Sliding Mode Control and Applications

Dr. Shyam Kamal

Department of Electrical Engineering

IIT(BHU) Varanasi

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Lecture-60

So, welcome back. In this particular lecture, I am going to talk about one of the very, very important industrial problems. How to design a control system for the power converters. Obviously, in this lecture, I am just going to talk about DC to DC converters, but one can extend this approach to any class of converter. And this is the last lecture of this particular course on sliding mode control and its applications. So, I am also going to share a moment in which I actually expand and learn from this course with several well-established researchers in this field, including one of the founders, Professor Utkin.

So, I am also going to discuss that. So, the purpose of the discussion is that we are going to talk about the development of sliding mode control for power converters. And if you see the current era in the industry, everything is switching towards electrical vehicles, so the application of these kinds of converters is very useful for charging design, especially when batteries or similar things are involved. So, the application of converters is huge.

So, now if you see the operation of the converters, that is naturally based on the switching property. Whenever switching is present, first-order sliding mode control is quite applicable and comfortable for delivering performance in the presence of unknown uncertainties or disturbances. So, during the development of the theory of sliding mode control for power converters, we are going to talk about a very, very important concept that is the time scale separation property. So, actually, if you see the design of the power converter system, they basically have two dynamics. So, one dynamic is fast dynamics and another is called slow dynamics.

And due to fast dynamics and slow dynamics, similar to sliding mode control, if you observe carefully, sliding mode control also basically shows two different kinds of properties. So, we have two phases; one phase is called the reaching phase, and another phase is called the sliding phase. So, reaching phase I have to complete as soon as possible because system become insensitive with respect to disturbance once sliding phase comes into picture. So, somehow this represents the fast dynamics, and this represents the sliding that will represent the slow dynamics. So, somehow this is coming from the theory that is well known as singular perturbation theory.

So, Professor Kokotovic has made a lot of contributions in this particular domain. So, during the development of sliding mode control, I will also utilize the property of the time scale. Due to this time scale separation property, it is possible to show that the whole control structure is basically the cascaded control structure. It means that you can deploy classical control as well as sliding mode control on two different loops, and you can achieve

the required performance. So, since fast dynamics is associated with the inductor current, we have an inner loop for that reason.
So, in the inner loop, I am going to deploy the sliding mode control to control the current, and after that, in the outer loop, one can design some kind of PI control such that one can achieve voltage regulation. Direct implementation is possible because sliding mode control is inherently based on discrete switching or some kind of switching-based property. So, obviously, here switching should be very high, and if you have a physical circuit that is capable of doing that, then you can easily get the required performance if you employ the sliding mode control. So, this is one of the alternative solutions to the pulse width modulation-based technique. So, a lot of power converters or power electronic circuits are based on this particular design methodology, but if you apply or deploy the sliding mode control at that time, it is possible to show that you can get several advantages over the PWM.

So, if you see some classes of the power converter, it is possible to show that one class of the power converters is actually satisfying the property of a bilinear system. What is the meaning of a bilinear system? If you see this structure, then both control and state are involved here. So, this is linear with respect to u as well as linear with respect to x. If you maintain x as a constant, then it is linear with respect to u. If you maintain u as a constant, then it is linear with respect to x .

So, somehow we are also trying to understand how to design sliding mode control for a bilinear system. A power converter is going to fall on that particular class. So, we have an advantage with the power converter because we only have to operate it either on or off. So, on or off, I will represent them by 0 and 1 . And it is possible to show that one can design control.

Here, control design is how you are going to control the operation, the on-off operation. So, gain is not going to play any role in this particular design. So, topology plays a very, very important role whenever you are going to design the power converter. So, the power converter has one part that is topology, another part that is the control design, and both are equally important. So, now we are going to deploy control like this, and what is the physical interpretation? When s is equal to 0, s is greater than or equal to 0 ; at that time, this is nothing but the signum of x being positive.

So, if this is positive, then 1 minus 1 equals 0 . So, u equal to 0 . So, you can able to do off operation. If signum s, s is less than or equal to 0 , then 1 minus and minus 1 . So, basically, you are getting 1 plus 1 .

So, basically, u is equal to 1 . So, in this way, I can deploy this kind of discontinuous control, and then I can control the switching phenomena. Obviously, I am able to design a sliding surface like c transpose x , and here c is nothing but the one that can easily be calculated from this particular expression. So, here I am assuming either positive or negative, and x may be more than 1 ; due to that reason, if you do this kind of calculation, then you will be able to get the value of this matrix c. Now, let us try to see how one can achieve sliding mode control.

