Sliding Mode Control and Applications

Dr. Shyam Kamal

Department of Electrical Engineering

IIT(BHU) Varanasi

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So, welcome back. In the previous class, I was talking about the super twisting algorithm and what we have seen is that one can develop the convergence proof of the super twisting algorithm based on Lyapunov analysis. And whatever Lyapunov analysis we have actually done in the previous class is a strict Lyapunov function. It means that the same Lyapunov function can be valid in the absence of disturbance as well as in the presence of disturbance. We have also actually proved that the Lyapunov algorithm is nothing but a kind of non-smooth PI algorithm, which is capable of rejecting certain types of ramp disturbances, because any condition that comes into the picture is in the form of the derivative of the disturbance. I am going to talk about another convergence proof of the super twisting algorithm that is based on the measurement curve method proposed by Professor Levant.

So, for the purpose of the discussion, I am going to talk about the super twisting algorithm, which is nothing but a kind of second-order algorithm. We have already seen in the previous class that this algorithm can retain

$$\dot{x}_1 = -k_1|x_1|^{1/2}\,\mathrm{sgn}(x_1) + x_2, \qquad \dot{x}_2 = -k_2\,\mathrm{sgn}(x_1).$$

So this is in algorithmic form, and it is possible to show that if disturbance is not present, meaning if disturbance is 0, and if you scale x_1 by λ^2 and x_2 by λ, then this whole algorithm again becomes homogeneous with degree -1. How can that be established? You can write here that λ^2 and the degree of homogeneity of time is -1, and here I have λ^1.

So, you can see that both sides are actually matching. So, you can also write like

$2-1$ here. So, somehow the degree of homogeneity is -1. Similarly, here you can see that I have scaled x_2 by λ into x_2. So, λ comes here and here λ^0 because of the signum function.

So, x_1 and λx_1 have the same sign because $\lambda > 0$. So, on this side I have λ^1, and on this side I have λ^0. So, I am able to write $1-1$, and in this way, due to time, I am able to write $1-1$. So, in this way, I can show that the degree of homogeneity for the super twisting algorithm is -1. So, basically, the super twisting algorithm is nothing but some kind of secondary algorithm.

And that is also provided by some kind of homogeneous differential inclusion. And since in form of control that is going to generate some kind of continuous signal. Why? Because if you observe in the form of control, then I have $k_1|x_1|^{1/2}\,\mathrm{sgn}(x_1)$. So, this is continuous, and after that, the integral of the discontinuous term. So, obviously, that is also continuous.

So, the addition of two continuous functions is a continuous function, and that reason is responsible for the mitigation of the chattering. And obviously, in the previous class, we established finite time convergence based on the strict Lyapunov function. In this lecture, I am going to talk about a geometrical proof, and this geometrical proof is given by Professor Levant.

Professor Levant is actually one of the very pioneers in the field of higher order sliding mode control. And he has actually coined the notions of twisting, super twisting, and then some notion of homogeneous differential inclusion.

So all the work is actually done by Professor Levant, and after that, this work is extended by several other people: Professor Leonid Fridman, then Professor Morino, and some other professors. And after that, you can see that in order to apply the super twisting algorithm, we just need the information of x if we have only a first-order system, and these are the gain conditions.

I have told you that this gain condition is basically given by Professor Levant, and this will work very well whenever you are applying super twisting control to any practical plant. So, let us start with exactly the same system that we discussed in the previous class. So, I am going to take the first-order system.

I am assuming that whatever disturbance is differentiable, and one more important thing I am assuming is that the derivative of the disturbance is also bounded by some number d_0, which you can assume, and this is the supremum over all t.

So, what is our assumption? It might be possible that the disturbance is unbounded, which means I can take some kind of ramp-like disturbance. So, this is unbounded disturbance, but you can see that their derivative is bounded.

So, if you substitute here and if you simulate this algorithm, it is possible to show that $x = 0$, $\ell = 0$. So, x is equal to 0 and ℓ is equal to 0 in finite time.

So, from the first equation, you can also tell that $x = 0$ at the same time. So, in this algorithm, x and $\dot{x}$ both equal 0 in finite time.

If you select output $y = x$ or $\sigma = x$, it is possible to show that $\dot{\sigma}$ is actually $\dot{x}$, and $\dot{x}$ is nothing but this expression. So, this is continuous. So, that will not satisfy the property of second-order sliding mode control, and for that reason, I am going to take one more derivative.

So, $\ddot{x}$. So, what is the meaning of $\ddot{x}$? That is the derivative of this term plus the derivative of $\dot{\ell}$. So, the derivative of $\dot{\ell}$ contains some kind of signum function. So, due to that reason this super twisting algorithm represents the second-order sliding mode control.

I have already discussed that for a first-order plant, this algorithm is going to generate some kind of continuous control and that continuous control is responsible for making sure that $x = 0$ as well as $\dot{x} = 0$ in some finite time $t \geq T$.

And after that, in order to implement control, the only requirement is that you should take the information of x. No derivative information is required, like twisting, if you are planning to implement it on the first-order plant. But you have to know that whatever disturbance should be differentiable as well as bounded, and you should also be able to select the gain easily.

Now, it is also possible to show if you have a second-order plant. Suppose I have a plant like

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = u + d(t),$$

and if you are willing to implement the super twisting algorithm.

So, how do we implement it? So, it is possible to show again that you can design a sliding surface like

$$s = c_1 x_1 + x_2,$$

similar to the linear combination I have designed; after that, what you can do is take the derivative of this surface. So, that is

$$\dot{s} = c_1 \dot{x}_1 + \dot{x}_2.$$

Now, $\dot{x}_2$ is nothing but $u + d(t)$. Here, basically, I have used this notion u or v. So, you can use the same notion v here.

So, that is basically

$$\dot{s} = c_1 x_2 + v + d(t).$$

Now, since I know that the super twisting algorithm is applicable for the relative degree one system as a control, so I am able to design super twisting like

$$v = -k_1 |s|^{1/2} \operatorname{sgn}(s) + z, \qquad \dot{z} = -k_2 \operatorname{sgn}(s).$$

Or you can equally be able to write in the form of the integral. In this way, it is possible to show that $s = 0$ as well as $\dot{s} = 0$.

Because if you see this whole algorithm, now in this whole algorithm, I have a structure like $\dot{s} = k|s|^{1/2}\,\mathrm{sgn}(s)$. So, here I have nothing but a dot. And after that, the $d(t)$ term will actually be reflected here as $\dot{d}(t)$.

So, $s = 0$ and $\dot{s} = 0$, and control is also continuous because the first term is continuous, and the integral of the discontinuous term has a discontinuity at the zero measure, which is also continuous.

So, overall control is continuous. So, using continuous control, I can able to slide along this line. So, in a two-dimensional plane, this is the straight line.

So, I can apply continuous control, and I am able to slide along the line, and what is beautiful is that $s = 0$ and $\dot{s} = 0$ exactly.

In the case of the first-order sliding mode control, what was our observation? $s = 0$, but $\dot{s} \neq 0$; the average value of $\dot{s}$ is equal to 0. But if you apply the super twisting algorithm, then $s = 0$ as well as $\dot{s} = 0$.

And due to that reason, accuracy is very, very high, but again, you are able to converge to the equilibrium point asymptotically, because $s = 0$ leads to some kind of differential equation,

$$\dot{x}_1 = -c_1 x_1.$$

So, the same kind of things in this way I have actually retained here.

Finally, with this remark, I am going to end this lecture. Thank you very much.