

## Application of Sliding Mode Observer in DC Motor Control

So welcome back. In the previous class, I talked about the sliding mode observers. And in this class, I am actually going to fill the gaps between theory and application. So, what is the main intention of this class? I am going to apply a sliding mode observer for the DC control problem. So, I am going to take the DC motor, and after that, I am going to design a sliding mode observer to solve various problems that are associated with the DC motor. So, let us come to the motivation.

So, this is an era of the electric vehicle, and most of the time, you might be aware that people are using DC motors for various purposes, and we need very high-performance operation of the DC motor. And as we know, there are several uncertainties that basically occur whenever we talk about the mathematical modeling of the DC motor. So, what is the main intention of this lecture? I am going to somehow design a sliding mode observer along with some kind of Luenberger observer to solve various problems that are associated with the DC motor. And in this way, you are able to understand the implementation aspect of the sliding mode observer.

And why is a sliding mode observer basically required for a DC motor? Because it is necessary to apply different control algorithms. Even if people are applying algorithms like on-off control, after that optimal control, several control engineers are designing. So, for that, we need some kind of information on angular acceleration, but in a practical situation, that information is not directly measured. So, what is one of the immediate solutions? You can use the numerical differentiator. But most of the time, it is possible to show that the numerical differentiator is very highly sensitive to noise.

What is the reason for that? You try to take a very simple example. Suppose that I have some noise like  $\sin(\omega t)$ . And for the time being, you can assume that  $\alpha$  is very, very small and  $\omega$  is very, very high. So, the magnitude of noise is very small, but if you differentiate, if you apply some kind of numerical differentiator here, then what happens?  $\alpha\omega\cos(\omega t)$  kind of expressions come into the picture. Now you can see the magnitude.

So, after differentiation, this noise has become further very, very huge. If you have very high switching frequency noise, then this kind of issue comes into the picture. Due to that reason, people are avoiding the numerical differentiator. And what are people doing? They are trying to estimate the acceleration signal with the help of observers like the Luenberger observer or the sliding mode observer. So, this is the topic of this particular lecture: how to solve the observation problem related to DC motor control.

So, if you see the DC motor, I have already discussed that this represents some kind of electromechanical system. So, we have some kind of electrical part and some part that is coming from mechanical engineering. So, actually, the mathematical model is the fusion of these two. Electrical engineering discipline as well as mechanical. So, in order to model this

electromechanical system, I need some kind of experimental evidence, and what the experimental evidence is suggesting is whatever back EMF.

So, that is proportional to the angular velocity. So, this  $\lambda_0$  is nothing but the back EMF concept, and after that, I will actually move towards this loop, and after that, I will be able to formulate this equation. And similarly, it is possible to show that torque is proportional to the current, and based on that particular phenomenon, I am able to do the mathematical modeling of this mechanical part. So, once mathematical modeling is over, I have already used and discussed that the DC motor is applicable for several practical problems. Now, what happens in practical problems most of the time is that the load is unknown

So, one of the immediate problems is how to estimate the unknown load. And it is possible to show that using the observer design, I can easily estimate the unknown load. So, for that, what I am going to do is show you that I have two different, actually differential equations: first-order differential equations in terms of  $i$  and  $\omega$ . And the load is directly related to the mechanical part of the mathematical modeling. So, here I am going to assume something in order to solve the estimation problem: what is our assumption? We are assuming that torque changes very slowly.

So, when something changes very slowly, it means that its rate of change is equal to 0. So, that kind of assumption I can incorporate. And why is this assumption justified? Because in any DC motor, there are two parts. One is a mechanical part, another one is an electrical part. The time constant of an electrical circuit is obviously much higher than that of a mechanical circuit.

Due to that reason, since this part is associated with the mechanical arrangement, this assumption is justified. Now, what am I going to do? I am assuming here that I know the information about  $\omega$ . And what is our goal now? Our goal is to measure the information about  $\dot{\omega}$ , which is the angular acceleration. So, what am I going to do? I am going to take the derivative of  $z$ . So,  $z$  is the combination of the load torque as well as the angular velocity, and this  $l$  is nothing but the gain of the observer, whatever observer I am now going to design.

So, here for simplicity, I am just going to design the Luenberger observer. So, this is the gain of the Luenberger observer, and after that, what am I going to do? I am going to take the derivative. Since  $l$  is constant, due to that reason,  $\dot{z} = \dot{\omega}$ . Now, from this equation, I am going to substitute the expression of  $\dot{\omega}$ , and then I have dynamics like this. Now, I have to design the observer.