So, in order to achieve sliding mode control, first I have to design the sliding surface. So, the sliding surface is proposed as c transpose x. c is simply some kind of constant matrix you have to select. Now I will take the derivative. So, once you take the derivative, x dot comes into the picture and from the bilinear dynamics A x and u , I am going to substitute into Bx, then I will get this kind of system.

$$\dot{s} = c^T Ax + \frac{1}{2}c^T Bx - \frac{1}{2}\operatorname{sgn}(s)c^T Bx$$

And what is our goal? Our main goal is to make s equal to 0 attractive. So, what is the condition of attractivity? It means that $s\dot{s}$ should be less than 0 .

$$s\dot{s} = s\left(c^T Ax + \frac{1}{2}c^T Bx\right) - \frac{1}{2}|s|c^T Bx < 0$$

So, somehow you can think of it like this: if s is greater than 0 , then $\dot{s}$ should be less than 0 . If s is less than 0 , then $\dot{s}$ should be greater than 0 . So, here is what I have done: I am assuming that s is greater than 0 .

So, now I am going to calculate $\dot{s}$. So, if s is greater than 0 , then obviously, the signum function here takes a value equal to 1 . And in this way, this and that are going to cancel out. So, I just have to maintain $c^T Ax$ that is less than 0 .

$$\dot{s}_{s>0} = c^T Ax < 0$$

Now, assume that s is less than 0 . So, once s is less than 0 , this becomes positive and positive and positive, then I have $c^T Bx$. So, now, $\dot{s}$ I have to maintain since s is less than 0 . So, $\dot{s}$ should be greater than 0 , and then this condition comes into the picture.

$$\dot{s}_{s<0} = c^T Bx < -c^T Ax$$

So, in this way, I can maintain or attract all trajectories towards s equal to 0 . Once s equal to 0 comes into the picture, in order to maintain trajectory along this particular sliding surface, I required the equivalent control.

So, using Utkin's method, it is possible to calculate the equivalent control. What can you do? You can put $\dot{s}$ equal to 0 . So, if you put s equal to 0 and consider that at that time signum s equals 0 . So, whenever you are applying Utkin's linearization, you are defining signum s, s equal to 0 as 0 , and due to that reason, you can see that this equivalent control should lie between 0 and 1. Why 0 to 1 ? Because we are doing just two operations: 0 and 1 .

$$0 < u_{eq} = -\frac{c^T Ax}{c^T Bx} < 1$$

So, control whatever equivalent control or whatever average control comes into the picture to maintain a trajectory that will also satisfy the exact same property. Now, if you see the converter, I am going to consider a very simple structure or simple topology. So, I am talking about the DC to DC converter. There are several types of converters. I have a buck converter, a boost converter, a buck-boost converter, and after that, I have a cuk converter.

So, in this lecture, I am going to talk about the buck converter and the boost converter. So, what we have seen is that whenever we are talking about a buck converter, we are giving some voltage, and we have to actually drop that voltage down. So, I have to somehow maintain a voltage that is less than the given voltage Vd. So, now we are going to explain how the cascaded control structure comes into the picture. So, here in the case of the converter, we have two different rates involved.

One is the inductor current and the other is the capacitor voltage. So, from here, you can easily see. So, this is the structure of the buck converter, and here I have elements like a resistor, capacitor, inductor, and this is responsible for switching, and this is some kind of voltage source. So, what is our main goal? If I provide some kind of voltage here, then I have to drop that voltage here. So, any output voltage that is less than the E is called a buck converter.

And here you can come again. So, in this converter, you can see that I have a resistor and a capacitor here again. Again, I have inductor, but switching initially, I am implying on the another place. Now, we are after inductor, we are placing this switch. So, this converter is called a boost converter. Why is it called a name boost? Because whatever voltage you are going to apply here, finally, whatever voltage you are going to get here is greater than this.

So, in this way, you have a buck converter and actually a boost converter. So now I have two things because I have a capacitor and an inductor. I have two energy-storing elements, and due to that reason, the order of the system is two. So, a state is current as well as voltage. So, current is always faster than the output voltage, and for that reason, I am able to deploy the property of singular perturbation that was actually coined by Kokotovic.

