Adaptive Sliding Mode Control for Industrial Emulator

Welcome back. In the previous lecture, I talked about the fusion of adaptive control techniques with another robust control technique, which is sliding mode control. And we have discussed four different methodologies. And it is possible to show that by using different combinations, we can achieve both real sliding and ideal sliding. In this lecture, I am going to show you how to implement adaptive control in some practical systems. So, the main purpose of the discussion is to control the industrial emulator.

I have covered this setup in previous lectures, and actually using this particular industrial emulator makes it very easy to check the control algorithm or the implementation of the control algorithm because this setup is directly compatible with MATLAB. So, now whenever we are talking about industrial emulator control, several challenges come into picture because there are several performance specifications associated whenever we are discussing some kind of industrial plant control problem. So, in every industrial plant control problem, our requirements or performance specifications are like this. We want a fast response.

It means that our settling time should be as short as possible. And obviously, precision should be very high. We also want robustness against the disturbance and control that I want to minimize. So, here sliding mode control is responsible for these two things: robustness as well as minimum control effort. If you fuse the sliding mode with the adaptive control, then one can simultaneously solve these two problems.

Now, these two problems still need some kind of actual improvement in this sliding surface design. So, what am I going to do for that? I am going to discuss a new kind of sliding surface design called a non-linear sliding surface. So, this sliding surface design is actually not like the integral sliding surface design or the terminal sliding surface design. So, both integral and terminal it is possible to show that they fall in the nonlinear sliding surface design methodology; it is possible to show that you are able to design the nonlinear sliding surface in such a way that you can achieve a fast response as well as precision, meaning low overshoot. It means that your settling time is short and the overshoot is very low.

So, for that, actually, Professor Bijnan Bandopadhyay and Professor Phulwani have given one of the methods, sliding mode control based on the non-linear surface-based design. What I am going to do here is add this technique to the adaptive control. So, this work is actually done during our PhD. So, this work is actually done by Professor Phulwani, and we are actually assisted by the master's student so that we are able to show the efficacy of the adaptive sliding mode control. on some kind of industrial plant.

And what is the key feature? Once you design a non-linear sliding surface, the variable damping ratio comes into the picture. So, initially the system is lightly damped, and after that it is possible to show by designing some proper function that once we are in a steady

state, at that time it becomes overdamped. And obviously, dynamic gain adaptation comes into the picture because we are also talking about adaptive control. And due to the idea of adaptive control, no barrier uncertainty bound knowledge is now needed. So, I am going to discuss the linear system in this lecture, but the same kind of philosophy can also be extended to non-linear systems.

So, here I have $z \in \mathbb{R}^n$, and I am assuming that I have a single-input single-output system. It means that this control belongs to $\mathbb{R}$, as well as whatever output also belongs to $\mathbb{R}$, and this uncertainty ϕ_d; I am assuming that this is matched uncertainty. What is the meaning of matched uncertainty? It means that I am able to express this uncertainty in terms of some kind of uncertainty that is going to enter this system through the control channel. Since I am going to design the control, I am assuming here that b is also controllable. Now, one of the easier ways to design sliding mode control is to express the original system in terms of the control such that the control will explicitly appear, where the dimension of the control is exactly the same as the dimension of the state.

So, here is what I am going to do. I am going to apply some kind of transformation T such that now $z \in \mathbb{R}^n$ and $u \in \mathbb{R}$. So, basically, in this particular transformation, now $x_2 \in \mathbb{R}, x_1 \in \mathbb{R}^{n-1}$. So, in this particular process, the design of sliding mode control becomes very simple since $y = c_1 z(t)$. Here, again, I have to express everything in terms of x, and for that reason, I can define $z = T_r^{-1} x$, and if you substitute that here, then obviously you will get $y = c_1 T_r^{-1} x$, and for that reason, this kind of expression comes into the picture.

In regular form, it is possible to show that this transformation, and I am assuming that whatever transformation I have constructed here is also invertible, then this A matrix is separated like $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$. So, in the course of classical sliding mode control, we have already discussed this, and after that, $T_r B = 0$, and B_2 and the disturbance are basically matched, and due to that reason, it is also separated, similar to B_2. Now, this is the nonlinear sliding surface design. So, why is this surface non-linear? You can see here that this system is linear. But whatever surface I propose will contain some kind of term, and this term is $\psi(y, r)$, y is the output, and r is the reference signal.

