Integrated Photonics Devices and Circuits
Prof. Bijoy Krishna Das
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 27
Integrated Optical Components: Microring Resonator (MRR): Passive Characteristics

Today we discuss about microring resonator passive characteristics because microring resonators can be designed for various active functionalities. So, first we need to know what is its passive characteristics and what all figure of merits, we need to learn that we will be discussing first. And then we will give some practical device design and a couple of experimental results that how it is related to theory that will be discussing today.

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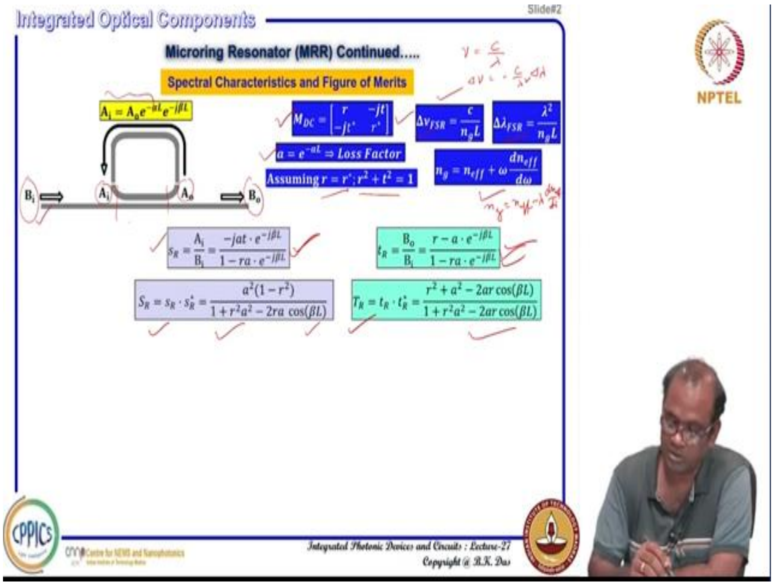

So, we have already introduced ring resonator the theory already developed. So, if you have a bus waveguide and coupled ring waveguide is there and there are input field output field annotated and inside field to we annotated with 2 different components A i and A naught. So, based on the analysis based on the ready state condition, we actually defined 2 parameters one is the field enhancement factor inside the cavity ring resonator that is nothing but A i / B i.

B i is the field amplitude in the bus waveguide and A i is the field amplitude in the inside the ring, it could be A naught also that does not matter they are related to A naught and A i related to this equation, this is actually called field enhancement factor inside the cavity and then this one we derived that is the transmission characteristics B out B o to B i what is the transfer function for the microring resonator resonators.

So, in this derivation we have used this directional coupler transfer matrix we define it as a M DC that means transfer matrix for directional coupler, we define like that, and then we have already defined one more parameter called loss factor which is actually A = e to the power - alpha l where alpha is the loss coefficient. And then we have also derived that these 2 things delta nu FSR and show called free spectral range we will discuss that a little while later that is nothing but c over n g L.

Where n g is the group index and derived that can be again converted into delta lambda FSR because we know nu and lambda is related by this. So, delta nu can be written as – c / lambda square delta lambda. So, once we know delta nu we can actually relate delta lambda FSR also were group index of the waveguide we define like n effective + omega dn effective / d omega of course this can be also converted into lambda function n g = n effective if you convert omega to lambda then it will be written as dn effective / d lambda that is one way.

And then this is amplitude field enhancement and that is the transfer function for the amplitude in the all pass configuration and if we take just complex conjugate and multiply then we will get this intensity enhancement inside the cavity that means, it major the power how much power it is amplified or it can be stored inside the microring resonator that can be expressed in this way just to take complex conjugate and multiply that then you get the we call it as a power transfer function.

Similarly, for transmission we will be having this t R dot t R star, so you get our transmission coefficient transmission parameter function we call we can define like that, you know in this process in this derivation we have just assume this r = r star that is actually considered r and directional coupler is lossless r square + t square = 1 but inside ring whatever loss is there that has been attributed to A that is the loss factor. So, this much we have learned in the previous lecture previous class.

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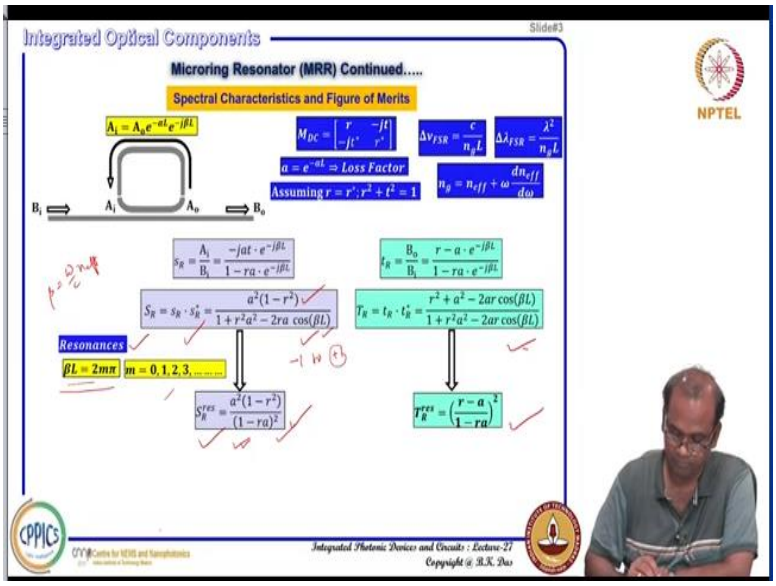

So, we also learned that in this function if you say this is the field inside or power inside that actually gives the value how much power is inside from this expression and if you carefully look into it then we can identify that in the denominator you have one factor called cos beta L, so this cos beta L can vary from minus 1 to plus 1. So, when cos beta L = +1 that means, denominator is minimum and cos beta L = -1 then denominator is maximum. So, maximum to minimum we can have.

So, in that case maximum power storage cavity power will be possible when cos beta L = +1 that means denominator equal to minimum, if it is minimum then only you can get overall maximum. So, in that case if cos beta L = 1 that means beta L has to be integer multiply 2 pi, m is the integer, m 0 1 2 3 4 so on. So, in this case it can start from 1. So, in that case I can say that power inside the cavity can be quantified by this expression when cos beta L = 1 that means it will be 1 - r square and numerator is a square multiplied by 1 – r square.

So, that is the thing that the resonance is we call it as a resonance condition resonances and that resonances can be expressed beta we know beta has a relationship with omega / c n effective so beta can be converted into omega. So, in that case we can find frequency. Frequency is discrete frequency solutions which can give you higher power storage inside the cavity and that particular corresponding frequency will be called as the resonance frequencies. So, at resonance frequencies the field or the power inside the cavity is quantified by this one.

At the same time, when we put beta = 2m pi transmission characteristics here also cos beta L = 1 so that we will call that at resonance what is the power transmission characteristics we can quantified r - a whole square 1 – ra.

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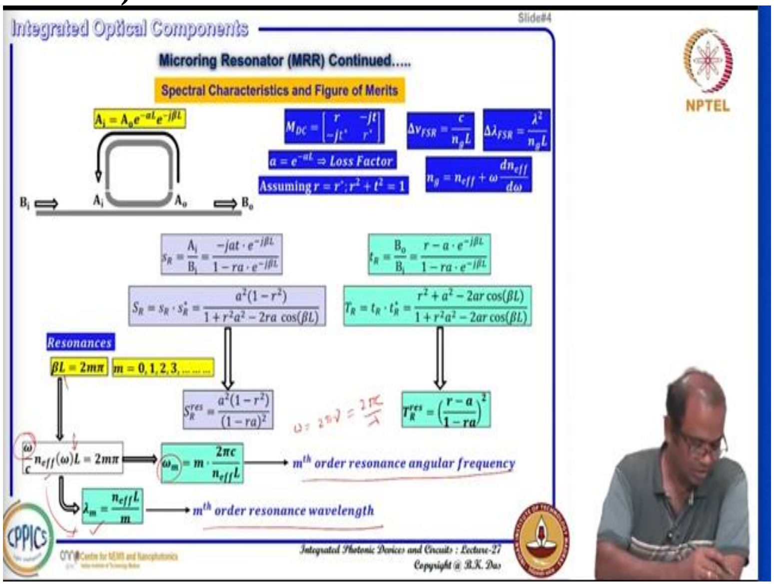

So that is straightforward and as I mentioned that beta can be represented as omega / c n effective and multiplied by L that is written that is equal to 2m pi from here for different m values I can get a discrete solution for omega that is omega m we can call it as a mth order resonance and angular frequency. Of course omega to linear frequency we can relate like this omega = 2 pi nu. So, we can get instead of omega m we can actually derive linear frequency and also we can convert also omega relationship with the lambda omega = 2 pi c / lambda.

So, in terms of lambda we can find the wavelength corresponding to resonance order m. So, mth order resonance wavelength we can define that thing. This is all about the theoretical aspects.

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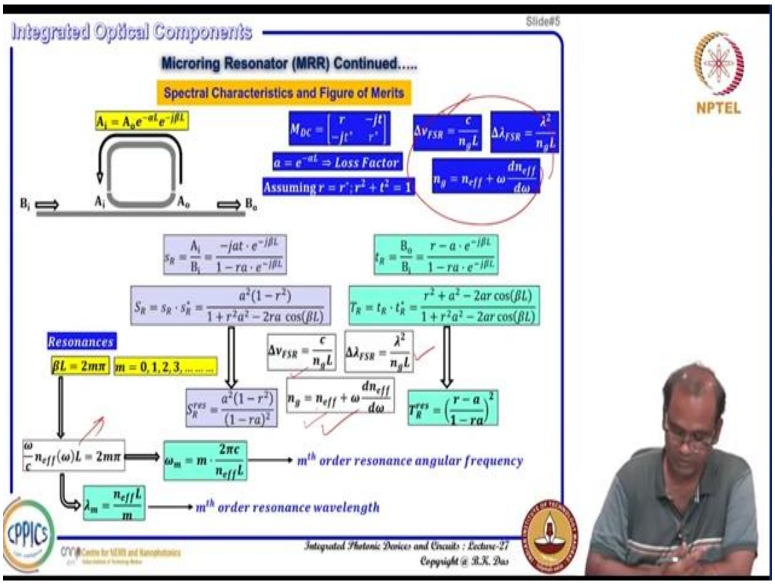

So, now whatever we have discussed that considering this delta nu FSR free spectral range and in terms of lambda in terms of frequency we have expressed this one also and group index whatever we have written here we repeated here for completeness. So, we can get because this is easy to get once we know this relationship from this relationship we have derived that if we go for successive resonance is mth to mth + 1 its resonances and their separation if we find the frequency separation that will be called as a free spectral range.

