, this is nothing but constant power loads and then when your a 1 is equal to b 1 is equal to 1. And, all other constitutes that is a 0 is equal to b 0 is equal to a 2 is equal to b 2 is equal to 0, in that case we are getting P which is Va, which will be equal to Va divided by V 0 magnitude multiplied by your P 0 and Q Va, which will be equal to Va divided by V 0 into your Q 0.

So, in this case since the power of proportional to your voltage change, this load is nothing but your constant current loads. And, in third case when your a 2 is equal to b 2 is equal to 1 and a 0 is equal to b 0 is equal to a 1 is equal to b 1, if there are 0, then your P Va is becoming equal to Va divided by V 0 it is square into P 0 and Q, which is function of Va a square into Q 0.

So, we can see that the powers are becoming proportional to your voltage square and that is nothing but characteristics of your constant current constant impedance load. So, this is nothing but your constant impedance kind of load. So, depending upon your coefficients the proportion of different types of load will change and depending upon proportion of loads in that particular node, we can calculate the current injected.

Now, another type of load model, which is again famous, which is called as exponential type of load model so, in this case we represent load like this.

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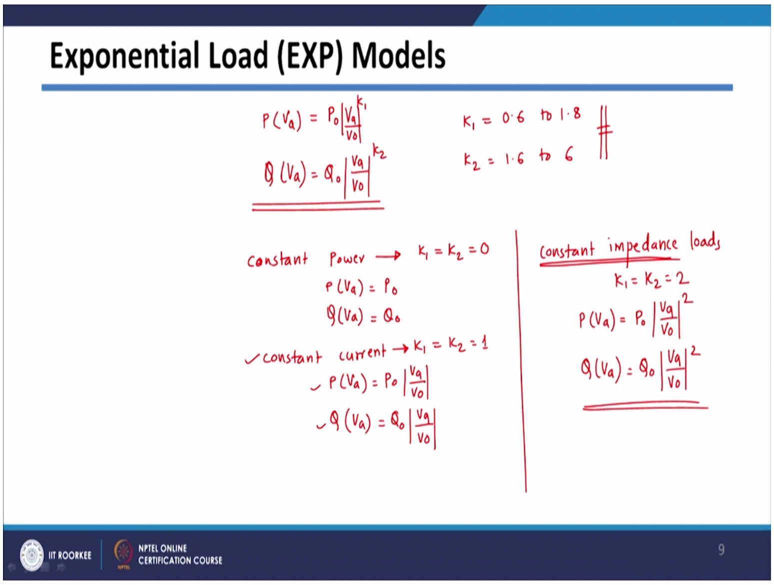

So, your P, which is function of Va I can this is Va is actual voltage, which will be equal to P 0 multiplied by V raise to some constant or can say Va raise divided by your V 0 raise to some constant k 1. And your Q, which is like an function of Va will be represented by Q 0 multiplied by your Va divided by V 0. And it is some constant another constant k 2 as it, since k 1 k 2 will not be same, because as I told you your behavior of Q and P will not be same with respect to your voltage means, change in P and change in Q will be very different when the voltage is changing.

So, k 1 k 2 coefficient similar to earlier cases I can depending upon k 1 and k 2 values your proportion of p and q will change. So, general values of k 1 and k 2 they are generally between say 0.6 to 1.8 for general kinds of load. And, then k 2 is generally between 1.6 to 6. And depending upon your load pattern it can be also be estimated based on your if you know the variation of power with respect to variation of voltage, if those curves are available by curve fitting technique you can get this coefficient k 1 and k 2. Now, this for constant power loads this k 1 k coefficient will be equal to 0. So, when this k 1 and k 2 they will be equal to 0, this load will be has constant power, because in this case PVa will be equal to just P 0 and QVa will be just equal to your Q 0.

Similarly, when k 1 is equal to k 2 is equal to 1 this loads will be proportional to your voltage ratio, in that case in case of constant current. So, this will happen in case of constant current loads when k 1 k 2 will be equal to 1. So, in that case P Va will be equal to P 0 multiplied by your Va divided by V 0, this to 1 and Q Va will be equal to Q 0 into Va divided by V 0. And in this case we can see that with these loads are proportional to voltage that is why they are constant current load. And in case of constant impedance load your k 1 is equal to k 2 will become equal to 2.

So, in this case your p Va will be equal to P 0 into V actual divided by V nominal square and Q Va will be equal to Q 0 into V actual divided by V nominal it is square. So, this is characteristic loads are proportional to your voltage square. So, that is why it is characteristic of your constant impedance load.