So, how do you design an observer? You can just copy this particular model and remove those parts that are not available. So, here in this particular expression, you can see that all parts are available. So, I am going to put the estimate of  $\hat{z}$ , the estimate of  $\hat{\omega}$ , and  $i$ . Here in this particular observer design, I am assuming that two things are known to us:  $\omega$  and  $i$ .

These two things are known, and for that reason, I am saying that I am going to substitute the exact value of them.

So, if you are going to substitute the exact value and then calculate the error between the estimated value and the original value, you will be able to get that  $\tilde{z}$ , and in terms of  $\tilde{z}$ , you can see that you have a linear system. And this is somehow  $\tilde{z} \in \mathbb{R}$ ; you have just a one-dimensional system. So, now you can apply Newton's philosophy, and you can be able to design  $l$  that is some kind of positive number; then it is possible to show that this  $\tilde{z}$  is tending towards 0 as  $t$  tends towards infinity, and in this way I can be able to estimate  $z$  equal to, or  $z$  tending towards  $\hat{z}$  because this process is going to end after a very, very long time; due to that reason, I have written it like this. Now, you can see that since you know this, you know  $\omega$ , you know the gain, so now you can calculate the load torque.

So, in this way, using the simple Luenberger observer, if you apply it, you will be able to calculate the load torque provided. You have to assume that the load torque is changing very slowly. If sudden changes occur, then we cannot apply this kind of philosophy. Most of the practical problems that actually change are very slow. For that reason, this assumption is justified.

So, what have I done? I have simulated the same philosophy. So, you can see that  $\tau_l$  is a nominal torque, and this is the estimated torque. So, both perfectly agree with each other. Now, what is our second goal? Now, I am going to design the Luenberger observer again, and after that, using the design of that observer, I am going to estimate the shaft acceleration, and our assumption is exactly the same. Now, what am I going to do? We are going to assume that I do not have any knowledge of the load torque.

So, that is the kind of assumption I have. So, again I am going to look into the dynamics of  $\omega$ . And what am I going to do? I am going to take the second derivative of this. After taking the second derivative of this, you can see that, and after that, I am putting that  $\tau_l$ , which is actually varying very slowly. So,  $\dot{\tau}_l = 0$ , because whenever you take the derivative of this particular term, the derivative of  $\tau_l$  will also appear.

So, I am assuming that is equal to 0. So, now you have  $\ddot{\omega} = \frac{1}{Jk_t}$ ; these two are parameters. So,  $i$  and what is  $i$ ? So, from this electrical equation, I am able to substitute the value of  $i$ , and in this way, I am able to get one model that is in terms of  $\ddot{\omega}$ . After that, I am going to define  $z_1 = \omega$  and  $z_2 = \dot{\omega}$ . And what is the meaning of  $\hat{z}_1$ ? That is the estimate of the  $\omega$ , and  $\hat{z}_2$  is the estimate of the  $\dot{\omega}$ .

And after that, I am going to copy the dynamics. So, if you copy the dynamics, then this term will appear, and after that, this particular term will appear. So, this is from copying the dynamics. And what is the significance of these two terms? These two terms are coming from the Luenberger observer. So, I am assuming that I have just information about  $\omega$ , and obviously, using that information, I have designed the correction term, and these two are

again  $l_1$  and  $l_2$ .

So, I have to design  $l_1$  and  $l_2$  properly now so that I can guarantee that some kind of error will occur between  $z_1$  and  $\hat{z}_1$  that tends towards 0 as  $t$  tends towards infinity. Similarly,  $z_2 - \hat{z}_2$  will tend towards infinity as  $t$  tends towards infinity. So, in this way, I can force  $z_2$ , and what is the physical interpretation of  $z_2$  is that it is angular acceleration. So, angular acceleration is actually going to converge to the true value.

So, in this way, by designing the Luenberger observer, this is the MATLAB simulation for that. Now, in order to show what kind of feasible value one can select so that one can be able to guarantee that the estimation error dynamics are stable. So, for that, I have to calculate the estimation error dynamics. So, I am going to calculate that dynamics using this. And here now, this is a very simple problem, the problem of eigenvalue placement or pole placement that you can do.

And by placing the pole, you will be able to design  $l_1$  and  $l_2$ . So, in this way I can basically design a Luenberger observer for this system, and then by designing the Luenberger observer I can estimate the angular acceleration. Actually, I have already told you that in several practical applications, whenever I am going to design the control, at that time some information or exact information about angular acceleration is required and how one can calculate angular acceleration. So, one of the ways you can apply the numerical differentiator, but the numerical differentiator always causes a lot of difficulty, and due to that reason, we are going via the observer design way. Another important philosophy up to this point is that we have just applied the Luenberger observer; first, I estimated the load torque, and after that, I estimated the angular acceleration.

