Newton's Law - Inspired Tuning of P controller - 1

So welcome to the second lecture of Model 1. What we observed in the previous lecture is that in order to design a control, we need the information of the present, past, and future, and based on that, the PID controller comes into the picture. We have also seen that present information is relatively more reliable than past and future information. So, in this particular module, we are going to explore control design using information from the present and what kind of limitations or what class of uncertainty is actually compensated by the proportional control that we are going to explore in this particular module. So, the purpose of the discussion is to understand the role of present state information; in this lecture, I am going to tell you what the meaning of a state is in control design. And two classes of disturbances.

That is commonly defined in the course of modern or non-linear control as vanishing and non-vanishing. We are trying to understand their definition as well as how a proportional controller can act whenever these kinds of disturbances are acting on the system. We are also going to explore the limitations of proportional control. And during this investigation, we have actually come up with a very nice idea that is the bondedness, stability, and ultimate bondedness.

It is possible to show that in practical situations, one cannot compensate for all kinds of uncertainty. And due to that reason, during the implementation, most of the time we are happy with the bondedness or ultimate bondedness of the behavior after applying some kind of robust control technique. Obviously, sliding mode control gives us some more attractive features that are insensitivity with respect to matched disturbances. What is the meaning of match disturbance? Those kinds of disturbances which enter through the control channel. So, if the disturbance is bonded, we can see that our system performance is insensitive with respect to that class of disturbance.

We are going to talk about that in several lectures, several subsequent lectures. And what is the outcome? So, obviously, one of the beautiful outcomes of this course is to understand the uniform bondedness and ultimate bondedness after applying the control system. So, we are talking about a closed loop control system. And we are also trying to explore the limitations of proportional control, as well as some kind of variant of proportional control, which I am going to discuss in this particular course. And after that, we are trying to understand the need for the PI controller.

So let us start the second lecture. So, you can see here that I have already told you in the previous class that I have to control an underwater vehicle and our objective is very, very simple. I have to stop somewhere in between the flow. So, the underwater vehicle is moving in the Ganga, so we have uncertainty, and I have to stop that. So, this kind of problem is called the stabilization problem.

In the previous class, we also saw that if I have to track some kind of velocity, then we can convert that into a tracking problem. So, in the error coordinate frame that is converted into a tracking problem, both problems are somehow equivalent, and for that reason, we are trying to solve a simple problem that is called a stabilization problem. So, I already designed the feedforward control in the previous class. And once you substitute the feedforward control, whatever system you are getting is represented by this kind of system. Now, what is our objective? Our objective is to design this feedback control system.

What is the meaning of feedback? It means that you should be a function of v. Those are the kinds of things we have to design. And for simplicity, I am assuming that the disturbance is not present. And you can see that since I am just going to use the information from the present, I have scaled the present information by some number k_p, and I am going to utilize this. And once you substitute this value here, since d is 0, this is the closed loop system.

So, this system is called autonomous system. Why is it autonomous? Now, you can see that in a closed-loop system, if you talk in terms of the block diagram, I have input as some kind of initial condition, and I will get some kind of v(t), and this guy is actually situated here. So, basically, there is no external interference, and due to that reason, this class of system is called an autonomous system. Since I am assuming that velocity is just a scalar quantity, I have to move only in a straight line; that is our problem, and for that reason, I am going to represent the evolution of this velocity using this real line. Now, suppose that our initial condition v(0) is positive.

So, if v(0) is positive, you can see that at each and every point I am able to calculate the rate of change of the velocity. What is that? So, $\dot{v}$ is nothing but that becomes negative; why kp is positive, v is positive, and due to that reason, due to this negative sign, now at this point, $\dot{v}$ is less than 0. So, what kind of things happen here is positive, but $\dot{v}$ is negative, and due to that reason, we will move in this direction. Let us try to see here on the left-hand side of the real line. So, here v is actually negative, and k_p is already positive.

So, v does nothing but positive, and what is the positive direction? It means that I will move in the direction of the velocity. So, it means that if v and $\dot{v}$ are less than 0. So, from Newton's law, I can always move in the direction of the decrement of the velocity. That is the observation. So, it means that I have to select k_p positive.

