Integrated Photonic Devices and Circuits
Prof. Bijoy Krishna Das
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 23
Fundamentals of Light waves: EM Waves Wave Propagation in Metals and Semiconductors

Hello everyone, today in this lecture, we will continue integrated optical components. In the last lecture, we have discussed about Y junction power splitter combiner and also Michelson interferometer based on y junction and also the transfer matrix of the Michelson interferometer. So, now, we will discuss directional coupler, sometimes directional coupler also used as a 3 DB power splitter to construct 2 / 2 Michelson interferometer and many other components also like ring regenerator etcetera design directional coupler plays a major role.

So, basically directional coupler is nothing but a coupled 2 wave guides, 2 wave guides, they can be parallel, they can be unparallel they can have different type of configuration as long as they are coupled somehow they can transfer energy between them, then that will be called they will be called as a coupled wave guides and in the coupled wave guides if you have a suitable design control, then you can use them as a directional coupler and in this lecture.

We will be first discussing about conceptual working principle that will help us to understand visualize how a directional coupler works. Little bit mathematical model we will be considering spatially super mode theory super mode principle we will be using, but we will finally we will see that, that type of conceptual understanding is not enough to quantify particularly amplitude and phase when light coupled from one waveguide to another waveguide.

So, for that actually, we have to solve really coupled differential equations, we have already discussed earlier coupled differential equation between modes, but those modes are within one waveguide that those modes can be propagating in the forward direction can be propagating in the counter propagating direction, but this would be belong to the same waveguide but here we want to construct a coupled differential equation between waveguides waveguide modes 2 different waveguide modes.

One mode from one waveguide another mode from another waveguide and how they are coupled that type of differential equation we have to develop to understand directional coupler and quantify the amplitude splitting and phase relationship between the splitting amplitudes splitting waves. So that is very important little bit differently, but almost same way we will be constructing these coupled differential equations, that is what we will learn today.

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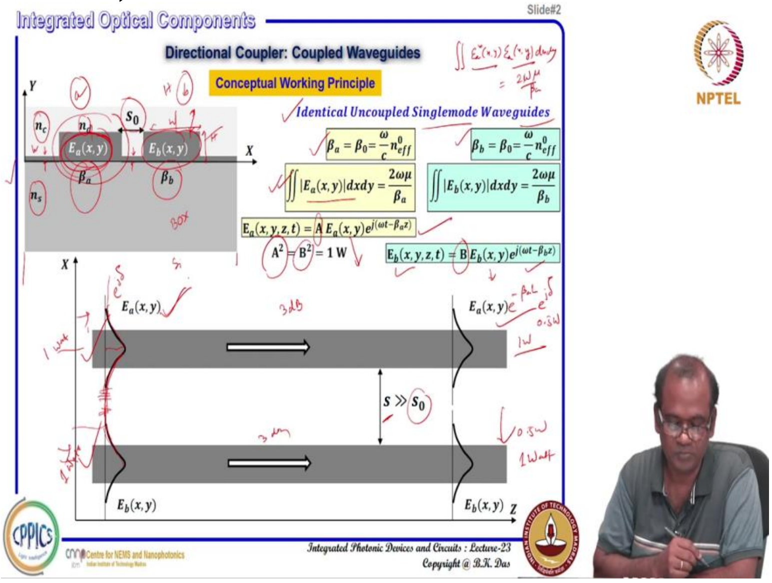

So, let us take example of this figure, let us try to understand step by step you have this. This is the box I have not so, the substrate below there will be a substrate silicon substrate and silicon on insulator. This is the device layer H which is fixed about 220 nanometer nowadays it can be anything but fixed and we have this is your H and this one is your waveguide width and this is totally when you say H, then a little bit of slab.

You are keeping h and that h also maintained in between 2 waveguides and they are separated by a distance a S not separation S not we are just considering and as you know, we can just define this box refractive index silicon dioxide refractive index and cladding here can be oxide can be nitride and then n d that means the device layer refractive index that means silicon refractive index and we can just think of that.

Whenever you are just solving this mode thinking that this is not present this waveguide this can be considered as a waveguide a and this can be considered as your waveguide b, a and b when you solve electric field distributions using standard full vectorial methods and you can consider E a is the dominant electric field components you can use the major scalar say T a like or T m like when dominant I that along the X direction or Y direction.

That field if you concentrate that field distribution if you solve that is the mode we define like this and we can assume that this waveguide is designed such that it only support fundamental mode. Similarly, another waveguide the w, h, H you design such that it also gives you fundamental mode, it is a single mode supporting it is not a multimodal guides individually, they are multimodal guides if you solve individually thinking that when you are solving for a you are thinking that b was not there.

And when you are solving for a then a b then you are thinking waveguide a was not there. So, if you solve like that, that is the field distribution, corresponding field distribution is this one, corresponding field distribution for waveguide b is this one, E b, E a, they will be if they are identical waveguide they will be identical field distribution only they are field distribution will be laterally shifted, that is it depending on the S not.

So, whatever field distribution you are getting here, you just shift S not distance, whatever the center to center distance if you just change, then you should be getting the field distribution like that if they are identical and corresponding propagation constant beta a and beta b will be getting if they are identical beta b equal to say you are beta not omega / C n effective not and beta a also should be equal to beta not omega / C n effective not.

So, we are if we are considering identical uncoupled single mode waveguides. So, they are uncoupled, they are far away or you are just considering they are not interacting at all or you are considering they are just simply decoupled top view you can see here this is the top view and the field distribution, I am just considering this S is selected much higher than S not where S not maybe the actually DGR waveguide separation you want for that so that they can be coupled they can interact.

So, you whenever you are putting a separation far away as if they are individual identity, there are no coupling no interaction between them, then we can consider. One field top view I can show, E a x, y only x distribution shown here Y direction it is same we are considering Y direction, the field mode will not be change. So, it will be like a whatever shape you will be getting in the cross section just I am just getting in the x z projection X direction what is the electric field strength in the central.

It will be maximum and slowly it will be decaying evanescent tail evanescent tail will be there for waveguide a similarly for waveguide b. So, if they are identical again you can see that they are shape will be identical, but they will be positioned at different X coordinate Y direction the field distribution will be similar particular direction. So, following the orthogonally condition, you know that orthogonally condition.

We write that E a star x, y E a x, y dx dy they are actually modes from the same field and that will be actually we will be writing 2 omega mu / beta a because only one mode is there. So, I can say that this is nothing but this thing dx dy and if you are defining that this field has your amplitude specific amplitude normalization constant and then propagation constant beta a. So would be beta not beta a = beta not.

They are identical and for waveguide b also E b x, y omega t – beta j not beta b z and so on and you have the constant normalization constant such that a square equal to b square equal to 1 watt individually they are carrying 1 watt. That is what do we keep on considering all the time. So, again you should not forget that this E a x, y whatever we are writing E b x, y they are in principle their distribution profile is same but coordinate while join we just consider we relative position of 2 waveguide.

