Observer Design in Control Theory

Welcome back. In previous modules, I talked about the sliding mode control for the linear system as well as the non-linear system. And what were our core assumptions? That all state information is available. It means that I am assuming that if the order of the system is n, then practically I am keeping $n$number of sensors such that I can get $n$different pieces of information, and I am going to manipulate all the information to design control. However, what is the practical issue? Practically in several systems, we do not have all measurements of the state. It means that due to several constraints, I cannot put $n$number of sensors if I have an order equal to n, and for that reason, I need some kind of extra arrangement, which is nothing but the observers. So, in this lecture, I am going to talk about why an observer is required, after that what the different possible observers are that are actually available in the linear and non-linear literature, and why we basically need the sliding mode observer

So, here first and foremost, I am going to explain to you why we need observers. So, I have already told you that several practical systems have several unmeasured states, and for that reason, I need some way to estimate. Estimate means I cannot directly measure the unmeasured state, but I need some kind of extra arrangement. This extra arrangement in terms of a computer program or some kind of extra circuit.

You can easily see in this particular diagram, and this is actually the foundation of almost all observer designs. So, I have taken the model of the system. So, for simplicity, I have taken the linear time-invariant system. And here I have a sensor, and that sensor is going to estimate this output, and here I am assuming that the number of states in this particular system is n. I am assuming that control belongs to $\mathbb{R}^m$, so I have $m$controls, and $y \in \mathbb{R}^p$.

It means that I have just $p$measurements, and here I am assuming that $p < n$. Now, here in this particular diagram, you can see I have created an extra arrangement, and this arrangement basically runs inside some kind of hardware, some kind of macro processor, or inside the computer. What are they basically doing? They are taking control from here, output from here, and after that, we are assuming that even if I have x, I am going to measure all estimates of all other states $\hat{x}$. And that $\hat{x}$ I am going to feed here, and then I will be able to design the simplest one, that is state-feedback control. Similarly, you are able to design any kind of PID controller; these are the reference signals, and this is the gain.

So, in this way, I am practically assuming that physically I have $p$number of sensors. Now, using the information from p number of sensors and the control input, I can estimate all states in several realistic applications. So, one of the realistic applications is an automotive control system. And you can see that whenever you are driving your car, you are always seeing the measurement of the velocity that is reflected on your dashboard. So, using an observer, it is possible to show that I am able to estimate the velocity.

There is no need for any kind of direct physical hardware that will actually measure the speed. Now, the next question is why an observer is basically required and what set of problems it is going to solve. So, the first problem I have already discussed is that in several practical applications, I have incomplete state measurements and several times it is very difficult to place sensors. Suppose that you have to measure the temperature in the center of the nuclear reactor. And in that particular situation, you can see that you cannot place the sensor in the middle of the nuclear reactor, because the temperature is very, very high.

And due to that reason, practically it is very difficult to realize that kind of sensor, or even if I do realize it, I cannot place it. And due to that reason, several problems I am defining make it impractical to assume that I can directly measure the state. Now, I have a certain amount of data. So, using that data, I am going to estimate whatever remaining thing. This is just like whenever you went to the doctor; the doctor was just taking the information of your temperature.

Your SpO2, some kind of BP measurement they will do, and after that, they will tell you what kind of disease you have. So, they are not directly going to check your disease; they are taking some extra information, and from that pattern, they will actually try to extract the exact information. In exactly the same way, the observer is going to work. Now, suppose I have electrical motors; nowadays, electrical motors are very useful because this is the era of electric vehicles, and it is possible to show that motor position can again be estimated using observer design, which is often very inexpensive. Now, you have to put one on the onboard computer, and if you have a fewer number of measurements, using that computer you will be able to get the rest of the measurements.

So, for that, you have to design an extra dynamical system, similar to the previous slide. So, this slide is nothing but a dynamical system. Why is this a dynamical system? Because this is changing with respect to time, So, this is the dynamical system you have to design. So, in this whole module, I will talk about how to design this dynamical system based on the statement of your problem. Obviously, whenever we talk about control design to solve a stabilization problem, a tracking problem, or some kind of specific trajectory tracking problem in the case of robotics, we have seen it in the previous class.