So that frequency separation between successive resonances and this is the wavelength separation between successive resonance wavelengths. And in that case, we need to introduce this group index that is standard for because whenever we are considering resonance, we will dependent things that time group index actually matters.

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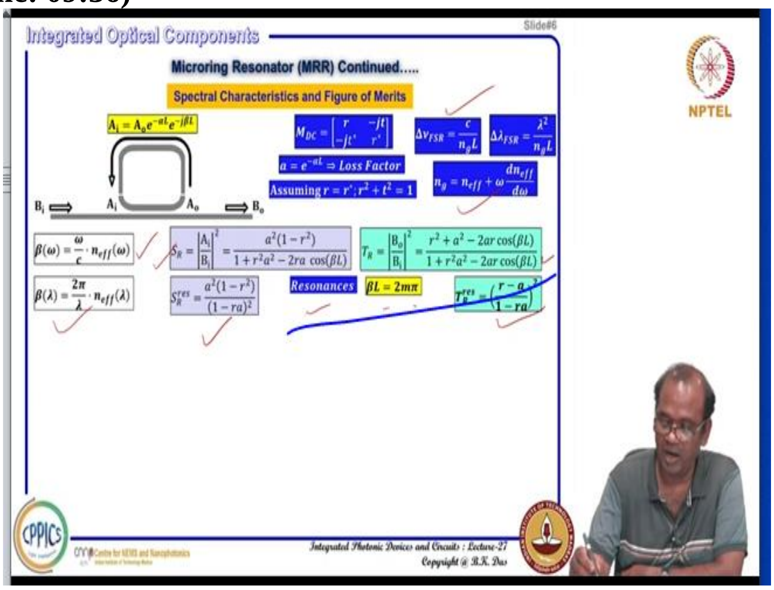

So, we actually summarized here about all the important equations, so delta nu FSR given here, group index given here and then power inside the cavity can be quantified by S R that means A i / B i that is the input field enhancement square of that actually power enhancement inside the cavity, this is the power transfer function maximum power storage at resonance beta L = 2m pi resonance whatever discussed so far repeated here. And beta omega relationship beta lambda relationship, so all these will be needed to discuss about the overall spectral characteristics and the figure of merits of a microring resonator.

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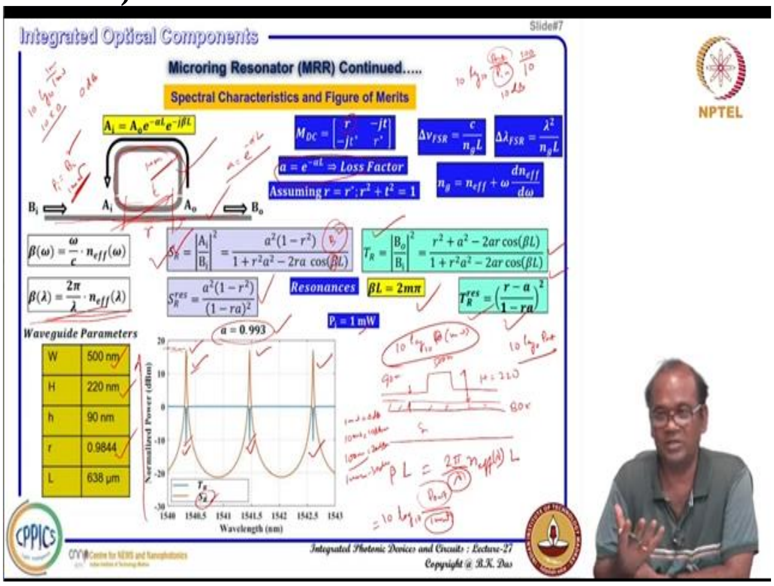

So, we consider a waveguide dimension of 500 nanometres silicon on insulator you know that if it is a substrate and this is your silicon substrate and then this is your oxide Si to BOX this is the box and then the top you can have your re-waveguide structure and this re-waveguide structure w equal to this one equal to 500 nanometre width and this height equal to h = 220 nanometre here consider w equal to this one and h that is the slab example of so the waveguide will consider that is actually 90 nanometre.

Using this waveguide we constructed this ring and bus waveguide and if we choose a suitable length here coupling length then we can actually consider how much it will be self-coupled and how much it will be cross coupled self-coupling parameter is r and cross is t, so r value in this case we have considered we have adjusted this bus waveguide and ring such that they r coefficient here that means r is self-coupling, self-coupling means whatever you give in a directional coupler, whatever you get in the bar port that is actually responsible because of the r.

So that value is almost that means almost 98% amplitude electric field strength is appear in the output and L that means total perimeter length it is actually 100 it is basically something like the ring type configuration you can consider 100 micrometre is that bending radius. So, 100 micrometre bending radius that is considered sufficiently large so that bend induced losses as minimum as possible such that we can get A this loss factor whatever we have defined here loss factor this one.

A = e to the power - alpha L loss factor is should be close to 99% so this alpha value should be as minimum as possible so that A will be close to one so alpha tends to 0 then A tends to 1, alpha is very large than A will be dropping so that is the information. So, alpha even if it is per 1 dB per centimetre loss we can get that one round trip you will lose maybe less than hardly about 1% or less than 1% power you will be losing.

So, per A if we consider like this an input power here P input, P input is nothing but B i square we consider accordingly according to our convention so far is 1 milli watt if we consider, then we can plot both T R and S R that means with respect to 1 milli watt how much I am getting in the throughput and how much I am getting in the inside the cavity and that I will be using with wavelength as a function of wavelength.

This beta I will be converting expressing this beta here beta L we will be writing as 2 pi / lambda times n effective corresponding lambda effective index I will be computing and A i using. And then if I tune this lambda value. Whenever this beta L will be integer multiple of 2 pi I can say that this will be maximum and this will be minimum. So, exactly whatever we got this blue curve here you see at certain wavelength if you see transmission is dropping, transmission is dropping, transmission is dropping certain wavelength.

They are actually resonance frequency, at that particular frequency if you calculate FSR you see the power or intensity increasing here. And in this y axis if you see, this is actually we call it dBm you know what is dBm? dBm is actually you have to consider P out / P in log of 10 to the base 10 that is actually the gain or loss in dB, whatever P out you express in unit and P input you express in unit that if it is watt, it is watt then you can get the watt watt cancel then you can get whatever gain you get.

So for example, you are getting for example S R maybe 10 dB if you are getting that means, if you have inside S R output will be about 10 and then 100 and this one maybe 10 P input is 10 that means it will be 10 so in that case 10 dB will be the inside more gain will be the inside and correspondingly if you will get P out / P input negative that means it will be dropping. But sometimes it is considered suppose milli watt power, power you can express in dBm how it is that suppose, whatever power you launched or available to you.

You just take log of that power m milliwatt whatever power you are getting power m milli watt you take a log to the base 10 and multiply 10 whatever value you get that will be called as a dBm that means if I consider my input power is 1 milli watt, output power whatever value is there P output log of 10 to the base 10 that will be actually your output y axis I am plotting this normalized power in dBm.

If input is 1 milli watt and then whatever value I am getting are the output according to this one because B i square I will be getting 1 milli watt then I can calculate A i square putting all these value I can calculate A i square, A i square if I put here B i square that means it is 1 watt that means A i square I will be getting 1 milli watt corresponding A i square whatever in milli watt I will be representing that really what if you just get that milli watt log of that whatever output you are getting and 10 that is actually plotted in y axis.

So, in dB scale you can actually see then signal to noise very clearly that is why normally power density or amplitude ratio normally we represent in dB scale. So, here you see if I launch 1 milli watt power, 1 milli watt means that means I can say that 1 milli watt if you are expressing in dBm normally, you can get log of to the base 10 1 milli watt, 1 milli watt / 1 milli watt 10, so that means log 10 1 that means actually 0 10 times 0, so that will be called as 0 dBm.

So, we can say that 1 milli watt corresponding to 0 dBm. Similarly, if we have a 10 milli watt then we 10 milli watt means you will be 10 log 10, so that will be 10 milli watt that will be 10 dBm. So, similarly you can have 100 milli watt that will be 20 dBm and so on. So, 1000 milli watt that means 1 watt that means 30 dBm, so milli watt to dBm that unit conversion you can actually express. So, my point is that if I consider at the input, it is actually equal to 1 milli watt you are launching in the bus waveguide.

Then inside if you just follow S R that is reaching close to just this is 10 dBm. So that will be actually about 18 dBm or something like that. So that means inside field it is enhanced at resonance. Power inside the cavity that is all now it is increased many folds close to 20 dBm that means 20 dBm means how much that is about 100 milli watts. So that means you are launching here 1 milli watt and it is passing but inside in steady state, it is storing 100 milli watt power flowing inside all the time 100 milli watts it is flowing if it is 20 dBm.

So that is the beauty of a microring resonator and you can store energy at resonance when resonance wavelength reached you can actually enhance to the power strength inside you can store the energy inside the ring. And in the transmission you see that at that particular wavelength that much energy it is dropped here, how much exactly not 20 dBm because that actually follow this rule according to this transfer function. So, you will see it will reach up to this level.

So, it will follow exactly at resonance you know the r value a value 1 - ra whatever value comes that you take log scale so you get this one another resonance length, so all the resonance you see inside field enhancing and in the transmission it is dropping perfectly it is dropping this one dropping, dropping, dropping and it is enhancing inside. So, red curve corresponding to S R, blue curve corresponding to T R.

So, when transmission drops at that particular wavelength that is the resonance wave length at that particular resonance inside the ring the power density increases power level increases. So that is the beauty of a micro ring resonator that is why it is called a resonator.