There are different positions that is it and since they are not coupled this field when it is propagated, that will be propagated here and if it is launched 1 watt you will be getting 1 watt if it is lossless 1 watt if you are launching here, 1 watt lossless. Again you will be launching maybe 1 watt here 1 watt. This is just a reference it can be 1 milliwatt, 1 watt, 1 watt it will be coming there.

There is no interaction they will be propagating if it is lossless suppose that you can imagine if there is a 3dB loss because of the scattering etcetera, then you will be getting here 0.5 watt and this is also 3dB we it will be 0.5 watt some losses there that means, it will be half. That is it, otherwise they are independent propagating. So far so, good and we are considering that whatever frequency we are launching here the same frequency here that is pi.

And whatever phase will be their initial phase e to the power j delta that traveling distance e to the power you will be getting beta a l, l distance traveling and then initial delta you will phase change you will be considering same will be here. So, it is as good as you like as an individual waveguides 2 identical waveguides separated at a distance when they are not at all interacting when we will call them not at all interacting when the evanescent tail.

You know the evanescent tail will be decaying and it will be 0 only at x = infinity or this around infinity and here also evanescent tail it will be decaying, but at infinity it will be 0, but if you see after a death skin so called skin death or something like that, the evanescent tail will be decaying very fast and nearly 0 you will be getting here. So, as long as they are nearly 0 that means it will not see this mode will not see the presence of the second waveguide and vice versa.

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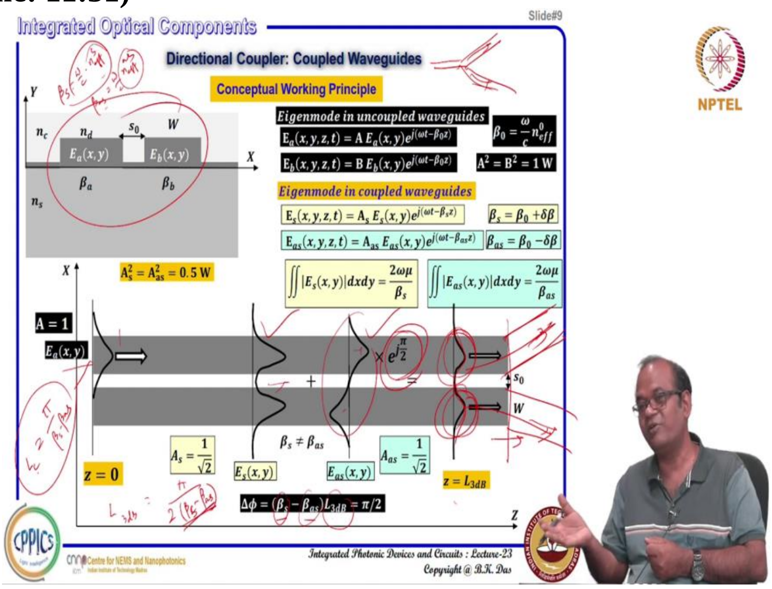

But, when you see that the separation reduced to S not that is our desired one when you see that evanescent tail of the waveguide a is penetrating through the separation gap and entering into the it sees some presence of the evanescent tail C is the presence of the secondary waveguide and vice versa. A mode this one it sees is the presence of the waveguide the mode in waveguide b sees the presence of waveguide a experiencing the presence.

So, when evanescent tail C is there this evanescent tail here supposed to see not waveguide maybe if it is some slab is there supposed to see slab instead of slab the evanescent tail it is going like this and it is coming like this. So, if this waveguide was not there this waveguide is, was not there the pillar would not see this refractive index the additional height presence of secondary guide. So, that means these waveguide presents what it will see some type of kind of perturbation this waveguide will see some kind of perturbation.

So, that is how we can consider this perturbation theory or coupling some way of coupling it you will see. So, in that situation we cannot really consider something like this type of mode and this type of mode individually because they themselves is no more following orthogonality condition rather they are interacting each other. So, orthogonality condition is broken and instead of these 2 modes will lose their identity.

Rather what you will see, you will see 2 super modes, we call them as a super modes, how these super modes are constructed. One is called symmetric super mode, that is the waveguide field distribution per waveguide a field distribution for waveguide b linear combination that is simple superposition and if you do this adding the field amplitudes here for example, you will be getting this type of shape that is called symmetric super mode.

But you can also imagine another type of superposition that is subtracting that means, you see E a, E b they are different position, but you will know they will be oscillating all the time oscillating back and forth depending on the time according to the frequency of the electromagnetic wave. So, what you could do see that they are linear combination plus and minus they will be jointly giving another type of super modes and that will be called as a asymmetric.

This type of concept normally used in solid state physics also in molecular science were actually 2 way function of electron if one atom wavefunction is this one another atom wavefunction is this one electronic wave function. If they are coming close by then electron from one atom and electron from another atom they are state they will be interacting each other. So, when they will be interacting each other the energy even though suppose 2 hydrogen atom you are considering they are being they are brought together closer.

Then one is our electron and one is electron here, they will overlap after overlapping their energy level will be splitted and it will create 2 different wave function. So, that is called actually in solid state physics or molecular spectroscopy call it as a linear combination of atomic orbitals. You can imagine also here one photon state wave function is like that and other protons function, wave function is like that and these 2 when it is coming closer.

They will be actually interacting each other and that will give you some kind of splitting of the 2 different wave function and in case of sometimes you know, electronic states in a crystal, it is a normally splitted because of the presence of some kind of perturbation, here also this perturbation actually causing some kind of splitting we call them as super modes and when they are splitted what you would see they are properly some constant.

That is the eigenvalue field distribution you get as a solved solution you get a eigenvector and beta solution you get a eigenvalue. So, originally individually when they are separated you are getting beta naught there also solution was beta naught, but as they are interacting each other you will be getting 2 different solution one will be delta beta added this propagation constant another minus exactly will be reduced.

So, some value will be added here. So, you will be getting one higher propagation constant another will be lower propagation constant it is like that if one energy is there because of the field or maybe magnetic field you know G man effect etcetera or if some magnetic field is there some energy level splitted like this. So, since this type of energy will be splitted this one will be beta not and this will be beta s and this will be beta a s. Beta is equal to beta not plus delta beta and this will be beta not - delta beta.

So, that way it will be splitting is just qualitatively here we can assume that these 2 modes they can linearly they can have some kind of linear combination of modes you can get symmetric mode you can get asymmetric mode and you can have 2 different propagation constants. So, called beta s beta as obviously beta s greater than beta as. Normally we will be getting.

So, they are called super modes and this super modes can be again normalized with a constant normalization constant as for asymmetric and for symmetric it is as and this is a as this a as subscript is there and then electric field also profile symmetric and asymmetric you can just define symmetric means this one coming profile just specially you are just adding at a distance.

But here specially we are adding but one will be opposite face that is actually 2 types of superposition you can expect 2 wave functions it was giving 2 super modes if it is 3 than you would expect 3 or more different combination of linear combination you will be absorbing fine and accordingly you can normalize this one this normalization like your standard you can consider this 2 as the Eigen mode of the combined system even if you solve full vectorial methods complete this structure.