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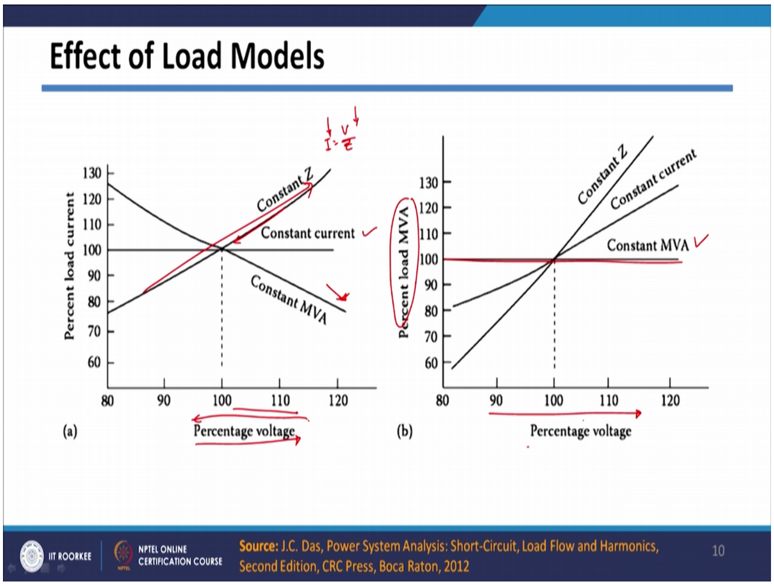

And, let us see the effect of this load models on current as well as your power. So, in case of current we have seen that in case of constant MVA load, whenever your voltage is increasing to keep the power constant your current has to decrease. So, whenever voltage is increasing current will decrease, in case of constant power or constant MVA load in constant current load by characteristic load will current will remain constant.

And in case of impedance load impedance load current will be calculated we have seen just V divided by z and Z is constant. So, when V is decreasing your, I also will decrease. So, when V is decreasing your, I also will decrease or when V is increasing your I also will increase.

Then power relations if you see. So, here I am plotting power relation. So, when the percent voltage is changing. Since, in case of constant power load your load should remain constant so that is why for constant power load is load is remaining constant. However, in case of constant current load as the voltages increases and current is remaining constant. So, current multiplied by current is increasing and voltage remaining current is remaining constant and voltage increasing means power has to increase.

So, the change in constant current load will be something like this as we increase the voltage, since the current is constant your power will increase. In case of constant impedance load we have seen that it is proportional to voltage square. So, there is in that case when, whenever your voltage is changing your power will change with respect to voltage. So, even need to model the load accurately by taking different types of model.

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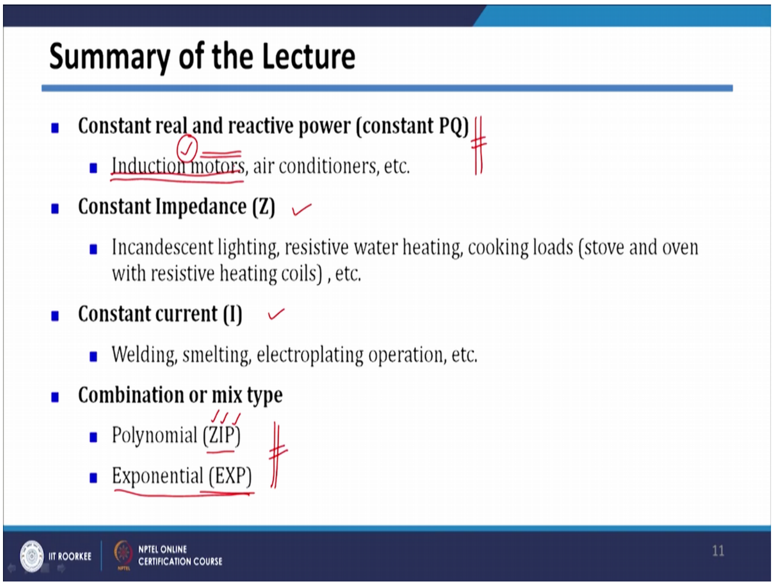

So, we have seen various types of model into this particular class that is those are constant impedance, constant current separately we have seen. And then we have seen two different mixed type of load model those are polynomial model, which is again called as zip model because zip, which is basically represent constant impedance constant current and constant power. And it is representing as polynomial of those three combinations and then we have seen exponential load model.

So, next time will see accurate modeling of induction motor.

Thank you.