It is possible to show that in the inner loop, which is the current control loop, I will deploy the sliding mode control, and in the outer loop, I will be able to design any PI controller. So, this is a sliding mode control loop, the innermost loop and outer loop where I can deploy any kind of controller, and finally, what is our aim is that I have to maintain some kind of Vd. So, whenever you are going to design, talk about the control design: what is the first step? First, you have to create the mathematical model. So, the mathematical model of the buck converter is not so difficult. What can you do? You can apply Kirchhoff's current law and Kirchhoff's voltage law.

$$\dot{x}_1 = -\frac{1}{L}x_2 + u\frac{E}{L}$$
$$\dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{RC}x_2$$

And I know that I have only two operations. So, both operations are denoted by U , and U can take any value between 0 and 1 . So, first, what am I going to do? I am going to apply the idea that I have this switch, and here I have some inductors. So, what am I going to do? I know that the current is flowing through this inverter.

So, I am going to do L dIL by dt. So, IL is the current. So, this is nothing but some kind of voltage, and I know that here voltage, whatever voltage comes into the picture, is V . So, positive to negative. So, finally, if you look carefully, I am going to apply the E voltage here. So, the voltage E is nothing but either here E or E that I have written here.

Now, either E will come into the picture or not come into the picture, depending on the switching sequence, and for that reason, I am going to multiply by E. U into E, so that will cover both cases and after that now this and minus this voltage, whatever voltage across the capacitor that is equal to the voltage across the inductor, this inductor. So, using the same equation if I apply it here, and after that, if I adjust L and I , I will place L as x 1 and V as x 2 ; then I will get this equation. Please check it. So by application of Kirchhoff's voltage law, I can able to get this equation.

Now you can apply Kirchhoff's current law within this loop. So here you can see that the IL current is coming, and now this current is going to distribute across the capacitor as well as the resistor. So, voltage is V. So, current is V divided by R.

So, x_2/R comes into the picture. Here, I have Q equal to CV and $\mathrm{dQ/dt}$ equal to $\mathrm{CdV/dt}$. And if you put everything in, then you will get this equation. So, in this way, you can be able to get the second-order mathematical model, which is justified because I have two energy storage elements. Now, what is our main goal? I have to maintain a voltage Vd that is less than E. And I have to justify why that is less than E, with the help of sliding mode control.

So, this loop is slower, so I am assuming that $\dot{x}_2$ is equal to 0 , because the first loop, the current loop is very, very fast. So, by putting $\dot{x}_2 = 0$ and $x_2 = V_d$, and by applying singular perturbation theory here, $\dot{x}_2 = 0$, and what I am going to do is put $x_2 = V_d$, then you are going to get x_1.

$$x_1^* = \frac{V_d}{R}$$

From this expression, the second equation, you can easily get this. Now, what is our aim? By applying sliding mode control, I have to force x_1; x_1 is nothing but whatever current is coming from this first differential equation, which is exactly equal to x_1^*.

So, if this phenomenon occurs, then obviously, I will achieve the V_d, required value of $x_2 = V_d$. So, how do I do that? So, for this, control I am going to apply because I do not have any choice to apply other form of control. This is a bilinear system, and this is the attractivity condition; finally, it is possible to show that this kind of condition comes into the picture. So, in detail, I am going to show you. What is the required condition? I have to maintain $s\dot{s} < 0$.

$$s\dot{s} < 0$$

So, if you want to do that, it is possible to show that x_2 is always going to lie between 0 and E . This means that whatever value of V_d is going to lie between 0 and E . Due to that reason, by construction, you can see that I will get a lesser voltage than whatever we are going to apply, because x_2 is nothing but the desired voltage. So, that is going to lie, and this kind of thing you can be able to maintain $s\dot{s} < 0$ throughout the interval; you have to maintain that.

Now, I have written the same kind of things. Naturally, this operation comes into the picture due to the sliding mode and s equal to 0 ; now I have to maintain it. So, once s equals 0 , at that time $x_1 = x_1^*$, and what is x_1^* ? That is V_d/R. Now, once x_1 is equal to 0 , the sliding surface is also equal to 0 . And whenever the sliding surface is equal to 0 , then the equivalent control comes into the picture. So, how do you calculate the equivalent control? You can put $\dot{s} = 0$.

