Integrated Photonics Devices and Circuits
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Lecture – 13
Optical Waveguide: Theory and Design: Guided Mode Solutions for Slab Waveguides

Hi welcome so, we will continue discuss on guided mode solutions for slab waveguides.

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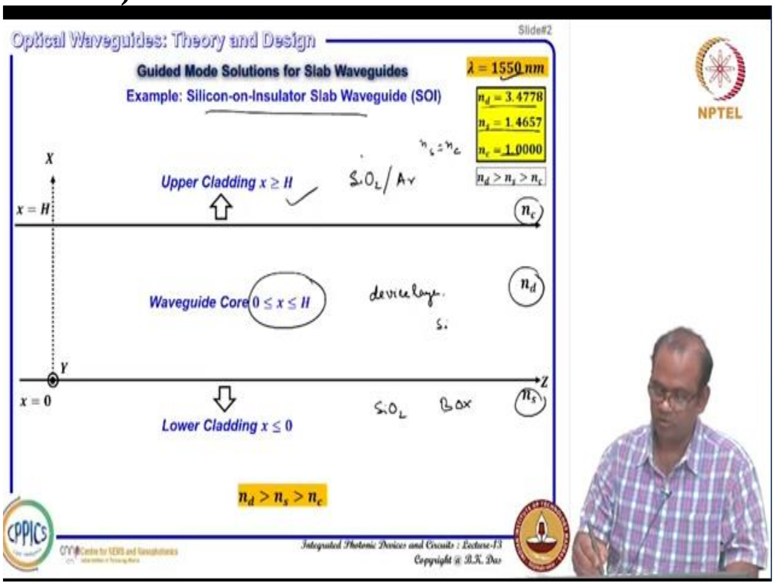

So, same reference we have this is your z axis, x axis, y axis just you consider as the perpendicular to the screen. Now, this region from x greater than 0 to H we are considering waveguide core, in case of silicon on insulator, we call that as a device layer and upper cladding it can be silicon dioxide or Ar that is greater than H and this is typically silicon dioxide, we call it as a barium oxide, refractive index n s and device layer refractive index n d.

Typically this is silicon basically crystalline silicon and n c is the refractive index of the top cladding n c defined. So, it is obvious that at any wavelength particularly you are considering lambda = 1550 nanometres that is the communication wavelength and third generation communication wavelength at that particular wavelength, the device layer refractive index is 3.4778 and silicon dioxide or box layer we call it as n s is refractive index at 1550 nanometres as well 1.4657. And top cladding in this case we are considering Ar.

So that I can have asymmetric cladding in general any point you can put n s = n c also you can cover can be oxide also so far so good.

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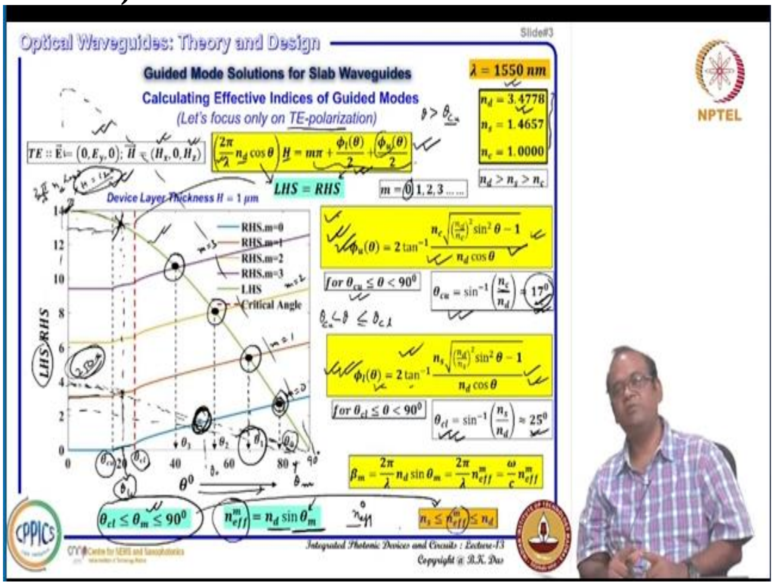

So then, we discussed the how to solve by using the ray optics model we could arrive to this so called Eigen solution. And we in this discussion today's discussion specifically, we will be discussing only on TE polarization we know that there are 2 types of polarization TE and TM specifically for TE polarization Y component will be dominating and magnetic field will be in the z plane in the plane of the screen we have seen earlier figure and that is why you have H x component, H z component.

So that is how we will be calling TE and TM polarization also, you can because they are basically identical situation. So, we do not need to worry about just concentrate focus on only one polarization and understand everything then we can go for we can easily extract what will be the result for the TM polarization. So, and in this case you see this n d refractive index coming like a device layer and lambda is the operating wavelength and H is the device layer thickness as mentioned.

And phi l is the phase change happens at the lower interface when this theta is greater than lower critical angle. So, lower interface critical angle theta cu and this theta must be greater than theta cu then you get certain kind of polarization that is phi u I am talking about phi u is the upper and phi l is the lower phase because of the total internal reflection and they are defined phi u upper interface you can define for TE polarization this one and for TE polarization again lower interface it is as a function of theta.

But of course, whenever I am talking about upper interface, then theta must be less than 90 degree but greater than the critical angle at the upper interface then whatever phase change takes place in the upper interface, this is the case and for lower interface similar expression only thing is that n s and n c will be interchanged. So, here also we see that theta must be greater than the critical angle at the lower interface.