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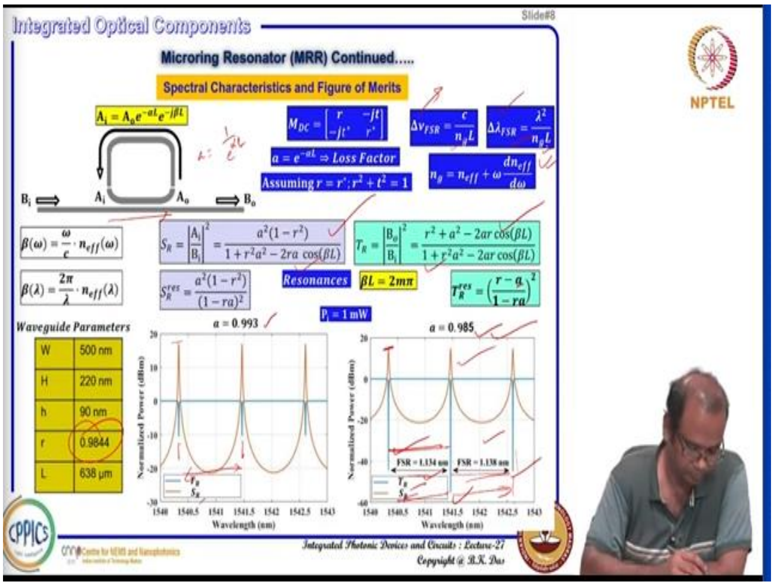

Now, here we saw that if inside the loss factor is decreased, loss factor is that means a is decreased, a is decreased means typically when a dropped that means round trip a drops that means alpha value is actually increasing, when alpha value is increasing that means a =1 / e to the power alpha L. So, when alpha value increases in that case a will be dropping so that means loss waveguide propagation loss increases I will be getting this type of characteristics.

You see when loss increases inside the cavity you see this storage is here it is much higher and here red curve if you see it is slightly dropped, but at the same time depending on the value because A is increased here also the value will be there for a given r, r value always we have mentioned this one this we have used r that self-coupling coefficient here that is r. So, in that case you see this storage will be dropped and extinction will be more here. Here in the resonance in the transmission if look up it is a T R.

So, it will be it is going more and more extinguished, so throughput is reduced at that resonance wavelength. At the same time your storage power throughput reduced does not mean that your storage power will be inside whatever store S R that will be increased because this is throughput is reduced because of the a loss in the waveguide. So, loss is there, if loss is there inside power also cannot be increased too high, so that is what it is evident here.

So, this is how we can just express we can just plot directly this expression, this expression we can plot as a function of omega S R and T R the function of omega T R where T R will be minimum there S R will be maximum. So, T R minimum means it is resonance and at resonance energy will be store power will be stored inside the cavity and inside the cavity energy will be stored more if loss in the waveguide in the ring if it is less.

So, this is very important and in this case if you see FSR if you try to say FSR means free spectral range so that means one resonance wavelength to another resonance wavelength what is the separation, these to these separation if I see I will be getting 1.134 nanometre per all these parameter. So, for example here 1541.5 nanometre one resonance and is there and if you go for next resonance 1542.5 plus something somewhere here.

So, if you see that different separation the difference in resonance wavelength to successive origin and trembling that is what your delta lambda FSR and that is actually this site is 1.138 slightly higher because, you see in delta lambda FSR it is proportional to lambda square more the lambda higher lambda value if you are trying to measure FSR there will be more. So, this is higher you are going towards higher wavelength.

So that is why delta lambda FSR that means FSR will be more and this side is you are going for the shorter wavelength that is why FSR will be lower. However, if you see the in case of free spectral ranges in terms of harch frequency in that case c / n g L that is what frequency dependent so that is actual depends on group index. So, as a function of group index is frequency dependent then only it will be you will be seeing that delta nu FSR will be varying resonance to resonance; otherwise they would be remaining constant as long as n g is constant.

So that means you see relatively free spectral range in terms of frequency it is more regular because it does not depends on frequency, but in terms of lambda if you try to see free spectral range then you can see it is square proportional to the square of the wavelength that is what it is reflected also here separation, whatever from the simulation whatever we are getting we just calculated exactly that way and it actually matches this formula also, so this is free spectral range. So, the all this figures of merit actually very important if you are planning to use such micoring resonator for various applications.

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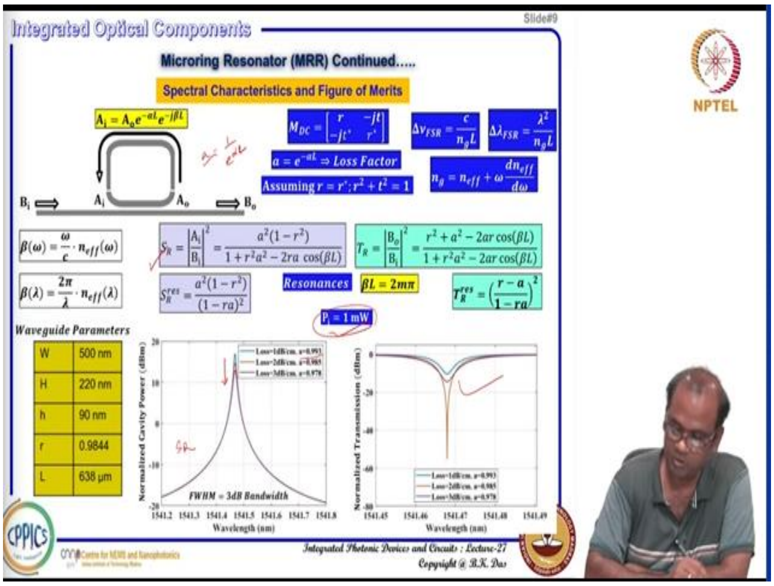

Now, another one again your P input is 1 milli watt if you are considering as I mentioned that if you are just increasing loss 1 dB, 2 dB, 3 dB then loss factor actually if you are increasing loss, then loss factor will be actually reducing because that is again a = 1 / e to the power alpha L if alpha it is neper it is 1, but you can convert neper to decibel we have discussed how to convert that. So, if alpha is increasing, a would be dropping.

So, if a drops that means loss is high a drops means that means one round trip how much it is coming it is reducing means loss is increased, a is reducing means loss is increased. So, when loss is increased you see they pick this is actually a S R we plot again normalized cavity power. So, when we plot S R for B i = 1 milli watt B i square = 1 milli watt then whatever power I get that is actually power inside the cavity power plot.

So, as loss increases the it drops the power will be dropping that is what if it is 1 dB per centimetre or 2 dB per centimetre 3 dB per centimetre then lost will be B i, so that means if you can fabricate your waveguide loss it is low well designed and technology is very good. So that you do not get much scattering loss is etcetera in the waveguide then you can store more energy inside the micro ring resonator.

So, at the same time if you see the transmission also it will be varying. So, when loss is actually increasing, you see when the extinction in the transmission that can also be controlled depending on the loss factor so far so good.

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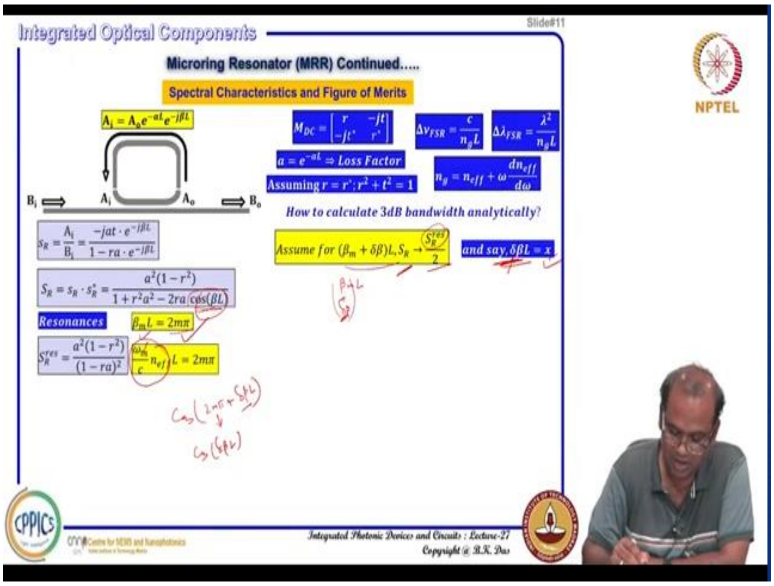

Now, let us move on so we have almost discussed whatever things are required. Now, how to calculate 3 dB bandwidth, analytically? We know that they are resonance wavelengths certain wavelengths. You see here according to this thing, this is the resonance wavelength the energy inside how much it will be storing. Now, I want to know what is the 3 dB bandwidth. 3dB bandwidth from maximum if you come down 3 dB below and then you see the spectral width of the resonance spectrum this is the resonance spectrum.

So that 3 dB bandwidth how to calculate analytically we can numerically simulate this curve and this curve specifically this is very important. So, this one if I plot I will be getting this one and then I see this is maximum, maximum is happening about 18 dBm. So, I come down 3 dBm about 15 dBm then here I see that whatever this omega one and this lambda one and then this one if I plot this on I estimate that will be your bandwidth 3 dB bandwidth, but can we calculate that analytically?

Analytical mean can we express or can we derive an expression without simulating this I can directly put some parameters so that I can predict what is the 3 dB bandwidth of this resonance let us try that. Now, we just consider S R amplitude field enhancement inside the cavity A i / B i, B i is launching A i inside field that is expression we have derived and this is the power enhancement inside.

So, I know that if I am launching 1 milli watt then whatever value comes that milli watt will be inside at resonance that is a maximum once again I repeat that resonance frequencies can be expressed 2 pi times integer. Now, what we do let us assume that beta m the resonance corresponding to resonance beta m means this one, this one in that case this cos beta L term will become 1. Now if cos beta L term if it is one then this will be 2 m pi cos beta is there or not that depends on what is the value of beta L, if beta L = 2 m pi this term we do not need to use.

Now, if it is beta L is slightly higher corresponding instead of beta m L, you have just adding additional delta beta with beta m and then L this delta beta how you can additionally include, it tuned frequency rate from the resonance wavelength if you tuned resonance wavelength a bit then you can see that that you are beta L instead of 2 m pi it will be slightly increased. So, in that case you will be writing something like that cos 2 m pi + delta beta L this will be 2 m pi + delta beta L.

Because you; have detuned these 2 m pi corresponding to be considered mth frequency now, you frequency a little bit detuned, so you suppose to get a little bit of delta change because omega is there. So, in that case I can write this one exactly equal to delta beta L because 2 pi + theta cos 2 pi + theta cos theta. So, we can write that one if that happens I can find that suddenly beta change whatever happening that we define as x and this delta beta is changed such that S R value.