So, s equal to 0 means $x_1 = x_1^*$ will give you because s is nothing but $x_1 - x_1^*$, and this is constant. So, $s = x_1 - x_1^*$. So, if you keep this equal to 0 , you will get the equivalent control and that equivalent control you have to place here. And if you solve it, then you can see that as $t \rightarrow t_h$, then perfectly $x_2 \rightarrow V_d$. So, some kind of exponential convergence comes into the picture.

$$x_2(t) = V_d + (x_2(t_h) - V_d)e^{-\frac{1}{RC}(t-t_h)}$$

So, in this way, one can achieve the required voltage, and in the simulation, I have taken E equal to 20, Vd equal to 7 , and you can see that one can perfectly achieve this, and other parameters are given like this. Now, if you understood the previous operation, so this is also very easy to understand. Now, here again, I am going to develop the mathematical model of this system, and this is a boost converter. And why boost? Because we are going to show you that whatever voltage is on the output side that is greater than the input voltage.

$$\dot{x}_1 = -(1-u)\frac{1}{L}x_2 + \frac{E}{L}$$
$$\dot{x}_2 = (1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2$$

And how does this equation come into the picture? So, if you look carefully, whenever u equals 1 , at that time both C and R are disconnected from the circuit, but whenever u equals 1 , then it is actually connected to the voltage source.

Again, I am going to apply KCL, Kirchhoff's current law, and Kirchhoff's KVL, Kirchhoff's voltage law. So, in the first equation, I am going to apply KVL, Kirchhoff's voltage law. So, here I have Ldi/dt, x_1 is the current, and after that, you can see here that once u equals 1 , at that time the capacitor is not connected. So, that kind of condition is automatically coming from here, but at that time, E is here. So, E is equal to Ldi/dt, but once this is connected, then what happens? I have this voltage; after that, this voltage drop also comes into the picture, and due to that reason, I have x_2 and u equal to 0 .

So, basically, minus x_2 comes into the picture, and Ldi/dt is going to come into the picture by division. Similarly, what do we have to do now? I have to apply Kirchhoff's current law. So, once I am in position, the current is going to enter here and that is going to distribute.

So, in this way, this mathematical model comes into the picture. So, this mathematical model will represent a kind of system called a non-minimum phase system.

So, what is the physical interpretation of a non-minimum phase system in the case of linear and non-linear systems? In the case of a linear system, if our zero is in the open right half-plane, then that system is called a non-minimum phase system. Similarly, in the case of a non-linear system, if you keep the output and its derivative equal to 0 , and if the required dynamics are unstable, then it is called a non-minimum phase system. So, controlling a non-minimum phase system is not so easy. Here, control becomes more difficult than the buck converter because control also appears on both loops. Means towards the current as well as the voltage, but here again they will satisfy the motion separation principle; means the voltage loop is always slower than the current loop, and due to that reason, we are going to apply the principle of singular perturbation theory again.

So, I have a fast current loop and a slow voltage loop. So, I have two loops. So, inside the inner loop, I am going to deploy the sliding mode control. An outer loop, I am going to apply the voltage regulation principle. And in that particular way, it is possible to show that our overall performance is robust. So, again, what is our aim? So, our aim is that whatever voltage is here on this side should be higher, and that should be lower than the output side.

So, now again what I am going to do is maintain $x_2 = V_d$ and $\dot{x}_2 = 0$, and based on that, I am going to calculate the value of x_1. Here you can see that x_1^*I have adjusted in such a way that R is the resistance, but here V_d/E. So, one term is the desired voltage by the error. So, I want to maintain that this is always greater than or equal to 1 . So, if this is greater than or equal to 1 , it is possible to show that whatever output voltage is always more than whatever we are going to give as the input.

$$x_1^* = \frac{V_d^2}{RE}$$

And how to do this job? Using sliding mode control, I am able to do it. I can define the current loop error as $x_1 - x_1^*. x_1^*$ is nothing but $V_d^2/(RE)$. An equivalent control during the sliding I am going to apply this. So, how do you calculate equivalent control? I have to maintain $\dot{s} = 0$. So, I have to maintain $\dot{x}_1 = 0$, because V_d, R, and E are all known to us.

$$u_{eq} = 1 - \frac{E}{x_2}$$

Now, what I am going to do is show you the outer loop by substituting the equivalent control; then this kind of dynamical equation comes into the picture.

$$\dot{x}_2 = -\frac{1}{RC}x_2 + \frac{1}{C}\left(\frac{V_d^2}{x_2}\right)$$

So, briefly, I have written everything, and after that, I will explain that $x_1^* = V_d^2/(RE), s = x_1 - x_1^*$, and $s\dot{s}$. It is possible to show that this kind of expression comes into the picture now. You can easily maintain this kind of condition by observing the dynamics, and x_2, x_2 is derived from the solution of this. Again, if you look at this solution carefully, then x_2 will converge to the V_d as $t \rightarrow \infty$, because we have some kind of exponential asymptotic convergence, and the same kind of things you can see here, that now the given voltage is 20 , but what kind of voltage do I want to maintain? I want to maintain twice the applied voltage, and that kind of action can be easily achieved by deploying sliding mode control.