Then you would be getting 2 more solutions, one will fundamental mode will be looking like that and next higher order mode will be looking like that. So, you can consider they are as if they are Eigen modes and those Eigen modes this will be the first order Eigen modes, so, called symmetric super mode and that will be asymmetric super modes. So, this type of solutions you can expect.

So, now, you just take this cartoon help of this cartoon. Suppose, you consider that you are launching suppose in the beginning suppose this waveguide starts from here second waveguide, waveguide b you are just launching here some mode in the first waveguide, waveguide a you are launching and you are launching supports 1 watt power A square = 1 watt. So, this is the when they that is not coupled A square = 1 watt.

So, A = 1 we are just launching at z = 0 just when it is starting 2 modes to coupled suppose it is extended here and you are launching like this and it started coupling here this is actually z = 0. This z = 0 what happens you know the combined waveguide has a 2 solutions. So, it is like that whenever you are considering a y junction like this y junction waveguide a fundamental mode is coming and you have 2 paths here.

So, if it is lossless adiabatic one if the amplitude a it is coming here then it will be a / root 2 it will be a / root 2 because of the 2 paths. So, similarly, you can think of that whenever a mode is coupled with amplitude say 1 or 1 watt power amplitude 1 we can consider 1 watt square with A square is 1 watt that means A = 1. So, in that case what you got you have also 2 options, it is like y junction you have 2 path but here it is a coupled waveguide but you will have a 2 options.

So, in this 2 options if you see the symmetric mode asymmetric mode if you see that overlap here this part integration of this one you will get some value excitation here and this one with this one if you see the overlap also you can get some kind of value. So, it will be also coupled, but in that case you will be getting asymmetric and symmetric mode propagation, but amplitude will be now A / root 2, 1 / root 2 here and 1 / root 2 here.

That is what it is given as will be 1 / root 2 and this will be 1 / root 2 that means As square will be 1 / 2 to A as square = 1 / 2. So, if it is A = 1 , 1 watt this will be 0.5 watt this will be 0.5 watt that is what your answer. So, that means, if I launch 1 watt with in one of the waveguide in fundamental mode, when they are coupling to each other, then this power will be instantly at z = 0 it is like a y junction it will be divided into 2 amplitudes one will be 1 / root 2, another will be 1 / root 2.

So, 50% power will be associated with this mode and 50% power will be associated with this mode. Once they are excited they will be traveling with a different phase velocity because they are propagation constants are different they will be traveling with a different phase velocity. So, that is the idea. Now, let us consider this one. If you adjust for instead of this one we are launching here in the second waveguide you will be giving and you are giving one watt so that this be amplitude is 1.

A = 0 for example, I am not giving anything in waveguide a. So, in that case also at z = 0 you see, you will be getting 50% excitation to 50% power will go to symmetric mode, because these are the solutions available these are the paths available in the waveguide any electromagnetic wave if you are launching in this system and if they are interacting, you have 2 options for their excitations, no other excitation is there even if it is there, you are ignoring that we are just considering that.

There is no other possibilities there light cannot tunnel to any other radiating mode. So, in that case, here also you will be seeing that it is excited super mode symmetric super mode will be excited this one and this will be excited like this one field distribution, field distribution, amplitude, it is either you are launching here or launching here you will see similar type of situations.

So, it will be 2 super modes will be excited now. Now, interesting. Suppose you are launching first waveguide also 1 watt second waveguide also you are launching 1 watt both waveguides you are giving amplitude like that, what happens? It is interesting, the first waveguide you will see this is corresponding to waveguide a this is corresponding to waveguide a.

So, from waveguide a you will be getting these super modes and this is the super mode symmetric super mode anti symmetric super mode and from waveguide b you will be getting also another super modes symmetric and other super mode symmetric. So, that means, if you see, I know that this mode will give 1 super modes here symmetric and anti-symmetric this one also be symmetric anti-symmetric.

So, there will be 4 super modes now, because 2 modes we have excited here. So, I would expect 4 super modes and out of these 4 super modes, these 2 supposed to be same right and they are degenerate, their propagation constant will be same because their solutions are different but you see 2 solutions are there. One coming from waveguide a another coming from waveguide b, launching waveguide b.

So, 2 types of meaning they are actual ultimately same, but since propagation constant is same, but you can fill them individually, they can be considered as degenerate, but they can that is actually one mode solutions you can think of. Similarly, here also degenerate as symmetric super modes are there, let me depress. So, this is a coming from a and this is also coming from a, this is b and this is coming from b.

So, if you see this one a and a if you superimpose this side positive this is positive this will give you this one and this one positive, this side negative that means waveguide excitation will actually result into this one add these 2 you will be getting this thing add this 2 you will be getting this thing. So, that is how it is decomposed into 2 degenerate super modes symmetric super modes 2 degenerate asymmetric super modes.

That type of situation can happen fine. Now, I come back to again this situation that where you are launching only in the waveguide a. Waveguide a you are launching here and you have this one 1 / root 2 and this one 1 / root 2 and that will be traveling with beta s and beta as suppose you know what is the propagation constant and this type of situation at z = 0 you can see whatever though I am just sketching at arbitrary z point, but this type of excitation happens exactly at z = 0.

So, half watt you can excite to symmetric super modes and anti-symmetric super mode and you allow them to propagate here. From z = 0 to you just think about some z = L c I am now looking into the super modes at certain length where the super modes this one they are phase difference this one and this one if you see previous one to this one now, what I see at z = 0 I see this one because if you just add this one you are just reproducing this one this negative plus this will be canceled and now.

If after traveling z = L c I am just considering at L c what is happening. This one will be in phase after traveling it will acquire some phase. So, for example, beta s L c it is something 2m pi phase shift is there. First symmetric super mode and anti-symmetric super modes you are getting something like that beta s L c you are getting 2m pi + pi additional phase shift is there. So, when it is additional phase shift is there that means, if you just compare this one, so, this one and then this one, this only this much flipped.

So, once it is flipped after propagating L c distance, that means, asymmetric super modes s pi phase ahead of symmetric super mode. So, in that case, if you just add them together this minus and this plus they will cancel each other this plus this plus. So, your energy major energy will be here plus plus in this way here. So, at this L c distance exactly at L c if you start decoupling your waveguide here for example, 2 waveguide be coupled.

Then what happens these waveguide will not carry any energy whatever mode is there you will get better overlap here it will be coupled to the waveguide b you are launching here in waveguide a but lying to be coupled to the waveguide b after traveling L c. L c is the distance where phase difference you are getting beta s – beta as L c 2m pi and beta as L c 2m pi + pi. So, if you subtract so, then you will be getting this one normally this one will be - pi it will be because this will be lower beta as is lower.

So, 2m pi - pi that is why, you will be getting pi. So, if you allow to travel then after traveling at a certain distance if they were initially their phase was same it has 0 phase with respect to here. Whatever excitation is there excitation is there at the time we are considering their phase differences 0. Now as they propagate and we said that whatever phase relationship is there they are probably and then after certain L c distance curve traveling.

You see pi phase shift. In that case superposition if you see we will be getting like this. Now see. If I instead of pi phase shift if you allow distance such that you get their phase difference is e to the power z pi / 2. So, pi / 2 means, what is the difference beta s - beta as L 3dB / 2 instead of pi, pi / 2. That means, just half the length whatever L c was there, when light was launched here, you are exactly everything you are getting here, you just go just half distance.