For our silicon operating at Silicon on insulator operating at 1550 nanometre, if we consider these refractive indexes then critical angle at the upper interface is 17 degree if we just calculate simple formula Snell's law coming out of cu Snell's law. So, theta cu = sin inverse of n c divided by n d so, that is the critical angle for SOI slab waveguide or the upper interface when upper cladding is Ar, n c I have considered here one, n d are here.

So, similarly for lower cladding the critical angle will be n s / n d. So, this is again since n s is higher than n c here. So, this lower interface the critical angle will be higher than the upper interface critical angle here we find this 25 degree, approximately 25 degrees less than 25 this is also slightly less than 70 degree we just rounded it. So, now we know that we have to have some solutions for theta and this theta solution come this is transcendental equation both sin theta is there phi l theta expression is this one and phi u theta expression is this one.

So, both sides theta is there and if I consider this m value that can run from integer 0, 1, 2, 3 so on and once we put 0 we get one solution once you get 1, we get another solution once we get 2 another solution and so on. How we did her? So, we try to consider this one is left hand side and entire 3 terms that are the right hand side, we can plot the left hand side as a function of theta and right hand side also function of theta for each value of n, then intersections will be the solution.

For example, left hand side if I plot here this is the curve this is a cosine function as a function of theta this is your theta in degree per swing 0 to 90 degree this is your 90 degree. Then you see the left hand side will be cosine function at theta = 0 that will maximum, so this value is nothing but whatever value you get 2pi / lambda n d and cos 0 and H in this case we consider about 1 micrometre multiplied by H, so 1 micrometre whatever value comes.

And lambda if we put 1550 nanometre, whatever value comes, this is the value y axis, it is coming left hand side I am just talking. Then cosine function or the function of theta it will be reducing to 0 at 90 degree. So, the left hand side is fine and the right hand side what we can get? We can get a series of expressions, series of curves just by changing m values m = 0 we get this one this expression this is m = 0 that means this first term will be 0 whatever value comes here that is the total value we are considering here.

And m = 1 this one m = 1 it is written here, this is actually m = 2, this is actually m = 3 so, we have 4 curves m = 0, 1, 2, 3 that is there are 4 curves and we clearly see that the left hand side and right hand side is intersecting at this point that means when m = 0. So, corresponding theta is that theta naught that is the one solution. Similarly, whenever m 1 we get this curve, so we get corresponding theta value solution theta 1 here, m = 2 corresponding theta 2 value, and m = 3 corresponding theta 3 value.

And in this curve you have seen here we have shown where is the position of theta c l and where is the position of theta cu. Theta cu 17 degree theta cl 25 degrees so, 25 degree between 20 here somewhere theta cl and theta cu and if you see this expression this phi l expression comes when theta greater than theta cl. So, greater than theta cl we are adding this one, but in between theta when starting from theta is greater than theta cu to theta cl only this expression right hand side will be 0, this one will be 0.

So, in this range when theta cu less than theta less than theta cl in this range, because this theta is less than theta cl so, this will be 0 this phi l part will be 0. So, only this part so, we get only this part corresponding to upper interface. So, that phase and once theta cos theta cl then you will be adding one value here and values also here both the values will be adding that is why you get a kink here.

So, exactly our theta = theta cl that time actually you are adding the phi l term, that is why you are getting a kink and before that you have only this term. So, everywhere when m = 1 that means you are adding phi here phi to this curve so, you are getting this one, this m = 2 again 2pi, 3pi, so, we are getting. So, what I understand that for H = 1 micrometre we have one solution for m = 0, second solution for m = 1, third solution for m = 2, forth solution m = 3, we could solve one more m = 4.

But in that case, if I just add maybe somewhere this one you are getting somewhere here, you will be getting something like this type of curve. So, what we get this solution, if you see this solution coming is lower than this whatever theta value theta 4 would be coming that is lower than the theta cl. So, that means at that particular angle, you would not see total internal reflection or the lower interface. So, that is not a guiding mode that is why we rejected. So, we clearly find that there are 4 solutions are there, 4 theta naught; solutions will be there.

So that means, there are 4 modes will be you will find 4 modes for different types of standing wave will be generated along the vertical direction in the slab waveguide we have soon here. So, you could do that thing, why you need to consider H = 1 micrometre, you could increase more if H you are increasing. So, this curve will be coming somewhere here it will be starting from somewhere here. So that means, you would add more number of modes. So, thicknesses more means, you are allowing more number of modes.

And if you can reduce the thickness, you can actually reduce the number of modes. For example, if you consider this is suppose somewhere putting suppose you are considered say 4 times lower. So, about 3 point something somewhere here you are considering. So, H equal to about 250 nanometres for example, if you are considering 0.254 times, then this value will be 4 times lower. So, you will be getting some curve left hand side and will be like this.

So, left hand curve will be this, then you would be getting one solutions here that is corresponding to your theta naught. And next solution you would have been getting here that is actually lower than the critical angle. So, if you are considering 250 nanometre device layer thickness, then you would end up only one solution here only one mode will be supported. So, that is the reason whenever you are talking about silicon photonics on silicon on insulator platform.

You consider around 250 nanometre device layer thickness where you can actually at least support one mode particular mode and if it is lower than that also you will be getting some mode that is also find lower than 250 nanometre also you will be getting solutions also. So, with this once we know these solutions, all the theta naught values theta m we are finding theta m must be somewhere greater than theta critical angle or the lower interface between 90.

