Sliding Mode Control and Applications

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Week-09

Lecture-39

Welcome back. In the previous class, I talked about higher order sliding mode control. Particularly, I was discussing twisting and super twisting. So, both algorithms belongs to second order sliding mode control. Now, one question is very, very important: how to extend the same kind of algorithm for third order, fourth order, or any higher order? The answer is not straightforward. It is possible to show that if you have a third-order system and the same twisting, if you take two gains and two switches, and if you extend to three gains and three switches, then convergence becomes very, very difficult.

Due to that reason, some alternative solution is required. In this particular lecture, I am going to show you the alternative solution. So, the main purpose of the discussion is to introduce one of the ideas where it is possible to show that by using continuous control, you can realize any order of sliding mode control, because one of the important requirements whenever we introduce higher order sliding mode control is to generate sliding mode directly to the equilibrium point while maintaining continuous control. So, several discontinuous controls that we are going to discuss in the subsequent lectures are proposed by Professor Levant, although the algorithm proposed for the higher order is very limited in the literature. Now, in this lecture, I am going to show you that just using a very simple idea proposed by Professor Levant, one can realize continuous control for any-order systems, and this is particularly the contribution from the Indian side under the guidance of Professor Bijnan Bandopadhyay on how to realize continuous higher sliding mode control for any other system. And, these classes of sliding mode control are also called fifth generation sliding mode control. And, somehow it is able to solve several problems that are actually commonly occurring in the industry due to discontinuous control. So, let us start the lecture. So, the main idea is to introduce the notion of continuous integral sliding mode control.

So, you have already understood what the meaning of integral sliding mode control is. So, the main idea of what we are basically doing is that whenever we have a system, we are going to start sliding from the initial point onward; there is no reaching phase. Suppose I have a second-order system; I am going to start sliding from here, and I am going to maintain. In this way, the role of the sliding mode control is to reject the disturbance, and now whatever control we have learned during the previous modules, integral sliding mode control, is discontinuous because it is based on the first-order sliding mode control. In this lecture, I am going to first tell you how to actually extend that algorithm so that by using continuous control you can realize the same kind of phenomena.

And obviously, I am going to utilize the beautiful property of the super twisting algorithm, and it is possible to show that the super twisting algorithm has the same property as first-order sliding mode control. It is also able not only to reject the disturbance, but one can also estimate the disturbance. So, that property I am going to utilize, and after that, I am going to design any arbitrary order sliding mode control. And obviously, we have verified the algorithm on some kind of electromechanical system, which is actually available in the lab of IIT Bombay, specifically the Systems and Control Lab of IIT Bombay. So, using this particular method, what is the learning outcome? The learning outcome, obviously, is that we are trying to understand the limitations of classical sliding mode control and, after that, we are trying to introduce the notion of arbitrary order sliding mode control based on continuous integral sliding mode control.

So, obviously, whatever algorithm I am going to utilize is actually a super twisting algorithm. So, before we go on to understand the further concept, let us try to revise the notion of higher-order sliding mode control. So, what we have seen in the previous classes is that I have a system like

$$\dot{x} = f(x) + g(x)u,$$

and after that, if I have

$$y = \sigma(x).$$

And suppose that if I want to maintain sliding along this output variable, then what are we basically doing? How are we basically defining order? We are taking the first derivative. So, after taking the first derivative, if control does not explicitly appear, then we are going to proceed to higher-order derivatives.

And in this way, whenever we are taking the rth order derivative, at that time discontinuous control appears. So, now what is our goal is to design some kind of discontinuous control u such that σ_1, which is the first derivative up to σ_{r-1}, all equal to 0 at infinite time, and after that, σ_r should be a solution of σ_r or a solution of the whole set understood in the sense of the Filippov.

In order to maintain second-order sliding mode control, what do we have to do? I have to take a second-order system, a kind of second-order uncertain system, which will be represented by ϕ and some kind of ψ, where both ϕ and ψ are uncertain. Now, I have to design some kind of discontinuous control system.

So, based on that, we have basically designed a first control called the super twisting algorithm, which is only applicable for relative degree one systems; although this is a second-order sliding mode algorithm, it is applicable for first order systems.

It means that if you take a system like

$$\dot{x} = u + d(t),$$

and if you design u, the first term is this, and the second term x_2 is the integral of this, and obviously, here I am assuming disturbance $d(t)$. Finally, this is converted into the form of ρ like $\dot{d}(t)$ after taking some kind of transformation.