If L c distance it provides pi phase shift half of the travel length it will give pi / 2 phase shift. So, in that case, you cannot really define very nicely like the pi positive and negative, clearly, you have to add you have to have a factor like this e to the power j pi n. So, in that case, after adding this phase and if you do some superposition and then this superposition, we will see field strength like this it will be 50% field strength here and 50% field strength here and at this point if you just separate decouple then this fraction will lead to this waveguide.

This fraction will lead to this waveguide. Suddenly, you are actually breaking the orthogonality condition here orthogonality condition for super modes, asymmetric super mode starts here, at z = 0 as long as you maintain the coupled waveguide cross section and then they will travel like a orthogonally modes. Now, once you see field strength is reasonable reasonably good set in first waveguide total field strength and second waveguide they are then in there you just do separating them.

Then you get exactly 50% power here in the waveguide 50% power here and in that case, we will be calling that as a 3 dB power splitter, it is exactly equivalent to y junction power splitter in y junction, you have one waveguide and you have 2 branches. So, you just get like this. So, in this case, 50% goes there 50% comes here, here also you see if you just maintain if you know beta s and beta as you just define a link find a link you can find out what is the L 3 dB here, L 3 dB equal to actually pi / 2 times beta s - beta as.

So, if you can solve this propagation constant for symmetric super mode and asymmetric super modes you know what is the length required to get a 3 dB possibility what is the length required to be complete power switching from one waveguide to another waveguide or vice versa. So, that type of L c will be exactly L c then it is like pi / beta s - beta as. Beta s beta as can be also you can define beta as = omega / c or 2pi c / lambda into n effective as and beta as will be omega / c n effective as.

So, if you do more solving of this entire structure then you can get 2 super mode solutions you will be getting also from full vectorial method if you do then you can get symmetric super mode we will have a n effective s and a symmetric super mode n effective as once you know that then you can find out this one and omega / c if you put lambda / 2 lambda by so on. Then you can get also very nicely once you know the effective index provide indices of the symmetric and asymmetric.

Then you can calculate what is the length required for full coupling what is the length required for 3dB coupling and then you can scale the length and try to find out actually, what is that link required to split power with a DGR amount or a DGR splitting ratio you can find out. So, we know that if 2 wave guides are there, if they are interacting each other evanescent tail is reaching to other waveguide.

Then there starts interacting and power can couple through a super mode excitations wherever as long as you are maintaining that type of coupled parallelism parallel waveguides then the super modes are Eigen modes they will be carrying energy individually independently, but when you see the superposition effect is suitable to have certain percentage of power in second waveguide you terminate their braid the orthogonality condition then you get a DGR splitting ratio.

This is actually somehow we can conceptually some mathematical help we took mathematical model super mode excitation etcetera. But we understand that you can really indeed you can design a complete power coupling, directional coupler or 3dB power splitter or maybe 6 dB power splitter that means 6 dB will be that means 25% will be coupled to the second waveguide if that 75% will remain in the first waveguide that so on you can do that you can design the club now.

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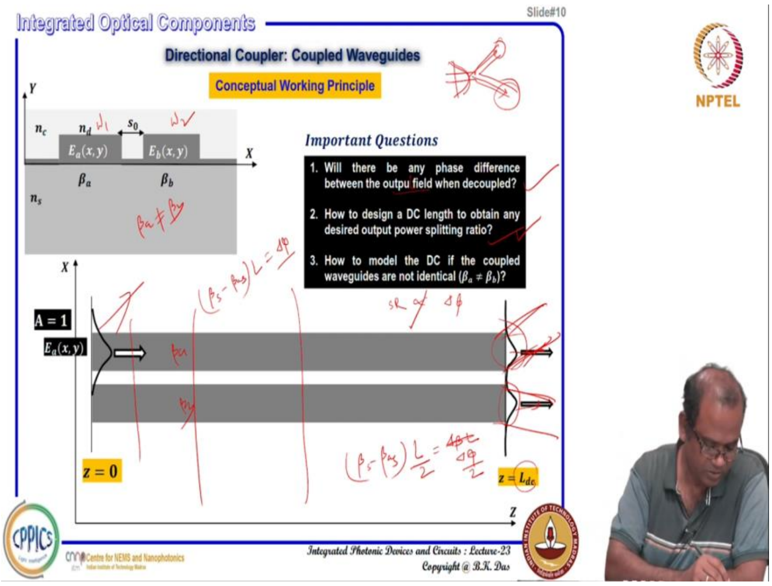

You have certain problem to design if you want to use this type of directional coupler, to use to design a complex circuit, like magenta, or maybe ring resonator or maybe cascaded wave interferometer or more complex circuits we will learn in course of time in that case, we would like to understand a little more just not the power splitting ratio is enough, you need to learn written more, what are those things?

Let us consider will there be any phase difference between the output fields when decoupled? So, when you add just decoupling here, suppose 3 dB power splitter L dc is there some power is here some power is here. Now, question is that I just decoupled here they are originated from the same source. Now when you are separated them what would be their phase difference. That is the interesting point normally in y junction what do you see y junction you see your mode is coming like that.

And it is identical situation here identical situation here with respect to this mode. That is why whatever phase it will be acquired in the junction, the same phase will be acquired in the secondary wave guide, but in this case, if you see some power is retained in the first waveguide and some power actually tunnelled to the secondary waveguide. So, when it is tunnelled to the secondary waveguide, do we expect their phase will be same it may be different.

But that thing we will not be able to learn from the super mode that working principle whatever the conceptual discussion we did so far. That gives us actually pretty good understanding that the early coupled waveguide if it is there, you can really couple light you can tunnel light according to your things, but if you want to use them for not for just splitter, you want to use them for an photonic indicator circuit in that case, normally you have to interface another device, whenever interfacing one device to another device.

You do not work with the power intensity rather you work with the amplitude and phase because integrated photonic circuits entire circuits you can analyse in terms of amplitude because there is a phase relationship that can cause some kind of interference. So, that is one first question that we will not be we need to know that not only the power, because whenever you are just measuring power, you are losing the phase information basically.

Because whatever put a detector if you are just using also just measuring power you cannot recover the phase. Now how to design a dc length to obtain any desired output power splitting ratio. So, suppose I just consider 3dB I need this much and I need to 50 % 25 % I can just cut linearly I can cut is that the right thing that if you just do half and half, if you just make a if this is the L c complete coupling here.

You just make you have 50 % and another half then it will be 25 % and another half then it will be 12.5 % that way it will it works really is that linear that power will be linear according to the phase difference, because normal total phase difference it will be beta s - beta as L that is a total phase difference, if this is supposed delta beta. Now, according to this thing, it is true that phase can be half that is delta beta L.

That is the delta phi, this would be delta phi. So if instead of L, L / 2 then phi will be delta phi / 2. So, you can actually phase linear L, L is a function of length phase is linear, but output power also will be linear. It is not proportional to phase different this power in the output is not at all proportional to the phase. Phase is proportional to length that is 2 beta s - beta as L that is actually phase difference between symmetric that is true.

