Integrated Photonics Devices and Circuits
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Lecture – 25
Integrated Optical Components: Directional Coupler: Design and Modelling

Hi everyone, we continue our discussion on some of integrated optical components continuing also the same thing directional coupler. Today will be discussing about some design aspects and how we can go for modelling the structure directional coupler, so that it can be useful for circuit simulation. So, first thing what I will do you know we have derived coupled equation for a coupled waveguides differential equation and we know how to solve analytically with some approximation.

And we found that this kappa is the major factor which you need to know for a coupled waveguide structure so, that we can use those analytical solutions. So, we will discuss today how one can extract this coupling coefficient from some numerical calculations. So, that you can use an analytical formula and then we will be discussing about some kind of design of practical directional couplers.

So, you have to go for designing such that it can be fabricated compatible to the technology and also you should get some your desired specifications you want. So, anything you design that should be possible to fabricate in the foundry or in the fabricate in the lab, laboratory university labs etcetera you should be able to fabricate. So, third think I will be just discussing I will just briefly I will be showcasing some device architectures that can be demonstrated that can be designed using a directional coupler using one directional coupler.

Or combination of directional couplers. So, that is why it is DCs, DC means a directional coupler here we are meaning.

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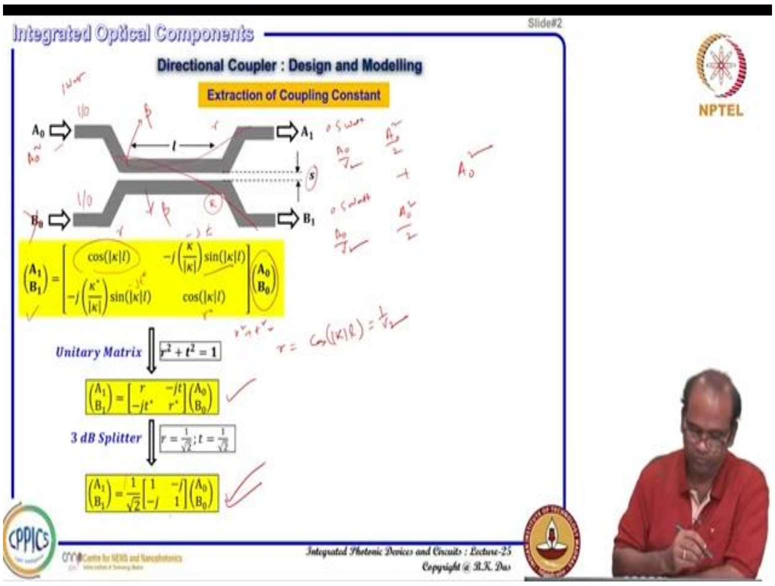

So, let us start with a simple structure that we have this is your directional coupler, this is input waveguide, input output you can consider this one. One optical input and optical output here we can consider and we can just launch from these 2 ports A naught B naught and then in the output we can get A 1 B 1. And we maintain a distance between parallel coupled waveguides in the central region of length l with a separation of s.

So, in that case we know the transfer matrix form we can write. If we are launching with amplitude a mode with amplitude A naught and B naught are the inputs and then the output A 1 B 1 we can find out in case of identical waveguide structure both these waveguides are identical if this is beta also this is also propagation constant beta is giving then waveguide with the dimension everything they all respect they are identical.

Then we can simplify our transfer function with cos kappa l that is kappa whatever coupling constant we are talking about. And then this is actually another thing which is cross coupling part and normally we call this one as your r we represent this one as r and this one is t and then this will be minus jt, this will be minus jt and this would be minus jt star and this would be r star. So, the unitary matrix comes like the determinant of this matrix will become r square + t square.

And normally if it is lossless normally you know this reflection and transmission. So, anything going back we call this the reflection and this coming transmission sometimes it is mention like this is r and this is K some textbook you will find that whatever the value it is cross coupling that is actually K value we are considering. So, if we just represent this one r and this one t then we can reduce in a very simple transfer matrix of a directional coupler r - jt - jt star r star. So, that is what we have already discussed in the previous lecture.

Now, if we want to design this directional coupler so, that you get a 3 dB coupler that means, if you are launching here 1 watt then you would be getting 0.5 watt here and 0.5 watt here then it is called 3 dB power splitter 50 50 splitting is 3 dB power splitter. So, in that case we have to design kappa and l such that this r, r equal to so called cosine kappa l that if it is somehow giving you 1 / root 2 then what happened?

We can get here A 1 also you will be getting something like that whatever A naught it is if you are not launching here anything if A naught is launched then A 1 will be A naught / root 2 and B 1 will be also A naught / root 2. So, in that case total power will be here A naught square / 2 and A naught square / 2 if you add them then it will be A naught square which is your input. So, then it is the accordingly if you design your transfer matrix that is r = 1 / root 2, t = 1 / root 2, 1 / root 2 if you take a factor then you can represent 3 dB power splitter using a directional coupler this would be your transfer matrix.

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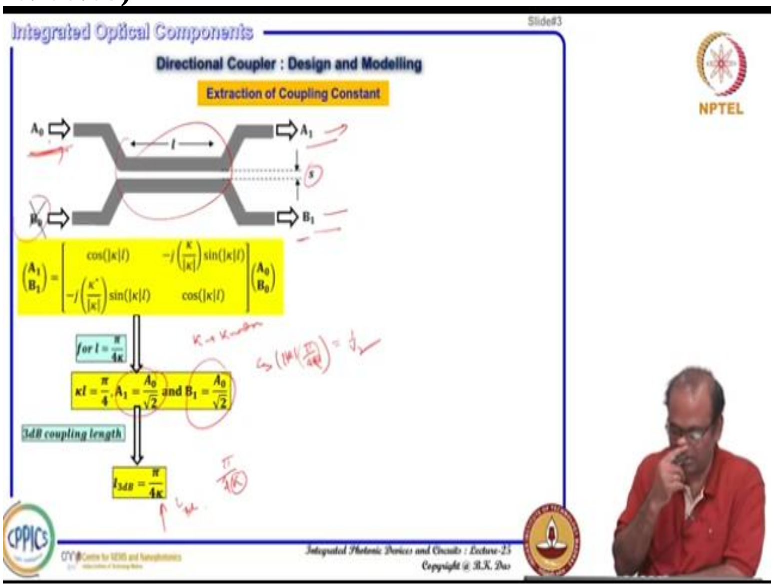

So, now let us clean up things for example, we can write this suppose for a given directional coupler, the separation everything is there and some constant s value is there and if they are maintaining a certain distance and the uniform that means kappa should be a parameter for this entire structure that is a constant that is a specification figure of merit of the structure. So, only thing is that if I maintain that waveguide dimensions then just controlling the length alone.

You can control how much power you are launching in the cross port if you are launching here let us consider B naught is nothing is there. So, how much power you will be getting in the bar port and this will be the cross port, B 1 is the cross port and with respect to this input, this is your bar port and with respect to this will be your cross port if you are launching here with respect to this one this will be your bar port and this would be your cross port that is vice versa.

So, if kappa is known and fixed some kappa value known some way we need to extract that value of course, if kappa known then I can just find a value of l length exactly pi / 4 kappa. So, if l = pi / 4 kappa that means cos kappa times pi / 4 kappa this is your l value. So, this kappa, kappa cancels then pi / 4, cos pi / 4 it will give 1 / root 2. So, that is how 1 / root 2 you will be getting A 1 will be A naught / root 2, B 1 will be B naught / root 2 that is what we have discussed.

So, I can say that if a directional coupler some kappa value is known given to you then you can easily model that if I need 3 dB power splitting then my length will be you just exactly pi / 4 kappa. You know dimension of kappa is per micrometre, per millimetre, per meter or whatever. So, accordingly one 3 dB you can find out. So, kappa value depends on higher the kappa value you can make compact 3 dB power splitter. Stronger kappa value is welcome to design a compact footprint smaller device structure.

But stronger kappa can cause a lot of other issues also it can introduce some kind of radiation scattering losses all those types of things will be there. So, you have to be careful that while increasing kappa you should not add some additional losses in the structure because of the structural non uniformity and so on.