Now, I am going to apply the reduced-order model philosophy. Then I am going to design the sliding mode-based observer. And here I am going to show you that whatever job I give is actually completed in finite time. So, now here is what kind of assumption I am going to take whenever I talk about the reduced order model. So, you can see that the overall system is an electromechanical system, and I have already discussed that the mechanical system is relatively slower than the electrical system.

So, I can always assume this particular expression. And now, what is our desired objective? Our desired objective is to force angular velocity  $\omega$  to equal  $\omega^*$ . So, how do we force it? So, I have to make sure that  $\omega_e$ , which is the error between the desired angular velocity and the original angular velocity, is the error that I have to force to zero. Now, these are the expressions and what we have done. Since I have the expression for  $\omega$  here, I have substituted  $\omega$  in place of  $\omega$  that I have to substitute.

So,  $\omega$ , I have actually kept this side. So, this becomes basically  $-\omega + \omega^*$  and the same kind of things I have substituted here. Now, what have I done? Since I have to express everything in terms of  $\omega_e$  dynamics, why? Because our ultimate aim is to force  $\omega_e = 0$  in some finite time.

So,  $t \geq t_r$  in the case of sliding mode or if I design the Luenberger observer. So, as  $t$  tends towards infinity, I have to force  $\omega_e$  to equal 0,  $\omega_e$  tending towards 0.

So, any one objective is okay. So, that is basically based on the philosophy of Luenberger or the philosophy of the sliding mode. Now, one theory that is called singularly perturbed theory. What is the meaning of this? If some coefficient is relatively very very small, it is possible to show that order of system is automatically reduced. So, during the high-gain feedback, we have already seen this. So, basically,  $l$  is very, very small, and due to that reason, we are assuming that this is equal to 0.

So, by substituting that kind of condition, I can generate  $i$ . And what is  $i$ ? It is nothing but  $\lambda_0/R$ ; you can take  $R$  on the left-hand side, and after that rearrange everything, then you will get this kind of algebraic equation. So, now I get it, so I am going to substitute here. So, what is the consequence of the singular perturbation? Initially, I have a first-order system due to the electrical component as well as the mechanical component, but the electrical component value of  $L$  is much smaller than  $J$ , and for that reason, I am able to obtain the first-order mathematical model now. Here, I am assuming that whatever  $\omega^*$  and whatever reference signal you have selected are differentiable or at least piecewise differentiable.

Otherwise, I cannot be able to write like this. Now, you can see here in the first-order model what happens with speed control without involving the knowledge of current and acceleration. So, here  $\omega^*$  is nothing but the derivative of whatever reference signal, reference angular velocity I want to actually force. But in this expression, nowhere is  $\dot{\omega}$  involved, and nowhere is current involved. And due to that reason, now I am able to design control just using the help of the  $\omega_e$ , and this control is on-off control. Okay, so now you can select  $u = u_0 \text{sign}(\omega_e)$ .

And what is the property of this control? When  $\omega_e > 0$ , then  $+u$  comes into the picture; when  $\omega_e < 0$ , then  $-u$  comes into the picture. So, based on your design specification, you can put a negative sign or a positive sign here. Now, what happens? That  $\omega_e = 0$ , but you have to maintain this kind of condition. Why is this kind of condition required? Because whenever you are going to design the sliding dynamics for this particular system, at that time I have to compensate for all other terms that act like the uncertainty.

So, this term is responsible to do that. Now, whenever you are directly going to implement this control. So, due to several unmodeled dynamics, what happens? That direct implementation of this control creates some kind of wear and tear inside the DC motor. That is called the chattering problem. So, I am going to look into the chattering problem in the next class.

How to avoid this problem. So, in order to avoid this problem, what am I going to do? I am going to construct one observer, and now, using whatever output the observer provides, I am going to design the sliding mode control. And in this way, in the main loop, I am not

going to apply  $\omega_e$  directly. What am I going to apply? I am going to apply the estimated value  $\hat{\omega}_e$ , and it is possible to show that by applying this, you can reduce chattering. That kind of thing we are going to look into next class. But here I just want to tell you that whenever I have to design some kind of discontinuous control, and if I have to reduce the chattering, it is good that you can design the observer directly; you cannot substitute the actual value, you can substitute their estimate.