But this phase; actually not contributing a linear power splitting ratio in the 2 output waveguides the power splitting ratio splitting ratio is not proportional to delta phi it is not. So, that information will not be getting here, what else how to model the dc if the couple of waveguides are not identical so far we have been discussing this is a beta a and this is beta b and then if it is identical.

Then 3 dB splitting or 3 dB splitter pulls switching everything we can design, but sometimes what happens if we want to make what is happening if the waveguides are different for example, this waveguide has been having w 1 and that is having w 2 that means beta a not equal to beta b. So, in that case how to deal mathematically how it was how to design.

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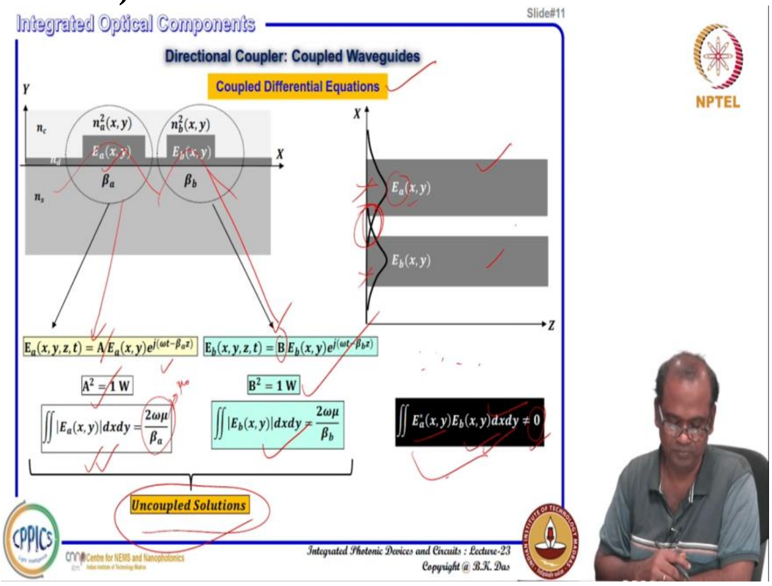

So, we need to understand first we need to develop a coupled differential equation. That is very straight forward, whatever you have learned to demonstrate to derive coupled equations in a single waveguides with multiple modes are excited and when there is a perturbation that can be periodic and so on. We have already developed the coupled equations same method we can use when the 2 modes are belong to 2 different waveguides.

I want to see if they are coupling then how they are interacting how power exchange will be there. It is almost identical, but not exactly whatever we have developed for a coupled mode equation for single waveguide, let us see how it goes individually waveguide A gives this solution individually waveguide B gives this solution A is constant normalisation B is normalisation constant.

A square = 1 watt we can consider B square = 1 watt, they can carry individually 1 watt and normalisation constant, according to the normalisation constant for wave guide A, this is E a star multiplied by E a that will be 2 omega mu / beta a of course, if it is non-magnetic material, you can always constant mu not, that is fine. So, similar would be when they are not coupled, this is good uncoupled solutions, we can write.

And in this case, when they are uncoupled, if you try to get this overlap E a star x y E b x y dx dy in that will be actually it should be equal to 0. Because this mode if they are uncoupled, that means they are overlap whenever E a x y is the E b x y will not be there. So, one of them will be 0 if they are coupled, if you change the coordinate to coordinate then you will be seeing when this is present, the other value is nearly 0.

And when this is present, this value will be nearly 0. So, when they are uncoupled in that case, you will be getting say 0 value, but if they are coupled if some area for example, in this region if let see 2 modes are there, E a x y, E b x y that region, if you see this overlap is not equal to 0. In some area, field a is also present field b is also present. So, both are present here we can say that field a is absent. Here, field b is absent, it E b is absent, but in this region.

Both were present so that means you can get if you integrate - x = - infinity + infinity y = - infinity + infinity you will be getting non 0 values. So, then if you get a non 0 value of these type of things, then you can say that these 2 waveguides are coupled.

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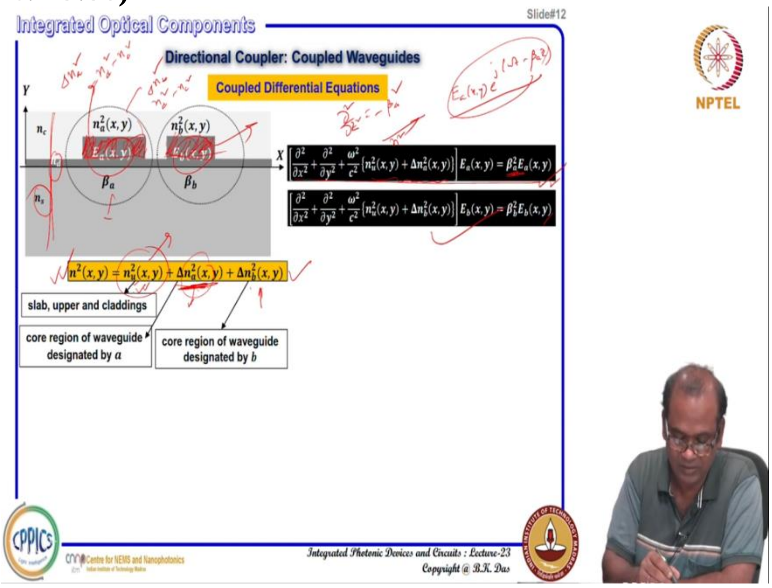

So, for that, what we will do, let us try when they are uncoupled, we can write their solutions differential equation how that is done, let us define the entire structure the cross section of the directional coupler, if you are considering individual their solutions are there E a x y E b x y beta a beta b that is known to you, you can solve this one assuming this is actually cut and again you can solve this one as you will cut then you can get this.

Now, you want to solve the entire structure entire cross section global you want to solve Maxwell's equation for the entire structure. In that case, I can say that the cross sectional refractive index profile can be written like that what is that I am just giving n square u x y n square u x y that means, uncoupled that means, without the waveguide what is the background refractive index profile that gives you the n x n d layer.

That means slab layer and then cover. So, these type of refractive index profile wherever is there that means, this rib is not there, this rib is not there that is actually we are calling as a n u square x y. So, in this region it will be n c in this region it will be n c square n c square it will be there. So, that is actually that type of profile that means the rib other than rib whatever the profile background.

Because only rib gives you the sense that waveguide is present. So, this region actually gives you this region silicon is there that means you have a waveguide this region silicon is there, then you have a secondary wave guide. So, if these 2 regions absent then entire cross section and if you map whatever the refractive index profile you get, that is actually n u square x y. Now, I can consider delta n s square that means, only the additional part that means.

This part the region this region what is the additional refractive index you have n d square – n c square that is actually your delta n a square n d square – n c square only this region that actually that presence actually gives you wave guide a similarly this one we can say that n d square – n c square in this region in this location in this coordinate system in this particular region, that will be that presence of that one gives you the definition of refractive index profile for the waveguide b.

