Sliding Mode Control and Applications

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Week-04

Lecture-19

Welcome back. In the previous class, I was talking about integral sliding mode control, and we have also applied integral sliding mode control to the cart pendulum system, considering both kinds of uncertainty: matched uncertainty as well as unmatched uncertainty, and slowly sliding mode control has become very popular. Due to several practical applications. And one of the very practical things that is evolving in India as well as in other parts of the world is called robotics. And it is possible to show that the dynamics of robotics are highly non-linear, highly complex, and several kinds of uncertainties are going to enter that system. Obviously, the concept of robotics is actually coming from mechanical engineering, but now it is not limited only to mechanical engineering.

So, robotics is truly now interdisciplinary subject and due to that reason, what I am going to do in this particular course, I am going to take several examples from the robotics and going to design sliding mode control, different kind of the sliding mode control. And due to that reason, I am going to give you this very, very primitive lecture where I am going to define some notion which is frequently used from the robotic side in the control system. So, let us enjoy this lecture. So, robot control.

So, I am going to address the challenges. As well as the approaches. So, I have already told you that whenever we are talking about robotics, this is nothing but some kind of mechanical system. And this system is highly nonlinear in nature. And obviously, in order to develop complete robotic system for some practical application like medical application, agriculture application, defense applications or some kind of other classes of applications.

We truly need people from electrical engineering, electronics engineering, computer science, aerospace, and almost every discipline. since this is came from the mechanical engineering and due to that reason several time people are not familiar with several terms and due to that reason first we have to familiar with some kind of term as well as property that is coming from the mechanical engineering and then we are going to apply the sliding mode control or other classes of the control, robust control or some kind of optimal control. So, whenever we are talking about the control of any robot, the first step is the model; a dynamic model is required. So, whenever we are talking about robotics, we have two different set of model, one is kinematic model and another is dynamic model. So, dynamic models become very helpful whenever we are talking about sliding mode control or any kind of optimal control.

In this particular lecture, we are going to understand how to obtain a dynamical model of any kind of robot, whether it is a manipulator or some kind of mobile robot. I am also going to give you an overview of why non-linear control is required, because these systems are very complicated and their dynamics are also not simple. And due to that reason, I need some kind of non-linear control which will also work in presence of uncertainty such that I will get or achieve some kind of high performance during the operation. For example, if I am going to use a robot for surgical purposes in medical use, you can think about the precision. If our precision is poor, then what will happen? I cannot able to use that for the surgery purpose and whenever we expect precision, obviously, we have to pay more and it is possible to show that sliding mode control is quite applicable to give several feasible solution for that critical applications.

So, now here whenever we are talking about the tracking problem that occurs during robotics, either based on sliding mode control or any other design, we have to follow some kind of prescribed trajectory. Here, the tracking problem is not arbitrary; we always have to actually follow some kind of prescribed sequence of the trajectory. And I have to maintain robustness also, because I cannot suppose that during surgery you will not be able to move your robot in any way; you have to always be very, very precise. Because we, it might possible we have to, suppose we have to move from here to here. In between, it might be possible I have several instruments and several obstacles.

First, I have to design the trajectory carefully, and after that, we have to move along that trajectory. And due to that reason, this problem, one of the subclasses that comes into the picture, is first how to design the trajectory or feasible trajectory and then apply some kind of very precise control. Now, I have already told you that the tracking problem here is not like the classical problem. So, one of the ways in which one can generate a path or some kind of tracking path based on the artificial potential. So, what are people doing? They are defining the artificial potential and after that by calculating gradient one can able to decide that which direction basically robot will move such that it is possible to avoid the obstacle.

So, several algorithms have already been developed in the literature that talk about this particular concept, which is based on artificial potentials. Now, we have to actually merge sliding mode control so that we can also be able to discuss robust control. And this technique is somehow capable of tracking in real time without causing any collisions. So, any complicated environment using this particular concept I can able to move. Now, there are several applications, obviously, but in this course, either through the homework or tutorial, I am going to cover at least these four possible applications.