Whatever you supposed to get at resonance for beta m = 2 m pi if detuned delta beta so that your maximum strength here according to our this formula this thing this is your S R max at resonance. Now, if this is your resonance frequency here lambda m wavelength now, you determine the frequency bit right hand side then once you detuned such that your maximum become half 50% what are you supposed to get at resonance if you detune a bit then your maximum will be dropped to half.

So, I assume that that delta beta we need to find out and delta beta if we find I know what is the relationship between beta and omega and delta beta we will be able to find, then we can find whatever the 3 dB bandwidth that is the whole idea. So, delta beta L x I am just considering delta beta L = x, so that means, you just not it.

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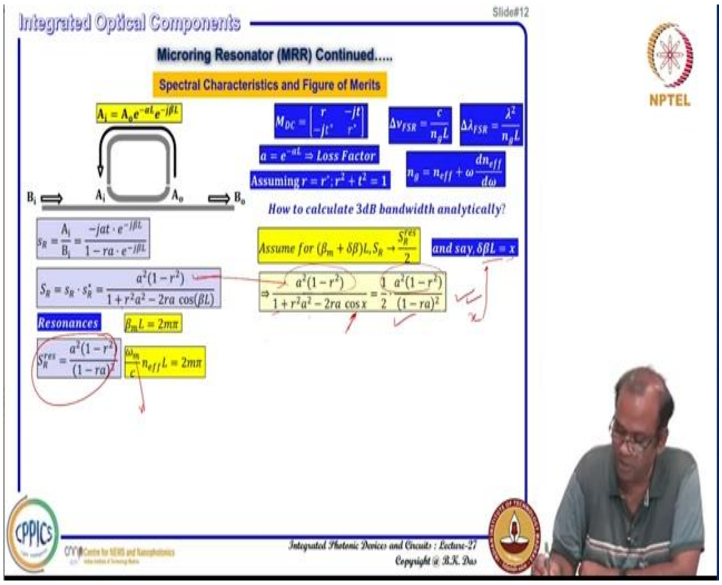

I can write a square 1 - r square that is numerator that I have written here and this one 1 r square + a square - 2 ra instead of completely removing cos delta beta L resonance I have detuned a bit detuned a value is delta beta L = x I just put down here x and it has been changed detuned such that the maximum is supposed to get this one because of detuning maximum the power inside the cavity will drop to half exactly half of that.

Now, from this expression our intention is that can we try to find out x, once we know x value then we can find out delta beta, once I know delta beta then we can find how much frequency is detuned from resonance then you can get at that particular frequency the maximum will drop to half field strength powers inside the cavity will be half at that particular frequency. So, our intention is to find out now x from this expression. So, if you just carefully seen numerator both sides same.

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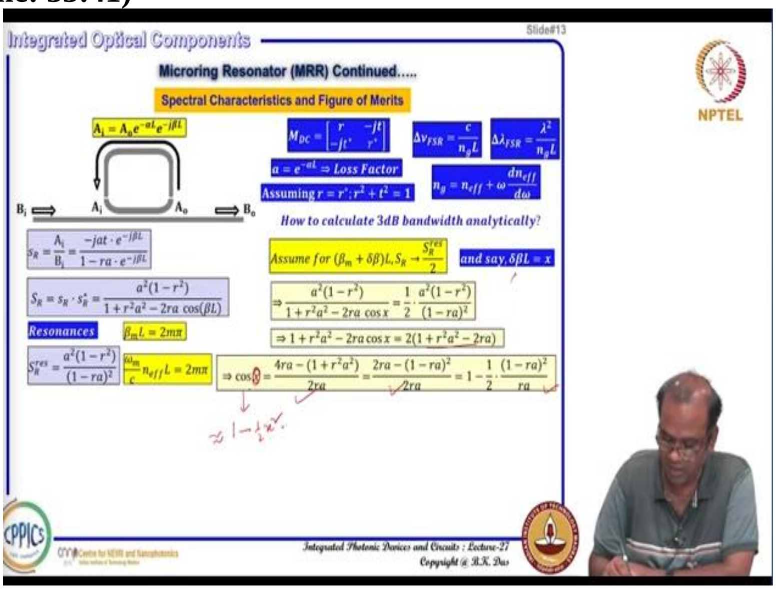

So, denominator will be you can compare the numerator denominator 1 + r square a square – 2 ra cos x and this is 2 times 1 - ra whole square that can be expanded as it is a - b square a square + b square – 2 ab we have written. So, if you just from this we can express cos x in terms of ra we have assumed that the values of ra everything are known from some other sources either by numerical simulation or by calculations we know that.

So, we can express the cos x, we can represent like this, we can represent like this and we can little bit simplify like in this form 1 - 1 / half 1 - ra square, so cos x I can define like that. Now you know this cos x value actually it is not very large it is just a little bit detuned from the resonance wavelength. So that you get a delta beta and then you get x so it is not you need not to change these delta beta very large value.

So, in that case this cos x you can just express as 1 – half x square cos x = 1 minus approximately equal to you can write 1 - half of x square then again you could write 1 plus or the Taylor series you can expand. so I can take first 2 terms.

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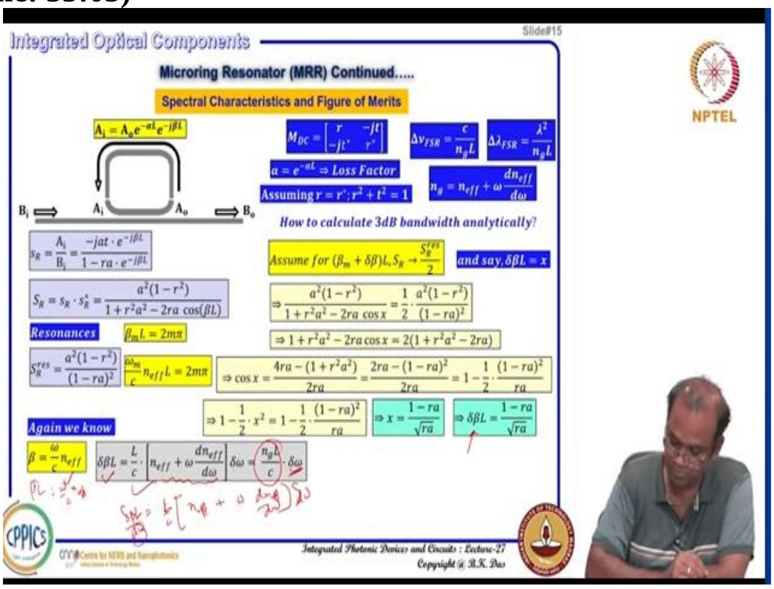

So, if I do so I have written that cos x equal to this one this I have written half x and you have this one right hand side, you see they are identical 1 minus something half and 1 minus half something, so that means x square equal to this one. So that means x = 1 - ra square root of ra, so that is very interesting x = delta beta L. So, delta L equal to this one I know now, if my delta beta L exactly to r and a known depending on the waveguide parameter.

What is the loss factor and what is the self-coupling coefficient for the directional coupler all these unknown then I know what is the delta beta L required so that your power inside the cavity drops to half, what would you expect resonance at resonance? At resonance we expect maximum storage, so it will detuned delta beta L so things will be dropped to half. Now you do one more thing we know that beta is equal to this one so we tried to find out you just make a differentiation delta beta L, L you are multiplying both sides that means beta L = omega c / n effective.

Now, I do derivative with respect to omega for example, delta beta L that means L / c I can write and n effective first you do delta beta = 1 / c you will just put omega if you just make derivative then it will be delta omega n effective I just write delta omega for example, d beta / d omega n effective and then you can write omega d n effective / d omega and this delta omega I can take this side, delta omega and if I multiply here I can multiply here L / c.

So that is omega delta here and this one we know that this is n g group index so that means n g L divided by c, n g L divided by c delta omega. So, I can write delta beta L equal to this one delta beta times L = n g times L / c into delta omega. So, I can just put here this is the delta beta L expression and then I can find out what are the delta omega.

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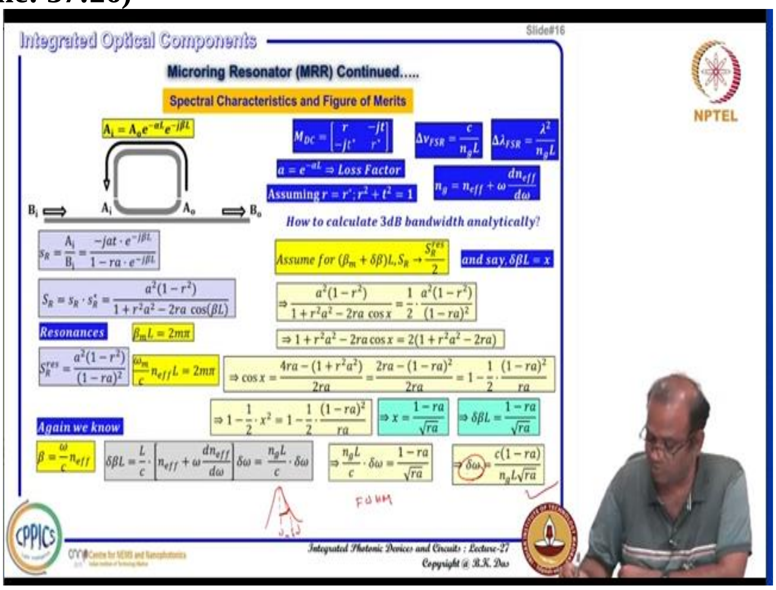

So, from here I can find the delta omega, so now you see I can say that resonance spectrum is like this and this is your some omega m resonance mth resonance frequency. Now, you move delta omega this side then it will drop to half, so I get this value is this one. So, if I want to find out FWHM Full Width Half Maximum, then we have to multiply delta omega times 2. So, FWHM will be 2 times you have to multiply with this.

