A Transition from PID to Nonsmooth PID

Hello everyone, welcome to the course on sliding mode control and its applications. I am Shyam Kamal, the instructor of this course, and in this lecture, we are trying to understand the connection between classical control, which is very famous in the industry, known as proportional-integral-derivative control. With the sliding mode control, we are also trying to understand how to formulate the control problems. So, this is the first lecture of the course, and as we know, in order to formulate a control, we need certain steps. So, control is nothing but a kind of logical mathematics. And due to that reason, whenever we are trying to design the control for any physical system, we first have to represent the physical system in terms of mathematical language.

Due to that reason, we need some kind of mathematical model of the system. So, suppose that I have some isolated part of reality and I want to perform some kind of study, or I want to design an isolated part of reality such that if I apply some input, then I will get the desired output. So, basically, this whole formulation isolates a part of reality with input and output, so somehow this boundary is distinguished between the environment and this closed space, and this whole thing is called the system. So, since our input and output measurements are both real, in this whole course I am going to assume that input and output both belong to some set of real numbers or some form of a matrix.

So, whenever we are formulating a mathematical model of some isolated part of reality that is called an object. So, you can see here that in infinitely many possible ways I am able to map output to input, and due to that reason, we have infinitely many mathematical models for the same system. So, whenever we are trying to design a control, I have to become very specific whenever we are selecting the mathematical model. Now, why are we basically designing this kind of system? So, most of the time, we have a certain kind of objective, and we have to satisfy that objective. So, due to that reason, in order to formulate control problem, second step, we have to specify the control objective.

So, in this course, I am going to show you that one can distinguish control problems into two subcategories, which are called the tracking problem and the stabilization problem. And this control objective is somehow tells you how to one can able to select the mathematical model. And now the philosophy of control or control logic. So, that is based on the combination of two things that are called feedback and feedforward. So, feedback is somehow the reactive action, and feedforward is the corrective action.

So, we are trying to understand what the meaning of feedback and feedforward is. And obviously, we are trying to understand why proportional integral and derivative is a very famous concept in the industry and how the notion of PID control or the motivation behind PID control comes into the picture. In the last part of this lecture, I am going to establish the connection between sliding mode control. And in this course, we are trying to show that basically sliding mode control is nothing but a kind of non-smooth control. So, we are trying to understand how PID control is modified, which ultimately leads to non-smooth PID and the outcome of this course in the selection of the practical model, followed by the physical interpretation of the PID controller.

So, you can see here that I have taken the underwater vehicle. So, an underwater vehicle has several components, and one cannot design an underwater vehicle based on a single concept. So, the concepts of electrical engineering, mechanical engineering, computer science, and materials require a lot of things. But in this course, I am trying to focus on control, and due to that reason, I am trying to see this particular object that is called an underwater vehicle from the control perspective, because that perspective will give you the way to model this system. So, you can see here that once this system is actually manufactured or built, we are trying to satisfy some kind of objective.

So, what is our objective? Our objective, which I am going to specify in the next slide, is to maintain a certain kind of velocity. Somewhere I have to stop that underwater vehicle as well, and I have to do this kind of job in the presence of the disturbance. And it is possible to show that these are two problems. So, one problem is, suppose that I have to track some kind of velocity, some kind of time-varying velocity, or some kind of constant velocity. So, and this is our initial condition.

So, this class of problems is somehow called the tracking problem. And now, whenever I have to stop, suppose that this is the initial condition and I have to stop. So, this is the time axis and this is the velocity axis. So, this class of problems is called the stabilization problem. And in this course, since we are trying to solve these two problems in the presence of uncertainty, this is called the robust stabilization problem or the robust tracking problem.

So, sliding mode control is somehow trying to solve the robust tracking and stabilization problems. So, how do we design and formulate the mathematical model of the underwater vehicle? We know the objective: I have to control the velocity. And what is the control input? So, this is a thruster motor. So, by using that, we are producing the thrust, and after that, we are controlling the velocity. And due to that reason, we are going to apply a very, very simple law, Newton's law, to mathematically capture this particular system.

And after that, we are trying to select the thruster motor control or force such that I can satisfy both objectives; either, suppose that if I have to move the underwater vehicle with some constant velocity, then I will move like this in the presence of disturbance, or if I have to stop somewhere, then I am going to stop here; this is the t-axis and this is the v(t) axis. Now, let us talk sometime about this mathematical model. You can see that this mathematical model is generated based on F equal to mass times acceleration, according to Newton's second law. So, here for simplicity, I am assuming mass is equal to unity, but you can take any value of the mass.

m and dv/dt. So, I assume that this underwater vehicle is actually moving in a straight line.