$$x_2(t) = \sqrt{V_d^2 + (x_2^2(t_h) - V_d^2)e^{-\frac{2}{RC}(t-t_h)}}$$

Now, another main area of sliding mode control is that people have found that if you deploy some kind of dynamic sliding mode control to the system, it means that in place of the actual current here, you can see what basically we are doing, whatever sliding surface we are defining. So, the sliding surface is based on the exact information of x_1 and x_1^*. So, instead of that, if you design some kind of observer and directly feed that information, one extra dynamic is required. So, if your system order is 2 , then the overall system order becomes 4 . And in that particular way, one can be able to improve the overall performance of the control loop.

So, this kind of feedback is called dynamic output feedback. And dynamically, we are going to actually apply I^* here. And if you look carefully, I have an observer system that is an extra system that comes into the picture by copying this particular system. I am going to give some kind of initialization. Again, the problem is that I am going to maintain the required value, and then based on that, I have to maintain the I^* current loop. So from there, I am going to deploy some kind of sliding mode concept based on some kind of observer.

And obviously, we are going to utilize a simple Luenberger observer here. So, now obviously, observer dynamics is exactly similar to plant dynamics. And, for stability analysis, asymptotic stability is enough. Here, you can able to deploy other observer also, sliding mode observer also, you can able to deploy. And sliding mode control is designed based on the observed current rather than the measured current. So, during, doing this, it is possible to show that overall, performance of control system is going to increase.

Now, what is the role of sliding-mode control? So, they are going to actually observe the current track with the reference current, and after that, using the concept of equivalent control, it is possible to show that one can maintain real current as well as real voltage. And during this particular process, reduced order dynamics comes into picture. It is possible to show that the original system order is 2 . Whenever you are going to design the observer, again order is 2 .

So, the total order is 4 . And during sliding, one order is raised. So, the overall order is 3 . So, in that particular way, reduced order dynamics comes into picture. We are going to see this through the mathematical equations. Observe the output voltage converge to the desired voltage that we are going to set. And obviously, the existence of the sliding mode control comes into the picture by selecting the proper initial condition.

If you are not going to select this proper initial condition, then some kind of energy mismatch comes into the picture and that will deteriorate the overall control loop. So, first I am going to develop some kind of observer-based control for the buck converter, and after that, I will move on to the boost converter. So, this is actually the structure of the current loop of the buck converter. What have I done? I have added the correction term, and I have copied the dynamics; due to that reason, I have used $\hat{x}_1$ here.

$$\dot{\hat{x}}_1 = -\frac{1}{L}\hat{x}_2 + u\frac{E}{L} - l(\hat{x}_1 - x_1)$$
$$\dot{\hat{x}}_2 = \frac{1}{C}\hat{x}_1 - \frac{1}{RC}\hat{x}_2$$

So, if you remember the design of the observer, the Luenberger observer. So, what is the first step? Whatever you know, you can copy it onto your computer, and after that, you can just add the correction term. So, here we are going to add only one correction term; the second dynamics we are going to copy as it is because somehow the sliding mode is going to deploy with respect to the first dynamics, but we have to show that $\hat{x}_1$ and $\hat{x}_2$ will both converge asymptotically to the original values x_1 and x_2. So, how do we prove that? So, for that, I have to actually analyze the error dynamics. So, the error dynamics I have calculated between x_1, where x_1 is the original current and $\hat{x}_1$ is the estimated current. Similarly, x_2 is the original voltage and $\hat{x}_2$ is the estimated voltage, and we are going to take the derivative of this; now it is possible to show that the overall dynamics are second-order dynamics.

$$\dot{\tilde{x}}_1 = -\frac{1}{L}\tilde{x}_2 - l\tilde{x}_1$$
$$\dot{\tilde{x}}_2 = \frac{1}{C}\tilde{x}_1 - \frac{1}{RC}\tilde{x}_2$$

Using the Laplace transform, you can also see that this polynomial is always positive, and due to that, it is possible to show that $\tilde{x}_1$ and $\tilde{x}_2$ both tend towards 0 asymptotically.

$$p^2 + \left(l + \frac{1}{RC}\right)p + \left(\frac{1}{RC} + \frac{1}{LC}\right) = 0$$

