Sliding Mode Control and Applications

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Lecture-37

So, welcome back. In the previous class, I was talking about the twisting algorithm. In this class, I am going to talk about another very, very popular second-order sliding mode control algorithm that is called the super twisting algorithm. If you check the literature on sliding mode control, it is flooded with the super twisting algorithm because one can use the super twisting algorithm for various purposes. For the control design, for observer design, for differentiator design, and for all other different kinds of purposes like parameter estimation and disturbance estimation. So, there are several huge applications of the super twisting algorithm.

In next lecture, we will also see that how one can able to talk about the integral sliding mode control just based on the super twisting algorithm. But in this lecture, I am going to focus mainly on the super-twisting algorithm. So let us discuss the super twisting algorithm. So, again I am going to justify the name, just like the twisting algorithm.

So, here word super, it means that obviously our trajectory is more twist than this twisting algorithm. Along the y-axis. Now, it is also very important to see that this particular algorithm, if you apply to first order system, then that will generate some kind of continuous control and it is possible to show that in that way you can able to mitigate the chattering. Obviously, you cannot be able to completely remove the chattering. So, in this lecture, I am mainly going to concentrate on the finite time convergence of the super twisting algorithm, and for that, I am going to do Lyapunov analysis.

So, this Lyapunov analysis is actually proposed by Professor Morino and super twisting algorithm is proposed by again Professor Levant. So, this is one of the very important contributions in the field of sliding mode control literature, because I have already told you that there are huge applications of the super twisting algorithm. So, what is the outcome? We are first trying to understand the super twisting algorithm, its control law, and after that, finite time convergence in the presence of uncertainty. So, in this lecture I am going to talk about a strict Lyapunov function because in previous class I have told you that whenever we have weak Lyapunov function then Lyapunov function only valid if there is no disturbance and meaning of a strict Lyapunov function or you can also tell a strong Lyapunov function that same Lyapunov function is working in presence or absence of the disturbance. We are also going to talk about the gain selection criteria for robust performance.

Some kind of optimal kind of gain is actually suggested by Professor Levant and during any practical application you can able to utilize that kind of gain and you are able to see that result is very very optimal. So, in order to understand the need for super twisting, I am going to connect it with twisting again. So, why basically notion of twisting algorithm comes into picture? So, our problem is to suppose that if I have a first-order system and if you design

$$u = -k\,\sigma(x),$$

where σ is actually in place of x and is the variable, and if you keep

$$k > \sup_{t} \max |d(t)|,$$

then it is possible to show that you can get $\sigma = 0$ as $t \to T$ or $t \geq T$, some finite time. So, in some finite time, you can maintain $\sigma = 0$, but what is the difficulty? If you analyze this particular control, then at $\sigma = 0$, this control is discontinuous. And we have already seen that discontinuous control can excite several unmodelled dynamics either inside the plant, in actuator side or sensor side.

Due to that reason, we need some kind of continuous control action. So, after that proposal of twisting algorithm comes into picture, and what is main motivation of twisting algorithm? To convert discontinuous control into continuous control. So, what was the strategy? The strategy is that you can take the higher derivative of this. So, higher derivative is

$$\dot{u} + \dot{d}(t).$$

So, here I am assuming that whatever disturbance that is differentiable as well as their derivatives are bounded. So, even if the disturbance is not bounded, that is not a problem; however, their derivative should be bounded.

At that time, I am now going to design $\dot{u}$ and explain how we are basically going to design $\dot{u}$. I am going to design the information of the variable σ and its rate of change. It means that σ and $\dot{\sigma}$ are both required to generate continuous control for a first-order system.

And obviously, some kind of comment comes into the picture. What kind of comment is that? This comment is made by Professor Leonid Fridman, who suggests that if you are saying that in order to generate continuous control, $\dot{\sigma}$ is required. So, the problem here is very easy since you know the control; you know $\dot{\sigma}$. So, $\dot{d}(t)$ is known, and if $\dot{d}(t)$ is known, then what is the need for sliding mode control? Directly you can design any PID controller, any other controller and due to that reason some new control strategy comes into picture.