So, the same kind of things I have written here. In a sliding mode controller system with unmodeled dynamics, chattering can be bypassed by constructing a sliding manifold using the observed state rather than the direct measurement. So, how do you do that? Again, you can see that I am going to play with exactly the same assumption that torque is actually varying very slowly and due to that reason its derivative is equal to 0. After that, now I have just first order system. Like this, and here you can see that I have this term.

So, I am going to actually cascade one more dynamic here. So, here I am going to write  $\dot{\tau}_l = 0$ . And after that, I am now going to add the correction term. So, this acts like some kind of corrective term. So, the original dynamics, if you just remove  $\hat{\omega}$  here, then these are the original dynamics. And after that, these two are the correction terms that are generated by the Luenberger observer.

Now, I have to design  $l_1$  and  $l_2$  such that I can force  $\omega_e$  to tend towards  $\omega_i$ 's original value, and similarly,  $\tau_l$  to tend towards  $\tau$ , which are the load torques. So, I have represented the load torque like this. How do I do that? So, what can you do for that? You can take the second derivative, and after taking the second derivative and absorbing all terms, I will get this kind of dynamics. And you can see now that this dynamics is free from any uncertainty. So, now I can easily design the gains  $l_1$  and  $l_2$  based on the second-order dynamics of a linear system.

You are free to apply any kind of pole placement technique you want, whether it be here or a frequency domain approach. And in that way, you can be able to design  $l_1$  and  $l_2$ . So, that is homework for you on how to select  $l_1$  and  $l_2$  such that you can guarantee that  $\tilde{\omega}_e$  tends towards 0 as  $t$  tends towards infinity. Now, once this part is over, so in place of  $\omega_e$ , now I am going to apply this kind of control. And again, obviously, at that time I had to keep this kind of control.

assumption. Then I can enforce the sliding mode control, and you can see that this is the MATLAB plot of it. I have also written the code. So, what can you do? I am going to share this slide with you. So, you can just copy this code and after that, you can run this simulation.

You are exactly able to see the same kind of graph. Here it is possible to show that the chattering that occurs here is not very high whenever you are going to use the observer-based feedback for the sliding mode control. Now, another important type of observer is used for the sensorless control. So, somehow this word is misleading because I cannot control any

plant where there is no sensor. Because what is the role of the sensor? In order to design a closed-loop system, I always need some kind of feedback from the output of the system. And how do you measure the output? So, the measurement of output requires the sensors.

So, the meaning of sensorless control is that on the mechanical side there is no sensor, but obviously on the electrical side we have a sensor. So, I am assuming that we have information on current and voltage. Then I will be able to design the control. Since this terminology is commonly used in the literature of machine drives or power electronics, we are going to continue with the same terminology. Now, what are we going to do? We are going to design some kind of sliding mode observer.

Based on the first equation. So, what kind of part is known to us here?  $L \frac{di}{dt} = u - ri$ , and after that, this term also contains some kind of  $\omega$ . So, what am I going to do? I am assuming that for observer designs, if  $\omega$  is not known to us because we are talking about some kind of sensorless control, it means that there is no feedback of  $\omega$ . So, since the feedback of  $\omega$  is not here, due to that reason, I am going to consider that as an uncertainty.

Now, this is the correction term for the observer design. So, now I have a measurement of current. So,  $i$  is the actual current and  $\hat{i}$  is the estimated current. And what is our goal? Our goal is to maintain  $i = \hat{i}$  as  $t$  tends towards infinity or, in the best case,  $t$  tending towards  $T$ . So, here I am going to apply the philosophy of sliding mode control. So, it is possible to show that in finite time I can achieve this objective.

So, how do we achieve that objective? What can you do? You can take the original dynamics that is

$$L \frac{di}{dt} = u - Ri,$$

and here  $\lambda_0 \omega$  is present; after that, this dynamics takes the difference. You can define the error dynamics by defining  $\tilde{i} = \hat{i} - i$ , and then now you can design  $L$ . So, basically, these two parameters are nothing but work like the uncertainty. So, if your gain is greater than this uncertainty, then I can give a guarantee that  $i = \hat{i}$  after a finite time  $t = T$ . So, here you can see that I am writing equal to because in finite time I will be able to show that basically  $\tilde{i} = 0$ .

$\tilde{i}$  is basically equal to 0. For that reason, I have written like this. Now, what are our observations? In this particular system, I am able to estimate the control and the current without the information of the angular velocity by designing some kind of sliding mode observer with a correction term. Now, one more important concept is the concept of equivalent control. What is the meaning of equivalent control? Meaning of equivalent control that once sliding occurs, and here you can see with respect to which variable I am talking about sliding.