So, what are our requirements? We need some kind of optimal control. Suppose I have a linear system, and one of the control methods for linear systems is the linear quadratic regulator, but this control philosophy requires the measurement of all states, and I am assuming that all state measurements are not available. So, how do you design optimal control? So, again, you need this kind of arrangement that is called the observer. So, I have again given the example of robotics because now robotics has become very popular for several applications. Now, it is also possible to show that, suppose you have a second-order system, you can assume you have actually placed two physical circuits to estimate the state.

So, suppose you have a mechanical system and a second-order system, so $x$is position and $\dot{x}$is velocity. So, in place of two physical circuits, suppose that if you just measure $x_1$and, using the measurement of x_1, you can estimate x_2, or you can measure both; you can estimate both states $\hat{x}_1$and $\hat{x}_2$, and if you utilize this information to design control, it is possible to show that your precision is basically improved. So, these kinds of things I am going to show you during the subsequent lectures. And due to that reason, in critical applications, such as in aerospace, which is one of the very critical areas, you can see here that Kalman filters are fused with the GPS system and they are using inertial sensor data to improve navigation accuracy. It means that if your controller is based on the information of the estimated state, you will achieve higher accuracy. Now, this is an era of the network control system, and faults are very, very common.

So, it is possible to show that by just designing a suitable observer, you can also be able to detect the fault. So, an industrial plant is very, very useful. Now, if you check the literature, we have several options based on the specification of the system. So, suppose that you have a linear system. Then Leuenberger observed, and this is the mother of all observers.

So, you are able to see a similar kind of structure in almost all observers. Another observer that is very famous in the industry is the Kalman filter and some form of the extended Leuenberger observer. So, that is called the extended Kalman filter for non-linear systems. So, we are also able to understand that kind of thing using this lecture, specifically how the Kalman filter is related to the Leeuwenberger observer. After that, obviously, uncertainty is one of the prime factors whenever we are doing mathematical modeling.

So, at that time, it is possible to show that the sliding mode observer will give very good performance. And after that, I have obviously discussed the extended Kalman filter and another observer that is very, very close to the sliding mode observer, which is a high-gain observer. And after that disturbance observer, this observer is also very famous. Now, you can see that this kind of observer can also be deployed whenever we are talking about self-driving cars. Again, this is a very good area of application.

So, you can actually deploy a Kalman filter, a sliding mode observer, or high-gain observers. So, what is the highlight of this lecture? So far, what we have is... Understand that whenever we have an unmeasured state, using an observer, I can measure it.

An observer design is also improving the system performance, and I am also able to solve the fault detection problem. And we have different choices of observers, and if I have more than one choice, then obviously I have to become a little more careful in selecting the unique one. And in biomedical engineering also, they need several different kinds of observers. So, mathematical biology is one of the very promising areas, and now people are utilizing the concept of the observer to solve that kind of problem. So, sliding mode is still very useful for that class of problems.

So, in order to understand the control design, whenever I have a fewer number of states, I am going to take one practical example. And this practical example I have already discussed in the other modules is that I have a DC motor. And here I am not going to tell you again how this mathematical model comes into the picture because you are already well aware. I have already discussed it three or four times. So here you can see that I have a DC motor, and here I have placed two sensors.

That is a tachometer and a potentiometer. And this load. So, suppose that I have to drive a robotic arm through this DC motor. So, these are the arrangement. So, what is our main goal? I have to control theta.

So, θ_r is actually given by the external input. And now I have to make sure that this motor will generate $\theta = \theta_r$. So, for that, what basically our philosophy is, I will generate an error, and then we will design this feedback with two gains $g_1$and g_2, such that I can achieve $\theta = \theta_r$; that is our objective. So, in order to formulate this problem, first I need the mathematical model, and in terms of $\theta$and ω, this is a second-order model. And where $\alpha$and β, which are the constants depending on the physical parameters, have already been discussed in the previous classes, they are given like this.