So, that is why I said the background and then additional region whatever the refractive index for waveguide a that is defined here that is for that is indicating waveguide a and that is actually indicating mathematically you can represent that background and if we consider this and if I consider this one equal to 0 that means, I am considering only wave guide b, if I am considering this one equal to 0, that means I am considering only waveguide a.

So, this delta 1 0 means waveguide b only n u square this one waveguide a and this one 0 means this is waveguide b. So, similarly, that means, I can actually form my differential equation for waveguide a for waveguide a we know the solution should be E a x y e to the power j omega t - beta A z that is the waveguide solution more solution for waveguide a when delta n b = 0, so, delta n b = 0.

Normal scholar wave equation if we write del square del x 2 del square del y 2 omega square by c times n n square we will have n squared that means, you know this is the omega / c square by n square that means, what is the refractive index and this side will be the propagation constant. So, that is why I have not put del del z 2 that should be equal to - beta square beta a square.

So, that is what right hand side was beta a square that we have used earlier also for waveguide a if only this mode is present waveguide b is not there absent then this is the differential equation you need to solve and when, individually when waveguide a is not there only waveguide b is there, we have to solve this identical equation for a waveguide b, that is when they are uncoupled as far as they are uncoupled, you can use that one. But I want to see they are coupled and this type of refractive index.

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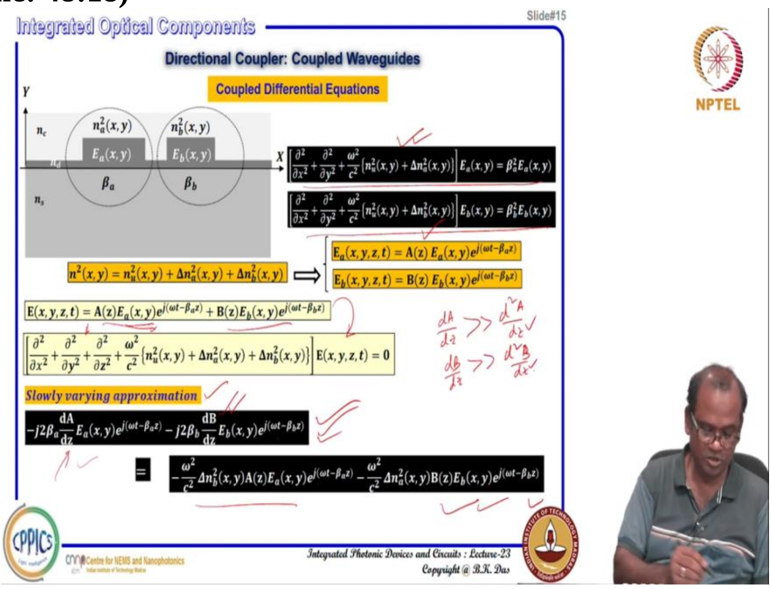

I define, then what I need to know I can define this they are actually interacting that means, the solution I consider earlier that A E a x y e to the power j omega t - beta aj that is actually E a = and E b = B E b x y e to the power j omega t if they are non-identical you can also consider something like this and A square = B square you can consider 1 watt and a = 1, b = 1 we can consider right normalisation constant you can consider.

However, when they are interacting each other you are making the orthogonality condition that means, like couple mode theory we consider that they are longitudinally their amplitudes keep on pairing because they are exchanging they are interacting power can exchange each other. So, you can get longitudinal z directional along with this phase you have longitudinal amplitude evolutions you are having.

similarly B z per waveguide b amplitude evolution you will see in the power direction and we are considering that this they are coming so, close that the perturbation of the second due to presence of second waveguide it is not really very high we are considering it is a weakly coupled we can consider both the waveguides are weakly coupled that means, the evanescent tail is almost going to 0 and there you see the presence of the secondary wave guide.

So, in that case this is the situation I can say that what is the total field in the system I can express that total field this one z dependent variation of the waveguide A z dependent variation of the waveguide B their superposition I can consider as the field strength and corresponding differential equation if I want to use that as a solution corresponding differential equation instead of just waveguide A.

You are considering this refractive index waveguide B you are considering this refractive index, we are considering presence of both the wave guide in the wave equation in the both the wave guide we are considering wave equation and this is the field and in that case you have beta a and beta b there combination is there along with that that is the z dependent phase but it can happen that this B z is an A z that can also be complex and that complex value can be adding here.

So, beta a and beta b will be modified then. So, z dependent function is something different now. So, I cannot write beta a square beta b square like individual Eigen mode solutions like a waveguide A and waveguide B rather I keep del to del z to here and omega square / c square epsilon r and total field I am considering where is same as a couple mode equations we have developed same process just we are just restricting ourselves that there are 2 modes and 2 waveguides.
There, they belong to 2 different waveguides that is what that is the difference. So, now, slowly varying approximation, if I just use here then I have to same way I have to find out delta 2 E by del x 2 that means only x dependent function delta 2 y / 2 del 2 same function del

to del 2 you have this term this term we will be getting so, you can consider that same way dA / dz should be much greater than d 2 A / dj 2 second order derivative is very small because the amplitude is evolving very slowly.

So, if you just go for second derivative, it will be approximately equal to 0 and similarly dB dz that is much higher than d2 d / dz 2. So, you use the same way whatever you used for coupled differential equation for a single waveguide having supporting multiple modes the same way if you do that and you come up with it with a simple few algebra you can use of course, some point you will be getting also this factor also will be appearing whenever you are substituting this one and little bit simplifying.

Then this type of factor will be there, then all this term will be cancelling each other because some portion we will be getting here some portion will be like this and that will be actually you can cancel because these minus this that will be 0 we will be putting and then ultimately we will be getting one equation like this side left hand side and right hand side you will be getting like that.

So, little bit of algebra I just kept you can do by yourself just not necessary, because only it will take lengthy equations only, but straight forward, you will be getting this 2 type of equation straight forward. So far, so good so, left hand side you see the evolution term of the field dA / dz in waveguide A also it is coupled another term is their evolution of the waveguide B and right hand side you see some kind of perturbation term delta n b square.

Because of the waveguide presence delta n b waveguide present delta n square this term says there so, this is something we get in this type of situation it is considered slowly varying approximation in this type of consideration in coupled mode equation derivation what we consider we used one trick dropping factors j omega t, because every term if you see here, every j omega t j omega t j omega t is there. So, you just dropped that one.