So that is what we mentioned here and we know we have defined n m effective index of the guided mode for the n mth solution n effective m is nothing but in the n d sin theta m. So, 1 you put theta value then you get effective index of the m = 0 if you put theta naught then corresponding you will be getting n effective 0, m = 1 theta 1 solutions you are getting you will be getting n effective 1 something like that.

And since this is the condition holds, we find they are directly theta cl if you just consider sin inverse something like that n s / n d, if you are putting that then you are getting n effective a must be less than or equal to n d or greater than or equal to n s. So, effective initiation of the guided mode when you obtain that must be less than refractive index of the device layer, but greater than the refractive index of the substrate or box layer that is what we have shown here, that is the substrate n d.

So, you get a mode propagating that propagating mode we will see effective index that will be less than n d and n effective and it should be greater than n s somewhere, n s solutions you are getting less than n s that means the light will be not confined it will be leaking that is what our understanding so far.

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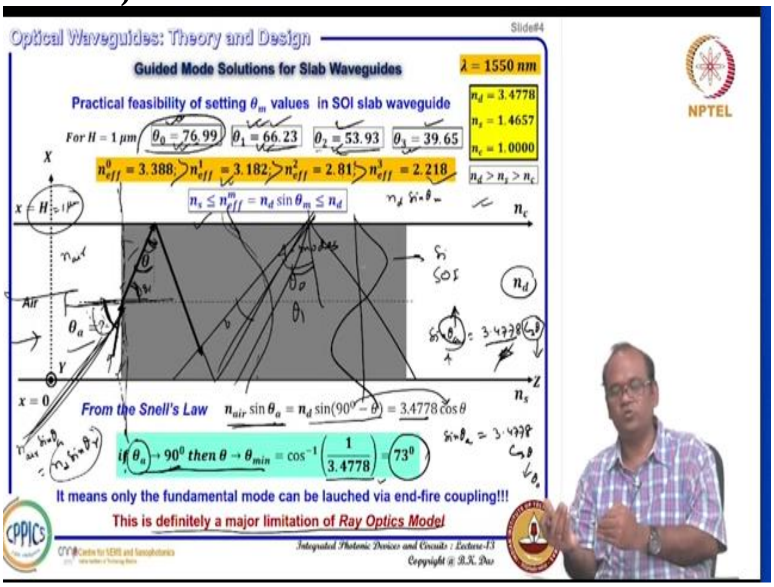

Now let us put down the numbers, theta naught value whatever solutions we get, according to their; this is 76.99 degree, theta 1 is lower as you see here for higher order mode theta naught is highest theta 1. So that means I can say that theta naught is higher than theta 1, it is higher than theta 2 and theta 3. So, that means n effective not will be higher and n effective 1 will be next higher and so on. So, accordingly all the theta values are there and it is descending order they are higher order modes.

And if you calculate n effective that is nothing but n d sin theta m. So, this is theta m we were putting n effective naught you are getting and then you are putting this value then you are getting this value. So, theta value reducing an effective value will be reducing, theta solutions will be reducing n effective will be reducing. So, different mode will have different refractive indexes, of course this is greater than this one greater than this one then this is so, as you go for higher order modes.

You can see that effective indexes will be dropping that is one thing we understand now, next thing so, here I want to discuss a very nice thing suppose, we know this is the H equal to suppose 1 micrometre device layer, this is your device layer, this is a silicon on insulator, it is a box below and top is the cladding something like that. So, this is some portion I am just showing the silicon or SOI silicon on insulator we are showing that is where refractive indexes n d.

And if I just consider that your wafer or device is ending here, this is the end point you clipped the side left hand side you clipped you palest, then right hand side you can say that your substrate is there. So, in that case it is possible that there are 4 different modes will be supported 4 modes will be supported and each of the modes they will give some kind of fundamental mode for example, that this type of profile and higher order mode it will be like these and something like this one we getting. So, higher order mode will be like this.

Suppose you consider like this, higher order mode will be like this, this is a fundamental mode and so, on modes will be there, but thing is that each mode actually assigned in ray optics picture this is standing where we are saying in the lateral direction, but in standing a picture you will be getting some kind of full distribution in the x direction, but we have a solid concrete solution for theta where you see that some kind of total internal reflection will be taking place for a certain mode that means theta naught.

If it is theta naught then it will be fundamental mode, but if it is coming like this theta 1 lidless. So, theta 1 is like this, this is your theta 1 then it is a higher order mode and so on. But, one interesting point you just consider suppose you are launching light from the side and you are interested to set one of these angle theta naught or theta 1, theta 2 and theta 3. Obviously, if I consider our ray optics picture, suppose I am assigning in this direction with a theta a angle.

Then what happens this is the interface and in the interface this is your normal and this is your incident angle for example, then what is the refracted angle that I will get from the Snell's law n suppose n air this side your refractive index is n air, n air sin theta a is suppose this is something like that theta r = n d sin theta r. So, this theta r is decided by the angle of incident theta a here and theta r obviously is nothing but 90 - theta suppose theta I have defined 90 - theta. So, that is why we had written this Snell's law n air sin theta a n d sin 90 – theta.

So, now if we put this in 90 - theta means cos theta comes cos theta so, that means, I have n air refractive index is 1. So, that means sin theta a = n d = 3.474778 that is what we have written here and cos theta so, this is the expression. Now if I try to calculate suppose I want to get a theta naught whether it is possible or not and it is clear that from this expression, so when I write sin theta a better it is written here 3.4778 cos theta, so this is a constant.

