

# Sliding Mode Control and Applications

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So, welcome back. In the previous class, I talked about continuous integral sliding mode control. And what was our main intention? Our main intention is to show how to realize higher sliding mode control based on continuous control. And up to 2014, this problem was actually unsolved. That is how to realize arbitrary higher-order sliding mode control using continuous control action. And this is our contribution during the PhD.

Under the guidance of Professor Bijnan Bandopadhyay and our colleague Dr. Asif Chalanga, we have proposed several solutions, and one of the solutions I am going to discuss in this lecture is continuous terminal sliding mode control. So, somehow we are able to actually generalize the super twisting algorithm for the second order system, and by algorithm-wise, that is third order sliding mode control, such that one can establish that third order sliding mode control can be achieved just using continuous control. And we have given a specific name.

Initially, the name is the generalization of the super twisting, but after observing the behavior, we have converted the name to a continuous terminal sliding mode. So, we are trying to understand why the name continuous terminal sliding mode control is given to this particular algorithm. So, let us start this lecture. So, this lecture is mainly focused on third-order sliding mode control. So, here I am not going to generalize for any arbitrary order; I am just going to concentrate on the third order.

But it is possible to show that using a similar kind of methodology, one can generalize it for any order. And if you are interested, then you will be able to see one of the works that is highly cited, which is the arbitrary order super twisting algorithm. So, we have generalized the super-twisting algorithm for any order. So, Professor Levant has proposed

a super twisting algorithm that is applicable for first-order plants in order to maintain stability or finite time stability based on continuous control, but suppose now if the plant is second-order:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u,$$

and suppose we have some kind of disturbance  $d(t)$ . So, if you want to apply super twisting control here, then again you have to proceed in some way, you have to design a sliding surface, and by designing the sliding surface, you will be able to maintain  $s = 0$ , and finally, you will achieve asymptotic stability.

So, somehow you are going to find out that both state  $x_1$  and  $x_2$  are still asymptotically stable, and somehow the charm of higher order sliding mode control does not come into the picture, because what is the main intention of higher order sliding mode control? If output is equal to  $x$ , then output and all its higher-order derivatives in the presence of uncertainty should be moved to both  $x_1$  and  $x_2$ , which are equal to 0, in finite time.

So, finite time stability is one of the core ideas whenever we are talking about higher order sliding mode control, but if you go this way, by proposing a sliding surface, then you can take the derivative and maintain an algorithm like

$$\dot{x} = k \operatorname{sign}(x_s),$$

and after that, we are going to add another term  $k_1$ , and here  $k_1$ , this is  $k_2$  and  $\operatorname{sign}(s)$  and

$$\int_0^t \operatorname{sign}(s) ds.$$

So, obviously, I can maintain  $s$  and  $\dot{s}$  both equal to 0, but the charm of finite time stability is lost in this process, and for that reason, this problem remained unsolved up to 2014 based on my knowledge.

And what is our contribution? What have we done? We have proposed some variants of the terminal sliding mode control. And obviously, control is continuous, exactly like the super twisting.

Due to that reason, we have coined the term continuous terminal sliding mode control to solve this kind of problem. So, by designing  $u$  as a continuous control, it is possible to show that in finite time, you can force  $x_1$  and  $x_2$  to be equal to 0. In this particular process, third-order sliding mode control comes into the picture.

So, similarly, whenever I have a plant like

$$\dot{x}_1 = u + d(t),$$

and if you design super twisting here. So, obviously  $x_1 = 0$  in finite time, but in this process, if you implement the super twisting algorithm here, then sliding mode is second

order.

A similar kind of phenomenon basically comes here. So, after this lecture, you are able to understand how to obtain some kind of algorithm, particularly the super twisting kind of algorithm for the second-order plant, and overall, the algorithm is the third order.

I will prove this, and obviously, in this contribution, it is possible to show that classical or conventional terminal sliding mode control has a singularity issue. So, some kind of non-singular terminal sliding mode is proposed by Professor Xiang Yu. However, it is possible to show that the accuracy of the control is not exactly like the accuracy of higher-order sliding mode control.

Accuracy is also one of the very important points, because whatever algorithm we are developing in continuous time, we are implementing in discrete time. And after that, obviously, control is discontinuous, and the third point is that control becomes unbounded in the case of singular sliding mode control.