So, I am going to talk about manipulators, robotic manipulators, how to control them, how to formulate problems about robot manipulators, and how to solve them, followed by collision avoidance for mobile robots. And one of the very good application whenever you are going to combine upper two problem, manipulator and top of manipulator or on mobile robot, on top of mobile robot if you put the manipulator, that is very good problem. Several agricultural problems can be solved using this methodology. Several defense problems one can solve using this methodology. I am actually discussing underwater robotics from the first class onward.

So, I am also going to address that, and after that, the aerial drone. So, it is possible to show that I have divided the problem into four parts. I am able to characterize the whole thing in just two parts. And that kind of terminology is actually encountered whenever you are going to do robotics. Due to that reason, it is better to become familiar first.

So, in this particular lecture, I am not going to talk about sliding mode control. I am just going to talk about the kind of terminology that is crucial to understand or design sliding mode control. Because, in order to design sliding mode control, I first have to properly understand the dynamics of the system. And during the definition of dynamics, several terms come into the picture. Because these are truly practical problems, truly interdisciplinary problems.

I have already told you that whenever we have a practical system, what is the first step? The first step is mathematical modeling, and I have to check the dynamics. So, due to that reason, a large number of control problems for the mechanical system, robotics is one of them; I have to control the position or location. of a mass based on force and torque. So, I am able to manipulate force and torque, and then I am able to control the position as well as velocity. But I have already discussed when I started this lecture that the position here is not a simple position; it is always some kind of trajectory, and a specific trajectory acts like a position.

okay and whenever we are talking about robot then we have coupling of several masses okay and it might possible each masses has some kind of control so this is truly multi input multi output problem okay so typical example that is robotic arm you can see that robotic arm generalized robotic arm has n links and joint and each joint has some kind of force okay you can also think your hand as a some kind of robot and then you can able to see the number of links and joint. So, now dynamic modeling is essential for controlling the mechanical system; after that, obviously, trajectory tracking means a special path or some kind of prescribed trajectory we have to generate first. So, basically first step of problem is generation of trajectory and second step to track that trajectory. And we also have to follow all kinds of constraints because several joints cannot move in an arbitrary way. Workspaces also have several limitations.

So, those are the kinds of things we have to address. Since this is a truly practical system and in order to apply control, we need actuators. So, most of the time in robotics, the actuator nowadays is an electrical actuator. Obviously, you can apply some kind of pneumatic or hydraulic actuator based on your application. Most of the current robotics contains this kind of actuator that falls into the electrical actuator category, and each electrical actuator has some kind of dynamics.

Actually, the model also contains some kind of flexible structure. So, at that time another kind of uncertainty. So, what I am going to assume during this development is that I am not going to consider the dynamics of the actuator, and obviously, whenever you are not going to consider the dynamics of the actuator. And you are going to apply sliding mode control, then what happens? You are experiencing some kind of phenomenon like chattering. This is called high-frequency switching oscillation inside the mechanical system.

And sometimes the actuator is not very comfortable with the discontinuous control. So, obviously in this course, I am going to tell you how to actually remove or minimize the chattering. Now, this terminology commonly occurs during the course of robotics. So, you are able to see terminology such as holonomic system or nonholonomic system. So, although the terminology looks somewhat difficult, the meaning of this terminology is very, very simple.

They are stating that constraint because in robotics, constraints always come into the picture. So, constraints can be represented using some kind of algebraic non-linear equation where only time or q; q is nothing but the generalized coordinate. I will give you the examples of the generalized coordinate whenever I am going to talk about some specific form of robot. So, it is possible to show that these constraints are integrable and these constraints do not involve the rate of change and due to that reason, most of time this type of constraint, it is easier to model and control. So, if you have holonomic system where constraint will come in form of the algebraic equation, it might possible non-linear algebraic equation, then it is relatively easy to design control.

So, whenever you are considering any rigid body system, robotic arm, with fixed joint, but suppose that if you have robotic arm with flexible joint, so at that time you have to become more careful. It might be possible that at that time you are going to lose the holonomic structure. So, another thing is called a nonholonomic system or nonholonomic structure. What is the meaning of a nonholonomic structure or constraint? In this particular class of mechanical systems, constraints can also appear in terms of velocity. So, apart from the generalized coordinate, generalized velocity is now also involved, and somehow we lost the integrability.