If you see carefully, so previous controller which is known as the twisting controller is some form of PD controller. Why I am telling this is some kind of PD controller or switched gain controller because you can see that I have just used the information of sign and gain and here I am going to take the information of $\text{sgn}(\sigma)$ and information of derivative of $\text{sgn}(\sigma)$. So, just this is the modification of PD control.

Now, this control, which is called super twisting control, is nothing but a kind of control known as smooth PI control. You can see here how to propose a PI. I am taking k_1 and information on x. So, basically, now our plant looks like

$$\dot{x} = u + d(t).$$

But obviously, you can talk in terms of the same variable. So, just for the sake of proof, I have changed the variable, but you can be consistent with this variable. At that time, you can see that I have $k_1\sigma$ here. The PI controller I am talking about has

$$k_2 \int_0^t \sigma(\tau)\, d\tau.$$

Now, how is this basically different from the PI controller? So here I am going to just change the state σ to x and $u + d(t)$. So, now you can see here that what we have done, x I am going to scale by some nonlinear relation $|x|^{1/2}\,\text{sgn}(x)$ and after that I am going to multiply with $\text{sgn}(x)$. And here what I am going to do, in place of the exact integral of some state, I am going to take their sign integral.

So, in this way, this is nothing but non-smooth PI. Why is it non-smooth? Because you are able to take the derivative of this. σ is differentiable and the integral of something; integration is obviously the reverse process of differentiation.

But here, what kind of restriction? Since I have $x^{1/2}$, at $x = 0$, this is not differentiable. Let us check the continuity. So, this function is continuous. Why is this continuous? Because this continuity only appears at $x = 0$.

At all other points, this is basically continuous. And if $\text{sgn}(x) \geq 0$, you can see the graph, and if $\text{sgn}(x) < 0$, then the graph is like this. And at $x = 0$, even if $\text{sgn}(x)$ lies between -1 to 1, but that is multiplied with $x^{1/2}$ and $x^{1/2} = 0$ and due to that reason this will coincide here and in this way this term is continuous.

So, first term is continuous and second term is integration of a discontinuous term. And discontinuity lies only at measure zero, $x = 0$, and you can actually remove that area because whenever we are taking an integral, the physical interpretation of the integral is the area under that curve.

So, $x = 0$ is just contributing zero area, and for that reason, this is also continuous, and for that reason, super twisting generates some kind of continuous control.

Now, I have to justify why this is second-order sliding mode control. So, in order to justify that, what am I going to do? I will take the second derivative of $\dot{x}$ because what is the definition of second-order sliding mode control? That supposes that if you have output $y = x$, then if you take second derivative, then control will appear and that control should be discontinuous.

But what happens here? If you start with this system, a first-order system, and if you take $y = x$, then $\dot{x}$ control explicitly appears, but control is continuous. That is no longer discontinuous.

Due to that reason, I have to take one more derivative. And if you take one more derivative, then $k_2 \operatorname{sgn}(x)$ comes into the picture. And due to that reason, this is applicable for first-order systems as a control, but in algorithmic form, this is basically a second-order sliding mode algorithm.

And due to the second-order sliding mode control algorithm, x and $\dot{x}$ are both equal to zero in finite time.

Now, again, I am saying that the disturbance is differentiable. So, super twisting control will work only if you have some kind of differentiable disturbance, and a boundedness condition is also required such that this $k_2 \operatorname{sgn}(x)$ is going to handle this disturbance. However, there is no restriction on $d(t)$; just continuity is enough, and for that reason, super twisting can handle some kind of ramp disturbance as well.

Because ramp disturbance looks like kt, and if you take its derivative, then its derivative is bounded.

So, that is the beauty of the current proposal: the classical PI controller is not able to handle the ramp disturbance, but this one is able to handle the ramp disturbance. This is also able to handle some kinds of time-varying disturbances.

Finally, we conclude that super twisting produces continuous control, finite-time convergence, and only information of x is required for a first-order plant.

So, with this whole remark, I am going to end this class. Thank you.