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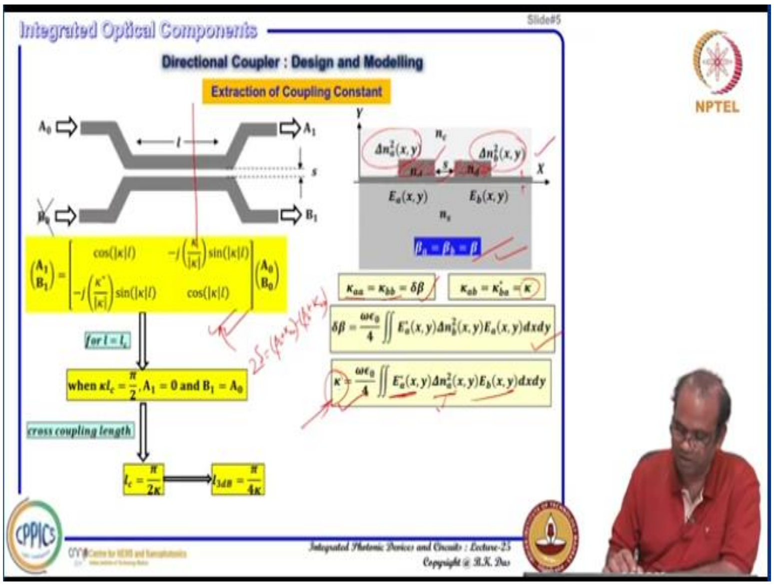

Now we know given kappa I can design length for 3 dB coupling. Now let us come back to our original cross section structure suppose if you take a cross section here this is top view cross section here you have one waveguide and other waveguide, the separation is s given here and you know that is this delta n b square is actually presents of second waveguide delta n a square this presents of the first waveguide and this is your slab height.

So, if you just consider n s and then slab and then n c together it is a background and only additional part of this one actually giving you waveguide b and this one gives you waveguide a. If you are missing if you are removing that one that means you have only waveguide b and if you are removing this one then you have waveguide a. So, delta n a square delta n b square just that additional in the total n b’s, additional refractive index additional material property you want to include so that you can get some kind of waveguide structure there.

And we just we have assumed that this type of transfer matrix we have derived when we consider that beta a = beta b = beta that means identical waveguide structure. If it is non-identical then we need some additional term would come here that is actually delta. Delta = beta a + kappa aa – beta b + kappa bb we have derived earlier this will be 2 delta. Now since it is identical we have in the coupled equation you remember that we have introduced kappa aa.

That is should be kappa bb and that will be actually delta beta at that expression will be like this that is derived using coupled equation in the last lecture and kappa ab kappa ba star that is actually kappa that actual contributes the coupling constant so called thing so, one thing is clear that this kappa expression that once you know dominant electrical profile waveguide a and waveguide b and what is the additional delta n b square if you are n a square or n b square you use here.

Then if you integrate then you will be able to get kappa you can just simply calculate your kappa value by solving the more profiled, more field distribution independently of individual waveguides and if you just introduce and then integration that is one way of calculating kappa. However, you know sometimes each time different type of structure you may not need all the time, this type of calculation, this type of solution it would be very difficult time to time. So, we need some kind of alternative approach to extract kappa value with our knowledge so far we discussed based on that we following.

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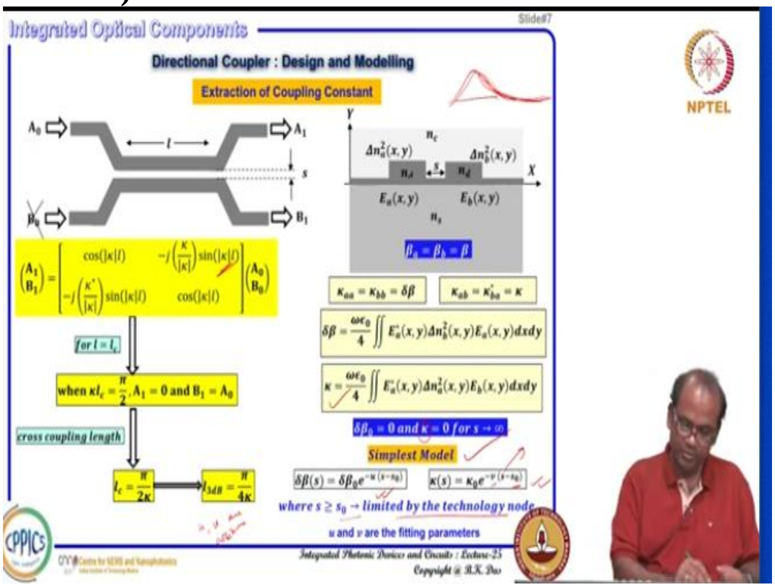

Now suppose delta beta naught = 0 and kappa will be equal to 0 if you just look into this one this expression or look into this expression as long as their single mode waveguide electric field distribution for the first waveguide, electrical distribution of the first waveguide delta n b square that is additionally refractive index whatever in the second waveguide that is your delta beta when that will be 0.

Obviously, when if you just have your structure such that one of them will become 0 at some regions or if you integrate everywhere either this one or this one or this one entire xy plane you will find one of these 3 will become 0 entire plane then this delta beta will be 0. And that is only possible when s tends to infinity suppose they are at least infinity means practically it is a very large distance if they are separated with a sufficiently larger distance is maybe called silicon waveguide it can be several micrometres is sufficient.

Because waveguide dimension are in some micron level so, when s tends to infinity then we can say that delta beta naught will be equal to 0 that means, this propagation constant modification of the first waveguide or second waveguide due to the expression of the first waveguide that will not be that much significant. Similarly, when s is large then we can say that this kappa coupling strength will be 0 only when s is equal to large.

Because, you will get everywhere because if it is a large distance E a will be 0 or E b will be 0 or delta n a square will be 0, if they have some value delta n a square will be 0. So, in that way you can say that you can ensure that for a large distance one value we are ensuring that both delta beta and kappa will be 0 and in that case we can say that they are decoupled completely and they maintain their orthogonality condition individually. So, that is what we learned so far.

Now, we will just try to make a simplest model if you see normally you know electric field distribution if you just say something this direction if you just plot fundamental mode, it will be almost a Gaussian and if you see that almost it is Gaussian at infinity will be 0. So, just following that the field will be dropping like one edge if you see it is almost similar to like exponential term.

So, for our compact modelling purpose for the directional coupler we can say that delta beta and kappa they must be depends on s as you increase the obviously you know when s tends to infinity kappa value also 0 delta beta will be 0, as you decrease your kappa s value then kappa will start growing and delta beta also will start growing. So, in that sense, we can model with the analytical expression you can say that empirical formula we can consider delta beta s, s mean space dependent waveguides are identical.

We are maintaining that cross sectional refractive index profile, only there s lateral in the x axis we are just changing if we change then we can say that now that u some constant s - s naught and kappa will vary kappa naught e to the power v, u v are positive constants, u v are some positive constants we are considering positive constants. And s naught we maintain that you know it is clear that smaller the s coupling will be more but how small you can make that depends on your technology.

Because 2 feature all our CMOS technology whatever device structure you define you make you can fabricate using your CMOS foundary that depends on your technology node. Suppose for example, if you are using technology in order 180 nanometre then probably this feature this gets separation you can maintain with a great yield good yield that would be in the order of 150 nanometres.

So, if you are go for it if you are going for advanced technology like 45 nanometre even for 25 nanometre technology CMOS technology then probably this gap can be reduced further. So, we mentioned that depending on the technology node s naught is fixed so, that is the minimum value of s naught is fixed. So, we can define that what would be the maximum delta beta or kappa you can acquire.

Actually speaking this delta beta has no meaning as such for modelling a directional coupler kappa is very important because kappa depending on the kappa you can define what is the length required for 3d power splitter or complete postulating. So, I can model like that as s increasing from s - s naught then this kappa naught will be decreasing exponentially with some kind of constant v beta similarly, some constant u s – s naught.

So, when s = s naught then obviously delta beta will be delta beta naught, kappa will be kappa naught. So, that means at s naught I know what is the constant value what can be the kappa value I can think of I can calculate and then I can plot that way so, we need to know for a given s naught technological things. So, once you know that kappa naught then you can just model if you just increase s then how it will be varied obviously, you need to find some fitting parameter even v. So, s naught is only limited by technology node that is what we can model.

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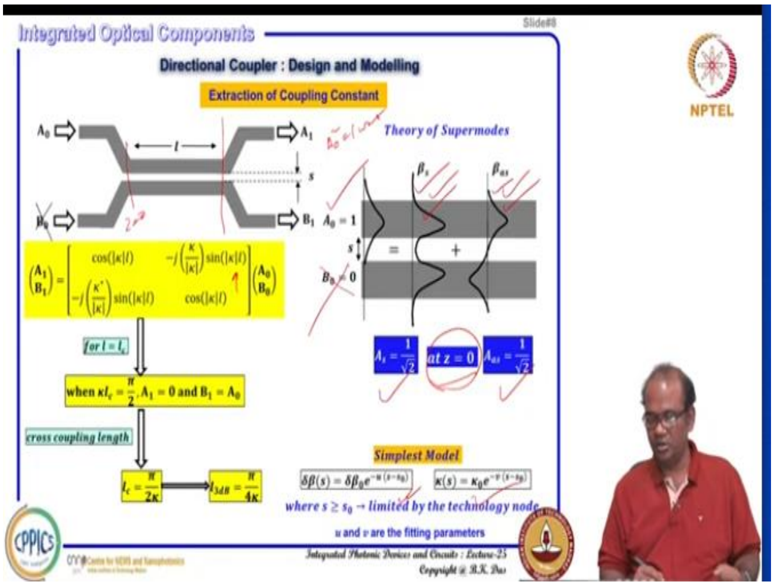

So, now you will see normally coupled to waveguide we have seen that there are 2 different types of solutions we can get symmetric solution and asymmetric solution. So, when z = 0 in the propagation direction when coupled start from here this is actually z = 0 for example, and this is actually z = l dc or whatever you can say. At z = z naught if you are not launching anything in the second waveguide only first waveguide you are launching A naught = 1 meaning A naught square you can consider that is actually 1 watt are so, just normalized n = 1.