Now theta a is more than this cos theta term when theta a is more according to this relationship, this theta will be less theta a if you increasing, theta will be decreasing if this angle is keep on increasing this theta a this angle according to this Snell's law that will be decreasing. What is the maximum possible value of theta a? Maximum possible value of theta is 90 degree. So, theta a can be 0 to 90 degree.

Because I can sign directly and I can increase increasing I can go 90 degree theta can be maximum 90 degree. So, when theta a is maximum whatever solution I will get from Snell's law theta will be when theta is maximum then theta will be minimum that is what I have written if theta a is 90 degree maximum then theta will be minimum and what is that minimum value let us consider according to this expression theta 90 degree we are putting.

So, this theta value will be cos inverse 1 over 3.4778 according to this expression writing and that is 73 degree. So, I can consider theta a value as a maximum as 90 degree as high as 90 degree not more than that and I know that higher the theta a lower will be the theta value. So, highest will be the theta x value lowest will be the theta value highest value 90 degree. So, lowest value of theta is 73 degrees. So, I can have if I shine a light from here to create some total internal reflection in the both interface.

So that light can enter and then once it enters it should undergo some total internal reflections but practically what we see that this theta cannot be less than 73 degree. So, it will be a minimum value is 73 degree when minimum value is 73 degree can we achieve 66 degree? No, can we achieve 53 degree theta 2? No, can we achieve theta 3? No, you will be only achieving theta naught so, that is a puzzle, your device layer that can support 4 modes according to ray optics model.

But what do you see practically, if you are launching at an angle from the air, so that you can excite selectively some theta that is the particular theta value, it can actually support some guided mode according to our ray optics model. But unfortunately in this example, we cannot actually excite harrowed our modes in spite of their solutions are there. So, this is something it is a limitation of it is kind of I mentioned here, this is definitely a major limitation of ray optics model.

In principle in practice, we can coupled all the modes if you shine light, you can couple laser light if you sign from this side, it is obviously possible you can excite all the higher order modes, but whenever you are shining from the left whatever angle you get, bringing your light, it can be diverging source, it can be some Gaussian or whatever, whatever k wave vector is there from the source. It cannot be theta a this angle cannot be greater than 90 it will be between 0 to 90 degree plus minus 90.

If it is plus minus 0 to 90 degree, the maximum possible angle you can get 73 degree of course, the 73 degree here whatever you get that is higher than the critical angle both at the upper interface or the lower interface, total internal reflection takes place, but those values all the angles will not give you standing wave only selective angles. So theta not be possible somehow you can change you can reduce a bit this theta a value then instead of theta maximum 73 degree it will be a little bit higher 76.9 degrees.

So, this is the minimum you can little bit make higher than 73 degrees 76 degrees that is possible this is for a certain example, if you just consider your H that device layer thickness is different the scenario will be different. So, in that case I would say that ray optics model we can explain that there are modes there are effective indexes and propagation constant and they will be having different phase velocity they can propagate support everything fine.

But, if you just consider how to couple all these modes, this ray optics model that is actually something is difficult to achieve excitation, but practically it is possible that is the reason this ray optics model is failing for explaining everything. First of all, this somehow you can think of that whenever you are launching a light you are considering a plane wave is coming and plane wave is going it is a macroscopically homogeneous medium that is why it is a coupling happening Snell's law, Maxwell's equation everything considered.

But ultimately what happens practically you can think of well electromagnetic wave comes from air switch space from another medium and reaching here. In this medium light matter interaction happens actually internal constituent of the atoms molecules, they actually respond to your incoming electromagnetic wave that starts their electrons are kind of thing electrons will be a little bit loosely attached to all nucleus.

And they will start oscillating and that oscillation that means electron oscillation around a nucleus that will create some kind of dipole and those dipole oscillations they will be depending on their orientation, they will actually radiate like antenna electromagnetic wave all around all directions inside the material. So, you can in principle get all the theta angles inside because of this scattering dipole antenna or light scattering inside the k star that can be considered.

Simple ray optics model if you consider you will not be able to explain whether higher order modes will be excited or not in this particular example, that is one kind of limitation I wanted to explain. So, we need some kind of alternative solutions. Alternative solution is it is better to again go back and consult with your Maxwell's equations and see if there are some additional thing you can think of consider so that we can explain all the modes how some more solutions as well as how light will be coupled directly into the all the excited modes all the supported modes. So, this is one important thing you should keep in mind.

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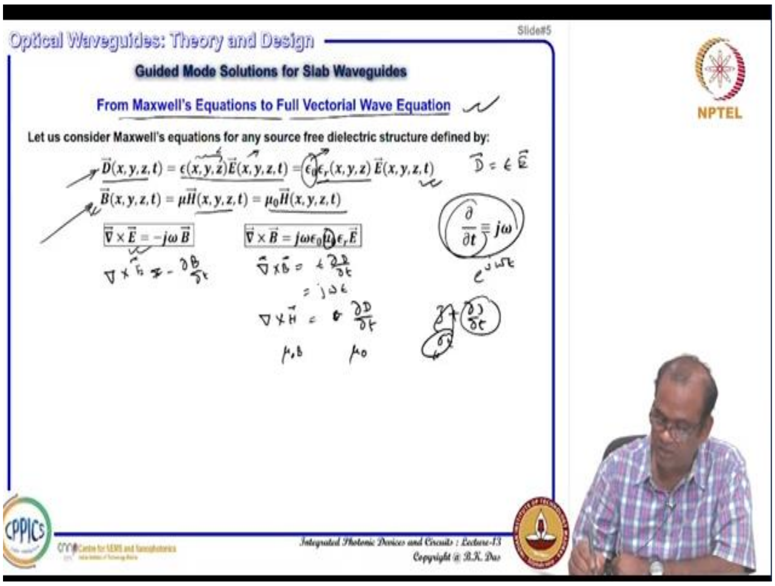

So, as I mentioned that we have to go back to your from Maxwell's equations and from Maxwell's equations to full vectorial wave equations, we just try to derive I will just go through very quickly because I assume that all of you, when you take this course, you have some kind of basic knowledge of Maxwell's equations, I have discussed already earlier, I just repeat again that when you consider Maxwell's equation, you have to consider displacement vector and you have to consider magnetic field induction, magnetic field strength B.