If I select k_p equal to negative one, then what happens? You can just select k_p equal to negative one. And if you are here, then k_p is negative. So, negative, negative that becomes positive. And if this whole quantity v becomes positive, it means that I will move in this direction. So, we will converge towards infinity; in a similar kind of situation, we will move in this direction if k_p is infinity.

And due to that reason, I can design k_p that should be greater than 0 if there is no disturbance. So, in this way, I can tune the proportional controller. You can see that several nice observations basically come whenever you are giving the geometrical interpretation of this particular differential equation. Suppose now that you start from here. This is v equal to 0.

So, if v equals 0, it means that $\dot{v}$ also equals 0. So, for simplicity, you can just assume v as some kind of quantity, a position-like quantity; $\dot{v}$ is a velocity-like quantity, so you can just assume. So, now, I am able to apply Newton's first law: the rate of change of some quantity is equal to 0, so that quantity will remain in that state. So, I have a very unique point; if I start from this point, I am going to take a step; however, if there is no external disturbance, that is the idea of the equilibrium point. So, how do you calculate the equilibrium point? if system is expressed in terms of the first order derivative.

So, suppose that now I have a system like $\dot{x}$ equal to f(x) and suppose x contains x_1 and x_2, it means that it belongs to two dimensions. So, two-dimensional people are represented like that. Why? Because x_1 is a realistic measurement and x_2 is the realistic measurement, it means some kind of real number comes into the picture. So, now here you can also see how to calculate the equilibrium point; I am just going to keep $\dot{x}$ equal to 0 because x is some kind of position vector. So, I just have to maintain the velocity vector equal to 0, then I will be able to apply Newton's law and I will get the notion of the equilibrium point.

Now, what we have seen in the previous slide is that if I start from anywhere except this equilibrium point, I am going to converge towards this unique point where the rate of change is actually equal to 0. So, the same kind of things, actually, I have written here that all trajectories are converging towards the equilibrium point. What is the idea of asymptotic convergence? You might have seen in the previous class I was talking about asymptotic stability. You can see what the physical interpretation of that is. As you move toward the equilibrium point, what happens? The rate of change is going to decrease.

And nearby the equilibrium point, you can see that the rate is very, very slow. So, it requires infinite time to converge if you are in the vicinity of the equilibrium point, and that idea is nothing but the idea of asymptotic convergence. And now, if you have some kind of non-zero disturbance, obviously, I do not have this kind of observation; I cannot preserve the equilibrium point, and I do not have asymptotic behavior. The asymptotic kind of behavior we are going to see in the next slide. Now, what we have seen in this particular slide is that if I have to control some plants, I just scale our present information and then apply that present information here.

So, you can think, suppose that if I represent information in some other way: this is the signum function; v greater than or equal to 0, this is plus 1; v less than 0, that is minus 1. And then, try to apply the same kind of geometrical observation to what I have told, that I am explaining Newton's law, because that is the consequence of Newton's law: if some quantity and their rate of change have opposite signs, then the quantity will move in the direction of the rate of change. So, that kind of idea you can apply and do this kind of homework for a clear understanding that you will be able to design proportional control, not just what classical proportional control suggests; you are also free to select this kind of control. Now, here you can see that whenever we are talking about any system, I have already told you that continuity. Continuity is one of the very beautiful properties, and what continuity is telling us is that if you have some system and if you have information about the past and information about the present, then you can be able to guess or somehow control the information about the future.

And most of the time, what is our objective? I want to follow the velocity. I want to control the velocity. So, somehow I am trying to control the futuristic value. It means that in order to apply the continuity property, I need information from the past. And this notion of a state is responsible for it.

So, we are trying to understand what the meaning of the state is. So, I have taken a very simple electrical system during 10 or 10 plus 1; you have already learned about this. I have input and output. And now, I have two switches, switch A and switch B. So, the system can switch between A and B.

If the system is in the switch A state, you can apply Newton's law, and you can see that if I apply v, then I will get v by 2. But if you are at B switch position, at that time, if you apply v, then you will get v by 4. So, it means that for the same input, if the internal condition is different, then you will get a different output. So, this idea is generalized for any class of systems, and we have defined the notion of a state. So, the notion of a state of a physical system is any property of the object that relates input to output.

such that the knowledge of input for t greater than equal to t_0 and the state at t equal to t_0 completely determine the unique output of the system. So, that is the physical interpretation of the state in block diagram form, but we are saying that if you know $u(t)$ and $x(t_0)$, that $x(t_0)$ is nothing but all the information from minus infinity to t_0, then you will be able to get $x(t)$. So, this information is nothing but the state of the system. Now, even if you do not have any physical system, you are working towards a very abstract system; the same definition is applicable. Whenever you are dealing with any system, after the application of control, you have already seen that I am talking about an autonomous system.