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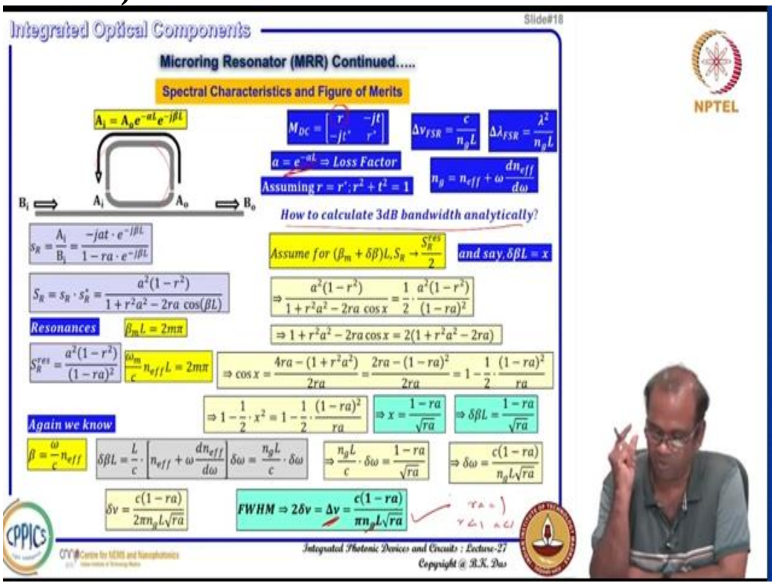

So, accordingly I get delta nu because delta omega I can convert to pi this one I can write delta omega = 2 pi times delta nu. So, 2 pi is has to be divided this side that is we have written and then we do FWHM 2 times delta nu that is I represent as capital delta nu, so capital delta nu I can get here. So, FWHM that means 3 dB bandwidth 3 dB means 50% down. So, how to calculate 3 dB bandwidth if we; know r value, directional coupler and a waveguide loss a value here and if I know the perimeter length.

If I know the group index of course c and pi are constants then I can find how much 3 dB bandwidth will be there for the resonance. So, this is interesting so you can find out from the waveguide design parameter, you can design a ring resonator to x split certain full width half maximum if you just increase for example, L value the value of the ring resonator if you are increasing then delta nu will be the full width half maximum will be narrowing down. And if you for example, if you are just somehow reducing L then it will be broaden.

So, you if your resonator perimeter length is shorter microring resonator then your bandwidth that means FWHM will be broader. And apart from that, for example, you just consider a is the waveguide loss if waveguide loss is increased, then you see here square root of ra. So, in that case if it is increased normal loss is increased, then this will be reduced and this will be also reduced. So, ultimately delta nu we will see that will loss is increased delta nu will be increasing full width half maximum.

Whatever the result you can just find out depending on the for example if you just consider ra = 1 for example, somehow if you can manage to get r = 1 never possible because r is also less than 1, a is also less than 1. So, ra cannot be 1 if only ra = 1 then you can say that delta nu will be 0 that means perfect line delta function you will be getting perfect delta function at exactly that particular frequency you will be seeing that as resonance. So, normally that is never possible that is your full width half maximum will have some width bandwidth practical device will have some kind of resonance bandwidth that is the take a message.

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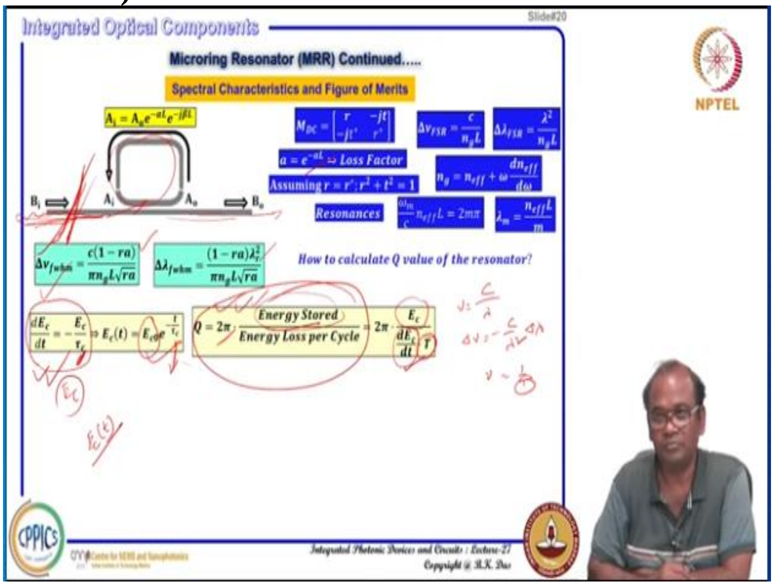

Now how to calculate Q value of the resonator? So, we have learned that it is a resonator. So, resonator means you know in any electrical circuit also resonator means it can store some energy. So, if it can store energy, we can quantify a figure of merit is called quality factor Q value, the Q value actually defines how much capacity you are storing energy at resonance. So, Q value we introduce just to justify how good is the resonator.

So, higher the Q value means the resonator has a capacity to store more energy, so that is actually how we define that in principle, normally we consider that Q value is the let us concentrate on this. So, we have already defined that delta nu in terms of frequency full width half maximum, this is the expression we have derived and if you convert into delta lambda you know c always c / lambda = nu, so delta nu = - c / lambda square so delta lambda.

So, I can write delta lambda r square by the resonance wavelength we have written. And Q value as I mentioned if concentred one this that is defined by 2 pi times so that is just a scaling factor we use ultimately this ratio matters energy stored / energy loss per cycle, per cycle means at resonance frequency, the time period how much time it takes to oscillate. So, time period is the corresponding to resonance frequency that is called cycle.

So, per cycle that means, that is the time period normally you know nu is the frequency then normally nu = 1 / T, T is the time period. So, within that time per 1 cycle of oscillation at resonance frequency how much energy getting lost and how much energy is stored inside the cavity if you take this ratio and multiply by 2 pi that actually measures the quality factor, ultimately energy store you are taking ratio how much loss is happening that loss is certain time we are considering that is the per cycle how much energy getting lost.

So, this actually we define as a Q value, we just keep in mind we will come to that point later after understanding this thing. Let us consider that inside the cavity E c amount energies store. total energy store inside the cavity the E c. Now suppose your this input you just kept input whatever the continuous wavelength was there at resonance and energy was stored and something you are getting out you consider that this is suddenly stopped and what would happen?

The stored energy inside the ring they will start losing because of the loss and also coupling to the bus waveguide again so slowly it will start losing if this E c is the energy and we consider E c is cavity energy as a function of time at instant of time once you have got it the energy will start losing inside the cavity. So that can be a time independent and that time independent energy loss that means energy loss per unit time can be written at E c / tau c, where this tau c can be expressed as a cavity lifetime that means, if you solve this expression you get this expression.

At t = 0 when you are cutting down that time if you E c0 was the energy total cavity energy in steady state. Now, whenever you are cutting your input, then it exponentially decays whatever energy stored inside the ring that will be exponentially decaying and at t = tau c that energy will drop to 1 / E that is why it is called cavity lifetime. So, even if you can imagine if you are considering electromagnetic wave light you consider one photonic is inside and you can try to understand how long that photonic will survive in that cavity.

So that is actually characteristically that in tau c you can say that within tau c that photonic will be lost, basically the classical it is 1 / E we say that is our lifetime it will be slowly to decaying this energy stored inside, in steady state whenever continuously laser was on per resonance to bring the energy was stored and that energy is much higher than whatever your power is much higher than circulating power, much higher than whatever you were launching because of the storage capacity of the resonator.

So, in that case, what do you have? You will be getting if you cut that one slowly to be lost and that lost energy lifetime is tau c. So, it is this one now, I can say that stored energy E c I have written here stored energy and then we are writing here that energy loss per cycle per unit time energy losses dE c / dt we can say that that is E c / tau c we have written and per cycle it is time, time period I said so Q value can be expressed like this.

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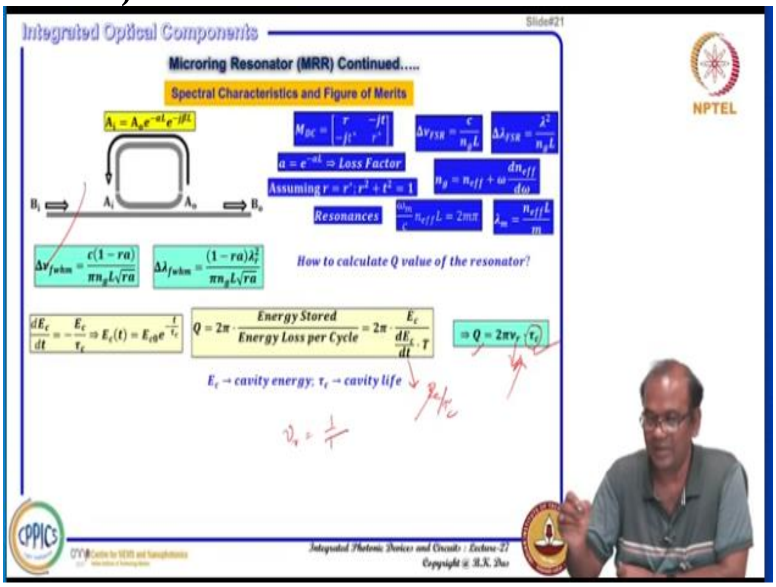

In that case, what we get Q = 2 pi E c / dE c / dt this one I am writing E c / tau c then E c E c cancel and 2 pi / tau equal to we can write 2 pi and 1 / T = nu r resonance frequency I have set nu r = 1 / T time cycle. So, 2 pi nu r and dE c / dt = E c / tau c E c E c cancel, so 1 / tau c that means this one, so Q will be 2 pi nu tau c. So, if we know this tau c then you will be able to find out the Q value that means what is the lifetime in the cavity, so longer it can stay any photonic inside the cavity the Q value is higher.

So, it is clearly it can be understood that Q value depends on how long energy it can hold inside the resonator when your source is cut, so longer the light time your Q value is with more.