And they are related with a material medium, if it is epsilon is the dielectric also here in this case, we will be considering this epsilon as a permittivity and that permittivity can be space dependent. And that permittivity actually will be associated with the electric field. And electric field if it is also space dependent x y z and time dependent and we consider at this point that epsilon is the just only space dependent not time dependent thing, the simple constraints and we consider here then D = epsilon E basically.

When E is x, y, z, t I can consider D also x, y, z, t that is nothing else. And then this is permittivity if we just consider epsilon 0 as the constant permittivity for the free space, then with related to free space, what is the permittivity of the medium we just scale it like epsilon r called the relative permittivity or dielectric constant. And basically, since this is a free space thing and you can assign epsilon r as a space dependent and this one, so, this is D equal to basically we write D = epsilon E that is what I have written.

And similarly, B = mu H and mu naught H because most of the optical waveguide dielectric medium we consider they are nonmagnetic. And if we just consider like curl E = -del B / del t del B del t we know that all the electric field and magnetic field if they are just oscillating with e to the power j omega t. So, del del t operator we can just simply write j omega.

So, first curl equation we write curl E = - j omega B and another is curl B dielectric medium sigma = 0 you considering that is actually epsilon del B del t time dependent del del t and D = epsilon. So, we can write j omega epsilon and curl B is equal to I think this is one mistake is there, this mu 0 should not be there, epsilon del D del t not normally that is curl H that is corrected then mu 0 will be there.

So, basically we know curl H = epsilon and del D del t, J + del J del t. So, J means sigma E sigma E = 0 that is why consider that is not there only del D del t. So, del D del t del del t omega, so, whenever I m writing B here, that means, I am just multiplying mu naught this side. So, this side also I have to multiply mu naught that is why mu naught is there, so, j omega comes because of the del D del t and epsilon E this expression comes.

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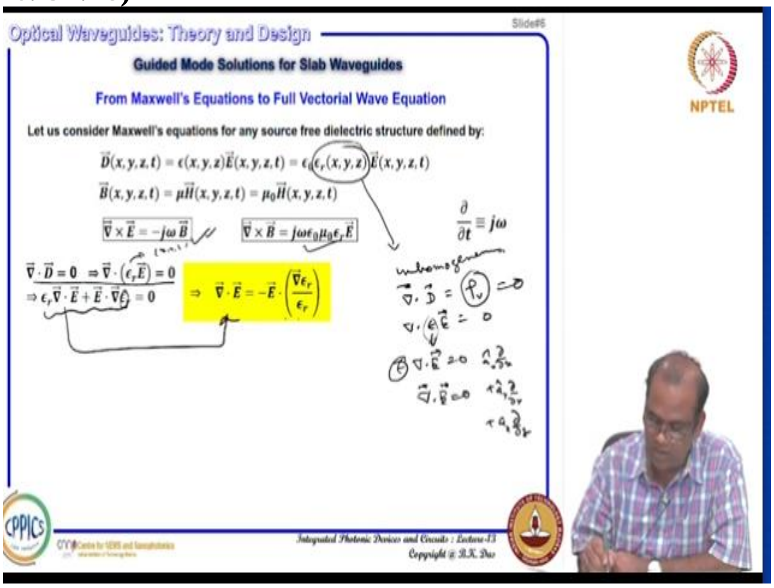

Little bit face it make it face so, I have this one curl B and this is the curl A now, we assumed that for waveguide consideration, this epsilon r it x, y, z dependent This is not homogeneous medium, I can consider this is inhomogeneous somehow because for waveguide you know 3 layer structures are there. So, I can consider homogeneous structure at least in x direction in 1d waveguide in inhomogeneous structure.

So, simply divergence D we know that is actually rho v, if it is charged free then we consider 0 and divergence D epsilon E it is 0, if epsilon is constant I could take del means normal you know del del x + a y del del y + a z del del z you means partial derivative required if this is a constant homogeneous medium then epsilon could be taken outside then you could get epsilon del E = 0 we could easily write divergence equal to 0.

Because epsilon is a factor that would; be 0, but in this case because we will be considering waveguide structure etcetera. So, epsilon r will be space dependent etcetera will be there. So, that is the reason we cannot take this epsilon outside instead rather we write like this epsilon r inside and if it is inside suppose a function disappeared function this is also a x, y, z dependent and E also x, y, z dependent then we can just write from the vector identity easily we can write this one.

So, epsilon r divergence of E + E dot del epsilon r you know any scalar quantity, if it is a space dependent if you take a gradient that will become vector. So, E dot del epsilon 0 so, that is equal to 0 and from here if you just try to find out this we will be getting easily divergence of E equal to say E dot del epsilon 0 / r we are writing like this. Let us clean a bit.