And what is our primary objective? Our prime objective is to maintain error equal to 0, $e_1 = 0$, as well as $e_2 = 0$. How do I select F? So, in order to select F, I am going to maintain A_{11}, A_{12} multiplied by F with a negative sign, and whatever eigenvalue I am going to construct should basically be strictly negative, and after that, their damping ratio should also be low. And after that, we are now going to construct $\sigma(y, r)$. So, there are infinitely many choices for $\sigma(y, r)$. I am going to discuss one of the choices in this lecture.

And what is our main goal? I have to maintain the damping ratio. That is actually from ζ_1 to ζ_2. So, the damping ratio will actually increase from some value ζ_1 to ζ_2. I will explicitly tell you once I fix σ, then how basically ζ_1 and ζ_2 come into the picture. After that, you can see that I have involved one more matrix, which is the design parameter P, so that it is going to

satisfy the Lyapunov equation.

So, here in this Lyapunov equation, P and W are both positive symmetric definite matrices. So, what is the meaning of a positive symmetric definite matrix? That is $P = P^\top > 0$. It means that all eigenvalues of the P matrix or W matrix are strictly positive. Now, here you can see that the non-linear term, whatever we have designed based on the output and reference signal, can be expressed like this. So, what is the physical interpretation of this? If $y = y_0$ at that time, if you look closely, then this whole term e^{-1} and e^{-1} become 0.

So, now, $\zeta = 0$. But once I actually track the reference signal, it is possible to show that once tracking is achieved, I have $y - y_0$ and r also becomes y, or y becomes r. For that reason, this becomes $1 - 1 = 0$. So, $1 - \epsilon$ is the same as this, and at that time, the second kind of damping comes into the picture, that is, ζ_2. So, in this way, this particular term is responsible for varying the damping, and smoothly, it is going to vary the damping. Why? Because this term is actually differentiable with respect to y. So, this kind of thing is basically taken from the work of my supervisor, Bijnan Bandopadhyay, and he is a professor at IIT Jodhpur, and he is one of the professors in Korea.

So, what am I going to do now? I am going to show two things that you should consider when designing sliding mode control. By keeping this kind of non-linear surface, then $s = 0$. So, once $s = 0$, at that time from this definition, it is not difficult to show that e_2 can be expressed like this. Just you have to do the manipulation and after that, now whatever resulting system, so you can substitute here x_2. So, what can you do first? You can express everything in terms of the error dynamics.

How can I express it in terms of error dynamics? You can actually define this. So, $e_1 = x_1$ and $e_2 = x_2$. Suppose that these two are constant; then you can design this sliding surface, and after that, you can express it in terms of the resulting dynamics. And now we know that damping is going to change. So, once sliding comes into the picture, at that time this becomes β.

So, I have to show that this whole equation is somehow Lyapunov stable. That kind of thing I have to show. Now, I am going to talk about how to design controls. So, at the start of this lecture, I already told you that in an industrial plant, most of the time, I do not know the uncertainty bond.

That is the challenge. So, for that reason, I have to adopt some kind of adaptive methodology, and obviously, if a certain plant bound is given, then I can deploy another methodology that is provided by Professor Utkin, which is an equivalent-based, control-based adaptive methodology. Now, what am I going to do? I am going to select controls like this. Now I have written, if you see here, then here I have used v and suddenly I started using v. So, I will tell you what the relation is between v and v in the subsequent slide. So, this kind of control I am going to propose, and after that, $\dot{k}$ I am going to maintain by this particular

differential equation, where s is the bound of the sliding surface.

What we are going to show again here is exactly like the previous lecture: once you design this kind of control. And obviously, by the Lyapunov method we have seen in the previous lecture that if I have some system like s and v and some disturbance here, then ϕ_{d2} is something you can assume. So, if you design a control like this, where the adaptive term is like this, then you can easily design a Lyapunov function in terms of s and k, and then it is possible to show that $s = 0$, and that will be maintained throughout the time interval. In the second approach, you can redesign the gain based on the low-pass filter concept, and at that time, you have to maintain some kind of gain condition, such that it is possible to show that once we achieve the sliding mode, at that time this gain is responsible for maintaining the sliding.