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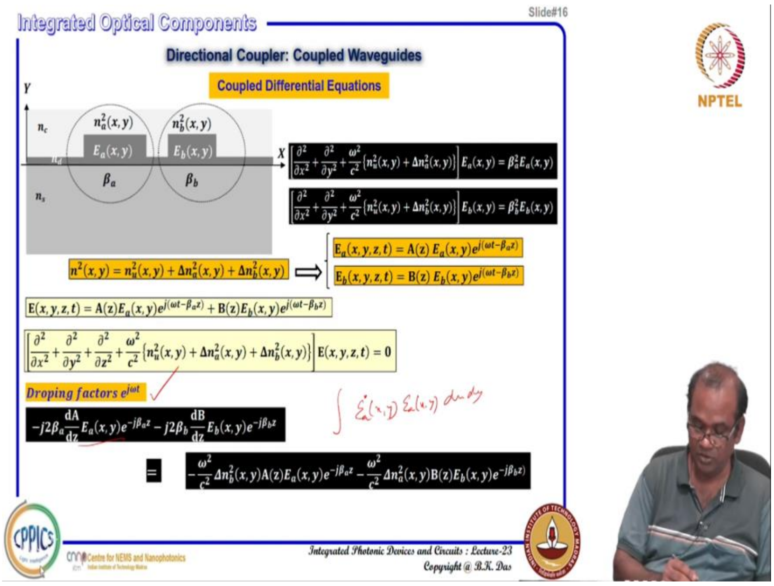

That will simplify a bit and then what you do multiplying E a star x y e to the power j beta z, so, E a x y e to the power - j beta a z complex conjugate of this one you multiply both side and integrate over waveguide cross section xy entire structure we get this one first term. So, if you see first term this one E a star x y means, this will be coming like additional E a x y star E a x y and E to the power + z beta a – z beta b that will be cancelled dx dy that is the first term and second term E a star E b E a star E b that is what you are getting E a star E b, E a star E b here.

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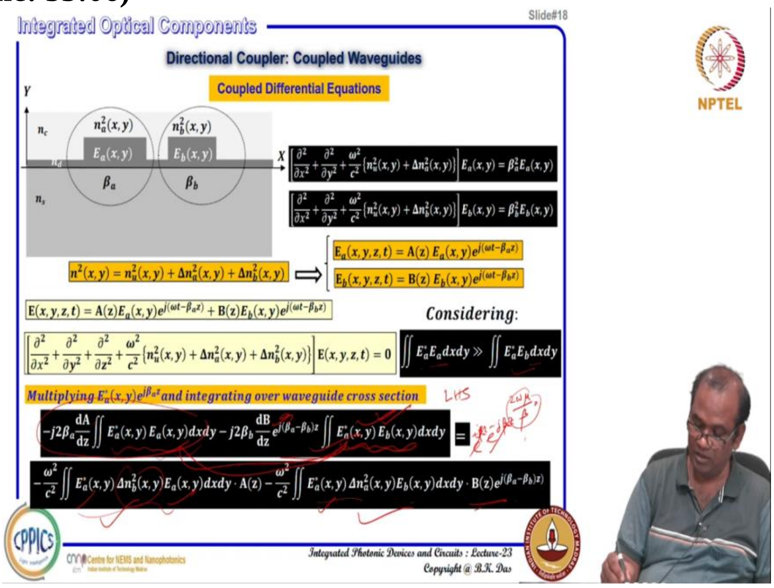

And right hand side also E a star you multiplied here E a star will be multiplied just straightforward you just multiply this one and integrating over waveguide cross section that is it. So, left hand side this is equal right hand side. So, this is your left hand side and this is your right hand side. Now, what is the here if you see this term and this term if you compare left hand side just compare the left hand side.

Only concentrate on the left hand side 2 terms are there in these 2 terms if you see E a star E a that means waveguide mode in waveguide A that is actually you know that should be first term will be 2 omega mu / beta that is known to you if they are normalised A is initially normalised to 1 watt waveguide A. So, that should be 2 omega mu / beta mu not you can put simply, but here this one if you see E a x y.

That means profile of waveguide A and profile of waveguide B. So, if this is non 0 no doubt about it, but you compare with this whatever value you will get that wherever sometimes some majority region you will be getting E a = 0 majority region you will be getting where E b is there E a 0 where E b will be 0, but E a star, E a star, so, that is why this comparing this to only we just considered this one E a star, E a is much greater than this one.

So, we are just ignoring with respect to this one entire structure we can ignore this one we can concentrate this one then we can see how it is evolving only A I can forget about B and right hand side I kept as it is whatever is there I just kept A z first term you do not have any exponential term because E to the power j – beta a and e to the power – j beta a y is there z is there and you multiply j to the power b beta a z.

So, they will be cancelling and star will be there E a star a is there and this E a star E b and obviously you see that now, this is slightly different E a, E a in between delta n b square is there then you are integrating. So, you cannot just simply use your orthogonality condition 2 omega mu / beta because in between delta n B square is there E a star, E a star but delta n b square is there.

So, it is orthogonality condition omega mu you cannot write if this was is one then fine you could write now, right hand side you see this one. So, you just simplified it now, after removing this one that ignoring that part.

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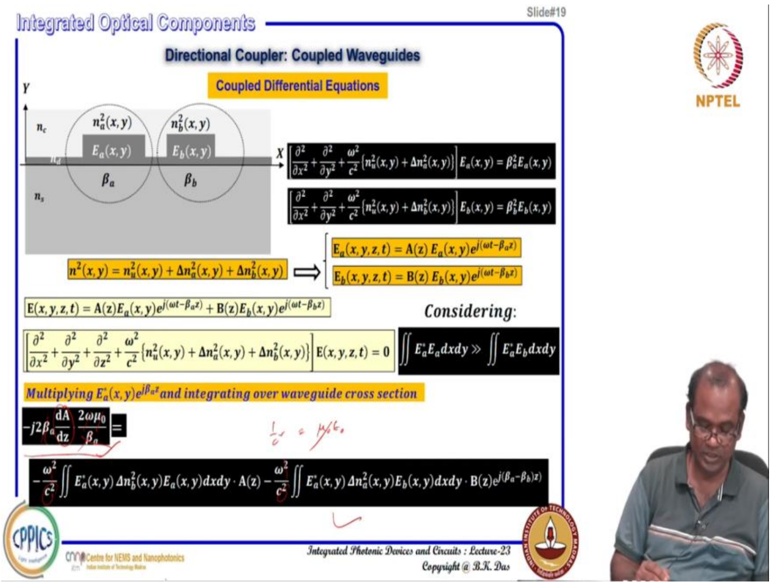

And applying the 2 omega mu not / beta a left hand side will reduce first term only. So, that means, what you have done you have just put to omega mu not / beta times this one and this one you removed, then you are getting this one left hand side. Right hand side I kept as it is. Now, left hand side I want to keep this one and this beta a beta a cancel and c square, c square means 1 / c square you know that is actually nothing but mu 0 epsilon 0. That means the left hand side mu 0 right hand side mu 0 will be cancelled.

So and this omega square omega square will cancel 1 omega. So, that means, right hand side will be omega and 2 2 times 4 beta, beta we have to cancel - j will come they are in the denominator + j.