So, once $\tilde{x}_1$ is tending towards asymptotically, then it is possible to show that x_1 is tending towards $\hat{x}_1$. Estimated current is actually exactly equal to the real current, and similarly, estimated voltage is equal to the original voltage; vice versa, you can easily write. Now, on the sliding surface, you can see that I am going to design based on the estimated current. So, here control is also based on the estimated sliding surface, and now sliding mode is going to occur with respect to the estimated surface that is given like this.

$$\hat{s} = \hat{x}_1 - \frac{V_d}{R}$$
$$u = \frac{1}{2}(1 - \text{sign}(\hat{s}))$$

Due to that reason, $\hat{x}_1$ is going to converge to V_d/R, and at that time I have to calculate the equivalent control. So, how do you calculate equivalent control? So, from here, you can easily see that $\dot{\hat{s}} = 0$. So, basically $\dot{\hat{x}}_1 = 0$; if you substitute in the observer dynamics, then you can obtain the equivalent value of the control.

One more important point I am going to highlight is that whenever you are deploying the observer, the same control input that you apply to the plant must also be deployed to the observer running inside your computer. Now, after calculating the equivalent control, what I am going to do is, since I know that during $\hat{s} = 0$ and the average value of $\dot{\hat{s}}$ also equals 0 .

And due to that reason, this dynamics, first dynamics that is going to collapse. So, how many dynamics do I have now? This is coming from the plant: $\dot{x}_1$ and $\dot{x}_2$, and $\hat{s} = 0$, and $\dot{\hat{x}}_1$ also equal to 0 . It means that these first dynamics will collapse, and due to that reason, I have only the second dynamics, but at that time, equivalent control comes into the picture. So, you have to substitute the control by equivalent control. And after that, I will define the error like this: this is the error dynamics.

$$\dot{x}_1 = -\frac{1}{L}x_2 + \left(\frac{1}{L}\hat{x}_2 + l\left(\frac{V_d}{R} - x_1\right)\right)$$
$$\dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{RC}x_2$$
$$\dot{\hat{x}}_2 = \frac{1}{C}\frac{V_d}{R} - \frac{1}{RC}\hat{x}_2$$

And it is not difficult to show that x_1 is going to converge to $\hat{x}_1$ by doing the analysis of this differential equation $\bar{x}_1^* = 0$.

$$\bar{x}_1^* = \frac{V_d}{R} - x_1$$
$$\bar{x}_2 = \hat{x}_2 - x_2$$
$$\dot{\bar{x}}_1^* = -\frac{1}{L}\bar{x}_2 - \frac{l}{L}\bar{x}_1^*$$
$$\dot{\bar{x}}_2 = \frac{1}{C}\bar{x}_1^* - \frac{1}{RC}\bar{x}_2$$

So, by designing l, l is nothing but the gain of the inductor, which is always positive; R and C are positive. So, it is possible to show that this phenomenon occurs, and due to that reason, $\hat{x}_2$ is actually going to converge to V_d, and in this way, the voltage is going to match the true voltage. And how do we talk about existence? You have to maintain this $\hat{s}\dot{\hat{s}} < 0$. So, if $\hat{s}\dot{\hat{s}} < 0$, then this kind of condition comes into the picture, please check it. An equivalent control, how do you calculate the equivalent control? So, you can just keep this equal to 0 , then you will get this kind of equivalent control.

One important point: if you calculate the stored energy, once convergence occurs, the stored energy mismatch becomes 0 , because x_1 is going to converge to V_d/R and V_d is going to converge to x_2. So, in this way, you can check that your control is working fine. So, once sliding mode occurs in finite time, at that time x_1 is going to converge to $\hat{x}_1$, which is V_d/R that is set by us, and after that x_2 converges to V_d and energy tends towards 0 , and this is the simulation. So, in simulation what I have done, initial condition I have told you that whenever you are designing observer, then initialization of this particular system is very very important. So, proper initialization is required; if you do 00 initialization, some more difficulties come into the picture.

$$H(t) = \frac{L}{2}\left(\frac{V_d}{R} - x_1(t)\right)^2 + \frac{C}{2}(V_d - x_2(t))^2$$

So, more energy is injected into the system, and for that reason, I have not taken $\hat{x}_1$ and $\hat{x}_2$ equal to 0 ; I have taken some finite values, and it is possible to show that, using this, I can achieve the buck operation. Similarly, you can extend this concept for the boost converter. Again, what have I done? Here are the dynamics of the boost converter, and I have added some kind of correction term. So, during copying, I kept the $\hat{x}_1$ here and the $\hat{x}_2$ here, and I did a similar kind of copy for the $\hat{x}_2$. I have just added a correction term here because I know that in the innermost loop I am going to deploy the sliding mode control. Again, what am I going to do? I am going to define the error, and in the error coordinate, I will get this kind of equation. Using Lyapunov analysis, since this is a highly non-linear system, I have u , which depends on s , and for that reason, I have taken the help of the Lyapunov function, and it is possible to show that $\tilde{x}_1$ and $\tilde{x}_2$ both equal 0 .