Most of the time, this constraint is non-integrable, and due to that reason, we are going to face several kinds of difficulties in generating the state. It means that in an arbitrary way I cannot move from one point to another. Several times this kind of constraint comes into picture whenever we are talking about wheeled mobile robots, rolling without slipping cases, and underwater vehicles. So, these kind of constraint, non-integrable constraint comes into picture and that is going to create additional difficulty. So, in order to handle this class of system, extra care, extra modeling effort you have to pay.

In the next class, I am going to talk about some applications of sliding mode control for holonomic systems. In the next part of this course, I will also take some kind of example of a wheeled mobile robot. At that time, I will talk about the nonholonomic system. And I have told you that the first starting step for control engineers is to model the system. Due to that reason, we have to understand each and every terminology.

We have to become comfortable with the terminology. So, suppose that I have a mechanical system, and we know that in any mechanical system, there are two different forms of energy: one due to the position and another due to the motion, which are potential energy and kinetic energy. So, now, if you are able to calculate the kinetic energy and potential energy, then you can always do mathematical modeling in a very simple way. And this is actually nothing but the principle of the list, the principle of list action. Using that you can able to model the system.

So, this is actually a partial differential equation, where L is nothing but the difference between kinetic energy and potential energy. This will inherently satisfy some kind of optimality criteria. So, by solving this equation and Q is the generalized configuration that is actually depending on the types of robot and τ is the generalized force or torque, you can able to get the differential equation out of this partial by solving this partial differential equation. Under equation. What is meaning of under equation? Several times we have several passive joints inside the overall structure.

What is the meaning of a passive joint? It means that direct control is not available. It means that several times I have a number of configuration variables that are going to lie in a higher dimensional space, but control is going to lie in a lower dimensional space. It means that the number of controls is fewer than the number of states that you have to control, and that class of system is called an underactuated system. Non-holonomic system, I have already told you what the meaning of a non-holonomic system is. Those classes of systems where constraints come in the form of the configuration variable as well as the rate of change of the configuration variable can now be combined into one problem.

Suppose you have a non-holonomic system and you also have an underactuated system; then, obviously, modeling as well as control design both become more challenging. And several times it is possible to show that your system is fully actuated. So, we have seen two different classes of systems: some systems are underactuated, while some systems are fully actuated. So, suppose that if I consider some kind of robot manipulator, so robot manipulator suppose this has two link. So, I have now, it might be possible that in this joint I have one control input or torque input; here also, I have one torque input.

So, suppose this length is l_1 and this is l_2 and this angle is q_1 and another angle is actually q_2 suppose that it means that here in this particular problem with respect to each joint I have one control. So, this class of system is called a fully actuated system. Q is a generalized configuration variable. Here, I have considered Q as a θ, which is an angle, and Q_2 also as a θ. So, a generalized coordinate system can come from a translational or rotational way.

Rotational way means θ; in translational way some kind of position will appear and their derivative will appear. M_Q is dependent on the configuration variable. So, this is the inertial mass, and this will satisfy some specific properties, and we are going to actually discuss what kind of property that is going to satisfy. So, during mathematical modeling, if you will get some kind of inertia matrix that is correct or not, you are able to check from that property. This term depends on the coupling of forces, gravity, etc.

That will also come into picture during the mathematical modeling of fully actuated system. And I have already told you that during the mechanical system, I can control it in two ways: either by applying force or torque. So, this particular mathematical model, where M_Q is actually the inertia mass matrix, and $\ddot{Q}$ is the generalized coordinate acceleration. So, $\ddot{Q}$ is, you can think of it as an acceleration term, and this is again some kind of force, and this is equal to torque. So, by understanding each and everything, you can able to design the control action.

