Hybrid Chattering Suppression Approach

Welcome back. In the previous class, I was talking about how to suppress the chatter. And we have discussed the boundary layer approximation as well as classical observer waste design to suppress the chattering. In this lecture, I am going to continue the chattering suppression strategy. And for that, I am going to discuss some kind of control, some kind of composite control, which is based on the philosophy that I am going to utilize the knowledge of the mathematical model of the actuator, and based on that, we are trying to minimize the chattering. So, the purpose of the discussion is the hybrid chattering suppression approach.

What is the meaning of hybrid? Basically, we are trying to hybridize the control. So, hybrid does not mean continuous or discrete here; the meaning of hybrid is continuous control with a discontinuous sliding mode part. Another approach, in which I am also incorporating the information on actuator dynamics, is that you might have seen in the previous class; I have not incorporated the dynamics of the actuator during the control design. Just to show stability, I have used the knowledge of the mathematical model of the actuator.

Here during the design itself, I am going to involve the actuator dynamics, and obviously, in this way, we are already aware that sliding mode control suffers from the inherent chattering. complete disturbance rejection required the some kind of discontinuous part. So, now, we are trying to show that it is possible to somehow replace discontinuous control with some equivalent form of continuous control and then minimize the chattering. So, I am going to start with exactly the same plant. So, this is a first-order plant.

And again, exactly the same assumption. What kind of assumption do I have? That $a$is unknown, but I know the lower bound and upper bound. $b$is not going to change the sign, and for that reason, $b > 0$. So, that is the kind of information I am using. $\omega(t)$is again the control variable, and $d(t)$is uniformly bounded.

Now, this is the actuator model, and what is our assumption now? I am assuming that the actuator model is partially known to me. So, I am not assuming that everything is fully known to us, but that partially it is known to us, and here again, $\omega$is the unknown bandwidth and $\mu = 1/\omega$, and again our aim is exactly the same. So, $x(t)$is the output of the plant; now I have to force $x(t) = x_d(t)$, and at the same time, I have to apply sliding mode control and minimize the chattering. So, those kinds of things I am going to look into. So, the first solution is based on the regular form solution.

That addresses the chat ring by incorporating the partial knowledge of unmodeled dynamics of the actuator. You can see that I have specified the actuator here. So, this kind of strategy is not suitable if I have some kind of unmodeled dynamics on the sensor side. So, several times we have already seen how we can actually remove discrepancies on the sensor side by designing some kind of proper H matrix or some kind of gain matrix, which we learned during the chattering analysis. So, it is not difficult to tune the sensor.

Now, what am I going to do? I am also going to take care of the unmodeled dynamics which are associated with the actuator based on the regular form design. What is the meaning of regular form design? We are trying to understand this particular lecture. So, here the design based on the regular form is basically a cascaded control structure. And obviously, I have told you that it is suitable for a system with measurable actuator output. Because of that measurement, I am going to utilize it somewhere to design some kind of loop where sliding mode is going to apply.

So basically, cascaded control contains two steps. The first step is to design continuous control for the plant, assuming that actuator omega t is the control input. So, I have already told you that if I have a plant, how are we basically applying the control? Through the actuator, we are applying the control. So, this is actuator dynamics. So, whatever output of the actuator is the input to the plant.

Now, for the second step, I am going to design the sliding mode, and what am I going to do? I am going to design one more loop where I will force $\omega$to equal ω_t. In this way, I am able to design cascaded control that also involves some information about the actuator dynamics, and then I will obtain the control that is actually going to minimize chattering. Why? Because that control is not directly applicable to the plant. So, let us try to examine the actuator model. What am I going to do? I am assuming that the exact model of the actuator is not available, but at least I know the actuator bandwidth.

So, that kind of assumption is required. In the lecture, you can see that I am assuming that this $\omega$is completely unknown, but here I am assuming that I know the actuator bandwidth. Particularly for electrical circuits, this assumption is fully valid. $u_t$is the control input and $\omega_t$is the measurable actuator output. So, I have a way to measure $\omega(t)$.