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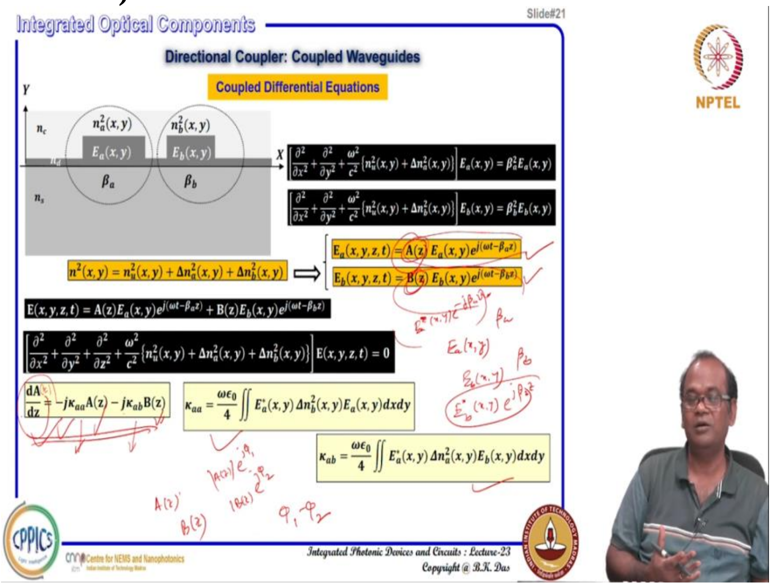

So, we can get this thing left hand side only I have left da / dz and right hand side I simplify then you are getting - j omega epsilon 0 / 4 this one and omega epsilon 0 / 4 this one and along with that you have this term also. Now, you know they are also in case of a coupled mode equation for single waveguide you have defined kappa. So, here also we can define a kappa, but there are 2 different types of kappa is there you see E a E b in between delta n b square E a in between delta n b square.

That means, it is a scattering of mode A because of the presence of second waveguide this is what it means. Scattering of waveguide A because of the presence of secondary waveguide delta n b square and this is actually scattering of b something adding to E a because of the presence of b field b. So, we define this one as a kappa aa why aa I have just taken these a and these a and in between when both are a in between you have to see the waveguide presence of delta n b that is what so, that is how the symbolised it is symbolised.

I define this one kappa aa and this one a and b when you are writing kappa ab you have to see refractive index of the waveguide a when you are considering both are a then refractive index of the secondary waveguide you have to consider waveguide b diffractive index perturbation of the secondary waveguide you have considered that is what kappa a and this is kappa b then if I this can be computed easily once you know the mode field distribution and then you can easily calculate delta n b square.

What is the presence of field structure there delta n square then you can calculate easily. So, once you know this calculation, this calculation your equation is very simple now, omega is frequency what is that we can write like this is. You see this is the kappa aa z kappa bb z that means, I have related a coupled equation of related to amplitude of the waveguide a this obviously, this is a function of z and kappa a is defined like.

That kappa ab is defined like that obviously, you should know that I have I know how to calculate using full vectorial method E b x y they are known and also we know that what is the propagation constant for waveguide a what is the propagation constant of b. Then I have defined that the mode field in waveguide a is like this mode field in waveguide b is like that, that is A z B z I need to solve then I can actually get every point along the direction what is the field strength.

Total field strength associated with the waveguide a what is the field strength associated my intention is to find out A z and B z yet we have to solve A z and B z depending on the kappa aa and kappa ab kappa aa and kappa ab. We know that and because of that presence because of perturbation presence, I can I will be able to predict what is the field and obviously, if I know that A z and B z their solutions will be complex that can be complex. So, according to this equation, you see they can be complex, this is A z complex.

So, once you know the complex value the complex relationship then you can write something A z something E to the power j phi 1 and this can be written as B z E to the power z phi 2, then I can find out what is the phase difference between them that is the ultimate goal phi 1 - phi 2 is the phase relationship. So, 2 waveguide when you are in separating at a particular point, I know what is amplitude I will be able to know the phase also that is why coupled mode equation is declared you cannot just simply use your super mode.

Principle working principle knowledge to find out what the phase is between the out coupled wave guide outgoing waveguide filed. So, this is what I have derived that evolution of amplitude in waveguide A, but it involves what is the field strength in waveguide A as well as waveguide B. Now, again, I need another equation for B z what I will do instead of multiplying E a star x y e to the power - j beta a z this 1 star and integrating what I will be doing.

The original differential equations we have got E v star you multiply and you multiply E to the power j beta B z. Then you will be getting another set of similar equation but belongs to that will give you the evolution of the waveguide that is B evolution. So, then I can get this type of equations.

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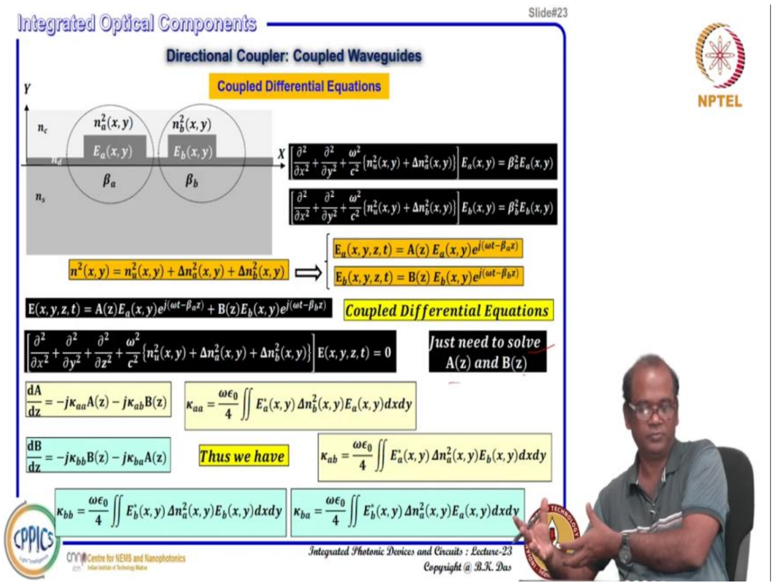

And say another set of equations we will be getting here I will be kappa aa you will be getting kappa bb here I am writing kappa ab you will be getting kappa ba some otherwise same. So, you get kappa bb similar types of kappa ac E a delta n b square E b E b delta delta n a square that is delta n b and this is delta n a because bb is there that is kappa bb. So, bb because of the delta n as and b a because of the presence of del n as square.

You see that is actually kappa ab and this is actually kappa ba b star a star so, a and b they are actually symbols will be interchange one mistake is here, this would be because a will become b and this b will become a, this a will become b also. So, if you see derive that one that will be also b that was an intentional mistake, so, that I can discuss one thing here. So, this would be delta n b square.

Thus we have 2 equations 2 coupled equation this can be computed this can be computed this can be computed this can be computed. Now, only thing is that in this equation kappa a known kappa ab known kappa bb will be known kappa ba unknown normally we can say that this kappa ab = kappa ba star you can get also and if it is identical then you can write also kappa aa star = kappa bb star also identical waveguide also in case.

Normal case this this thing always complex conjugate ABC they are actually complex conjugate almost nearly identical of course, this is ABC this is identical this will be the there because delta n squared delta b b square here this would be also identical then only you can see this one. So, couple differential equations we have we have this coupled differential equation and we can we just make to solve A z B z.

Once we know resolve A z B z as a function z what is the field strength what is the phase relationship, everything will be known. Now we will try to next lecture I will try to solve A z B z and of course, we will be considering that this waveguide are not identical first and from there we can just see if that is identical what is the consequence if not identical, what is the consequence. So, accordingly depending on the consequence I can design according to our design specifications, what do we need? I stopped here for this lecture. Thank you.