In the next class, I will do control, robot sliding mode control for this particular class of system. So, suppose that I take the example of a robot manipulator, whatever I have discussed here. So, now this contains some kind of fixed base with a rigid link connected by the joint, and the configuration variable is the joint position. And actually, you can see here that I have two things: forward kinematics and inverse kinematics. So, what is the meaning of forward kinematics? Suppose that if you have some kind of manipulator and if I provide this and this angle, then I can able to move anywhere in this work space.

What is the meaning of inverse kinematics? Suppose that if you know this, so I can able to make several combinations such that I can able to actually get this particular position. So, whenever we are talking about inverse kinematics at that time, I have multiple configuration, multiple way I can able to adjust angle such that I can able to get this particular position. I am assuming that this is rigid up to here; here, this is rigid as well. Due to that reason, it is difficult whenever we talk about inverse kinematics.

Because now I have several choices. So, which one is the optimal choice? So, those are the kinds of things we have to look into. Now, another class I have already told you about is mobile robots. Now, several applications are being worked on by people. So, now people are also thinking for cleaning purpose. People are also thinking suppose that for war purpose or some kind of civilian purpose like agriculture purpose.

So, a mobile robot is nowadays a mobile robot with a configuration on top that is very good for practical utility. And therefore, the mobile robot configuration variable is in terms of the position and orientation. So, whenever you are going to club these two, you can see that challenges are going to increase. Now, here you have to make sure that whenever you have to move mobile robot, then work space should be safe. Now, I have already talked about the holonomic robot and the nonholonomic robot.

So, in terms of the constraint, holonomic and nonholonomic is different, and whenever you are going to put some kind of manipulator on the top of the mobile robot, so now you can see that there is the fusion of these two combination. So, robot manipulator, omnidirectional mobile robot, so in medical field, particularly in dental field, you can able to see several application of the robot manipulator. And I have already discussed that we have several different kinds of robots; one is the car-like robot, and another is the differential drive robot. So, slowly you have to become comfortable with each and every terminology, then only you can able to design the control, because in order to design control, three things are basically required: mathematical modeling, and what is objective, what is our objective and what kind of constraint we have. Then, based on that, I can select the control algorithm and the type of actuator that is actually good for our application.

So, that kind of thing is required. Now, I have already told you that whenever you combine mobility with manipulation, you have a mobility constraint as well as a manipulation constraint, and you have to actually give extra effort whenever you are designing this class of system. So, that is called a mobile with manipulators. Now, a holonomic robot can basically be modeled using the Euler-Lagrange equation. So, in the previous configuration, I am going to talk about the fully actuated system, but this will represent the generalized mathematical model of the holonomic robot, where Q is the joint configuration either translational or rotational, M_Q is the inertia matrix, V_m is the Coriolis or centripetal force, $V\dot{Q}$ is the viscous friction, and τ is the generalized input or torque.

So, in this way, we are actually getting the equation. So, I am saying that the equation is very easy. What can you do? You can calculate kinetic energy, potential energy, and kinetic energy minus potential energy, and then you can solve the partial differential equation. So, during this particular dynamic, you can see that several terms come into the picture. So, one is the inertia matrix, another is the Coriolis or centripetal force, and there are friction force and gravity. So, once you develop the control, at that time you have to give a guarantee; some sort of guarantee you have to give, and it is possible to show that since the system is highly non-linear, you need some kind of analysis based on passivity or Lyapunov analysis.

And due to that reason, we have to explore some kind of property of this particular physical quantity such that construction of the Lyapunov stability or some kind of passivity based stability analysis is easier. I have already told you that a holonomic robot, one example of which is the manipulator, has several applications. I have already talked about all the applications, and due to that reason, I am not going to spend much time on this slide. Now, you can see here that for the analysis purpose or control design purposes, the property of the holonomic robot model is very, very important for us.

So, here is whatever mass matrix that is, M_Q, I was talking about. So, this matrix satisfies the property that it is symmetric, positive definite, and bounded. So, mass is always positive. Now, they will also satisfy some kind of property that is skew symmetry. We are going to prove that this property and dynamics remain bounded. If that is not bounded, then at least we can face difficulty if we are going to apply the sliding mode control because this is an inherent assumption of the sliding mode control that even if the disturbance is not known to you, it should be bounded.