Then we know that the symmetric mode and asymmetric mode they are amplitude will be CR / 1 over root 2, 1 over root 2 at z = naught that means 50% of power will go to symmetric mode and 50% power we go to asymmetric mode and symmetric and asymmetric solutions we have discussed also in the last lecture. So, our intention is that how to find out kappa and so that we can model like this at least we can find u and v and s naught.

So, that we can use this formula for any circuit simulation we will be able to know if s is reduce whether the performance is increasing or decreasing just with a single parameter you can think of the directional coupler for a given length or maybe length also of course, length will come here actually. So, this is symmetric and asymmetric how they will be coupled that is it is soon.

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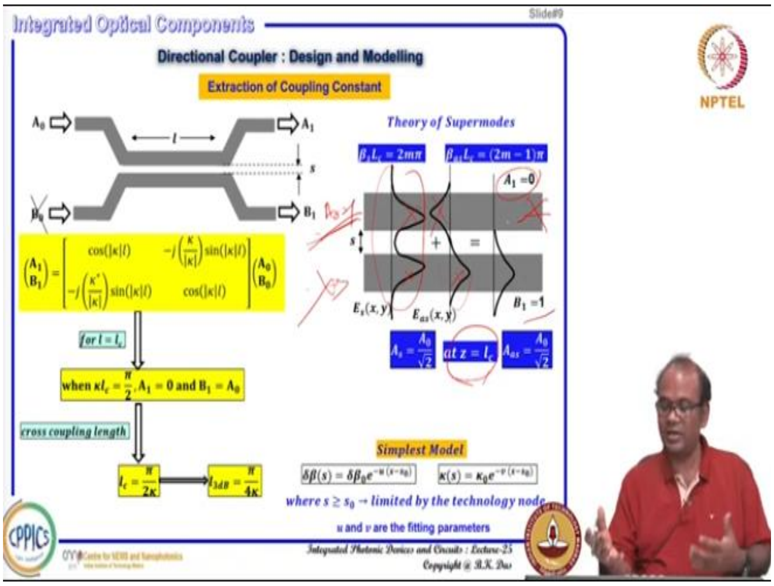

And when at z = l c where we want the length that after traveling power launched in here A naught = 1 already z = z naught and here you have done nothing is launched here. Then at z = l c I would see that this symmetric mode will travel with a pairs of 2 pi times integer value m. And suppose at that particular length = l c the asymmetric mode that will acquire phase of 2m - 1 pi, pi is less that means they are phase difference is exactly pi after travelling l c length.

So, in that case these one will be flipped here and if you take a superposition this negative sign, this positive sign this in the waveguide A you will see nothing and symmetric positive anti-symmetric also positive is second waveguide do you see the maximum. So, in that case if I launch A naught = 1 here I will get B naught B 1 = 1 A 1 will be equal to 0 here a nothing will be there. So that means complete cross coupling takes place we can explain that just considering symmetric and anti-symmetric supermodes and these supermodes can be actually solved.

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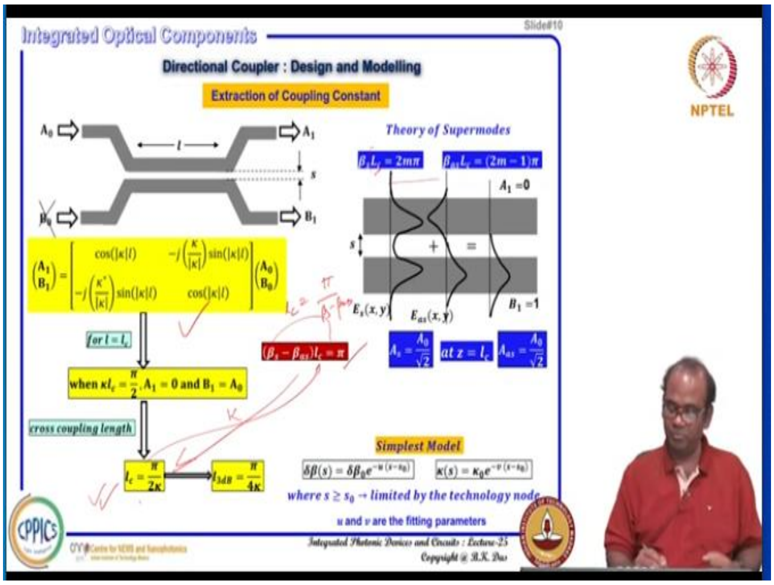

So, now I will do a trick actually you see I can say that beta s - beta as times l c that means, beta s l c multiply and beta as l c multiply if you subtract that means, after traveling this l c is smaller l c I have written here a L C here. So, they are same basically so, if you subtract for that length then you would get their phase difference is pi and that is the length you need for cross coupling.

Now, in another way from the transfer matrix we know that l c is required pi / 2 kappa, pi / 2 kappa is l c that is what we have discussed earlier that is why I am carrying forward just to use now, here if we just use l c what happened then I can express kappa in terms of beta s and beta as assuming that this symmetric and anti-symmetric mode solutions you can easily calculate we just give a more solver.

And then you will be getting 2 modes and you can find what is their Eigen values properties and constant for symmetric and anti-symmetric mode. Once you know that symmetric, anti-symmetric mode using these 2 equations you can find kappa value. Here l c is there, here l c = pi / beta s - beat as. So, these are these if you just make equal then I can get kappa equal to say whatever the value comes I have just written here.

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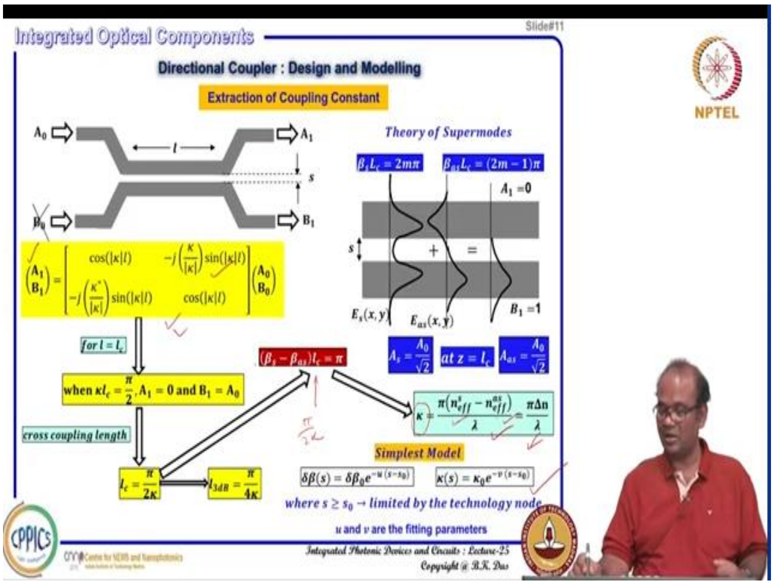

Just use this one then kappa will be you just compare this to use this l c here and then this value you know beta = 2pi / lambda n effective s beta s = 2 pi / lambda n effective asymmetric and this 2 because you have 2 here 2 kappa l c are putting here pi / 2 kappa and whatever 2 comes 2 2 cancel. So, pi pi both sides will cancel also then you will be getting kappa value is this one.

So, where kappa value can be expressed in terms of if you know the effective index of the symmetric and asymmetric mode obviously symmetric mode is a fundamental mode for the combined structure, asymmetric mode is the first order mode for the combined structure they can be easily solved using a mode solver commercially available or you can develop yourself as discussed in our earlier lectures.

So, once you get this thing then you can find this kappa value, this kappa value can be easily fixed with this expression. So, that; this expression can be useful for your analytical or for your compact model. So, here you see basically you know the effective refractive index change between difference between symmetric and anti-symmetric mode then if you know what is the operating wavelength support 50 and 50 nanometre and if you know delta and then you know what is the value of kappa.

Once you know kappa value then you can use depending on the length you can use this transfer matrix then you can find what is the ratio power or amplitude in the cross sport and bar port A 1 is in the bar port or B 1 is the cross port, so we can find out. So, main thing now is that this kappa instead of solving numerical things where delta n b is there you have to define this a little bit of program you have to do instead of that you ask your more solver give me the solution for this structure.

What is the symmetry mode, anti-asymmetric field distribution and mainly I am interested to know what is their Eigenvalue? That means beta s value and beta as value once we know that I can extract for a given lambda what is the effective index and then what is the kappa value. So, this is the one easiest way we can find we can use.