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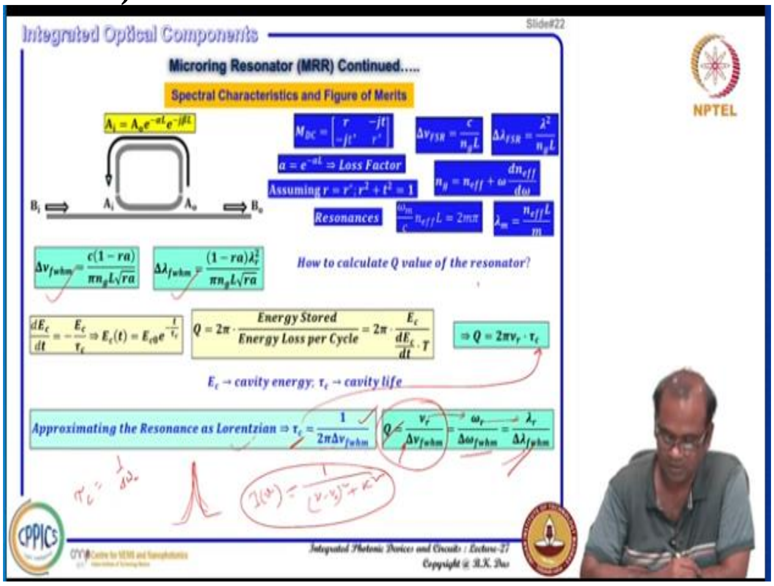

Now, what we can consider approximately resonance as a Lorentzian, so normal Lorentzian you know that around this one. So, normally 1 over nu - nu naught square plus some constant will be there. So, if some frequency if some spectral density if you are considering I nu is proportional to this one this is actually Lorentzian shape. And do we know it from Fourier analysis etcetera we know that if any spectrum is Lorentzian shape we can consider very good quality resonator the spectral nature is almost looking like a Lorentzian.

And in case of Lorentzian the decay time constant and full width half maximum is related with this expression. So, basically tau c = 1 / delta omega c angular frequency bandwidth, angular frequency bandwidth 2 pi delta nu we have written. So, tau c we know this one we can put this tau c expression here, 2 pi 2 pi will be cancelled, nu r will be there and denominated with delta nu r.

So that means Q value will be this one resonance frequency divided by full width half maximum frequency bandwidth 3 dB bandwidth and nu r can be scaled to omega r 2 pi multiply omega r 2 pi you multiply delta omega angular frequency bandwidth and omega r can be scaled to lambda r and delta omega can be scaled to delta lambda so lambda r / delta lambda. So, if you know the; resonance wavelength and if you know the FWHM 3dB bandwidth.

Then you can find out what is that you have this is the easiest way of estimating Q value I know that FWHM is expression known, this expression known and I know how to calculate by resonance wavelength that is coming from beta L = 2 m pi so beta if I write omega / c n effective L m = 2 m pi so I know resonance wavelength. So, mth order resonance we are considering we can take. So, once we know the resonance wavelength and once we know the expression then we can find out the Q value that is what it is written.

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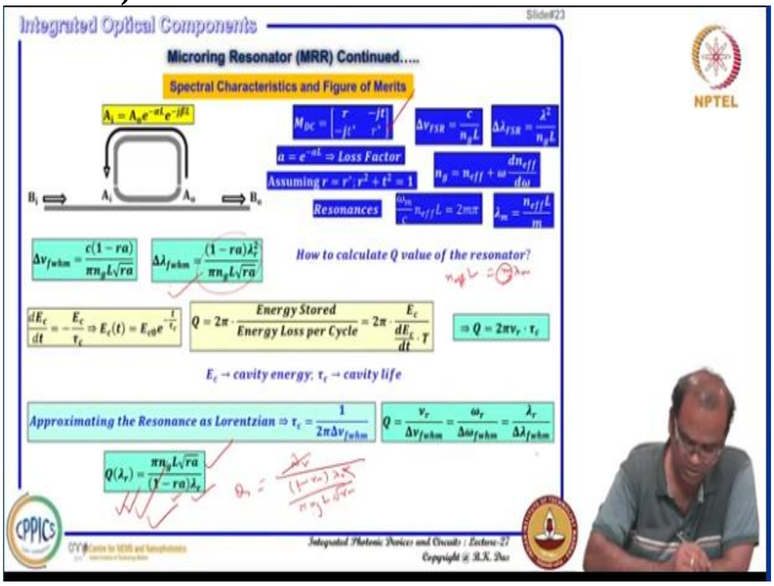

The Q value this one is the Q lambda r divided by delta lambda r if you are writing this one you are just writing this one lambda r this will be denominator and lambda r will be in the numerator. So, lambda r this one divided by this one, so I written like this lambda r divided by 1 - ra lambda r square divided by pi n g l ra. So that is actually your Q value, so then lambda r 1 lambda will be cancelled denominator 1 / lambda r 1 - ra written and pi n g L root over ra that will be going to numerator, so Q value we have expressed.

So, here n g that is the waveguide parameter we know, L perimeter length we know, r that means the directional coupler characteristics I know how much self-coupling coefficient is the round trip loss I know and lambda r I am considering the lambda r which resonance wavelength I can find out that n effective L should be equal to m lambda m. So, mth order if you I know which order of resonance I am considering and then resonance wavelength to be known.

Then I can find out Q value, at a particular wavelength what is the Q value I can express very clearly you see Q value is proportional to length and inversely proportional to lambda r you can express this Q.

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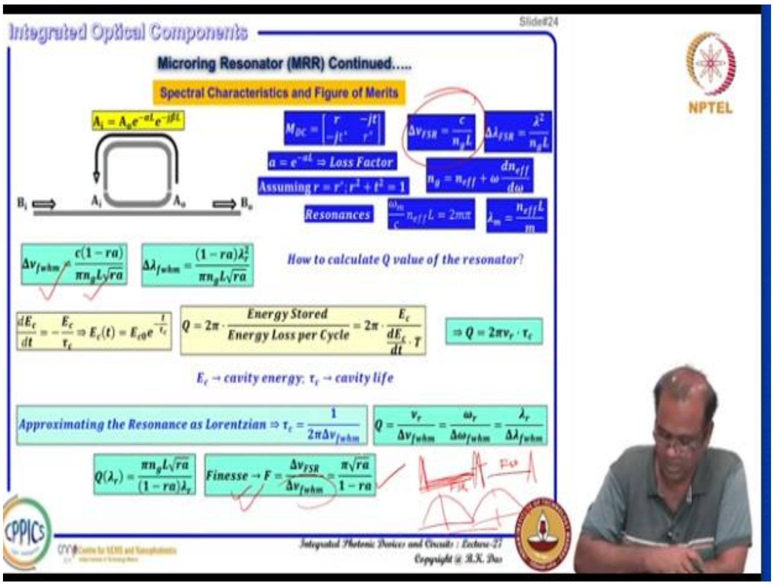

Another one figure of merit called Finesse, finesse is you have a resonance wavelength here and resonance wavelength here, this is the resonance wavelength this is called a FSR free spectral range this is called another FSR and here you can get the 3dB bandwidth. Now, this FSR and 3dB bandwidth if you take a ratio that is called finesse. Finesse actually gives how good you can resolve the resonance wavelength that means if one resonance wavelength another resonance wavelength this is the separation.

And if you see that, if the separation is like this, if the resonances are like this, that means this is your FWHM, this is your FWHM and this is your FSR and this FSR is exactly equal to FWHM then your finesse will be 1 that is a very bad resonator. So, finesse is high it means delta nu FSR, FSR will be larger compared to the full width half maxima that is another figure of merits called as finesse if and we know delta nu FSR this expression and delta nu FSR here, this delta nu FSR and delta nu FWHM here take ratio you get this expression.

So, if you know the waveguide parameters, ring parameters, ring resonator parameters and you can design r value you can design what is the you can you know what is your waveguide loss bending radius you can control a value, then you can get the finesse.

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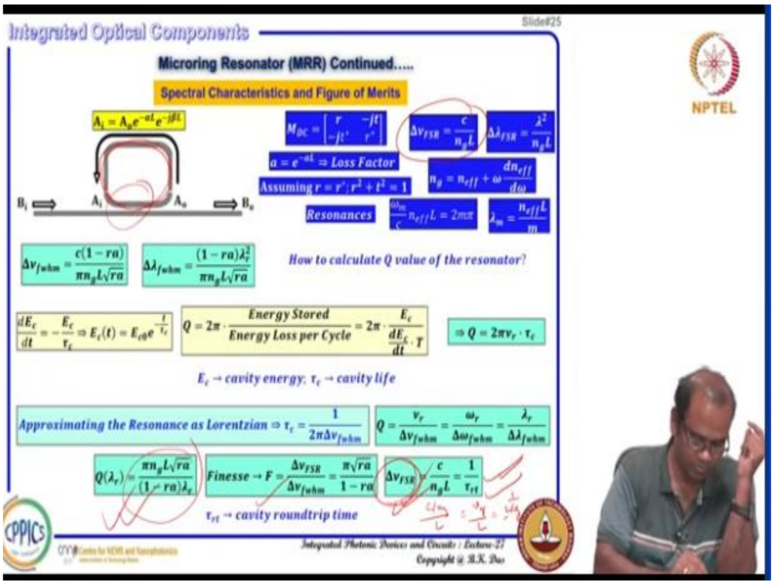

Then one more parameter is called if you see delta nu FSR if you just see one important thing c / n g L if you just consider then you get 1 over tau rt, what is tau rt? Just express this one, this c / n g / L equal to group velocity by L, c is the velocity of light, n g is the group index so c / n g L is equal to v g group velocity / L. So, in that case, you can write 1 over L / v g, L is what? L is the total perimeter of the ring divided by v g that means, group velocity how the information traveling how the data traveling inside the ring that is the velocity.

So, L / v g means total perimeter divided by velocity that means, how much time it takes per round trip travel that round trip travel we can write 1 / tau rt. So, once we know the spectral range of a ring resonator then I can find a data signal how much it will take time to get a complete round trip inside the ring. So, this is another important information we need to know for a ring resonator when we will be designing when will be using this type of ring resonator to design different type of delay lines, filter characteristics that time it will be helpful.

So, these are the figure of merits finesse, quality factor and FSR, for a particular design particular applications, we can look for these 3 figure of merits and depending on these 3 figure of merits we can actually use them, suppose certain application I need Q value is very high. So, we need to design accordingly L we can expect r value, what is the r value require what is the a value required.

We can assume that the n g group index that depends on the single mode waveguide design and what is the group velocity, what is the core material refractive index, what is the cladding refractive index is all those depends on the n g that can be considered as fixed. So, now we can design whatever the value of r, whatever the value of a r and a basically that means the DC directional coupler how you are designing and a what is the loss in the waveguide that actually responsible for storing more energy or less energy.

But this r and t they do not influence free spectral range, free spectral range is c / n g L that depends alone on group index as well as length. So, one resonance to another resonance in frequency domain what is the separation c / n g L harch you can find out. So, this figure of merits it is very important.