And where $k$is nothing but the motor torque, $J$is the moment of inertia of the load, and $r$is the armature resistance. Now, once I get the mathematical model, in order to design the control, I have to assume something. So, what am I going to assume? Initially, I will assume that I have two sensors. It means that I have information about θas well as information about ω. Now, since I have to maintain $\theta = \theta_r$in the previous problem, that problem is nothing but falls into the class of the tracking problem.

And how to maintain it? Philosophy is exactly the same. In order to control the error, you can control the rate of change of the error. That is Newton's philosophy. And so now, you have $\dot{e}$, and now you can express your whole system in terms of $e$and ω. And then you will see that this is a state space representation of that, and how basically this state space representation comes into the picture; if you calculate $\dot{x}$, then actually the differential is the linear operator.

So, linearly that will be distributed; you can substitute $\dot{e}$and $\dot{\omega}$, where $\dot{e}$is nothing but ω, and then you will get this model. Now, I am going to design u, which is nothing but $g_1$and $g_2$into ω. So, you can see here that these are two gains. Now, what is our objective? Our objective is to design $g_1$and g_2. So, how do you design? So, for that, I am going to adopt pole placement.

And what is the meaning of pole placement? So, in order to solve the pole placement problem, first you can specify your pole. So, I am going to specify my poles at $\alpha_1 < 0$and $\alpha_2 < 0$. Okay, and then I will substitute the control, and finally, I am going to actually equate this and this, and in this way, I can get the value of the gain. So, this is a very simple philosophy to design the proportional gain based on pole placement. Now, if you design that since both eigenvalues, because this is also an eigenvalue of the system, are strictly negative, then due to that, both the error and $\omega$are tending towards 0.

So, as the error tends towards 0, I can give a guarantee that $\theta = \theta_r$for all $t \to \infty$. And in this way, I can solve the tracking problem, or I will give $\theta_r$to the robot so that the robot can move in the desired way. Now, a similar kind of thing to what I have done, since these are mathematical things, is that you can simulate using the MATLAB environment. So, this is the simulation, and you can see that the simulation is also suggesting exactly the same. Now, what I am going to do is assume that I have just a tachometer, and using the tachometer, I can measure the angular velocity.

So, suppose that if I have just a measurement of angular velocity, now I am going to explore whether it is possible to solve this problem where $\theta = \theta_r$as $t \to \infty$. The answer is negative. You can easily analyze it here. Again, I have expressed everything in terms of error dynamics. I have calculated the eigenvalues, and you can see here that what happens is that ω, the angular velocity and its measurement, are available to us, which is tending towards 0.

But as $\omega \to 0$, you can see that I have this kind of dynamics. It means that $\theta - \theta_r$becomes constant. It means that I am not able to track $\theta = \theta_r$even if I wait for a very long time. So, what is the conclusion of this slide? It means that if I have just a fewer number of measurements than the order of the system, I cannot satisfy the desired objective. And what is the solution? You can design the observer.

You can see the same kind of things basically given by this slide. So, what is this slide telling? That the error is not going to converge to 0; that is going to converge to 2, somewhere between 2 and 3. Now, what you can do is suppose you have just placed one sensor that is a potentiometer. So, by using a potentiometer, you can capture the information of θ, and since you know what $\theta$is, you can calculate the error.

Now, suppose I design the control action based on the information of the error. You can see that due to proper dynamics, we are lucky that just by using one output of information, I am able to control the plant. But if you check the simulation, you can see that the system has a lot of oscillation. So, if I am going to utilize a fewer number of measurements several times, even if I am able to solve our objective—our objective was $\theta \to \theta_r$—but what kind of difficulty comes into the picture? You can see that the system dynamics is not basically proper for the application.

We always have some kind of oscillation. It means that θ is tending towards $\theta_r$and they are oscillating. There is some kind of oscillation. So, that is not suitable for practical applications. So, I have also written code, MATLAB code. So, please check this MATLAB code and implement whatever plot I have shown on this particular slide.