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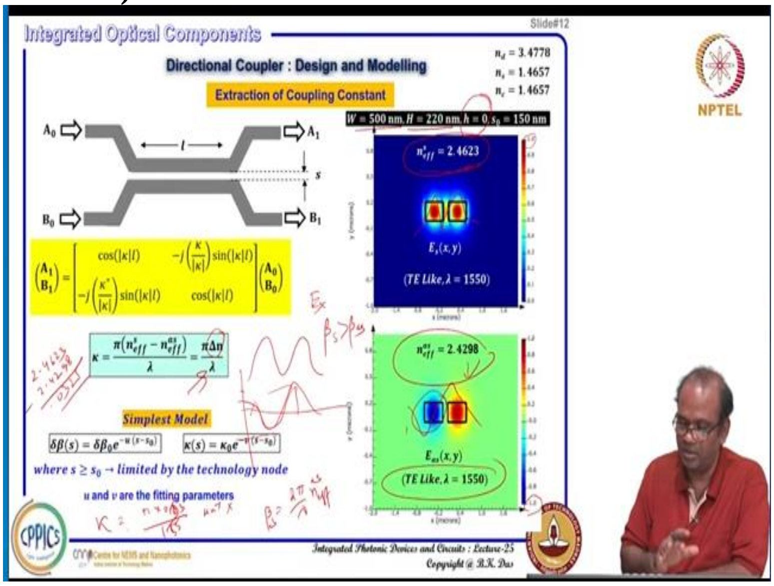

Now here it is here we go, we have used a waveguide dimension 500 nanometre device layer thickness of h = 220 nanometre and then small slab area slab we have considered 0 that means completely thing photonic layer and gap it as I mentioned found that in 180 nanometre technology one can easily fabricate 150 nanometre separation between 2 waveguides so here is the cross section this rectangle.

If you see that is actually 500 nanometre right one 220 nanometre the aspect ratio is different that is why it is looking like a square. But it is not really square it is rectangular dimension, this one also rectangular this one also will be a rectangle because 500 / 220 nanometre. Now, if you solve for TE mode that means, you are solving for E x component dominant component that E x component if you solve then you can see your electric field distribution will be looking like this.

So, then the center it will maximum the maximum will be 1 so, it is when you know we have already shown that this symmetric mode will be looking like this. So, one hump and other hump so, this is your symmetric mode you are getting one hump maximum in one first waveguide and other hump will be in the second waveguide. So, this is symmetry mode that is more solver giving and then go for higher order mode then you will see again you will be getting support something like this you are getting.

So, this type of mode you getting this side is negative values s is minus 1 colour this is the colour code this one is minus 1 and this one is plus 1 rate here plus 1. So, it will be if you are just giving the field strength then if you are just doing like that this would be plus 1 and this would be minus 1, so that means you are getting something like this type of mode. So, you know what is the mode solution that field distribution giving to you, it can be it is giving you maybe a dominant component of the electric field.

At the same time mode solver will give you what is the effective index for the symmetric mode and what is the effective index of the asymmetric mode, you see asymmetric mode is 2.4623 and anti-symmetric mode is 2.4298. So, that means this is lower so that means, it is your that beta s is greater than b as is your beta value is nothing but 2 pi / lambda times n effective if it is s we will call s if it is as we will call as that is it.

So, now I know n effective value symmetric and n effective value asymmetric so, I can easily find out what is the delta n value there is difference once you know the difference and lambda we are calculating at 1550 nanometre put down lambda value then I can find kappa if lambda is in micrometre for example, in this case delta n equal to say let me write down 2.4623 2.4298. So, if you subtract that whatever the value will be getting 5 and this will be 2 and then this will be your 3 and this will be 0.

So, 0.0325 is that delta n so, that means kappa value will be I can easily calculate kappa will be pi times 0.03 divided by 1.55 micrometre. So, that will be then micrometre per micrometre value will be getting. So, 0.03 so you can get this, this cancel 155 so, something whatever value comes micrometre you can multiply some 100. So, that is the kappa value and say depending on that kappa value you can find out what is the length required for a certain power splitting ratio.

So, this this is what we have calculated one of our students actually helped us to calculate this thing mostly any commercial software mode solver now it is available you can use them you can solve easily otherwise you can develop your own code as we have discussed in the earlier lectures how to develop.

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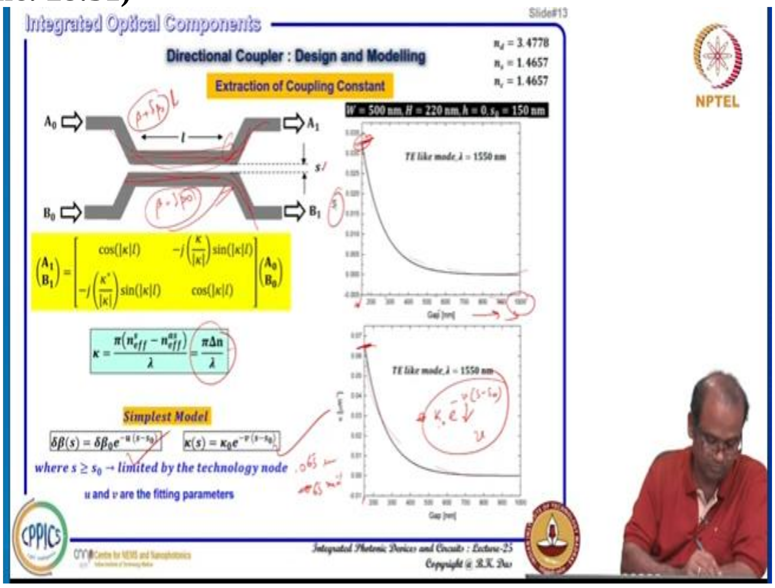

And then what we discuss we tried to find out as I say that, as I predicted earlier that if you just change the gap is increase the gap delta n value will be dropping because symmetric, anti-symmetric mode there will be slowly you will lose their coupled nature, superposition nature as if there will be like a independent identity. So, delta n that is their separation that is symmetric and anti-symmetric mode they will be propagating with the same phase velocity. So, almost 0 it is going when there as is very large.

For example, here we are considering 1000 nanometre meaning 1 micrometer, if the separation the separation is just 1 micrometre, you will see that they are almost decoupled just 1 micrometre, now you bring down, down and you are getting the 150 nanometre here, then you get as a calculator 0.32 or something like that exactly you are getting here also 0.03 and something very sharply falling exponentially looking like and coming there.

So, from here you can calculate kappa because kappa is once you know delta n you just insert delta n for every s value you can get a kappa it is something we consider here whatever value will be getting here delta. So, we can consider kappa naught e to the power –v s – s naught. So, s naught 150 nanometres that is the value so as you increase its value beyond 150 nanometre and kappa naught will be this one about 0.065 kappa value whatever calculated per micrometre.

If it is 0.06 per micrometre then per millimetre will be it is 65 probably 0.065 per example per micrometre. So, per millimetre will be 65 per millimetre. And you can consider per centimetre means that is about it again you have to multiply 10 so then it will be 650 per centimetre that way you can keep on calculating as you go along length that below and it is obviously exponential decay when fit with this data simulated data and you can get the parameter v for example, you can fit this v is a fitting parameter.

Once you get a fitting parameter then I can say that this is my analytical formula numerical semi empirical you can say that we can use that for circuit simulation. So, this is one type of way how to extract kappa and how to model in this problem, delta beta naught if you needed, obviously, this delta beta naught is needed because if waveguides propagates here you know what is the length, but you need to know what is the propagation constant so, that after traveling this much how much additional.

Because of the travel path how much pairs it will be aware that you will need to know by beta suppose your original beta naught or whatever beta value then this delta beta naught you have to add that will be your propagation constant and if the travel length is l then you know beta + delta beta naught times l that will be the phase for the whatever upper part going here that phase you have to include and the lower part also you can consider beta + delta beta they are common basically.

If they are identical waveguide this phase acquire whatever power propagating through when it is decoupling, then you can find out what is the additional phase it acquired here and what is the phase additional it is acquiring here that you can calculate easily.