A new proposal, whatever we are going to discuss in this lecture. So, I will get finite time convergence in the presence of uncertainty; obviously, there is no singularity, control is bounded, and we get smooth control. So, somehow this is also one of the classes of fifth generation control, where we are going to realize third order sliding mode using continuous control action.

So, let us just revisit sliding mode. So, suppose that I have a second-order system; how are we basically stabilizing  $x_1$  and  $x_2$ ? We are designing a sliding surface

$$s = c_1 x_1 + x_2,$$

and after that, we are defining a kind of law that is called the reaching law.

So, here  $\phi$  is called the reaching law such that in finite time I can reach from any initial condition to this surface, and after that, we are basically maintaining along the surface.

In this coordinate system, I have variables like  $x_1$  and  $s$ , and after that, what I am going to do since this term is known to us, I am going to keep control inside, and I am going to design  $\phi$ .

Once  $s = 0$ , I have dynamics like

$$\dot{x}_1 = -c_1 x_1,$$

with  $c_1 > 0$ , and it is possible to show that  $x_1 \rightarrow 0$  as  $t \rightarrow \infty$ .

Since  $s = c_1 x_1 + x_2$ , if  $s = 0$ , then  $x_2 = 0$ . So, linear switching surfaces guarantee asymptotic stability.

Non-linear switching manifolds have already been seen during terminal sliding mode control, where the sliding surface is non-linear. So, as we move closer to the equilibrium

point due to non-linear control, I will achieve finite time stability.

But control is discontinuous and might become singular. So, one of the limitations comes into the picture when we take

$$s = x_2 + x_1^{\frac{q}{p}},$$

where  $p$  and  $q$  are positive odd integers.

Reduced order dynamics is given like this, and it is possible to show finite time convergence, but the control contains singularity because  $\frac{q}{p} \in (0, 1)$ , and near the origin, the control becomes unbounded.

So, Filippov solution does not exist globally, and we have to restrict the initial condition.

Now, what am I going to do? I am going to take the help of the super twisting algorithm again.

So, ceiling and floor function is defined as

$$\lceil z \rceil^p = |z|^p \text{sign}(z).$$

We define weighted homogeneity using weights  $r_1, r_2, \dots, r_n$ . A function is homogeneous if it satisfies

$$f(\lambda^{r_1}x_1, \lambda^{r_2}x_2, \dots) = \lambda^k f(x).$$

If the degree of homogeneity is negative and the system is asymptotically stable, then finite time stability holds.

Now, consider the second-order plant

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u + \mu(t),$$

with  $\mu(t)$  bounded and differentiable.

We propose the algorithm

$$\phi = \lceil x_1 \rceil^{\frac{1}{3}} + k_1 \lceil x_2 \rceil^{\frac{1}{2}},$$

constructed using weighted homogeneity by scaling

$$x_1 \mapsto \lambda^3 x_1, \quad x_2 \mapsto \lambda^2 x_2.$$

We introduce the fictitious state

$$x_3 = z + \mu,$$

and obtain a third-order differential inclusion.

The weights are

$$x_1 \sim \lambda^3, \quad x_2 \sim \lambda^2, \quad x_3 \sim \lambda,$$

so the degree of homogeneity is  $-1$ , which guarantees finite time stability.

A Lyapunov function is proposed of the form

$$V(x) = \beta|x_1|^{\frac{5}{3}} + \gamma_1x_2^2 + \gamma_2x_3^2 + \gamma_3x_2x_3.$$

This function is continuously differentiable and homogeneous of degree 5.

Using Young's inequality

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}, \quad \frac{1}{p} + \frac{1}{q} = 1,$$

we show positive definiteness.

Taking the derivative and imposing sliding along  $\phi = 0$ , we show

$$\dot{V} \leq -c\|x\|^\alpha,$$

which implies uniform finite time stability.

Scaling arguments allow gain selection for arbitrary disturbances.

Simulation results confirm that all states converge to zero in finite time with continuous control, and chattering is eliminated.

So, the theme of this lecture is how to extend the super twisting algorithm. Extension is not obvious, and for that reason, this problem remained unsolved for decades.

Even today, stability proofs beyond fourth order remain an open research problem.

So, using this methodology, fifth generation sliding mode control has become popular for practical applications.

With this remark, I am going to end this lecture. Thank you very much.