And I have already discussed that the regular form of approach is not applicable if unmodeled dynamics are mainly introduced by the sensors. So, if that is actually introduced by the actuator, then only we can be able to design this kind of philosophy. So, this is the complete block diagram. Here you can see the sliding mode; what is the role of sliding mode? First, I will design some kind of $\omega(t)$that is applicable to the plant. So, that particular $\omega$is nothing but $\omega(t)$.

Due to the unmodeled dynamics, you can see that $\omega(t)$and $\omega_d$are not going to match. How can one basically decide ω_d? Because I know that $x(t)$is the output of the plant and what kind of desired trajectory I have to track. So, based on that, I can design x_e, and based on the information of x_e, I can decide ω_d. Now, by designing sliding mode control inside this inner loop, what am I going to do? I am going to actually force $\omega(t)$to be equal to ω_d. Now, $\omega(t)$can be selected based on the continuous control strategy as well.

So, in this way, I can able to fully utilize the benefit of sliding mode controller. For the system with an electrical actuator, we are using DC motors or some kind of BLDC motors most of the time. So, we know their dynamics. So, we can able to utilize that information to design continuous control for the plant. So, obviously, performance is going to be enhanced by incorporating the actuator dynamics.

And, in order to apply this kind of strategy, namely a cascaded or hybrid control strategy, I require the measurement of actuator output. If I do not know this measurement, I cannot apply or design this kind of state. So, I have already told you that we have two loops: one is the auxiliary control loop, and what is our goal? Our goal is to find ω_d, and this control, ω_d, is responsible for tracking $x(t)$. Here, I am assuming that control, since $\omega_d$is a continuous control. So, I can always assume that it is bounded by some kind of upper bound and its rate is also bounded.

Now, how I am basically going to select $\omega_d$is based on the state feedback control, and since I have to solve the tracking problem, I have selected $x_d - x(t)$, the difference between the desired trajectory and the trajectory that is generated by the system. So, if you come to the error dynamics, I have to calculate the error. So, in error dynamics, what actually happens? That this is the control action, but some kind of unmodeled dynamics also comes into the picture because whatever plant I have considered, this kind of plant I have $b\,u + d$ here. And all these things are uncertain, and for that reason, I have actually kept all parts here. So, it is possible to show that one can further improve the design if I know some parts of this dynamics; the nominal part I will design is called feedforward control.

So, if I keep that control, then again I can be able to improve the disturbance estimation accuracy in our control design. So, now, what is the second step? So, the first step is over. Now, since I know that if I design ω_d, then I can track $x(t)$tending towards x_d, what do I have to do? Next step, whatever the output of the actuator is, due to that design, I am stating that measuring the output is required in order to implement the sliding mode control strategy. I am defining the error between $\omega_d(t)$and $\omega(t)$, and now I am going to design the sliding mode control. And what is the sliding surface here? That is the linear combination of $\omega_e$and $\dot{\omega}_e$.

So, once $s = 0$ from this dynamics, you can see that this equation is nothing but

$$\dot{\omega}_e(t) = -\frac{1}{K}\,\omega_e(t).$$

And this is first-order dynamics. So, once $s = 0$, there is no other choice. What is happening? That $\omega_e = 0$, so you can also see that $\omega = \omega_d$. So, basically, $\omega_d = \omega(t)$.

In this way, by forcing sliding mode control, I can force the actuator to generate a continuous signal that is $\omega_d$and is applicable to the plant. So, basically, here sliding mode is actually forcing the actuator to generate some kind of continuous control. And once control is continuous, then I will be able to mitigate the chattering effect because there are several unmodeled dynamics that we have not modeled during the modeling of the original system. And what is M? $M$is the sliding mode control gain. So, this is the known part that comes from the actuator, and this is the sliding mode control.

So, overall control, you can see that I have two parts. Now, I actually have to analyze the error dynamics. So, how do we analyze the error dynamics? So, you can calculate $\ddot{s}$. And after that, you can substitute everything, then you will get this kind of dynamics. Now, if I rearrange, this is a second-order system, so by designing $m$to compensate for all kinds of uncertainty, I can force the sliding mode $s = 0$ through the design of the gain, because the gain is very, very important here; it should be positive, and I can directly implement the control.