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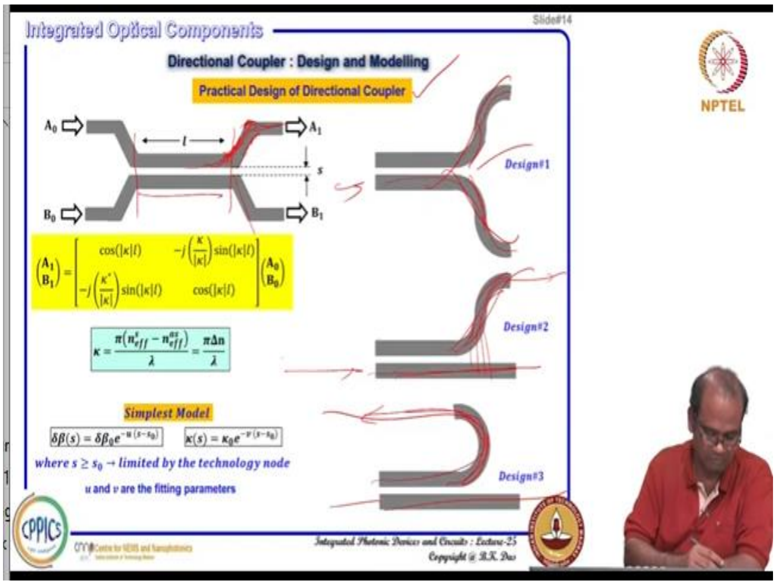

Now will go for some kind of practical design of a directional couplers you know, whenever we are just designing like these you know, this is a simple structure we just make separation decoupled here and it can be in here we said that it should be bending should be adiabatic so that you do not see any sharp transition. So, that light mode can be actually smoothly propagating from one type of waveguide structure to another waveguide structure when the propagating constant is changing, direction of propagation is changing.

So, you could get some smooth things, so that is actually very important that to make it smooth, but one more important thing is that you cannot just calculate the what is coupling happening in this length and say that this is my model because, you know in the previous slide if you see as increase the length it is not that the separation increasing it is not that they are coupling it is suddenly coming down to 0 it is not like that.

If you are just starting with a critical gap is s naught then if you see as you go in this direction their separation is increasing that is it, but that does not mean they in this Y-Junction type waveguide no coupling is happening we need to find out what is the coupling happening in this grain region how far they are coupling. So, you have to stated that match when it is coupling is almost 0 and then you can be in parallel so that you can x axis your output waveguide output things.

So, that is actually important to account what is the coupling happening in this junction regional transition regional also, so, whatever coupling here happening that is important in addition to that you have to consider what is coupling happening when they are slowly adiabatically they are decoupled that thing also need to model. So, for adiabatic transition we can have these types of 2, directional coupler they can be made decoupled.

Instead of 2 symmetric separations you can have one waveguide straight another waveguide make a separation so, that it can be decoupled. So, in that case, it will be this separation actually, less slow. Here are actually separation is abrupt almost 2 times separation is happening this is also moving this side this is also moving this side so, separation is it is being separated very fast.

And here separation is rate is half of that whatever you are getting here and then you can also think of that instead of taking output in this direction you can have some device structure output you want to take opposite direction also so, you can leave it like this also. So, these are actually so called directional coupler. So, if you launch like it here how much it will be coupled you can find here, if you launch here how much it is going there how much coming here, how much leaving here.

If you launch here, how much it is going in the backward direction here and how much it is passing this direction. So, in this way you can design and while designing that this bend structure you cannot really bend very sharply because you know any sort of bend is not adiabatic the mode sudden change in k value cannot be actually cannot very easily it can be accepted by a guided mode.

Because, when it is changing the k value you know while propagation the propagation constant also need to be maintained some kind of boundary condition if that is not maintained that means some kind of loss some kind of it is actually it will lose some kind of orthogonality condition and light can be scattered into the higher leaky mode. So, in that case bend loss will be there so, you have to bend that sufficiently large.

So, that bending loss is minimum per silicon waveguide this bending loss you can make around to 5 micron 10 micron which has some kind of loss if you are just bending radius is more than 5 micron 10 micron maybe 15, 20 micron I think bend loss is almost adiabatic and no loss is there. So, that way you have to design I will not discuss that bending related things, but if I get opportunity I will discuss whenever necessary, but for the moment we assume that this bending radius is tolerable. So, that bending loss is minimum that is what we are we will assume this discussion today.

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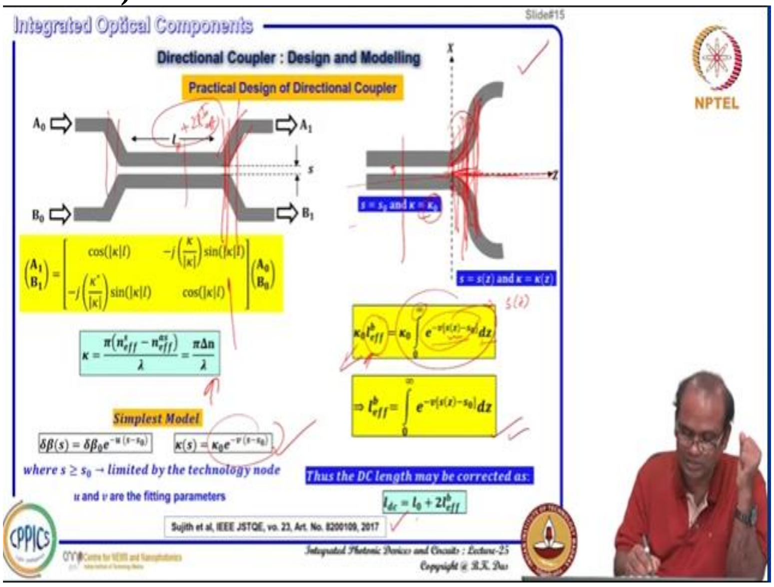

Now, let us see let us take this example of this type of directional coupler where actually the waveguides are getting decoupled very fast this is also going that direction and this is coming this direction and we maintain a coordinate like this, this is your propagation direction z and top view if you see this the x direction and vertical direction is y some way we are representing like that.

And then we know that here are in this region this gap we maintain depending on our so, called it is limited by the technology node. So, technology node can give you minimum feature 150 nanometre separation if you want 200 nanometre that is also fine. But here we are just considering you maintain s naught which is actually you can fabricate in some kind of lab or in the foundry with a good deal it.

So, when s = s naught I can calculate what is the kappa naught just calculating symmetric anti symmetric mode and using this formula I can find kappa naught s only I will be putting there while solving symmetric, anti-symmetric mode. So, will be just 150 nanometre or depending on the technology node or the l cd who can get in your lab fabrication laboratory. Now slowly this is going that means I can consider that no coupling is happening in this region.

They are still coupled as long as they are one tail is interrupting each other there will be still coupling. So, how to calculate that coupling in the bend region, so we can just simply say that this kappa here it is actually s dependent kappa value gap if it is widening I know that kappa value will be dropping exponentially and that exponential formula is like this and now the separation is also a function at z now.

So, if I just put this s equal to a function of z you can calculate depending on the bending structure the math you can do and then you can find out for a given z as it is going z increasing what is the separation between these 2 waveguide is happening as you go further what is this separation so I can get a function separation as you function of z. So, separation at a function of z and if I just subtract from s naught, because when s = s naught that is actually here it was s = s naught and from then there 1 over s is increased from s naught.

So, I can actually find I can model something like this, that whatever value is there, that is a z dependent I integrate over z. And I integrate 0 to infinity z in that z direction. I can integrate over z you do not need to integrate up to infinity it can be several micrometre probably as long whenever it as I have seen that if it is 1 micron separation, then no coupling is there. You can go up to 10 micron separation that up to that z you can calculate and you can integrate.

So, that exponential dz and you know that expression is k naught times this one and if you integrate then I can say that k naught times whatever integration value you are getting because this is a number whatever integration value you will be getting that we can say that n effective b. So, I can actually model this grain structure as if that is a parallel waveguide with the effective length of this one and having same kappa naught what is happening what is the kappa naught in that parallel section.

So, practically I have to design like this, but mathematically I can say that this l is now modified into l + 2l effective b that will be actually total directional coupler effective length and I can consider whatever kappa naught I am considering that will be kappa naught and then I can say that even though I am showing here it is something we can assume that the waveguide starting from here if I just include the original length here.

Then starting from here I can assume that the waveguides are not coupled anymore because the effect in the coupled region I have just included in the total length so, that I can use this transfer matrix perfectly. That is how we can say that the actual l dc what is the l naught in the central portion, whatever l naught will be getting I can say that this is l naught and then what is the effective in the bend region equivalent thing and that l dc I will be using here in this transfer matrix if you see.

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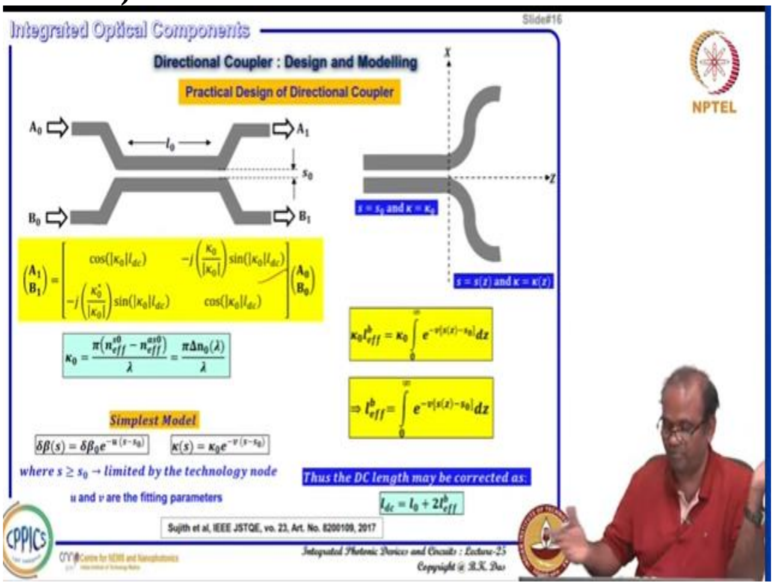

Now, I am writing kappa naught l dc, l dc that means bend structure both side bend structure will be there that I can include in the model, so this is straightforward. So, only thing is that you need to know what is the procedure stepwise you have to do and accordingly you can design your device.