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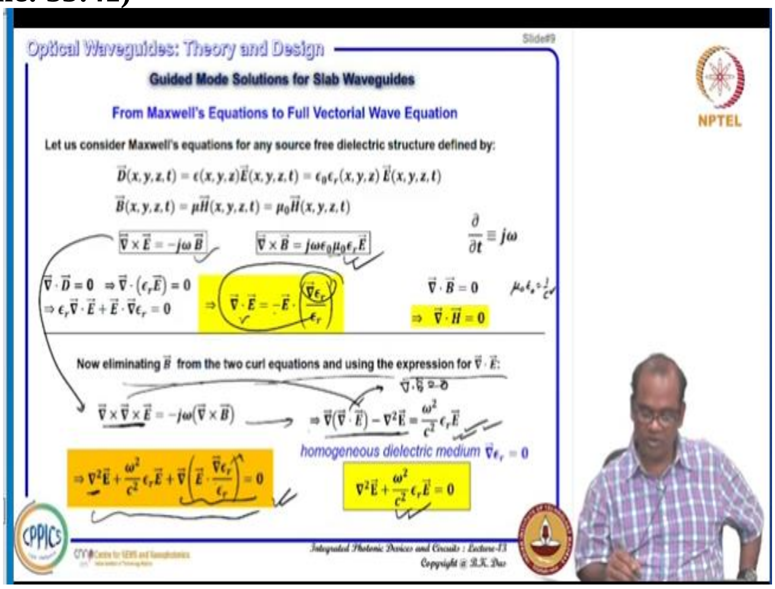

Now another thing also you can just think of divergence B = 0 always, so sensitive is a nonmagnetic material mean mu r = 1 mu r can be taken out. So, divergence H = 0 so, you go back. So now, we check what we get from this expression, this is the expression I take one more curl both side that is what we know taking curl from both sides we try to decouple. So, now we are eliminating B from the 2 curl equation and using the expression this one this expression will be using.

So, how to eliminate we have to take curl because if you take all this side will be curl of B this curl of B can be replaced here so, that is the goal objective. So, if you just simply follow it, this will be left hand side, left hand can be written like this, this one is like this. According to the A cross B cross c that vector identity you can write like that. And right hand side curl of B you are putting like this.

And here mu 0 epsilon 0 you are just writing like 1 / c square mu 0 epsilon 0 is there 1 / c square and omega j omega is there 1 more j omega minus sign minus sign plus omega square / c square. So, right hand side if you are putting that is actually only electric field expression for the electric field. Now, earlier for homogeneous medium we could easily put divergence equal to 0.

Now, we cannot because we will be dealing with inhomogeneous medium and that is why we have to put divergence evaluate according to this one. Obviously, this is a constant space independent that part will be 0. And whenever you can consider that that time you can put down 0. So, here what you are doing this divergence if you put this term will come and del square E will be there del square E write minus sign is there. So, minus sign will be plus the sign will come, this is minus sign and this is also coming this side backside.

So, ultimately all this thing you are just simplifying that straightforward all the terms if you put in left hand side, so ultimately we will get this one. So, for homogeneous dielectric medium this will be 0 we can write down this thing. Now, so, this is actually called vector wave equation for electric field, vector wave equation this is all are vector nothing is scalar it still will be question will show that anything we want to solve in inhomogeneous medium we have to start from here.

Because practically you have to deal with an inhomogeneous medium also so, sometimes this thing has to be directly started instead of so, far in electromagnetic theory any textbook we have already deal it with only homogeneous medium here delta epsilon = 0. Now, we have to deal with in homogeneous thing, so we have to start from here all the time.

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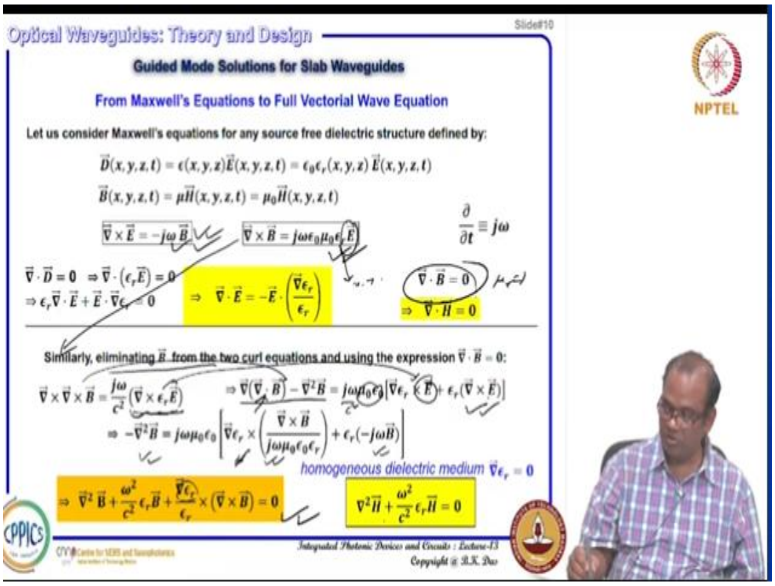

Now same thing if I eliminate B from the curl equations, these are the 2 curl equation again if you want to eliminate B, then what we have to do? You have to take curl off for the both side then once you get to curl on this side, you are getting with simple few steps we just put down here straight forward here. So, again left hand side will be like this and right hand side again you see curl off epsilon r E epsilon r is sensitives x, y, z dependent so, curl of E epsilon r cannot be kept outside.