Now, in order to understand the need for an observer, I am going to discuss another example. Several practical systems in ideal situations can be represented in the form of a chain of integrators. So, what is a chain of integrators? A second-order chain of integrators is given by $\dot{x}_2 = u$. Generalization is up to you and you are able to go further. Now, what am I going to assume? Suppose I have just information about x_1.

Now, I am going to look into whether it is possible to control $x_1$and x_2; again, the answer is negative. You can see that one pole, one eigenvalue or pole, is positive, while the other one is negative, so you cannot do it. Now, what have I done? I am assuming that I have only one piece of information. So, in place of position, I am representing $x_1$as a position and $x_2$as a velocity. I am going to measure velocity, and then I am going to design the velocity feedback.

Again, the answer is not satisfactory. So, for that, I have to design $k$in a proper way. So, for that reason, I need an observer. And I have already told you that all observers, as one of the fundamental papers for observers, you must look into this paper. So, this is a very, very basic paper. This paper suggests that if you have a linear time-invariant system, then how to estimate the observer.

So, what is the need for an observer for the double integrator system? I can define some kind of dynamical system by supposing that if I have information about x_1, I can generate the information about x_2. And then I can design any kind of control, any kind of optimal control, any kind of state feedback or dynamic feedback; this type of feedback is called dynamic output feedback. Why dynamic output feedback? By using the best of the information on the output, I will estimate the extra information and then utilize that information for the control design. So, this contains dynamics, and states always contain some kind of dynamics. So, due to that reason, that is called dynamic output feedback.

So, now, let us try to see the structure of the Luenberger observer. So, I have considered the linear time-invariant system (LTI system), and this system is in observable form. What is the meaning of observable form? The meaning of observable form is supposed to be that if I know the control $u(t)$, I know the output $y(t)$. So, if the system is observable, I can be able to give a guarantee that from the information of $y(t)$and $u(t)$, you are able to estimate all the initial conditions. Initial condition means you can always give us the guarantee that in finite time I can estimate $x(t)$.

So, for that, Kalman has given a very simple condition. You can simply check the rank of this matrix, where CA^{n-1}will appear, and you just have to check that this matrix has full rank. So, particularly for a linear time-invariant system, we have some kind of algebraic way to check whether the system is controllable or not. So, physically, I am saying that if I know the output $y(t)$and $u(t)$, then in finite time I can estimate all states or not.

So, this observer is called a full-state observer. Why? Because even if I have partial information that $y \in \mathbb{R}^m$, it means that some part of the state is actually reflected inside the output; using that measurement, I am now going to estimate the extra state. So, what am I going to do for that? I am going to copy. So, you can copy exactly. So, I have copied this, and after that, I am going to add the correction term. So, how do you design the correction term? Since I am assuming that $y(t)$is known to us, this $x$and this $\hat{x}$are basically different.

You can see that the dynamics are different. So, what is our main goal? That I have to force $y(t) = C\hat{x}$. So, once $y(t)$is equal to $C\hat{x}$, then this collapses and this dynamics is exactly the same as the original dynamics above. So, once $y(t)$tends towards $C\hat{x}$, which is the estimated state, you can see that this term tends towards 0or is exactly 0; then this dynamics collapses, and in this way I can guarantee that $x$is nothing but equal to $\hat{x}$. Either asymptotically in infinite time or finite time, in this particular place, I am just able to guarantee that $x \to \widehat{x}$ as $t \to \infty$. So, now how to give this kind of guarantee? So, what do you have to do for that? Now, you have to actually go for the analysis of the error between the original dynamics and $\hat{x}$.

Now, if you check this dynamics, I have actually just rearranged it, and after rearranging, this is the structure. So, it is possible to show that I have to force this pole $A - LC$to lie in the left half-plane; then I can guarantee that the error between $x$and $\hat{x}$is tending towards 0as $t \to \infty$. And if we are able to give this kind of guarantee, then you can see that as $e(t) \to 0$, and what is $e(t)$? So, $x(t)$is tending towards $\hat{x}(t)$. In this way, I can estimate the state just based on the information from the output. So, if you just look into the error dynamics, the error dynamics is nothing but an autonomous system, and a linear autonomous system.