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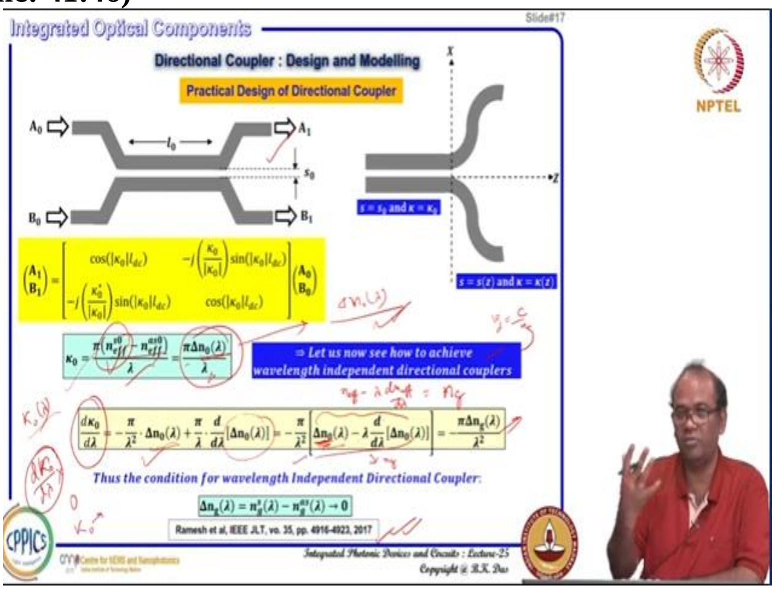

So, now let us now see if you see this kappa value exactly, that is actually kappa naught equal to that is actually effective index of the symmetric mode and effective index of the asymmetric mode and also lambda involves that means this kappa naught is lambda dependent, n effective also you know that when you calculate effective index they are also wavelength dependent, this is also wavelength dependent. So, that means kappa itself is your wavelength dependent.

Now our intention is that a waveguide designed for one particular wavelength it may not work a directional coupler, designed for one lambda it may not work for another lambda suppose I want to have a 3d power splitter a certain length. So, certain length it can give 3d power splitting at a certain wavelength. Now, if you want to operate same device if you want to operate in another wavelength, so instead of 1500 you want to use your 1600 nanometre wavelength.

So in that case, this kappa is wavelength dependent that means a directional coupler will not be 3d power splitter anymore. So that is a problem so what we do, let us try to see how one can get wavelength with the independent directional coupler that means at least a certain bandwidth, it should give you around that wavelength around maybe 100 nanometre 200 nanometre wavelength range you should see that.

If you change your wavelength for WDM purpose wavelength division multiplexing purpose DWDM purpose suppose I want to use a broad band all the channels I will be using 1500 to 1600 in that case around that wavelength I would see this kappa naught is not showing wavelength dependencies. So, how to get that let us try to find out d kappa naught / d lambda you know that if you see this d kappa naught / d lambda that means as a function of lambda if you find something this one is 0 that means kappa naught is constant over lambda.

If you just see that d kappa naught / d lambda tends to 0 that means k kappa naught will be tends to lambda independent I have the expression of kappa naught this one and if I try to find out d kappa naught / d lambda kappa naught means s naught what is the technology limited gap that for corresponding to that whatever the kappa naught value. Now, if I just try to do some kind of differentiation of this one.

Then I keep with respect to lambda first pi / - lambda square and delta n naught lambda kept there and then I want to do differentiation here pi / lambda d d lambda of this one. So, I know at least from our mode solver I will be able to know what is the difference n effective refractive indices of the symmetric and anti-symmetric mode how that is wavelength dependent that curve will be able to get from our mode solver.

I will tune lambda and try to find out symmetric mode, anti symmetric mode, what is the effective index for this symmetric, what is the effective index for the asymmetric and subtract that that will be the delta n for that particular lambda. Tune lambda another so, you get a function. So, from that function I will be able to find what is the value like this so, you just a little bit modified minus pi / lambda square.

If just a common then it will be delta lambda then minus delta d / d lambda and this type of thing earlier we have discussed that something you remember that n effective - lambda d n effective / d lambda that is actually nothing but n g group index that means, this group d index is very important to the estimate how information travels through the circuit, because information means modulated signal several frequency will be there.

So, several frequency as a group how they are traveling that velocity comes from this expression v d = c / n g. So, this is your n g here, since it is n g means delta n naught with respect to s to n effective you have delta n naught, delta naught means that is the separation of these effective indices of the symmetric and anti-symmetric mode. So, I can write this one in brief that is actually delta n g, this delta n g nothing but the group index of the symmetric mode and group index of the asymmetric mode.

So, if the symmetric mode and anti-symmetric mode they can have they obviously will have different effective in this no doubt about but if I find their group index is somehow identical almost close nearby then delta n g will be equal to 0 once it is delta n g = 0 that means, I can say that the kappa naught / d lambda = 0 that means your kappa naught is wavelength dependent. So, I need to find a design where delta n g is close to 0.

So, details will be found in this IEEE paper journal of light technology published in 2017. If you are interested you can go there we have discussed detail about how to design a wavelength independent directional coupler.

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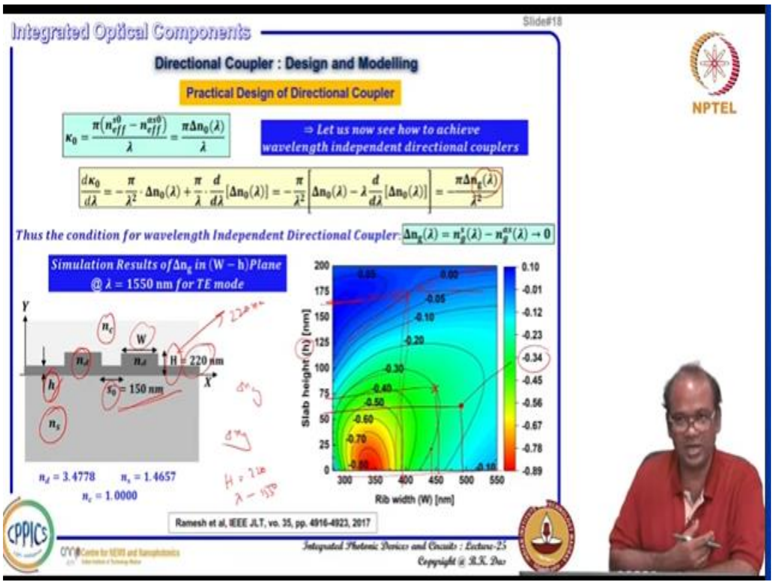

So, let us see how it is done you take in our hand what are the parameter for a silicon on insulator waveguide you have W you have capitalized that is fixed 220 nanometre that is the industry standard now, for example sudden most of the semiconductor foundry nowadays silicon photonics foundry they are using to 220 nanometre. And then another parameter you have which one this is h. So, in my hand I do not have any control over 220, I do not have any control over n d, I do not have any control over n s, I have better control over n c.

But that is either air or oxide, so effectively I have the control over W h and s naught, s naught as I said that smaller is better you will be getting compact. So, that is let us consider that this is s naught 150 nanometre which is actually technologically wave smallest one more is better more is not good you cannot get compact thing sometimes you need for example, you do not need high coupling or something like that in that time you can use this thing.

So, in this case what do we get we are getting W and h this is our freedom design parameters so, what we did, we have used this structure to solve symmetric mode, anti-symmetric mode by varying W and h and by varying that we tried to find out what is the value of delta n g, delta n g means whatever value I have started this value this calculation. So, I calculate symmetric mode, I calculate anti-symmetric mode, I calculate their lambda variations.

Then I know how to calculate this delta n g and I plot it here x axis is the waveguide width and y axis is the h for a given h value 220 nanometre and lambda around 1550 nanometres in the silicon you consider and in this plane they are these are the colour profile source that actually delta n g value if I vary so, if I just consider this point for example here that means the waveguide width is this much and slab height is this much as 50 nanometres about 50 nanometre.