We, apart from autonomous systems, generally apply control; another class of systems that comes into the picture is called non-autonomous systems. So, most of the engineering systems actually fall in the third category. Here I have written that it is a force system. What does it mean? I have a system; I have some kind of input, and using this input, I want to control the futuristic behavior of the system, and I am assuming that the information of a state is given at t equal to t_0. So, the information of a state is nothing but information from minus infinity to t_0.

So, now I have to design control based on the information from a state such that I will achieve objectives like stabilization or tracking. In the previous class, we have already learned about stabilization and tracking. So, now you can see that if you design u that is function of x, then system either convert into autonomous system. What is the meaning of an autonomous system? There is no forcing function here, and time is not explicitly present; it is implicitly present. Explicit means you try to understand implicit and explicit like this.

Here, for the first time, the independent variable is time, and then x(t) comes into the picture, and based on x(t), I will get f. But here, in order to get the rate of change of some quantity, explicit information about time is required. So, here x came from somewhere; I do not worry about the time, but here I have to worry about the time. So, that is the difference between autonomous and non-autonomous systems. In this particular course, most of the time I am going to talk about the autonomous system because by designing control actions, our system is going to be converted into an autonomous system.

And now, what is the meaning of the order of the system? So, you can see that it is coming from the geometry. So, the idea is that you can represent one dimension, two dimensions, three dimensions, but how do you represent n dimensions? So, in order to represent n dimensions, even if you cannot visualize it, just put n real numbers that correspond to the n dimensions. So, those are the kinds of things we are trying to visualize using this. Now, I have already told you about stability. Stability is one of the very crucial properties because somehow they are forcing us to understand that if you know the past and the present input, then you can characterize the behavior of the system.

Due to that reason, I now have to specify the notion of restability here. In the first part of the discussion, we have observed that for the one-dimensional system, if I start anywhere and design control based on proportional information, then I am converging towards the equilibrium point. And if there is no disturbance, then what happens? So, I will converge in the vicinity of the equilibrium point. I will not exactly converge to the equilibrium point because we have a sum total here. So, I have this kind of stability called asymptotic stability.

It is possible to show that if some class of disturbance comes into the picture, I cannot exactly converge to the neighborhood of the equilibrium point, but I will remain in this vicinity. So, that idea that if you start from the equilibrium point and there is no disturbance, you are going to remain there. But if there is a disturbance, then you are going to fall within the neighborhood of the equilibrium point if your system is stable. That is the idea. Due to that reason, we have to understand the notion of stability.

And in the previous class, I already told you that the notion of stability is inspired by continuity. And how does continuity basically come into the picture? That is coming from the concept of limits: left-hand limit, right-hand limit, and limit at that point. And whenever we talk about limits, a limit is nothing but a kind of mathematical microscope. So, in this particular system, what have we seen? I have given the initial condition; if our system is autonomous, then we are actually looking towards x(t) because whatever control I have designed is a function of the state, and for that reason, now the autonomous system can be represented in this form. Now, we also know that if I apply 0 input to this system, it means that if the initial condition is equal to 0, it means that I am forcing something like this.

So, I am assuming that x equals 0 is my equilibrium point. It means that if I give 0 as input, I will get 0. So, how do we apply the notion of continuity here? What will I do? I will zoom in on x(t), and I will see the distance of x(t) from the origin. So, x(t) minus 0, so this is the symbol for the norm, because I am assuming x(t) here as a vector. So, whenever you are putting an equality sign or an inequality sign, for the inequality sign you have to convert an n-dimensional quantity into one dimension.

So, for that, you have to use the notion of norm. Those who are not familiar with the notion of a norm, please just check anywhere on the internet. You are able to see how to calculate the norm. One of the simple ways to calculate the norm using the Pythagorean theorem is that if you have two quantities, x_1 squared plus x_2 squared, and the square root of that, something like this. This is called the 2-norm. So, what are we basically going to do? I am going to zoom the output, and after that, how do I zoom the output? So, you can see from this particular figure that.