And what is beauty? $x(t)$ converges to 0in infinite time because I have applied some kind of on–off control here, and $x(t)$ converges to x_d, and these are the simulation results you can see here. So, based on this, I can be able to design the solution that is based on the regular form, and if you see carefully, then actually this contains continuous control plus the sliding mode control. Practically, continuous control is applicable to the plant, and in this way, I can see that even if I have unmodeled dynamics inside the plant, it is not going to falter. Now, let us try to see the second approach, which is also called the disturbance rejection approach. So, what is the overview? Overview of this approach shows that the disturbance rejection solution combines continuous control and discontinuous control.

And due to that reason, we are also stating that as a hybrid control. And what is the main strategy? Again, I have to maintain tracking and reject disturbances; that is the goal. So, overall control is nothing but the combination of two controls, that is continuous and discontinuous control. So, this continuous control is actually capable of compensating for the uncertainty. And I am able to avoid the chatter in the main loop.

Obviously, I am not able to completely mitigate the chattering, but at least I can avoid it in some loops. So, you see control the overall system behavior and desired track and actually accuracy is maintained. And after that, what is the role of discontinuous control? Discontinuous control is going to suppress the disturbance as well as the parametric uncertainties. And how do you apply discontinuous control? Again, I am going to apply the classical sliding mode philosophy.

So, this is the block diagram. You can see here that in the block diagram, again I have an actuator and a plant. And what is our main goal? I have to keep $x(t) = x_d(t)$. So, this is the desired signal. So, we have designed two controls: one is discontinuous control and the other is continuous. This is the discontinuous controller and this is the continuous controller.

Why is this a continuous controller? Because I am going to assume that it is passing through a low-pass filter. So, the equivalent value of the control comes into the picture. So, the equivalent control is applicable here, and due to that reason, that becomes the continuous controller. So now, let us try to see the design part. So, here the desired trajectory x_d is known, as I am assuming that the disturbance is also known and b is known.

So, here in this analysis, our assumption is x; suppose our plant equation is like this:

$$\dot{x}(t) = a\,x(t) + b\,u(t) + d(t).$$

And plus $d(t)$. So, I am assuming $b$is known to us. So, it is easy to apply this kind of strategy. So, what are people doing? Whenever they have a disturbance, they are actually forcing that disturbance into the input and treating it as a matched disturbance with respect to the main disturbance.

And in this way, this assumption is always satisfied. So, what is our objective now? I have to design $u_c$continuous control and discontinuous control such that I can exactly track $x_d$in the presence of the disturbance. So, what am I going to do for that? I am going to design the continuous control. So, continuous control contains two parts. This is x_e; $x_e$is nothing but the difference between the desired trajectory and the original trajectory, that is, $x_e = x_d - x(t)$.

So, this part is somehow feedback, and this is the feedforward part. Okay, so I know $x(t)$and $x_d(t)$, and I know $\dot{x}_d(t)$, so I am going to utilize their information. Okay, so this control contains both feedback and feedforward parts. Okay, now what is our main goal? Since our main goal is to maintain $x_e(t)$tending towards 0as $t$tends towards infinity, that is our main goal, and due to that reason, I have to express everything in terms of the dynamics of $x_e(t)$. So, if I take $\dot{x}_e(t)$, then $\dot{x}_d(t)$comes into the picture and $\dot{x}(t)$comes into the picture.

So, basically, $\dot{x}_e(t)$is nothing but $\dot{x}_d(t) - \dot{x}(t)$, and I will substitute the $\dot{x}(t)$expression and the $u_c$control there. You can see that basically I have dynamics, and that dynamics comes like this. Okay, now what I am going to do is, this is the continuous design part. Now I am going to design the discontinuous part, the sliding mode part. Okay, so what I am going to do, if you see the block diagram, also things become clear to you that basically I have two things.