So, here whatever the colour is there that colour is corresponding to for example is somewhere here, so delta n g about minus 0.34. So, this is how we have mapped and some value contour plots is given. So, these lines source that around that any value of W suppose you want to consider these value so, this is the waveguide width and this is the slab height so, they are what is the group index that is minus 0.3 and also same thing will be happening if h is small and I can go to another waveguide width, so that is how we have calculated.

So, this is a kind of lookup map I can find that suppose I need a certain delta n g, then I can choose my waveguide do it and h for a given s naught = 150 nanometre and if you see the contour just look this contour this is showing 0 that means, if you use any value sitting in this curve that will give you delta n g equal to nearly 0. So, that means for example, if I use some value here suppose this value I am using here or some area this value and this value about 160 and this value that means delta n g = 0.

So, around that if I design a directional coupler with a gap of 150 nanometre then I can ensure the directional coupler will be wavelength independent. So, from this lookup table we can use because this is standard design. So, we can model accordingly compact model your device for circuit simulation.

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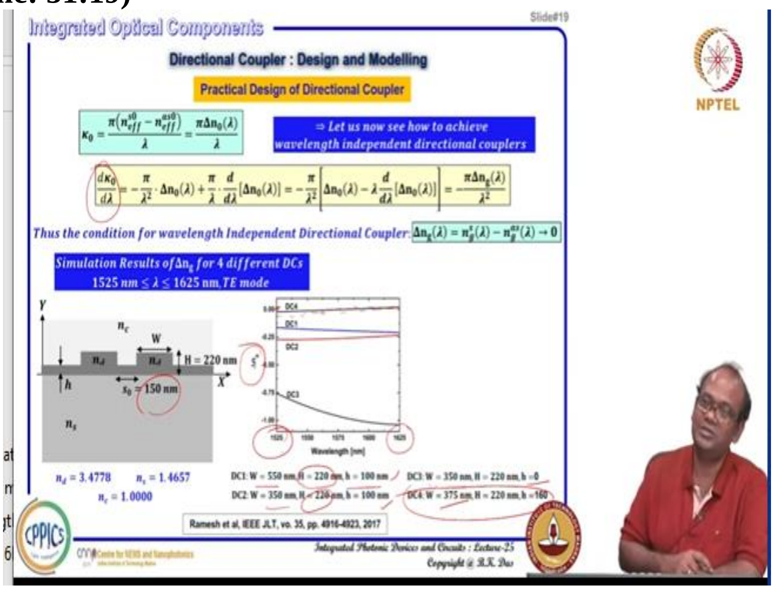

So here it is, here we go we have just taken 4 different parameters one is W = 550 H this is actually fixed for all is 220 and H = 100 nanometres that means, previous picture if you see 550 means around this one and then what I have 550 and then h = 100 nanometre. So, here so this one these value if I take 550 this one that is actually DC1 that means waveguide width is 550 nanometre and this is 100 nanometre. So, for that purpose what is our according to the colour bar if you see that is about somewhere 0.23 or something like that so that is called DC1.

So, it designed DC1 with this parameter DC2 another parameter and DC3 with waveguide width 350 nanometre, slab height 0, DC4 waveguide 375 160 things. So, all these 4 parameters to use then you see different directional coupler they will see wavelength dependent delta n g how it is varying, delta n g for DC1 it is less, a relatively less wavelength dependent but DC3 if u see highly wavelength independent the delta n g value. So, delta n g value highly independent means d K naught / d lambda is also wavelength independent.

So, kappa value will be wavelength independent, but if you just consider something like DC4 if you all the 4 if you compare DC4 if you see delta n g is close to 0 this is 0 value, this is just crossing close to 0. So, in that case 1525 to 1625 I can expect that if we use these parameters, then that device will be wavelength independent directional coupler, that coupling will be depending on your length how much length you are using.

But for this 150 nanometre gap and waveguide width is 375 nanometres slab, height 160 nanometre ensuring you that the delta n g that means the group index for the symmetric mode and group index for the anti-symmetric mode they are nearly equal they travel with the same speed, if they travel with the same speed then you can ensure that obviously, that the devices are wavelength independent.

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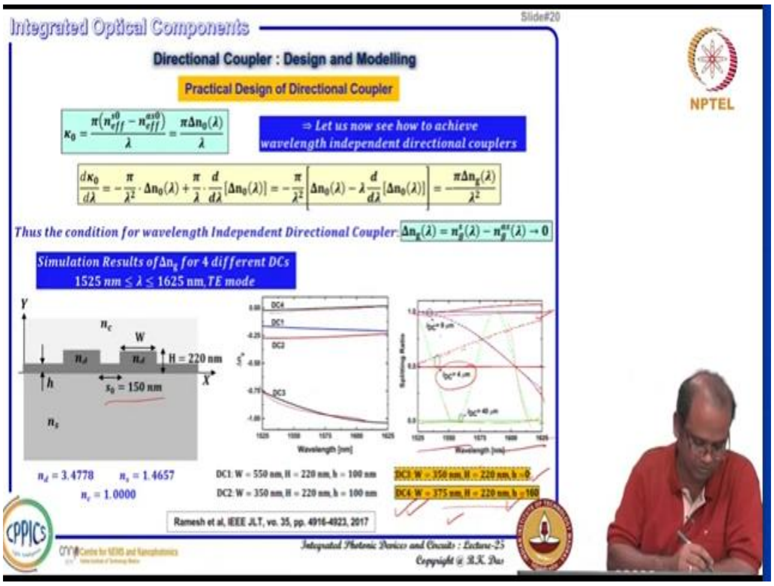

You check to see I have taken DC3 and DC4 DC3 is the highly wavelength dependent thing and DC4 is the less wavelength independent now, you see this is the dashed line. So, all the dashed lines are corresponding to DC3 that means wavelength dependent and solid liner wavelength independent now, you make a DC of length 4 micron and if you go for DC3 design that means, dash one this design you see splitting ratio between 2 output ports that is actually at 1525 it is nearly 50%.

As you increase the wavelength splitting ratio is increasing it is almost going 90% splitting ratio or something like that. However, if you use DC4 this dimension, you see 1525 also 50% 1625 also 50%. So, if you are using DC4 and 4 micrometre length if you use this waveguide design directional coupler design which s naught = 150 nanometre and length is exactly 4 micrometre. So, you can get 3d power splitting. So, 50% will go to cross port 50% will go to bar port.

So, this design ensures that actually. Now, if you go for same to structure DC3 and DC4 for 8 micron you see longer length then it is a 4 is 3 dB. So, double if you do that could be cross coupling completely. So, you say splitting ratio one that means one part will be completely taking a power of one of the port and other port will be 0, it is completely cross coupling to the second waveguide that is what it is showing and they are wavelength independent.

But other one DC3 you see how they are wavelength independent starting from cross coupling and the other wavelength it is almost 40% coupling ratio. So, that is wavelength independent they will even if you go for longer length then you see it will be more over because that is ultimately cosine square function sin kappa l cosine kappa l. So, that will be sinusoidal according to this design.

So, longer length means you can get more dispersion in coupling ratio, so just increasing the directional coupler length you can enhance the dispersion of coupling. However, if you design properly even longer length also you see the splitting ratio is fixed. So, that is how you can engineer your structure to make highly wavelength sensitive kappa or almost nearly wavelength independent kappa these packs are very important if you are designing a broad band for a device or a circuit to operate in wide band wide optical band.

For example, you want to use C band, L band of optical band so, C band is 1535 to 1565 and L band is 1565 to 1610 and something S band also lower 1500 to 1535 and so on. So, entire here we are considering C band L band you can get using DC4 design this is 4 microns if you 50-50 splitter, 8 micron that means cross coupling and then if you go about 40 microns 5 times then again it will work.

So, depending on the length actually you get whatever length you use splitting ratio you can control but splitting ratios using wavelength independent but another design if you see just controlling the length, you can actually get more dispersion so, this different application you can choose this design actually.

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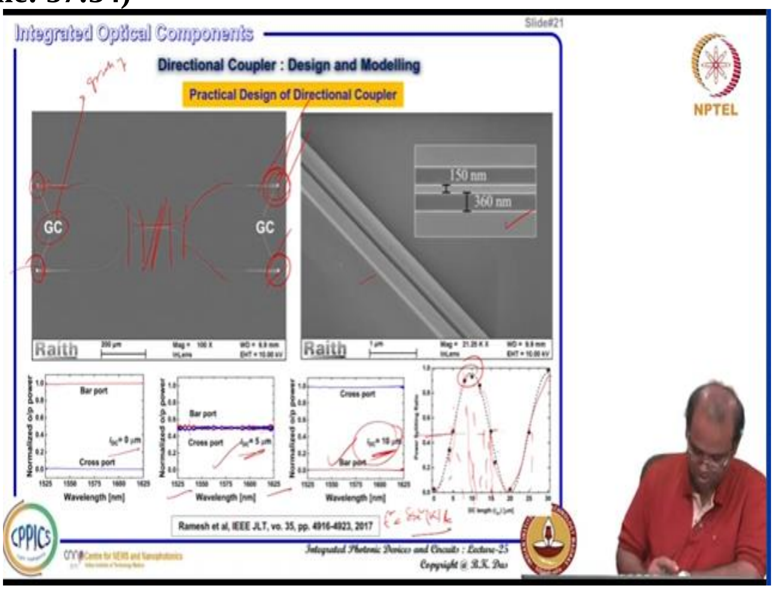

So, now I will show some experimental results based on this design and this type of device fabricated here in IIT Madras using our nano centre, centre name nano photonics you see here, this is a directional coupler simple directional coupler design, this is 2 / 2, 2 input ports and this is 2 output ports you can consider vice versa you can use also. You say this is GC, GC means grating coupler, grating coupler means, you know we have used a vertical grating coupler. So, that you can prove you can use a fiber being here light will be coupled to the waveguide.