So, this is the equilibrium point $\bar{x}$; without loss of generality, I can assume that it is 0. So, if you start from here, you can remain here if there is no uncertainty. But if you start in the neighborhood of this, you have several behaviors. It might be possible that you will remain bounded, or you may be able to converge to this equilibrium point sometimes with asymptotic speed and sometimes with exponential speed.

So, based on that we have different notion of stability. So, what are we basically going to do? First, we are going to select the output ball. What is the meaning of output ball here? Ball of x(t) and due to that reason, you can see that for all, this is a symbol for all there exists a delta such that now what we are basically enforcing is that if you start from this ball, the ball of initial condition or input ball. So, stability suggests that you can only remain on the output ball. So, the size of the output ball depends on the application. What is the meaning of asymptotic stability? I need one more condition that is called the attractivity.

What is the meaning of attractivity? That as t tends towards 0, this condition should satisfy means x(t) tending towards 0. So, this symbol has more meaningful significance than the norm of x(t) tending towards 0. So, here we are talking about some kind of bondedness; whenever we talk about stability, we are discussing convergence and bondedness, and exponential stability refers to how quickly you are going to converge. So, the rate of convergence is also associated. So, this kind of concept is very very fundamental in the non-linear or linear control literature.

So please go through that. Now, I have taken a very simple example here. So, you can also relate this to something like k_p into v(t). So, I have just replaced v(t) with x and I have solved it. And after that, I applied the notion of ε and δ.

So, I have chosen ε . And, if I choose δ equal to ε, then I can show ε and δ stability. So, how can one show ε and δ stability? First, you have to solve the differential equation. Then, you have to select an output ball, and after that, you have to show that corresponding to the output ball, there exists a ball of initial condition or input ball. Similarly, you can show the asymptotic stability. First, you have to show the stability, and after that, you have to show the convergence and exponential stability.

So, you have to take the absolute value because here this is just a one-dimensional system, which means $x(t) \in \mathbb{R}$, but if we have x(t) belonging to R^n, which means n number of states, at that time I have to take the norm here. So, if this kind of relation comes into picture then our system is exponentially stable. It means that if you design proportional control for this system then v equal to exponentially stable. Now, let us try to see the behavior in the presence of the disturbance. You can see here that disturbance can be modulated in different ways.

So, suppose that if the disturbance looks like this, what is the meaning of this? That as v tends towards 0, disturbance becomes 0. So, that class of disturbance is called a vanishing disturbance. And if you apply, if you have that kind of disturbance, it is possible to show that you are able to convert like this. What is the signum v here? v greater than 0, that is plus v less than 0, that is minus 1. So, now, if you design k_p greater than or equal to d sin v, signum v multiplied by v will give us the mod v.

So, if you design, and I am assuming behavior, I am just going to see the behavior on the right-hand side because the left-hand side behavior is exactly the same as the left-hand side. So, you can see here that what happens is that if I select k_p and that is greater than k, the d_0. What is d_0? That is whenever we are calculating this kind of bond. If this is the case, then I have exactly the same kind of behavior.

If I start here, b is positive, and b dot is negative. You can simply apply Newton's law. So, it means that if your gain is sufficiently large, then you can exactly able to compensate the vanishing uncertainty. So, the conclusion is that the equilibrium point remains the same because I can exactly converge to the equilibrium point, and at the equilibrium point, uncertainty is 0. So, the equilibrium point remains preserved; all trajectories will converge towards the equilibrium point provided this is the gain condition. So, if you have vanishing uncertainty, then you have to somehow estimate that uncertainty.

So, using machine learning, AI can also help you do these jobs. We have several different options. And asymptotic convergence still remains the same. Obviously, if D is non-vanishing, then I cannot conclude the same. So, I have already defined the signum function. So, you can see that if d is not equal to 0, then what is our behavior? So, I am assuming that d not equal to 0 is represented by $d \neq 0$.

Most of the practical systems, this kind of disturbance comes into the picture. You can just take an example of the mobile robot. So, the mobile robot is moving in the open environment. So, air is going to hit them, and you all know that air is some kind of periodic wave. So, you can be able to transform that wave in the form of a constant and sinusoidal function, and that is not equal to 0 at the equilibrium point.