Why am I assuming that that is moving in a straight line? Because this will simplify the mathematical model. Because in order for the existence of this mathematical model, this velocity should be differentiable. If that is not differentiable, I cannot write mdv by dt. And due to that reason, for the simplicity of the problem, I am assuming that this particular underwater vehicle is moving in one direction.

Now, you can see that second term on the right-hand side, which is $v \mid v \mid$. So, once the underwater vehicle is moved in some water, then what happens? Some opposition force comes into the picture and that force is proportional to the square of the velocity. So, from experimentally, people have already actually quantified this. So, I am trying to actually take advantage of that experimental result, and you can see here in a state of v^2I have written $v \mid v \mid$. Why? Because if you are going to write v^2, then either velocity is positive or negative; v^2always becomes positive.

Naturally, we know that if the underwater vehicle is moving in the forward direction, then the direction of the drag force is in the opposite direction. And for that reason, we are writing $v \mid v \mid$. So, $\mid v \mid$ will just give you the magnitude; v will also capture the sign. So, in this way, we are generating the mathematical model. So, for control purposes, you try to make the model as simple as possible because we are trying to talk about some kind of robust control.

So, whenever a mismatch between reality and the model comes into the picture, it is captured by the control or somehow mitigated by the control action, and this is somehow the unknown part d. Thus, we are assuming that d is bounded but unknown. So now let us try to talk about whatever things we have just discussed in terms of the block diagram. So you can see that I have a plant. So the plant is nothing but, in our case, the underwater vehicle, and after that, I need some kind of sensor.

So suppose that I have to measure or control the velocity of this underwater vehicle. So, I have to measure the velocity. So, we are trying to utilize some kind of physical sensor, which is actually responsible for measuring the velocity. And this whole course is basically about this particularly important part called the control algorithm. And what is the reference input? Reference input vs.

So, either $v_d = 0$for the stabilization problem and $v_d \neq 0$for the tracking problem. So, every time a sensor is going to measure the output, it will create some kind of error signal, and now we have to select the control algorithm. So, here you can apply a PID controller, a sliding mode controller, or any kind of controller. But we have to understand how, basically, this proportional integral and derivative control comes into the picture. This guy is the actuator, so once the control logic is designed, we need some kind of physical circuit to execute this particular command.

So, in our case, you can see here that we have a thruster motor. So, using a thruster motor, I can implement control of this particular plant. So, in this way, basically, the classical mathematical model of control comes into the picture. Now, I have already told you that I have two different problems: a stabilization problem and a tracking problem. It is possible to show that these two problems are equivalent, provided some assumptions are satisfied.

Obviously, both problems are not exactly the same. You can see here that in the error coordinate frame, if I try to... Make some kind of equivalence between the tracking and stabilization problem that is possible, provided that v_d is the reference signal.

You can see in the previous block diagram that the reference input v_d should be differentiable. If that is differentiable, now you can see that I am able to convert everything into the framework of the error. One of the very important philosophies here is that Newton gives us the idea that if you want to control some quantity. So, it is better to control its rate of change. So, if you control their rate of change automatically, that quantity is controlled.

So, in this particular lecture, I will tell you about the connection between control and calculus. So, almost all sliding mode control or all other forms of control are somehow connected with advanced linear algebra and calculus. So, what have we done? $\dot{v}$, $\dot{v}$mathematical model I have already formulated. So, I have substituted here and I am assuming that $\dot{v}_d$is not differentiable. Then these two problems are not equivalent, okay.

Sometimes suppose that if you have to track some kind of square wave, it is not actually always differentiable, but at least piecewise differentiable. For that particular problem, the tracking problem is equivalent to the stabilization problem, and for that reason, in this course, I am going to talk about the stabilization problem because once the tracking problem comes into the picture, I will convert everything into the form of the error and maintain the error equal to 0. So, if $e = 0$, you can easily see that $v = v_d$as $t \to \infty$, and due to that reason, somehow, these two problems are equivalent. But if $v_d = 0$, you can see that the error is exactly equal to v. Now, the philosophy of feedforward and feedback comes into the picture.

So, during the design process, we already know that I have to track this velocity. I have made some extra arrangements so that I can calculate the rate of change of velocity, and after that, I know that some kind of drag force is going to enter that system. And for this particular problem, I am assuming that the drag force is proportional to the square of the velocity. So, I can design these two terms in advance, and this control is called corrective action. So, corrective action is called; this is called feedforward control.

Okay, now this is a function of $e$that is called feedback. Okay, so what happens after the substitution of $\dot{v}$whatever term you know, because I am sensing v from the output using some kind of sensor. So, I know this term. So, before closing the loop, I can design this. I know, because once you design the control, you have to guarantee that the error is equal to 0.