So, $\tilde{x}_1$ and $\tilde{x}_2$ both equal 0 , and $\tilde{x}_1$, if defined as $x_1 - \hat{x}_1$ or vice versa, can be defined; based on that, the dynamics are going to change here; negative and positive are just going to change. So, is there any way you can define it? Now, what are we going to do? I am going to take this as a sliding surface.

$$\hat{s} = \hat{x}_1 - \frac{V_d^2}{RE}$$
$$u = \frac{1}{2}(1 - \operatorname{sign}(\hat{s})), v = \frac{1}{2}(1 + \operatorname{sign}(\hat{s}))$$

So, now, on the sliding surface, I am going to take the estimated value of the current. Here, since we are seeing the dynamics of the boost converter, current control explicitly appears with respect to current as well as voltage, and at that time, $1 - u$ comes into the picture. So, if you can define it like this, then the complementary thing that v comes with is that v is nothing but $1 - u$. So, one sliding mode occurs at that time by the definition of this $\hat{x}_1 = 0$. So, x_1 is tending towards $V_d^2/(RE)$. And V equivalent, how do I calculate the equivalent control? As you can take and their derivatives equal to 0 .

So, $\hat{x}_1 = 0$, and what is $\hat{x}_1$? This is the dynamic of the observer. So, you can just set $\dot{\hat{x}}_1 = 0$, and you can calculate the equivalent control that is given like this. Now, I have current that is actually the current dynamics; this is the voltage dynamics, and one dynamic is going to collapse because during sliding

I have to apply equivalent control. So, due to the equivalent control average value of $\hat{x}_1$ that is also equal to 0 , if I substitute the equivalent control inside the system, then the order of the system, which is 2 for the original system plus 2 that is running inside the computer, is maintained. And now, due to sliding, 1 has collapsed, and for that reason, minus 1 occurs, making the total order 3.

So, all 3 orders you can see from here. Now, I have to show stability. So, how do you show stability? Again, I will define the error dynamics. So, this is the current error; this is nothing but the voltage error, and again, this is a highly non-linear system. So, how can we prove that $\bar{x}_1^* = 0$ and $\bar{x}_2 = 0$ at least asymptotically? So, one can take the help of the Lyapunov function. Before doing that, what we are going to do is this kind of substitution: in place of x_2, we are going to put $\hat{x}_2 - \bar{x}_2$ because I have defined $\bar{x}_2 = \hat{x}_2 - x_2$.

Okay, so these are the simplified dynamics. Please do it by yourself, and after that, we can define the Lyapunov function, and $\dot{V} < 0$. So, $V > 0$, and $\dot{V} < 0$. So, in that particular way, it is possible to show that $\bar{x}_1^*$ and $\bar{x}_2$ are both tending towards 0 , and as this is tending towards $0, x_1$ is going to tend towards $\hat{x}_1$, which is nothing but the $\hat{x}_1$, the desired kind of current that comes into the picture. Once the desired kind of current comes into the picture, the desired voltage also comes into the picture. And you can also see that if you deploy this kind of transformation, then this is nothing but some kind of linear system.

And in a linear system, this is already 0 . So, there is no other choice. y is basically V_d equal to x_1; a square that kind of thing comes into the picture, and in this way, I can prove that as $t \rightarrow \infty, x_2$ is actually tending towards V_d. So, that kind of thing comes into the picture. Here, the definition should be $x_2 - V_d$; only then does y tend towards 0 , and this kind of condition basically comes into the picture. And in this way, I can achieve the desired objective necessary condition because you have to maintain $s\dot{s} < 0$. Here, the sliding surface I am making is based on the estimated current, and due to that reason, $\hat{s}$ comes into the picture.

After that, I have to substitute $\dot{\hat{s}} = 0$; then I will get the equivalent control, but the equivalent control should lie between 0 and 1 . Why? Because in the whole converter operation, I am doing the 0 and 1 . So, somehow, we are controlling the duty cycle. Now, the energy matrix, if you will see, is such that once convergence occurs, energy is equal to 0 . And again in simulation, I have taken very, very high gain, and it is possible to show that one can maintain the voltage by using dynamic output feedback or dynamic sliding mode control because the output is current.