So, you have to put whenever you are taking curl that epsilon r has to be multiplied with the E then only you can get epsilon r cannot be taken outside. So, if you do not take outside then you can actually this is also a function of x y z if you are thinking if you have then you can write down from the vector identity we can write like this j omega / c square we are writing here we could write also this mu 0 epsilon 0 c square here instead of c square here this c square should be there j omega / c square mu 0 epsilon 0 plus.

Now divergence B = 0 that is already there so, they are you do not need to bother because mu r again I said the mu r is not it is the nonmagnetic material. So, it is magnetic point of view it is homogeneous. So, again by simplification this one little bit simplification because here this E can be written like this E I can derive from here you have curl delta epsilon r cross E the E I can take from here I can replace here then you will be getting this expression and this one I will be writing here.

So, curl off E I know what is the value -j omega B that is -j omega B written simplification you get this and this is called actually your vector equation for magnetic field in terms of B E here written. So, anywhere you can just make it a homogeneous medium del epsilon r will be again 0. So, then you are getting the vector equation for the homogeneous dielectric medium.

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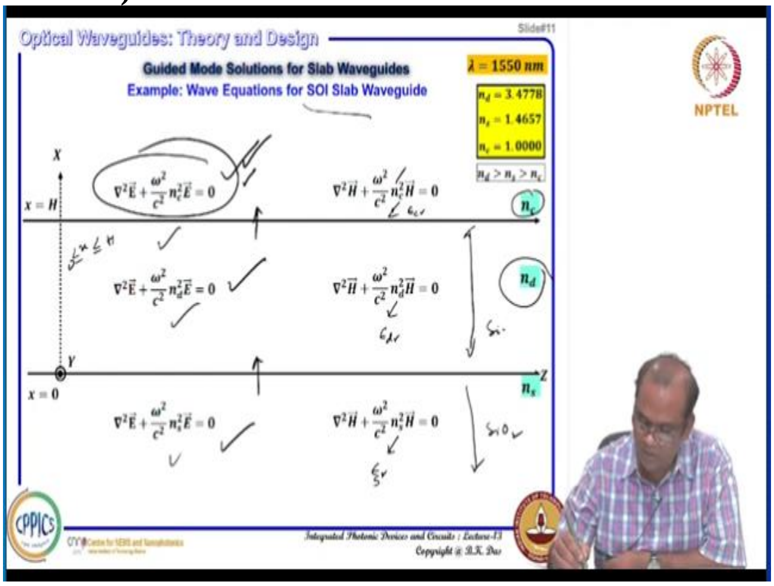

So, now come back if I consider the homogeneous medium 3 media. So, if I have silicon on insulator slab again here and so, device layer is actually this one x greater than 0 greater than H this is a silicon with respect to an n d. So, from here to here it is a homogeneous medium of silicon and lower also it is a homogeneous medium or silicon dioxide this is silicon dioxide this is silicon homogeneous medium and this is again your what you call that here with refractory n c cladding or it can be silicon dioxide homogeneous media.

So, vertical direction you have a 3 layer stacks of homogeneous media so, if I want to deal with our equation wave equation for homogeneous media, I can consider 3 different equation is satisfying this medium, this medium, this medium separately only in the boundary they are getting a dielectric constant change here our interface dielectric constant change is happening. So, for electric field why because I can write like this 3 medium.

For example, if you see here, this one epsilon r, this is the magnetic field and this is your electric field epsilon r is there. So, this epsilon r basically n square n = square root of epsilon r so, 3 medium is there. So, in this medium this is epsilon dr and this is epsilon cr cladding region and this is epsilon sr. So, it is instead of epsilon r we are just putting down the refractive index you know dielectric medium only whenever you are considering dialectics medium source free.

So, instead of epsilon mu sigma although timingly refractive indexes only the deciding parameter for electromagnetic wave propagation. So, we have just written down in 3 medium only thing is that this refractive index will be different then fine.

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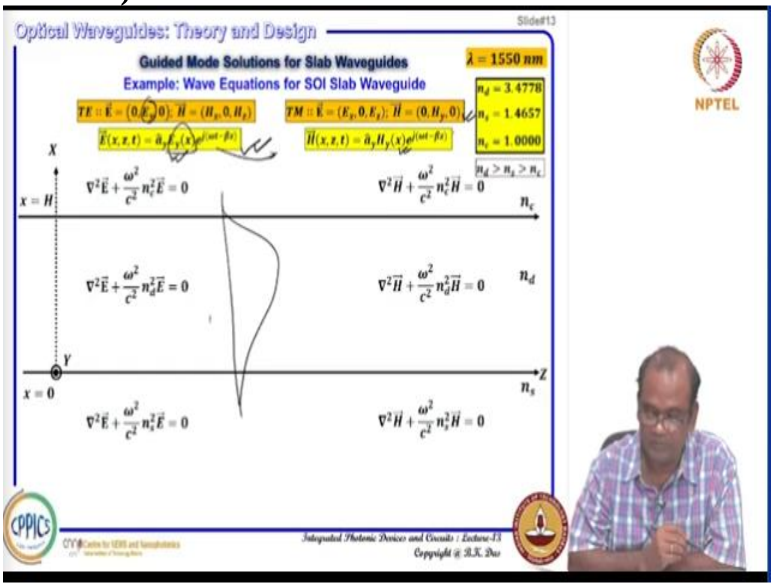

So, let us see what happens I consider that my solution for TE polarization the mode is like this, electromagnetic mode E x y z you have only y component TE polarization and you have magnetic field will be in the xz plane. Similarly, for TM polarization we know your magnetic field will be in the y direction tangential component and electric field in the xz plane in ray optics model we discussed that. And in this case if we just consider our electric field having only one component you will component and gamma ray optics picture.