And this approach obviously requires the knowledge of uncertainty because you can see that $\bar{k} > B^{-1}A$. So, here B and A are given, which makes it easy, but several practical plant B and A matrices are not actually known, or they will know that their bounds are only known. So, at that time, obviously, you have to calculate that the bound and low-pass filter are going to actually create some kind of difficulty during implementation. So, obviously, the delay comes into the picture. Some kind of transient response comes into the picture because, here again, a differential equation is involved.

So, the order of the system is going to increase overall, and the signal magnitude is attenuated due to the combination of two systems that are somehow coupled by the differential equation. Another approach that we have already discussed is the combination of both approaches: when $s_t \geq \epsilon$, you can apply the first methodology to increase your gain, and once you are in sliding mode, you can actually switch to the methodology given by Professor Utkin. And obviously, the fourth one I have discussed is the methodology of the real sliding. So, all four that we discussed in the previous lecture, I am going to implement in this particular lecture. So, you can select anyone whenever you have to implement classical sliding mode control.

The number of control input is unity. If you have more than one control input, then what can you do? Again, like in the vector case, you can do component-wise analysis, and you can achieve the same kind of theory again for the multi-input, multi-output system. So, let us discuss the sliding surface again. This is a non-linear sliding surface because this σ term is nothing but non-linear in nature, which is responsible for varying the actual damping ratio from some value of 0 to $-\beta$, as we have already seen. After that, once we are during the sliding, if I define ℓ, then this kind of substitution comes into the picture, and due to that reason, if you calculate s, you can see that control explicitly appears.

Now, I am assuming that e_1 is known and e_2 is known. So, whatever known term and whatever term that is during this sliding, I am going to write together. And after that, what I am going to do is design v so that this entire term will disappear. And finally, by designing

this, we can land up to this term.

And we have seen how to adopt this. So, how do we proceed? You can see that initially I have dynamics in terms of e_1 and e_2; I am going to express everything in terms of e_1 and s. So, this whole dynamics is expressed in terms of e_1 and s. Thus, this is somehow similar to the backstepping process. Now, I have to design v. So, what am I going to do? I am going to design two different parts, one part that has a nominal value.

So, it is possible to show that once sliding comes into the picture at that time of the discontinuous part. So, this is the discontinuous part. So, that becomes actually equivalent to the disturbance and the whole system is sliding due to the equivalent control u_0. So, what is the value of equivalent control? You can easily calculate it by setting $\dot{s} = 0$ and removing the disturbance. So, this kind of thing I am going to substitute.

And after that, obviously, you can see that I have substituted minus the new term here, which is responsible for inducing the sliding and maintaining $s = 0$ for all finite time greater than or equal to t, and maintaining $s = 0$ in the presence of uncertainty. And obviously, this term ψ, which is the damping term, I am going to design like this. And obviously, I have told you how to select F. So, F is basically involved during the L.

. So, P is designed based only on a function ψ, based on the damping, changing the damping. F, again, I have to design based on the pole placement or eigenvalue placement. And now, once I have this system and if you design adaptive sliding mode control here, it is possible to show that $s = 0$. So, once $s = 0$, this is the dynamics during the sliding, and I have to show that this dynamics is stable; obviously, due to ψ, this whole dynamics is non-linear. So, we have to prove using some kind of non-linearity.

Non-linear theory states that $e_1 = 0$, and since sliding is a combination of e_1 and e_2 equal to 0, $s = 0$ means $e_2 = 0$ is also equal to 0. So, the first approach is actually given like this, and the second approach, which is responsible for real sliding, is given like this. We have already discussed it in more detail in the previous chapter or lecture, and for that reason, I am not going to discuss it again. Now, I have to prove that once $s = 0$, then the error dynamics is given like this. So, I have to prove that the error is equal to 0 as $t \to \infty$.

How do you prove it? I will define a quadratic Lyapunov function. What is the meaning of a quadratic Lyapunov function? P is a positive symmetric definite matrix, where all eigenvalues of P are positive. And $e_1^\top P e_1$, that kind of form comes into the picture, and after that, if I calculate $\dot{V}$, then it is not difficult to show that I will get this less than or equal to 0. Now, I am going to implement this kind of methodology to the industrial emulator, and this setup is actually available in the lab of the IIT BHU System and Control Lab, and this educational setup is provided by California, USA.