So, how can we tell that $e(t)$is tending to $\mathbb{R}_0$? So, for that, $A - LC$, the eigenvalues of $A - LC$should be Hurwitz, meaning all eigenvalues are strictly negative, and we can force it because $L$is the design parameter and the system is observable, so we can do it. Now, you are able to select the eigenvalues by selecting L, and you can take the eigenvalues that you can force towards minus infinity as well. In that way, your decrease in the error is very, very fast. It means that you can quickly estimate the unmeasured state. Now, whenever you are designing a control that is based on the observation, I have this system and I am assuming that I have just the information of $y = Cx$.

Now, what I have to do is design the state-feedback control or dynamic output-feedback control, which you can also tell. So, in order to design this kind of control, obviously, I need the information on all estimations of all state variables. So, how to get that estimation? For that, I have to design the observer. This is the observer dynamics. Okay, and after that, I have to design the controller. So, the total dynamics consists of the plant dynamics and the dynamics that are running inside the computer, and all are running together.

You can see that the order of the system is further increased by n. So, if you are…

.. Working with an n-th order system. So, now, you have a 2n-th order system. So, what kind of practical difficulty comes into the picture? Now, you have to guarantee that this closed-loop system, where you have both system dynamics and observer dynamics, is stable. You have to show one more important thing: since you have two loops, one loop where you have designed the observer and another loop where you have designed the controller. What is the meaning of design? It means that you have selected B and L. So, if both loops are independent, then independently I am able to place the eigenvalues. And since our controller is based on the measurement, I am able to place the eigenvalues of the observer far from the eigenvalues of the controller.

And in this way, I can first estimate and then utilize that information for the control design. So, how can I show that I am able to independently design the poles of the observer and the controller? This property is called the separation property. So, in order to show the separation property, what am I going to do? I am going to design some kind of transformation between x, $\hat{x}$, z, and w. So now, in the coordinates of $z$and w, you can see that this is nothing but $x$and $\tilde{x}$. What is $\tilde{x}$? That is the difference between $x$and $\hat{x}$; I have defined $\tilde{x} = x - \hat{x}$.

Now, if you apply the transformation like this, it is possible to show that in that dynamics, our system looks like this. So, this is a block diagonal form. It means that you can independently design B here and L here, and you can put or force the eigenvalues towards the negative half-plane. So, Luenberger observed that for an LTI system where there is no uncertainty, it is possible to show that this will satisfy a very nice property, and due to that reason, now I can place the poles of the observer and the differentiator in an arbitrary manner. For that, I have to assume that the system is controllable, because I want to place the poles at an arbitrary location, and it must also be observable; only then can I do it

So, this property is very crucial. Obviously, for a nonlinear system, this property is not very obvious. It means that since the dynamics are nonlinearly coupled, you cannot show that both loops are independent, and therefore, you have to take extra care whenever we talk about observer design for a nonlinear system. Now, this is just homework for you. So, what can you do? You can take $\dot{x}_1 = x_2$ and $\dot{x}_2 = u$.
After that, what can you do? You can assume that you have information of $y = x_1$.

Now, you can design; first, you can prove that the system is observable. So, you can calculate $C$and CA. And then, since this matrix has full rank, you can design the observer-based control; please also verify the separation principle. This will give you an overview. So, now it is time to conclude today's development. What we have seen is that, particularly for linear time-invariant systems, where there is no disturbance initially, I am assuming that I can design a dynamical system. And what is the beauty of that dynamical system? That will give us the estimation of the unmeasured states by tuning some kind of gains.

I can also utilize that gain to design the control, state feedback control, and for that, I need one property called the separation property. Now, this philosophy is very useful for several applications; you can apply it here. What can you do? If you have vehicle dynamics, you can linearize it and apply this approach. Robotics, aerospace, or biomedical, in every field, if you have a non-linear system, you can linearize it; you will get a linear LTI system, and then you can apply this philosophy. In this way, you will be able to measure the unmeasured state, and after that, you will be able to solve several classes of problems. So, with this remark, I will end this lecture. Thank you very much.