I am talking about sliding with respect to  $\tilde{i}$ . So, whenever  $\tilde{i} = 0$ , it is possible to show that

$\tilde{i}$  and their equivalent value also equal 0. So, philosophy is exactly the same that in infinite time you are converging, and after that in order to maintain here equivalent value equal to 0. Since this is a one-dimensional system, you can visualize it like this: you can plot a one-dimensional graph of this particular differential equation; now you have to maintain the equilibrium point. How to maintain equivalent value or average value should be equal to zero. And due to that reason, I have actually substituted equivalent value, and after that, I am forcing 0.

So, now you can see that using this expression, since I know  $\tilde{i} = i$ ,  $\lambda_0$  I know,  $l_1$  gain I know, so I can also estimate the speed, angular speed of the mechanical part I can estimate. So, for that in order to calculate equivalent control, you have to design the filters, low-pass filter. One can improve the accuracy by designing some kind of higher-order low-pass filter as well. At that time, since  $A$  is an  $l \times l$  matrix,  $B$  is an  $l$  matrix, so at that time you have to maintain this. And then it is possible to show why that is actually going to tend to  $l_1 \text{sign}(\cdot)$  and  $i_{eq}$ .

I have actually substituted  $i$  with  $\tilde{i}$ . In this way, I can get the estimation of this particular term. So, it is better if you write it like this:  $\tilde{i}$ . And this is the simulation. It means that by using sliding mode control, you can estimate the current.

Using that estimated current, you can also estimate the angular velocity. And you can utilize angular velocity to somehow force your system to behave in a specific way. Now, you can see that I can also do the load estimation torque and load torque estimation with the help of the sliding mode. So, how do we do that? I am going to start with the mechanical part, the dynamics of the mechanical part, where I am assuming that the load part here has  $k_t$ . So, I know that I am assuming that the load part  $\tau_l$  is not known. So, during the formulation of estimated dynamics, what have I done? I have kept  $\omega$ , I have kept  $i$ , but I removed  $\tau_l$ .

Now, I am going to add the correction term. The correction term is  $l_2 \sin(\tilde{\omega})$ .  $\tilde{\omega}$  is  $\hat{\omega} - \omega$ . And now again what I am going to do is write the dynamics, and then I am going to design the sliding mode control. How do you design the sliding mode control? So, as you have to select, here I have selected that is  $\tilde{\omega}$ . What is  $\tilde{\omega}$ ?  $\omega - \hat{\omega}$ , that is nothing but  $\tilde{\omega}$ .

So, I am going to set this equal to 0 in finite time. How to force zero? Since this is discontinuous control, you can maintain  $\dot{s} \leq 0$  because we are in one-dimensional space. In this way, I can again be able to calculate the equivalent value because once sliding is established, at that time  $\tilde{\omega} = 0$ , and  $\dot{\tilde{\omega}} = 0$ . So, if you pass this term, a discontinuous term, with a low-pass filter, either a first-order low-pass filter or a higher-order low-pass filter, you can easily get the value of  $\tau_l$ , because the average value of this is equal to 0. So, I have only one choice: that the average of this is equal to  $\tau_l$ . So, by passing this through a low-pass filter and designing the low-pass filter properly, you can also be able to estimate the  $\tau_l$ .

So, this is the load estimation with the help of sliding mode control. One more important

point here is that you can do load estimation in finite time. In the case of the Luenberger observer, you can only do so asymptotically. So, that kind of difference comes into the picture. So, whenever you are going to apply sliding mode control, you will get some sort of advantage in terms of convergence time and robustness. Now it is time to conclude the lecture, which is based on observer design and its application to DC motors.

So, what have we seen? We have actually taken several problems that are related to load torque estimation and angular acceleration estimation with the help of the Luenberger observer as well as the sliding mode observer. And what kind of things have we seen? That using a sliding mode observer in finite time, I am able to solve these two problems. Okay, after that, what we have seen is that if you are going to utilize sliding mode control, it is better to go via a reduced order model, and then you can also design control based on the measurements coming from the Luenberger observer. So, somehow you are able to design observer based control to actually mitigate chattering phenomena, which is basically one of the very dangerous phenomena whenever we talk about electromechanical systems.

And we have also done the estimation of the shaft speed. So obviously, DC motors are very, very popular in the industry. So our main goal is to connect theory to the practical. So now it is your turn to do whatever concept you have learned, and you have a DC motor in almost all control labs.

So try to implement this kind of concept. So, with this remark, I am going to end this lecture. Thank you very much.