So, whatever input you want to give you can just couple here, here also you can couple and same way you can couple light output sides that we have discussed from the top you can bring your fiber, wave fiber and vertically with proper angle you according to your design you bring in then you can couple light. So, this is the directional coupler you have used and some directional coupler in the central region it is not resolved here in this picture to waveguides are coming and they are coupling and they are separating here like this.

And in the central portion, if you just zoom in and check how is the 2 waveguide porous silicon waveguide looking like 3d view it is a little bit tilted sample and SAM image you have taken and if you see the top if you see, we wanted to have 360 nanometre dimension and gap is 150 nanometre and accordingly we are getting in our lab. And then if you see that as a function of lambda when this l dc is 0 just you consider that this bend region coupling.

And central region nothing is there that coupling also you can give if you are launching so entire power will be in the bar port, cross port almost nothing 0. So, the bend region coupling that gives you light going to cross port and then back again to bar port. So, almost everything we are getting in the bar port, it is designed like that. And then if you are using l dc = 5 micron then you are getting 50 50 according to our theory it said that 4 micron.

But experimentally we found that it is should be 5 micron actually because a lot of approximation we have used for modelling purpose, but we can get your 50 parameter accordingly back and then you get 5 micrometre, you are getting almost 50% cross port and bar port which is 5.5 red colour is for bar port and blue colour is for cross port. So, as you function of lambda 1525 to 1625 they are wavelength independent.

Now if your length, directional coupler length increase back again then you see cross port you are getting completely earlier with l dc = 0 bar port you are getting this much and cross port nothing here cross port is maximum 1 and bar port 0. So, double wavelength that means this 3d power this is the complete coupling length 10 micrometre. So, here it is shown that as a function of length you know the coupling in the cross port or bar port.

What you say this is you are getting in the cross port and bar port, bar port cross port is sin square kappa times l. So, this is the l dc if you just increase l then coupling that means it is your t, t square basically t square equal to this one sin square kappa l as you increase it will be sinusoidally actually you supposed to get this is a dashed line theoretical model and this is the experimental results you are getting different length whatever the power you are getting in the cross port that actually plotted here.

So, that is actually so if you want that 5 micron then you are supposed to get 50% and again if you just go for around that wavelength 50 micron also they are also you would be getting 50% in the cross port, 50% in the bar port. And if you want to completely switch over then you go for 10 micron that is this value directional coupler length is 10 micron. So, this is the experimental results whatever actually proving find according to our theory, theoretical prediction.

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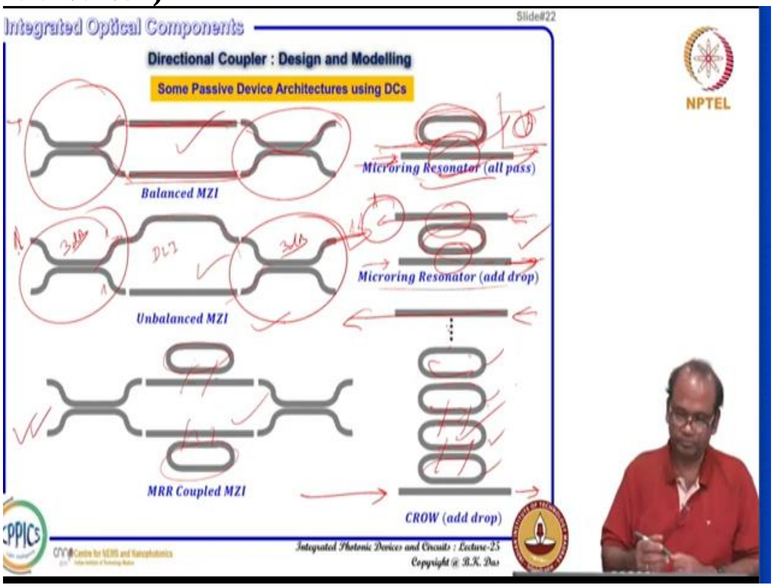

Now next is I will just discuss a few things that this directional coupler design is not just for power splitter, it can be used for developing different types of circuits, first I will be just going 4 different types of devices from component directional coupler component to some devices and then we will go slowly per circuit for different functionalities. For example, if we have a one directional coupler here another directional coupler here, if you connect them with a waveguide just back to back.

Then these actually up in launch here power will be splitted into 2 and it will travel and then you can interfere there. So, this is actually called 2 / 2 the balanced Mach-Zehnder interferometer, balanced why? Because these arm and these are they arm equal length. So, these types of Mach-Zehnder interferometer very important for developing active devices like modulators and also a lot of unitary operations for quantum photonic applications.

This Mach-Zehnder interferometer very, very useful we will be discussing that and another thing we can also use unbalanced Mach-Zehnder interferometer 3d directional coupler 3 dB this is also 3 dB and this is also 3 dB and then unbalanced that means one of the arm longer. So, that type of device also used for different type of band rejection filter, lattice filter design etcetera and sometimes it is called delay line interferometer DLI.

That means one if you have a suppose one signal is coming time dependent signal is one pulse is coming then what happens this pulse will be splitted into 2 half smaller pulse then after traveling here when they are mixing at any point you are getting back then one pulse will be faster another pulse will be just following so, you can actually copy 1 pulse into 2 pulse. So, this type of delay line interferometer it is also used for different types of quantum mechanical applications and also microwave photonics many applications you will see.

And you can think of that you can have some kind of structure directional coupler you can design like this microring resonator, what is this is your pass waveguide you launch here and because of the coupling some couple will be there and some wavelength will be dropped here and that will be missing here so you can get a filter that is called all pass filter when we will be discussing microring resonator wide it is called all pass will be discussing, but thing is that certain wavelength will be missing in the transmission.

If you just see lambda version this particular wavelength will this tool here that is knotch filter and then that is actually all pass filter, this is called microring resonator add drop filter, add drop filter means if you are launching here that whatever the missing part that will be stored here but that can be coupled back here you can get here that particular length, that particular wavelength will be missing here so that means that wavelength is dropped here.

And at the same time if you want in this channel back again that wavelength actual couple here and this will come here, come back here, so it will be adding them so it is that is why it is called add drop will be discussing how the function is there. But what I am saying that you need a direction of our special type of directional coupler here to design a microring resonator and you can again this directional coupler this directional coupler together gives you the add drop filters add drop type multiple resonator.

Remember that this design is nothing but it is a directional coupler one of the output you are be giving back to feedback here it is kind of directional coupler one port and another port is supposed to be here and another port supposed to be here what it is done this you bend back and coupled to the input waveguide. So that is some kind of feedback that is how the resonator is designed and this type of structure same design.

But you can have multiple rings to get a higher order filter good quality filter, high extents and filter you can design by coupling rings up the rings like this all these are directional coupler that means this ring is coupled through this, this ring again coupled to this ring, these are again coupled to this ring and finally you can bring as many as ring you want you can integrate people have demonstrated up to 235 microrings like this coupled they are called coupled ring optical waveguide structures.

So, you can get like a drop here at a we can launch here this will be adding port and this will be just bar port so you can have 2 / 2 input output ports and also you can just design a bit of complicated structure again very important for quantum mechanical application and filter applications. It is a Mach-Zehnder interferometer first you design and we use another directional coupler to use integrate one ring here another directional coupler you integrate another ring.

Then you can get a complex useful functions you will be getting particularly useful for photonics pair sources generation etcetera will be discussing it. So, that means what I meant to say that only directional coupler design that itself can be considered as one of the major building block you use this directional coupler to build versatile different type of passive structure and that passive structure in future also in course of time we can also make it active.

So that you can make also active devices like modulator switches all this type of thing we can get reconfigurable filter etcetera. So, with this I stop here for this lecture today. And next will be continuing that what is the transfer function of this passive structure, once you know the transfer function of the directional coupler, then you can actually extract what could be the transfer function for this structure, this structure, different structure transfer function we have to derive that is actually straightforward.

Once we start discussing will be and will try to find out what is the figure of merit of that particular device or circuit when they are operated as a passive component. Thank you very much.