And here, if you look carefully, I have two disks. So, this is one disk, a load disk, and

somehow here I have one drive disk. So, now, here we have a DC motor and a brushless DC motor. So, first we are controlling this disk, and after that, we can obviously control this load disc through this belt. So, I have a drive disc with a servo actuator, and after that, a load disc that is adjustable. Adjustable inertia means you can keep weight here, and after that, we have a timing belt and speed reduction assembly.

It is not difficult to show that in this arrangement of the brushless DC motor, we have an elastic belt, and after that, the secondary drive motor has an adjustable friction brake that is also present, and finally, a high-resolution encoder is there to measure the position. And the servo amplifier is also there. So, these are just the descriptions of this particular plant. And, if you are going to create the mathematical model, obviously, this whole system is driven by just one motor. And, due to that reason, the second-order model comes into the picture, where z_1 is nothing but the angular position and z_2 is the angular velocity.

And I am assuming that we have this kind of uncertainty. And the saturation limit, since this is a practical plant, is always given. So whatever adaptive control you are going to design should fall within this range only. Now, if I apply this, this plant, if you see carefully, is already in regular form, and due to that design, I am assuming that whatever transformation I am going to apply is the identity, and A_{11}, A_{12}, A_{21}, I can easily define, because I have not changed anything due to the identity transformation. And after that, we designed this kind of sliding surface, and our assumption is that I am going to consider 17.39, and after that, β; I am going to consider 336, and obviously, β is varying from 0 to 336, and P I have selected based on the Lyapunov strategy.

So, we have selected two sets of gains. So, this is the first set of gains, and this is the second set of gains. What are our specifications? So, we have to control the position from 0 to 50 degrees. So, that is 0.87 radians, and we are going to show you the simulation result as well as the experimental result if you apply the control.

So, here we have adaptive control. So, I have two controls, and if you design the non-linear sliding mode control. So, you can see that this is the simulation result and this is the experimental result. Both the simulation results and the actual experimental results look exactly the same. Here, we have applied adaptive control one and adaptive control two, so almost real sliding and ideal sliding look the same most of the time. This is the case whenever you are applying sliding mode control to a practical system, and after that, this is the adaptive control.

Position control. So, position control; this is the position control after which we are talking about the error. Error means the difference between x_1 and x_d. So, you can see that performance during the simulation, as well as during the experiment, is also very good, and after that, this is the sliding surface. So, obviously, whenever you are doing practical implementation, due to chattering, this kind of phenomenon basically comes into the picture.

And finally, you can see the controls. So, actually, what happens whenever you are implementing some kind of sliding mode control? So, you are not implementing directly. What are you basically doing? You are actually changing the signum function to a saturation function, and then you are applying it; due to that reason, whatever control that looks like appears to be continuous. And after that, this is the control effort during the practical implementation. So now it is time to conclude this lecture. So, what have we seen in this lecture? So, there are two things we have actually seen in this lecture.

So, the first important aspect is how to design a non-linear sliding surface such that it is implementable in a practical plant. Most of the time, I do not know the bond of the gain, and due to that reason, we have to incorporate the strategy of adaptive control. So, how do we redesign sliding mode control such that gain adaptation comes into the picture, and for that reason, we are going to take advantage of two different things? That non-linear sliding surface, as well as the adaptive control. And if you combine these two methodologies for most industrial plants, you are able to get at least four benefits. So, obviously, your overshoot is low, and the settling time is very short.

You can gain robustness with respect to disturbance. Chattering is reduced because of whatever gain you are going to apply. That you are going to apply based on real sliding mode control or a combination of sliding mode control. So, most of the time it is possible to show that gain over estimation does not come into the picture. So, once gain is small and the gain of the discontinuous term is somehow directly related to the amplitude of chattering, due to that reason, you can get less chattering. And obviously, control effort is also minimal because by using minimal control effort, I want to mitigate or minimize the uncertainty.

And there is no proper knowledge of the uncertainty because I am talking about adaptive control. So, with this remark, I will end this lecture. Thank you very much.