S contains Z and X_e, which is the linear combination of these two. So, for that reason, I have defined it like this:
$S = X_e + Z_t$.

What is X_e?
X_e = X_d – X_t,

and Z_t is nothing but some kind of auxiliary variable. Also, you can assume that as a sliding mode variable. And I am going to define that variable like this because I know that I have information of x_t dot and − b_m signum(x_t).

Now, here you can see that I am able to induce sliding mode. Based on this particular term m sign (x_e), how do we introduce sliding mode control? So, what do I have to do for that? I have to calculate the derivative of the sliding surface. So, what is the derivative of the sliding surface? That is $\dot{x}_e + \dot{z}$, and I can substitute $\dot{z}$ dynamics from here. And after that, now what is $b(t)$and $u(t)$? Because I know that $\omega(t)$ is nothing but the output of the actuator that is applied to the plant, and I know its mathematical model, I have substituted it here.

So, now I am going to simplify the dynamics. So, here is the third term. On the right-hand side, it represents the mismatch between actuator dynamics $u_t$and the output w_t, and this will actually decay rapidly due to the sliding mode. And now, $\omega(t)$is measurable. I am assuming that $\omega(t)$can be measured whenever I implement this kind of control. I have already told you that whatever chattering separation I am discussing today, I am assuming that the output of the actuator is measurable to us.

Then, the mismatch of $z(t)$ can be further minimized. It means that at that time, there is no need to define it like this. One can define it like this if we assume that $\omega(t)$is known to us. Again, I will do what I have to do. I have to prove that $s = 0$. How can I prove this? Again, you can take a Lyapunov function like this and calculate the derivative.

And after that, design the gain, and by substituting the gain, one can show that $x_e(t) = 0$. Now, in this way, I can confirm that somehow I am able to achieve $x(t) = x_d(t)$. Because, after that, I have two things here: $s$is the combination of $x_e$plus $z(t)$, both equal to 0. So, now these two are equal to 0; I have to somehow show that I will finally achieve this. How can I achieve this? So, the equivalent value of this should also be equal to 0.

So, I have to show that. Then only I can show that $x_e = 0$. So, what have I done for that? I have calculated the equivalent control, and finally, you can easily show that the dynamics is once

$$\frac{f(x)}{b} = u_{\text{eq}}.$$

So, u_{eq}will act, and then I will get this kind of dynamics. And what is the information of this kind of dynamics? I now have a first-order equation equal to

$$\dot{x}_e = c\,x_e.$$

So, basically, x_e is tending to 0 as t tends to infinity. In this way, I can be able to reject the disturbance. Now, how do we generate this term? Because this term, I am going to apply directly to the system. So, how do you generate this term? So, using a low-pass filter, you can take the average value and you can easily generate that term. You have to tune the time constant of the filter in order to make the continuous term $u_d(t)$ continuous because the control is continuous. So, now, $x_t = 0$is maintained for all time, and using this, I can be able to closely track $f(x, t)$.

So, in this way, you can also see that I can able to achieve the disturbance estimation. So, due to that reason, this term, this continuous term is fully able to compensate the disturbance and whole system is now actually governed by the continuous control. That is the idea. So, in this way, I can maintain the overall stability.

So now it is time to conclude this lecture. So, what have we seen? In this lecture, since continuous control is applied to the system, I am able to minimize the chattering. We have discussed two solutions: the regular form solution as well as the hybrid controller, which is also called the disturbance rejection waste controller. And after that, if we are able to apply any of these four methodologies discussed in the last two classes, one can minimize the chattering. You are able to see that somehow whenever we apply this kind of methodology, we are only able to get real sliding. Most of the time we are not able to get the exact sliding, and due to that reason we have to develop some new theory again, which is why some new concepts like higher order sliding mode control come into the picture.

So, in the next several modules, I am going to give a lecture on higher order sliding mode control so that one can mitigate chattering and maintain the equilibrium point in finite time; that is the main philosophy of higher order sliding mode control. So, with this remark, I am going to end this lecture. Thank you very much.