So, giving a guarantee is easier if the non-linear part is not present, and for that reason, that is also another motivation to select the feedforward part. This part, u_e, is nothing but the feedback. We are going to design the feedback, so feedforward means if some known part is there, then I am going to substitute it inside the control. Due to that design, the overall control will contain both feedback and feedforward, so if something is known, then only you will be able to design the feedforward. Feed forward is corrective action, so it is always better; however, in order to apply corrective action, I have to understand what kind of uncertainty and what kind of known term we have, so sometimes it is difficult to design.

Feedback is a reactive action, so once an error comes into the picture and your system has deviated from the objective, you apply control; due to that reason, feedback is not always better. Sometimes, if your feedback mechanism is not correct, then that might destabilize your system, and for that reason, you have to design the feedback carefully throughout this whole course. Now, suppose that if you have to solve a stabilization problem, then the error is exactly equal to v, as we have seen here: $e = v$, $v_d = 0$. So, in this way, we are talking about the tracking problem and the stabilization problem. So, in this course, I am just going to talk about the stabilization problem.

Some part I will also devote to the tracking problem. Now let us come to the very fundamental controller. And this controller is very, very popular in the industry. 94% of controllers are still PID controllers. And how does the PID controller basically come into the picture? You can easily see that the PID controller is fully inspired by whatever way we are making the decision.

Okay, so you can see here that I have been getting some kind of set point. So, this is the tracking point, and this is the output. So, whenever a mismatch comes into the picture, I have to actually design some kind of logic. So, this is the control logic part, and then we have to apply this logic with some kind of practical circuit. Okay, that is called an actuator, and overall, the actuator and processor somehow follow the set point.

So, suppose that I have, at some time t, an error generated. So, this is the present information about the error, and this is the past information about the error that is actually shown in this area, and this is futuristic information. So, as we know, whenever we are making a decision, our decision is based on the present, past, and future. So, how to represent the present, past, and future mathematically; that is the problem of calculus, and the whole calculus is developed based on that philosophy only. So, present information e(t), so we have scaled that information and after that we have applied, same like us.

Now, our decision is also influenced by the past, and for that reason, mathematically we are representing the past with the integral operator and the future with the derivative operator. So, somehow here some kind of attachment between logic and calculus comes into the picture. Okay, so I have philosophy; philosophy like our control strategy should function in the present, past, and future. And you can see that the same kind of thing is reflected, and we are giving different weights.

Okay, now several challenges come into the picture. We have too much information whenever we design the control. We are saying that, okay, I need the present information, past information, and future information. I need three different kinds of gains, and tuning these gains is very difficult. Another important thing to note is that our decision is based on the futuristic error, and since the future is not known to us, this block will create problems in several practical situations whenever we are discussing a system that is actually operating in an open environment. Similarly, now a lot of control systems are actually on the network because people are talking about autonomous systems, and for that reason, this integral part is also dangerous because hackers can hack this information, and somehow bad feedback can be entered into your system, causing your system to become destabilized.

Due to that reason, people are now starting to look towards advancements in control, and one of the very easy choices is sliding mode control. So, there is a very nice book that was written by Professor Wang, and you can see that this book actually talks about several practical systems and their applications. Now, let us try to understand what calculus is. I have incorporated this here because calculus and control are somehow fully related. And once you design control at that time you have to give guarantee.

Whatever objective I have that will satisfy our system is not going to violate some kind of property, and one of those properties is called stability. It is possible to show that the notion of stability actually comes from calculus, and for that reason, we have to understand calculus. Calculus with some kind of practical example, and some kind of practical philosophy. So you can see that calculus is not a new subject. You are able to see that if you look at classical Indian literature.

So, at that time, people were talking about the slope, a constant slope. The Pythagorean theorem also comes from the Indians, if you look at the old literature. So, somehow several geometrical structures are coming from the philosophy of this particular slope. But they are talking about a constant slope. Now, whenever Newton explored several physical phenomena, he found that things were actually not changing like that.

So, they tried to solve this kind of problem: how to calculate the slope of something that is continuously varying. So, what have they done? Actually, they are trying to convert this problem into that problem, because the solution to this problem is already available in Indian literature, and for that reason, calculus has thought that it is better to convert this problem into that problem. So, how have they converted? So, they are telling you what you can do. You can zoom in on this, and if you do infinite zoom, then you have this kind of structure, and once you have this kind of structure, you can apply whatever is already in the literature. Okay, in a similar way, now if you do infinite zoom, you can see that I can easily calculate your slope, and due to that reason, calculus is not a new subject; I just have to know how to zoom it.

Similarly, if you see the classical literature, people are already well aware of how to calculate the area if the curve is like this. Now, either we have a rectangle or a triangle, then we know from the classical literature how to calculate the area. But now, area is like this. And due to that reason, the notion of the integration comes into the picture. And what is philosophy? Again, what can you do? You can convert this into the form of a triangle and a rectangle.