So, that is represent the non-vanishing disturbance. So, in the presence of a non-vanishing disturbance, you can see that even if you take any higher value of k_p, d is non-zero. So, at some point this disturbance is going to dominate. So, if you start far from the origin, you are going to converge. But if you are in the neighborhood of the equilibrium point, what happens? Now, the sign of v is positive and $\dot{v}$ is also positive because the disturbance, suppose that the disturbance is positive.

So, you have some kind of divergent behavior. If you are lucky enough that d is negative, then that will support you. But most of the time, we have to do worst-case analysis. And this is nothing but the idea of an ultimate bond. So, what have we seen? We have seen that now whenever the non-vanishing uncertainty comes into the picture, I am not able to talk about asymptotic conversion.

At that time, I had to talk about uniformly ultimate bondedness. So, in the next 4 to 5 minutes, I am going to brief you on the meaning of a uniformly ultimate bond and how it differs from the ultimate bond. And now, whenever non-vanishing uncertainty comes into picture, then I have to explore different controller. So, I have to incorporate information from the past as well, and for that reason, we are going to explore the idea of a PI controller. So, here you can see that in order to illustrate the difference between bond ultimate or uniformly ultimate bondedness, I have actually taken a very, very simple example. So, you can see that this system is affected by some kind of not vanishing disturbance and I have this kind of assumption.

Suppose initial condition and d some quantity I have d whatever that is associated with this disturbance and if you solve this equation during your 10 plus 1, you have already solved using the exponent. Please do this exercise and after that if you calculate the bond, it is possible to show since exponential function is positive, a is positive. So, it is possible to show that for all t, this is less than equal to a. It means that In presence of this disturbance, v(t) is always remains in some band, this is 0 and this part is either minus a or this is plus a.

So, this kind of bond is called uniform bond because this is independent of the time. Now, it is possible to show that you can able to reduce this bond after some time. So, whatever bond we have calculated in previous So, I have actually created from t greater than equal to t0 onward. It is possible to show that since exponential term is also associated with the solution. So, we have not calculated the effect of exponential term, because after some time this become less dominated. And due to that reason, now it is possible to show this kind of inequality easily you can able to solve and show that v(t) is less than equal to b after this time and this time is always finite.

So, this is after some time that is called transient time. So, once transient time is over then we will actually converge to this bond and this bond is nothing but the ultimate bond. So, what happens that whenever we are starting some practical system. So, initially we have more fluctuation, but after some time some dynamics will die out and at that time we will get the true bond and that is nothing but the ultimate bond. And these are the standard definitions that is existing in literature, you can check the Khalil book, I have given the reference. So, if you have generalized system, so this is non-autonomous system, so this is the definition.

So, if there exist a positive constant C independent of T_0, so this is very very important because we are talking about uniform. If this relation will satisfy, Then we are telling that your trajectory x(t) is uniformly bonded. How to do that? You have to solve the differential equation and after that you have to check this. What is meaning of globally uniformly bonded? So, at that time you are allowing any large value of A. And after that I have already told you that once your transient is diode, then you will get the true bond and that is called ultimate bond.

So, here you can see that I need two constant and after that I will select A and that time is depending on A and B. In previous calculation, please check that finite time is depending on A and delta. And, after that you can able to establish this kind of relation. It means that I can calculate a finite time t after that our trajectory is remains bounded. So, time information also I can able to provide whenever we are going to talk about the ultimate bond.

This book is very good, extremely good in order to understand the linear as well as non-linear control system. So now it is time to conclude the today's lecture. So we have actually seen the effect of vanishing and non-vanishing perturbation. So if I have vanishing perturbation, then proportional controller is good enough.

There is no need to actually involve the information of the past. Because past information is sometimes dangerous. I have already shared that hacker can hack it. Uniform bondedness and ultimate bondedness concept we have seen because whenever you are designing practical system, so most of time we are working with the ultimate bond. And in next class, I am going to explore the PI controller and what class of the disturbance that is going to tackle.

So, what is suggestion, my suggestion now? You can take any first order system. even if second order system. And then you try to understand this concept means what is meaning of vanishing, non-vanishing, uniform bonded and ultimate bonded. So, in next class, I am going to tell you that how to tune PI controller using the Newton's philosophy, same like the today's lecture. Thank you very much.