So, now, I have already told you that if you initialize like 00 , then obviously, you have to wait for a long time such that these two mismatches, which are actually energy mismatches, are minimized, and for that reason, proper initialization is always required. So, what is the key contribution? We have talked about sliding mode control of power converters, and power converters are actually essential nowadays due to the boom in EVs. So, now you are able to extend this kind of concept for AC to DC , DC to AC , buck-boost, or cuk converters; any kind of converter you are able to extend this concept. We have discussed both the observer-based and voltage regulation and stability, based only on whenever that is required; we have actually used that, and I have also shown the theoretical and MATLAB implementation. Obviously, sliding mode control has very natural and robust performance, and one can also check the stability and energy-based performance matrix.

Additionally, one can evaluate if one is going to deploy some kind of sliding mode control loop as a current control loop. Now, before ending this particular course, this is the last lecture on sliding mode control and its applications. So, this is a picture of my supervisor, Professor Bijnan Bandopadhyay. So, now he is retired from IIT Bombay, and he is a visiting professor at IIT Jodhpur.

I was doing my PhD at that time; this picture was taken around 2012 or 2013, and this is Asif Chalanga. Several times we have developed a lot of theory with this, and this is Abhishek. So, Abhishek is now a professor at IIT, sorry, IIT Roorkee, and after that, this is Prasidh; we are calling him Prasidh bhaiya. So, he is again, almost all are professors, and he is working in the industry; he is a master's student. And in 2012, my supervisor sent me to Professor Utkin, and Professor Utkin actually arranged some kind of school to teach sliding mode control to young students.

And I feel very lucky because I have learned many concepts. Particularly, from the application point of view, the theoretical point of view, the HopfFilippov solution, and Utkin's regularization are different. So, every 6 days, he was taking a 6 -hour lecture. So, within 36 hours, he taught me classical sliding mode control and some parts of higher order sliding mode control.

I will again feel lucky; you can see this guy, a very big guy. So, this is Professor Leonid Fridman, and this is the group. Here, he is Professor Morino. So, a lot of Lyapunov proofs you have seen that in this lecture I have used. This is also one of the professors in the same university in Mexico, and these two are my friends. So, during my stay, this is actually an actor, and this is Tonio.

So, both actually had a nice discussion with them, a nice dinner, and every time. So, for two months I was there, and during this visit, I learned a lot about how sliding mode control is developed and what kind of lacunae exist within sliding mode control. So, every story I can actually hear from this professor, and he is always very motivated and always sharing a lot of things. So, in any VSS or variable structure control, you can see this big guy. Now, here is Professor Levant. So, Professor Levant visited India in 2012, and he is Professor Xing Yu, one of those who are working in terminal sliding mode control.

So, I have actually collaborated a lot; still, we are collaborating with Professor Xing Yu, who is now in Australia and is my supervisor. So, the supervisor is actually arranging this conference VSS. Levant is a very serious guy. So, he is very less talkative; it is very difficult to collaborate with him, but concept-wise he is very sound.

And here, this is basically Ferrara. So, a lot of work is actually done by Professor Ferrara. And recently in 2024, my supervisor organized an IFACsponsored school that is held at IIT Jodhpur. So, this is Professor Fulwani. So, the nonlinear surface kind of work is done by Professor Fulwani.

Again, he is my friend, Abhishek. I have already told you that Professor

Abhishek is a professor at IIT Roorkee. He is in IIT Jodhpur, and this is Sara Spurgeon. So, you can see a very famous classical sliding mode book. This is Ma'am Shailaja. So, this is just like my sister. So, during my PhD, she helped me do a lot of experiments, and during several presentations, I shared the experiments that are actually the contribution of this particular lady.

So, with this particular remark, I will end this lecture. Obviously, I am going to connect you virtually, and at that time, I will share more stories and more unsolved problems I am going to share. So, I have taken almost 61 lectures and most of the lectures are somehow between 30 to 35 minutes. So, please go through each lecture. Some lectures may be very, very fast, but what you can do is pause and go through it.

Whenever you have confusion, do not forget to contact me. You can just send me an email. So, my email ID is syamkamal.eeee@iitbhu.ac.in. So, I think that within 1 to 2 days I will reply to that email. So, with this remark, I am going to end this lecture. Thank you very much.