So, now who is going to do that kind of zoning? So, I need some kind of macroscope such that I can convert some kind of changing problem into a constant problem. So, a limit is some kind of mathematical macroscope. So, what is the mathematical macroscope doing? They are going to zoom in on it, and by zooming, they are converting some kind of tough problem into an easier problem, for which a solution already exists in the literature. So, what is my main intention in bringing this topic here is that I want to somehow correlate continuity and stability. So, in order to understand continuity, you first have to understand the limit.

So, what is the philosophy of calculus? Whenever you have some kind of curve and if the curve behavior is not like a constant slope, then what can you do? You can apply the zoom. And you can do that zoom such that now this problem is converted like this problem. So, how do you apply the zoom? So, suppose that you have some kind of function like $x$here and you are getting $y$here. So, you can only able to zoom the output.

Why? Because output information is available to you. And for that reason, we are applying zoom on the y-axis. And due to that reason, you can see here that the language is: For every epsilon, epsilon is related to the output side. So, first I am selecting the output side corresponding to the output. So, x is the input here and y is the output. There exists a delta, and you can see here that the delta ball is somehow related to the input side, which is x, and epsilon is related to the output side.

So, first we are selecting epsilon, and if there exists some kind of delta, obviously I have to generate this output. Due to that reason, first I am going to write x minus a, which is the distance from here to here, some kind of variation from here to here. Then, If I give a guarantee that I am going to live in this particular ball, then this is nothing but some kind of ball. So, in higher dimensional space, this absolute value is converted into some form of norm.

So, for that reason, I am telling this as a ball. If that is possible, then we are saying that some limit exists. So, somehow we are actually able to do Zoom. What is the meaning of continuity? If the left-hand limit, right-hand limit, and limit at this point are the same, then we are talking about the distance from here to here. So, if the left-hand side and right-hand side are the same at this point, we are saying that some function is continuous. And why is continuity very essential? You can see here that whenever we are designing a system.

What is our objective? So, after designing, we are trying to give some kind of input so that I can control the output. That is our prime objective for any system design. So, somehow we are trying to control futuristic behavior. So, how do we guarantee that futuristic behavior is okay? So, suppose that if the system satisfies the property of continuity, then the left-hand limit means that past information, present information, and future information, due to continuity, all actually lie in some ball.

So, we can guarantee that the future is not going to deviate much. So, somehow we are able to correlate the present, past, and future, and for that reason, continuity is very important in order to check stability. And if you see the definition of stability, it is exactly the same as the limit because their definition is exactly the same. This is the definition of the left hand if you are proceeding from here at this point, and this is the right side. If all are equal, then we have continuity. It means that the definition of continuity is also exactly the same, and the same kind of definition you can see.

In case of stability. So, when system is stable, you can able to see here that if I will give some kind of initial condition. So, suppose that our system is like this; it is an autonomous system because if I give some kind of initial condition, then our system is actually going to evolve. So, if I give $x(t) - x_0$, some kind of initial condition to the system, and if I get exactly the same behavior as the limit or continuity, then we are telling that system is a stepper.

So, now we are able to understand how continuity and stability are related. Now, this is the last slide of this particular lecture. You can see here that in this course, we are talking about several autonomous systems like airplanes or cube boats. So, this was developed by Quansar. This is actually a Stewart platform that was developed by one of our colleagues from IIT ISM Dhanbad, and this particular system is actually a pendulum system. So, in this course, we are going to learn about several systems and what philosophy is now, since classical control has several limitations.

And due to that reason, we are going to modify the control. If you look closely, then this control is nothing but a variant of proportional control. Why is it proportional? Initially, we are using k times y. Now, we are using y to the power of half. So, somehow a non-linear variation of proportionality is here.

And a similar kind of thing you can see here. Initially, we are using past information as a yield. Sliding mode control is going to be used like this. Okay, this is a generalization of the PID controller. Due to that reason, sliding mode control is very close to the PID controller, and in several practical applications, you can see that the world is also implementing this kind of control in missile systems.

Therefore, sliding mode control concepts are very close to the application prospects. That is also going to give one of the properties that is robustness. We are going to discuss this course in much more detail about how sliding mode control is actually beneficial compared to the PID controller and other controllers. So now, it is time to conclude this lecture. So, we have understood how to select the mathematical model, what the meaning of feedforward and feedback control is, and how the continuity property is related to stability because somehow we have to control future behavior. So, if I have continuity property, I can always able to control the system and somehow I have given you the very brief overview of ϵ–δ..

So, now whenever you are designing the control, please try to understand the tradeoff between feedforward and feedback, and try to appreciate the definition of stability, which is inspired by continuity. With this remark, I am going to end this lecture. Thank you very much.