And how basically we have proved the convergence of this algorithm? We have proved the convergence of this algorithm using some kind of state transformation, and now, using an algebraic equation, it is possible to show that if I design gains k_1 and k_2, then it is possible to maintain x_1 and x_2 both equal to 0 in finite time.

So, let us try to see what the idea of integral sliding mode control is. This is classical integral sliding mode control that was actually coined by Professor Utkin. So, the idea is supposed to be that if $x \in \mathbb{R}$, then I have a one-dimensional manifold. So, what am I going to do? I am going to design this kind of control

$$u = -k\,\sigma(x),$$

and in the design of this control, it is possible to show that $x(t) = 0$ after a finite time $t \geq t_s$, and beyond that, it is going to be maintained.

What does it mean? That $x = 0$, but the instantaneous value of $\dot{x}$ is not 0; however, it is possible to show that in order to maintain $x = 0$, the average value of $\dot{x}$ must equal 0, which is also called the equivalent control method.

Now, let us try to see the super-twisting control. So, I am going to start with exactly the same system:

$$\dot{x} = u + d(t).$$

Now, what am I going to do? I am going to design the super-twisting controller. So, the first term is

$$-k_1|x|^{1/2}\,\mathrm{sgn}(x),$$

and after that, I have

$$\dot{v} = -k_2 \operatorname{sgn}(x).$$

So, due to that reason, this is known as smooth PI kind of controller.

Now, you can see that since $d(t)$ is differentiable, the algorithm will converge. The condition on $\dot{d}(t)$ that comes into the picture is

$$\sup_t |\dot{d}(t)| \leq \Delta.$$

So, if you select the gains based on Levant's condition, it is possible to show that x and z both equal to 0 in finite time. Since $z = v - d(t)$, it follows that

$$d(t) = -v(t).$$

So, in this way, I can easily calculate the disturbance.

Now, let us come to higher-order sliding mode control. Suppose the relative degree of the system with respect to $\sigma(x)$ is n, then at the nth derivative, control explicitly appears. So, in an ideal case, it might be possible that a nonlinear system is completely feedback linearizable.

Suppose that I have a completely feedback linearizable system, then it is possible to show that you can design some kind of state feedback control like

$$u = -k_1 x_1 - k_2 x_2 - \cdots - k_n x_n,$$

and if the polynomial is Hurwitz, then

$$x_1, x_2, \ldots, x_n \to 0 \quad \text{as } t \to \infty.$$

But here our requirement is stronger: we want finite-time convergence. For that, we use weighted homogeneity. We scale the states as

$$x_1 \mapsto \lambda^{\alpha_1} x_1, \quad x_2 \mapsto \lambda^{\alpha_2} x_2, \quad \ldots$$

with $\alpha \in (-1, 1)$.

Using the Bhat–Bernstein framework, it is possible to show that all states x_1 to x_n are finite-time stable.

Now, consider a second-order system:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u + d(t).$$

In the absence of disturbance, classical feedback works. But in the presence of disturbance, equilibrium deviates.

So, what am I going to do? I am going to modify the previous control and add a sliding mode component. Let

$$u = u_{\text{nominal}} + u_{\text{discontinuous}}.$$

Define a sliding surface based on the last subsystem. Then,

$$\dot{s} = u + d - d_{\text{nominal}}.$$

During sliding,

$$\langle \dot{s} \rangle = 0 \Rightarrow u_{\text{discontinuous}} = -d.$$

Now, what am I going to do? Instead of discontinuous control, I am going to use continuous control via the super twisting algorithm:

$$u_{\text{STC}} = -k_4 |s|^{1/2} \operatorname{sgn}(s) + v,$$

$$\dot{v} = -k_5 \operatorname{sgn}(s).$$

So, this continuous control estimates and compensates for the disturbance. Hence, the whole system is governed by the nominal finite-time controller.

Thus, overall control is continuous, and discontinuity appears only in higher derivatives. So, it is possible to realize any higher-order sliding mode control using continuous control.

Finally, experimental validation was done on a belt–disk system. For rigid and flexible belt configurations, simulation and experimental results show reduced chattering and excellent tracking performance.

So, to conclude, combining Bhat–Bernstein finite-time control with super twisting integral sliding mode control allows realization of arbitrary-order sliding mode control using continuous control action, which is highly suitable for mechanical actuators.

With this remark, I am going to end this class. Thank you very much.