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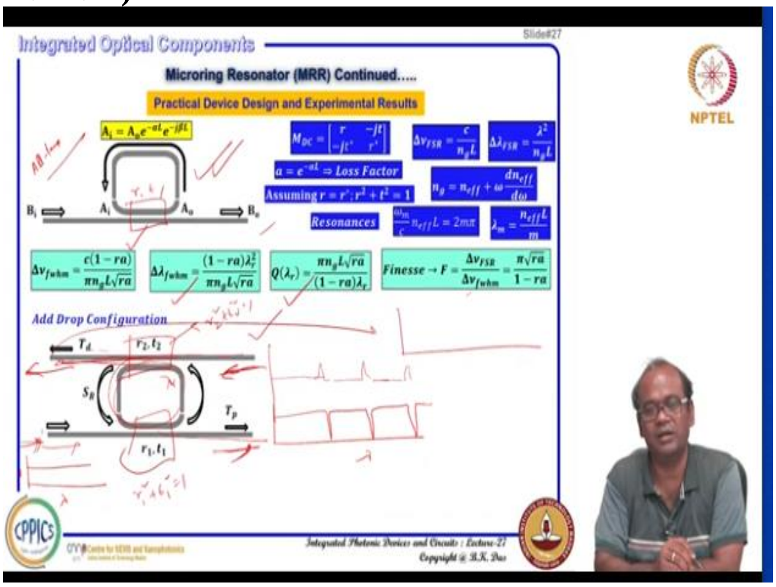

Now let us move on practical device design and experimental designs, so we know that what are the important parameter for a microring resonators and how to design that and these are the figure of merit one is the full width half maximum frequency domain in wavelength domain and Q value at a particular resonance wavelength lambda r and for a given perimeter I know finesse also I have derived starting from the your normal transfer metrics mechanisms.

Only thing is that we ensure that these value these transfer metrics are known, r and t is known, a as also known and L is known these are known parameters once you know those parameters then you can quantify what are these values. Now one more particular device you can think of this is what do you call that this bus waveguide and the ring, bus waveguide couples 1 is to 1 1 input 1 output it is basically a directional coupler one of the output you are just leaving a positive feedback and you are getting like oscillator resonator.

Electronically no positive feedback gives you as a electron oscillator like a (())(57:37) you can use as your oscillator if you give a positive feedback. So, you can have another type of device where you have one bus waveguide where you are launching light, light couples a resonance wavelength can store here if you bring another one bus waveguide in the top then the resonance wavelength who just stored here that can be tapped through these bus waveguide again by another coupling parameter so called r 2 t 2.

This one can be considered as these DC can be characterized with r 1 and t 1, so that r 1 square + t 1 square you can consider one if they are lossless this can be considered r 2 square + t 2 square that is 2 coupling points, I am launching here what happens the resonance wavelength will store here that will be missing here. But when it is stored; if you see that another waveguide is coupled neighbouring bus waveguide then that resonance wavelength will be dropped here.

So, if I have broadband lambda I am launching here in this here what I will see? Here I will see that this is lambda a certain wavelength which is resonance that will be missing in the transmission, maybe regularly to be FSR will be missing here, but the wavelength which one missed here that is stored inside the ring, but that stored energy can be taken out a bit here. So, in here in this output what you will see? You will see exactly here I can plot.

So, I can get something at this point maybe you can get a little bit high and here you can get a little bit high and here you can get little bit here. So, you can get that wavelength function. So, this is a very important device in add drop configuration resonance. Add drop configurations will see that this type of device not only used for a wavelength dropping add drop configuration just a particular channel you can drop not only that you can use this one as a so called modulator multiplexer demultiplexer.

And normally nowadays all the on chip transceiver circuit silicon photonics transceiver circuit they are actually playing they are actually designed with microring resonators. So, we will discuss that in future but at this point of time we will just try to understand this type of device from the knowledge of this simple microring resonator which is actually called all pass configuration when one bus waveguide during that is called all pass configuration that means everything passing through this bus waveguide only that is why it is called all pass.

So, whatever thing that either it is storing or it is passing here that is why it is called all pass but in this case everything actually it is not passed you have the drop port something you can drop here. So, this is called all pass, this is called add drop, why it is add it can drop add means if you again that whatever the wavelength you are dropping here that wavelength if you launch here that will again coupled to this ring and you can couple with here so you can add also.

So, I have one input port another second input port, one output port another output port, so 2 input ports, 2 output ports that is how add drop configuration with design. So, in this case how we can define what would be the bandwidth full width half maximum at resonance in terms of lambda, in terms of Q value what would be the Q value whether it will be increased or decreased finesse how we can modify this, one so remember that in this case you have only one r 1 t 1 and no other things are there. But here in this case you have additional things how to deal with it?

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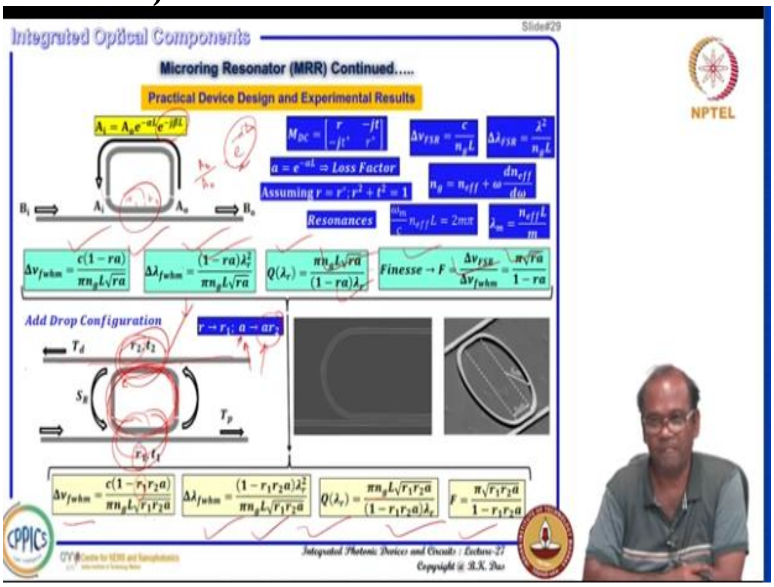

So, here are some pictures if say this is something bus waveguide here, this one this is your bus waveguide this fabricated device in all lab actually, so you see this is a restack waveguide something like this it is designed and these 2 waveguides are coupled so light you launch here to couple and one half it is shown here because you could zoom and see the waveguide things and another add drop configuration we have taken from a literature this is a 3D view a little bit tilted someplace little bit tilted and scanning electron microscopy image.

If you see this is a 5 micron bending radius and little bit of back force and restack design it escaped so that you can couple light and couple out here. So, these are the design but that is the fabricator device how it will be it is looking like after fabrication and for different types of applications. But our intention is now to find out equivalent formula for delta nu FWHM, delta lambda FWHM, Q value and finesse when the ring resonator region add drop configuration.

So, it is very easy, from here you can directly get derive the expression for add drop configuration, how it is? Let us try to see. What it will do earlier you have r 1 and t 1 here that is why you are getting everywhere r 1 r 1 and A you are getting but a is what? A is round trip loss. So, if it is round trip loss is A that means if I just consider A i / A o = e to the power - alpha L if I just forget about the phase that much is the round trip loss. So, a is that means ratio A i to A o ratio is the a, a factor I will be getting that that is a.

Now that is transmission coefficient transmission whatever transmission through this total ring, now in this transmission path, you have additional coupler is there where the transmission this will give you r 2 that means, you used to get a now instead of a you will be getting ar 2 because you have another coupler, your loss factor is not anymore, your loss factor is converted into ar 2 because of the presence of the second waveguide r 2 is coming up path that means maybe 90% is coming back 10% maybe going there it can happen.

So, that means I earlier I was getting 99% now, not 99% even I am losing something here, that is why nu ar will be ar 2 what was the original a and r 2 and original a instead of r I will have an r 1. So, these expressions you can directly use for add drop multiplexer just putting r = r 1 and a = ar 2. Wherever a is there, you just put ar 2 and wherever r is there you put r 1 then you can convert all this expression for actually add drop configuration you see what delta nu neper c 1 - ra r I am putting r 1 a I am putting r 2 a and ra r 1 and a = r 2 a.

Similarly delta lambda 1 - ra r means r 1 a means r 2 a lambda and this pi n g L r 1 r 2 a, Q value you see Q lambda r = pi n g L pi n g L is there and square root of ra r 1 r 2 a and ra r 1 r 2 a. So, just you put r 1 because originally it was r here that is why all these coming r now that place we have changed to r 1 t 1. So, r will be converted into r 1 and a because of the presence of second one a is numerator you have it is modified to here too. So, you just modify that one then you get the direct equation.

So, whatever once you solve you could derive we have derived already how to get all these expressions figure of merits just to convert just to use them for add drop multiplexer of course you can find easily. So that means only you need to design your directional coupler here for a desire r 2 t 2 for desire r 1 t 1 and perimeter you design for a particular resonance wavelength you want to store or you want to drop then it is fine absolutely fine.

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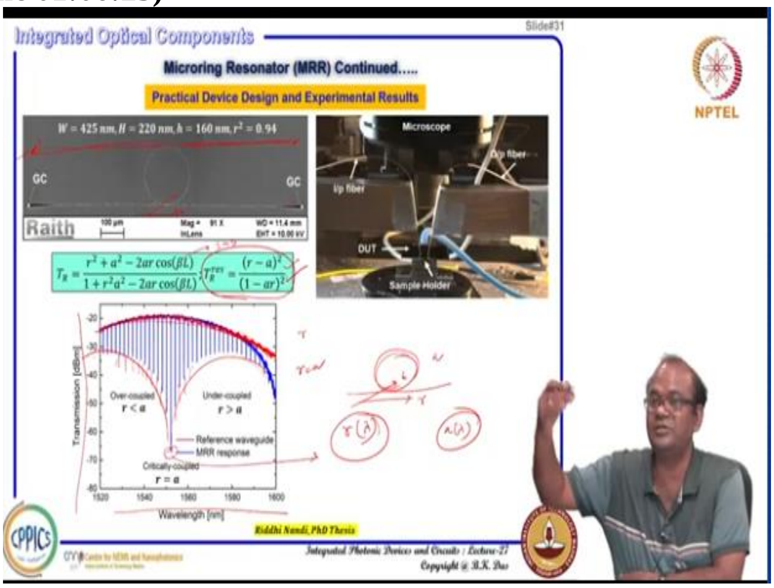

Now, we want to show some experimental results as I promised, so you see here is so one ring resonator, this is the ring resonator it is about it is 100 micrometre, you see this is the scale same image it has been shown input grating coupler this is grating coupler we have discussed earlier getting coupler is used just to couple light into the waveguide. So, you can bring fiber here if you launch here light will pass through the bus waveguide and you can take here

And this is the ring fabricated where you can expect some stored things energy will be stored power restored at resonance, and that transmission characteristics we know how it will be looking like at resonance and what is the transmission this is all pass configuration 1 input 1 output everything will be passed through that output only that is why it is all pass configuration. So, this is called all pass configuration other one is add drop configuration.