And most of the time, I cannot exactly calculate the mass or some kind of length, and due to that reason, there is always parametric uncertainty involved whenever we are talking particularly about robotic modeling. Now, the core property that is the skew symmetric property; I have already told you that this property is very, very important. So, how do we establish this property now? So, here M_Q is the mass matrix, and that is symmetric and positive definite. So, what is the meaning of positive definite? Something is positive definite provided that it satisfies this: you can take any vector of proper dimension.

Because M is some kind of square matrix and y is a vector. So, you can be able to select the proper vector. So, suppose this is a 2×2 matrix. So, this is nothing but a 2×1 vector that should be greater than 0. And what is the meaning of the symmetry? That $M_Q = M_Q^\top$, and it is possible to show that whenever I have some kind of positive definite matrix, I can apply the Rayleigh inequality, and I can show there exists a bound based on the eigenvalue. In combination with the previous class, we have already used, prior to the previous class, I have already used, whenever I am talking about the sliding mode control based on unit vector control. At that time, you will be able to see that I was talking about Rayleigh inequality.

Now, it is possible to show that $M\dot{Q}$ is related to the Coriolis matrix by this. So, without any proof, I wrote this. So, these kinds of things, one can establish from the higher dimensional physics. Now, you can see here that I am able to calculate $V_m\dot{Q}$ from this particular expression, and that is given like this.

And after that, I have to ensure that this is a skew-symmetric matrix. So, how do we ensure? So, we have already seen that it is given like this. So, for simplicity, I am not writing Q and $\dot{Q}$. So, this already contains Q and $\dot{Q}$. So, what am I going to do? I am going to subtract $-2V_m$ from both sides and it is possible to show that $V_m - V_m^\top$ is nothing but $-A$, and A is a skew symmetric matrix. If A is a skew symmetric matrix, because A I have defined like this, and after that if I proceed like this, then I will get $A^\top = -A$. So, that is a skew symmetric matrix. And it is possible to show that if you multiply y on both sides of a skew symmetric matrix, then this is equal to 0.

So, easily one can establish based on this, because if A is a skew symmetric matrix, then here this is also a scalar $y^\top Ay$, A is a matrix, and y is a vector. So, suppose that if this is a 2×2 matrix, then this becomes a 1×2 matrix. So, finally, this becomes a 1×1 matrix. So, this is a scalar, and this is again a scalar.

So, finally, you are able to see this. So, what is A? A is this, and due to that reason, we are telling that this matrix is basically a skew symmetric matrix. Whenever you are going to define a Lyapunov function, this term always gives you 0. Whenever this kind of term comes up during the analysis, you can simply put that equal to 0. In this way, you are getting very easy whenever you are proving the stability, and due to that reason, you have to understand this property. Now, I have already told you that the mass matrix is always upper bounded and lower bounded. So, several times, whenever you are doing practical application, at that time you have to calculate the bound of the mass matrix, and its inverse is also bounded.

So, using induced two-norm, you can calculate the bound of $M^\top M$, and after that you can calculate λ_{max}. It is also possible to show that whatever Coriolis force or centripetal force is present, that is bounded with respect to velocity. So, now whenever you are going to design control, at that time you have to become very, very careful. You have to understand in which space our robot is basically going to work, and based on that, you will be able to propose a semi-global kind of stability.

So, now this is the conclusion of this particular lecture. So, what we have done in this particular lecture, this is not related to sliding mode control, but I have given the very brief or crisp overview of the robotic control problem. So, in robot control problem, basically what we have seen is that I have two different classes of robots, one is holonomic, one is nonholonomic. One class of robot is fully actuated; another is underactuated. It is easier to control fully actuated robot than underactuated. After that, we have also seen how to do mathematical modeling, and during mathematical modeling, several key properties come into the picture, namely the mass matrix, which satisfies the property of positive definite symmetry.

After that, we have some kind of skew symmetric term, and whatever dynamics that are centripetal or Coriolis dynamics, they are always bounded. So, using all this information, it is possible to design the sliding mode control, which is the topic of the next class. Thank you very much.