And here how we characterized you see this is the fiber coming you see here, this is the fiber coming and this is your sample and fiber is actually vertically about 10 degree, this grating is defined vertical coupling purpose 10 degree inclination it is done. So, the grating coupler is designed we have learned earlier already how the grating coupler need to be designed, fibre is coming and you are taking out sample is this one. So, this is your characteristics result.

You see we have shown here 2 curve you see one red curve, red curve is one more additional waveguide structure is there as a reference where ring is not there, just you will launch grating coupler and grating coupler, just to see what is the wavelength dependency in coupling grating coupler and what is the loss in state waveguide you can find out. So, you could just normalize with a certain power then you see that this you get this red curve here.

Red curve means wavelength, this is your wavelength, wavelength dependent transmission you are seeing here in dBm scale. So, whatever transmission you are getting you convert into milli watt and then you take log 10 and log to the base 10 and convert milli watt and to take log value that is actually dBm. So, you get this one and in the state waveguide you do not see any resonances.

But when you are launching here, you see, it is polling same path, but at the same time, some wavelengths if you see those are resonance wavelength, those are missing in the output and they are having different extinction also. They are extinguished for different value, you see some resonances are extinguished very large up to 65 dBm. And some of them very little, what is the reason because at the resonance wavelength you see the transmission is equal to r - a square divided by 1 - ar square that means beta L = 2 m pi you just put 2 m pi then you get this result,

But if you see r is what it is the self-coupling coefficient r and a is the round trip loss. So, carefully we will look if somehow at some wavelength if r = a transmission at resonance will be 0. So, when r = a transmission output here at resonance wavelength will be getting absolutely minimum you see all the resonance spectrum here which is missing in the transmission a particular wavelength you see resonance wavelength it is minimum at the output. So that is happening because that is where you are getting r = a.

So that particular condition we can call it as a critically coupled, so when you have a ring resonator and you have a bus waveguide and this is going r and this is going your t now, if r value is exactly equal to the round trip loss factor, then your transmission throughput happens to be 0. Here still you are getting a little bit that may be noise level, so you can get 0 that is actually critical coupling. And right side if you go that means this r value somehow it is greater than the round trip loss and that is why your extinction is dropping.

So, r is greater than a your transmission will be reducing you are increasing transmission will be increasing and r less than a because it is a square value that will also be reducing. So that means I can say that this r is a function of lambda a can be a function of lambda but most of the time this round trip loss waveguide loss is not a function of lambda rather this r this coupling coefficient of the directional coupler that is actually function of wavelength that is the region as a function of wavelength.

You see extinction is different for all the resonance wavelength, so that is actually following this one. If you can design your directional coupler is actually wavelength independent directional coupler, then you would see all the resonances would be, extinction would be at the same level.

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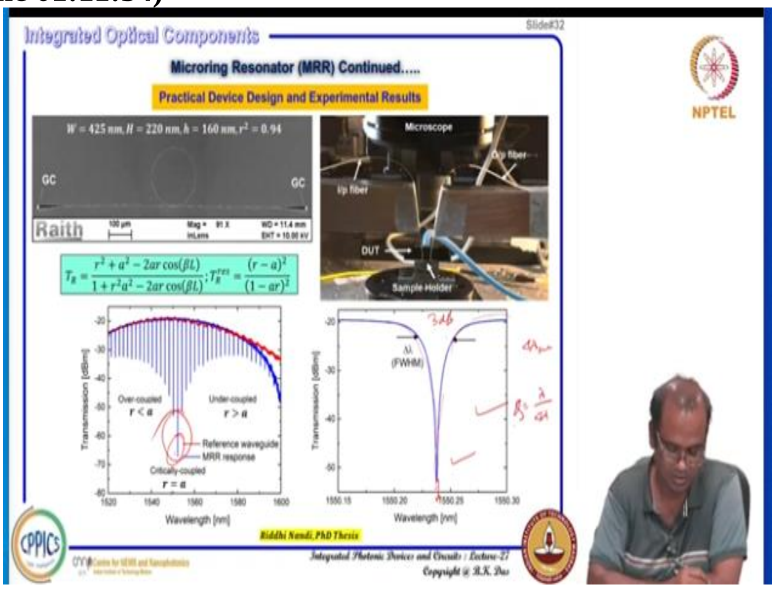

So, if you just little bit expand this one, one of the resonance if you see exactly at the critical wavelength around 1550 something like that one resonance you just pick you zoom into a resonance say with one of these resonance wavelength to zoom in. Then you see how it is coming down and then going up and you can find out maximum to minimum, 3 dB down. Then you can get 3 dB bandwidth experimentally 3 dB bandwidth in all pass configurations. Once you know 3 dB bandwidth; you know delta lambda FWHM and you know the resonance then you can find the Q value. Q values that lambda / delta lambda that is what we have seen earlier.

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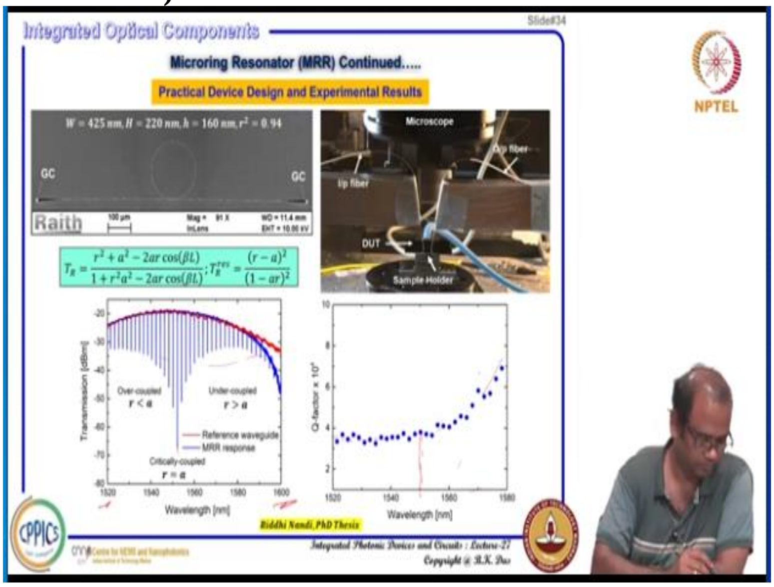

Now, we can actually find that I can calculate a FSR that means free spectral range between successive that means wavelength separation between successive resonances. So, I know that this free spectral range is lambda square / n g L perimeter length, if I experimentally get a FSR and I know L and I know resonance of length then I can find out group index. So that is how you see as a function of wavelength I can find group index is also changing that means, n g is also a function of lambda.

Group index of the waveguide that is also a function of lambda that can be also you can extract. How is the waveguide you will design whether that is a group index is also dispersive or not you can find out and from group index also you can model what is the effective index experimental you can extract what is the effective index. So, you just as a function of lambda you just calculate a FSR that means separation between 2 resonance wavelengths that is actually free spectral range that is actually FSR.

This plots all you calculated and once you know FSR you can find out the energy, so group index as well as free spectral range you can extract from the experimental result.

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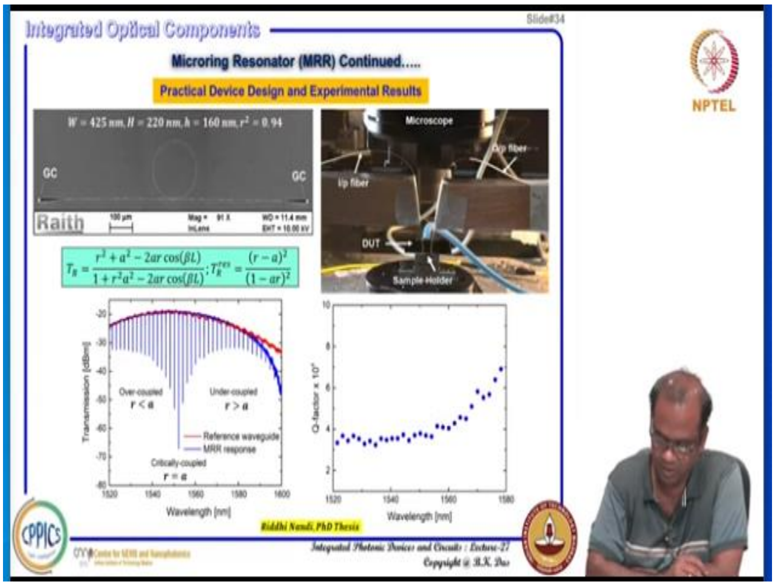

So, then once you know all these are everything 3d| bandwidth and lambda you can just see what is the quality factor, all the resonances you can find out the quality factor actually, you see towards the 1520 to 1600 nanometre we have the data and 15 initially Q value at low, low means it is about 3 into 10 to the power 4 30,000 and so on. And then as you go for longer wavelength in this region, so this is about this around critical wavelength.

So, around critical wavelength Q value is about 30,000, as you go then you can go up to 80,000. So, some of the ring resonator we have developed that is actually Q value is more than 10 to the power 5 means one lakh also. So, this is how the ring resonator we can design we can get whatever the design FSR, we can actually design whatever the Q value required, Q value actually highly dependent on what is the waveguide loss, what is the perimeter length, the volume of the cavity etcetera.

And depending on that we can think of various types of applications. And this ring resonator actually is heavily used for photonic integrated circuit to a devices that is Mach-Zehnder interferometer and ring regenerator they are heavily used and both of them are actually designed with a directional coupler. So, we have discussed this microring resonator, Mach-Zehnder interferometer and maybe probably one more lecture I will spend to understand distributed bag reflector that is also another important device for photonic integrated circuit.

Then we can go for more complex design of different type of functions like a reconfigurable filter and configurable delay lines all those types of things how can be designed using these fundamental bulling blocks that will be discussed, with this I will stop today. Thank you very much.