We know that this electric field will be somehow it will have some kind of distribution x dependent distribution will be there along the x direction. So, this is x dependent and it will be propagating along z direction we consider that z direction propagation constant is beta and here also for magnetic field only y component we are considering here that is also H y x dependent and beta z.

So, now next thing is that we know that this is the solution for electric field here and this is the solution for the only component magnetic field here. And other component for TE polarization is x axis and z axis we can directly use our curl equation to find I will show that later and here also similarly, once we get H y then I can get E x and z from the Maxwell's equations.

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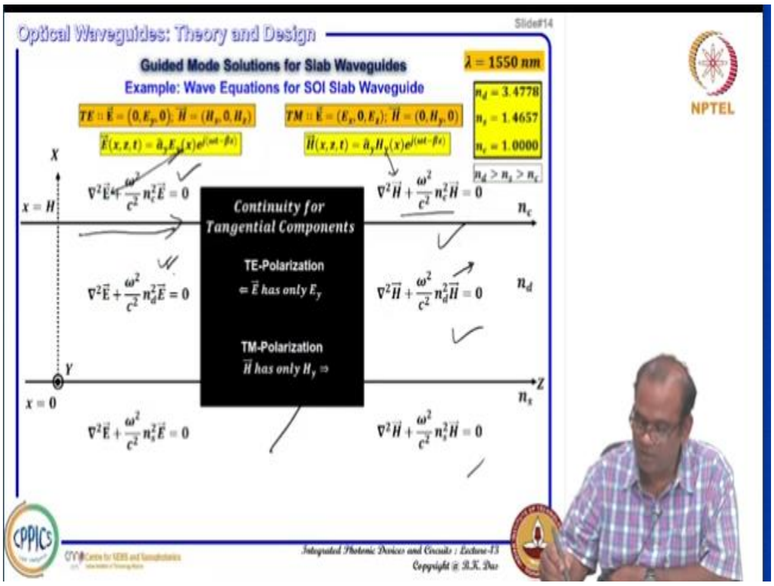

Now, continuity for tangential components TE polarization E has only E y component true TM polarization only H y component true know what else you just think about that if I just use the electric field equation here electric field equation I just only write there previously what I have written this E will be directly it will be written as E y component here. Similarly, here H I can write H y and TE polarization I can use this one.

TE polarization I can use this field on the electric field equation magnetic they are identical basically identical equation once you know how to solve this one you will know what will be the solution for all these? So, 3 medium I am writing separately. And these 3 mediums separately I am writing because if I get a solution here and I get a solution here I know that there will be this solution and the solution will be same in the boundary there will be continuity. TE polarization and E y is the tangential component they will be continuous.

And for TM polarization H y is the tangential component they will be continuous so, whatever solutions I will be getting here because of the H y I will be getting here H y if I put x = H they will be identical.

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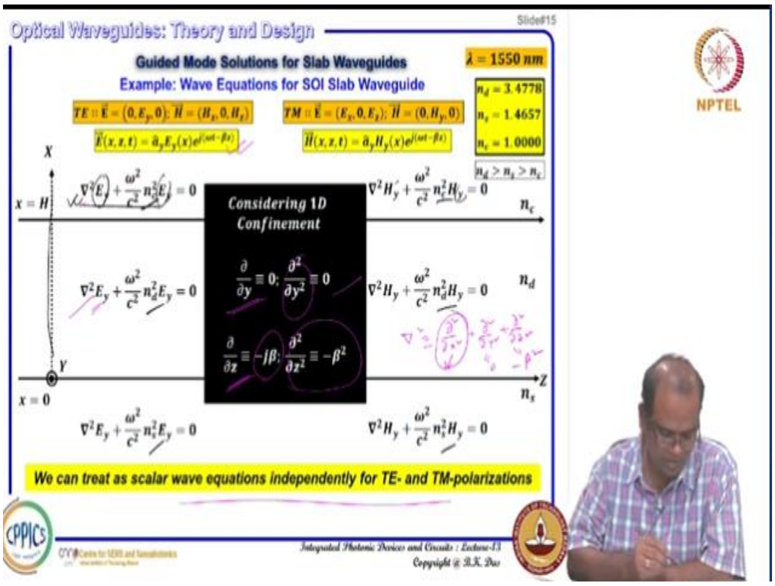

So, I have just written E y here del square E y and del square E y only difference n c, n d, n s 3 different media n c, n d, n s. So, now it is no more a vector equation it is like you see you do not need to consider only one component here only one component magnetically so you can treat them like a scalar. And in this scalar if you just consider this frame, this is the x axis this is z axis and along y direction it is infinitely extended.

So, in that case I can write nothing is changing in this direction so here if you see this one this one del y del del del y I am just conceder because y direction what is happening I do not need to bother I can consider everything is fixed in that direction I can assume. So, del del y = 0 that means del del 2 y 2 del 2 del y 2 = 0. And in case of del del z, because z dependencies is this minus beta z is there. So, del del z del E del z if you just put it will be getting minus z beta.

So, that is what we have written here and if you take double derivative with respect to z it will be beta square. So, we know that this del square = del 2 del x 2 + del 2 del y 2 + del 2 del z 2. So, this one equal to 0 I consider and this one I consider is minus beta square and this one will be staying. So, E y component I can finally this del square instead of del square I will be just only dealing with this one for one waveguide, this one I can put here and del del z I can put minus beta square. So, we can create a scalar equation because that is what I mentioned.

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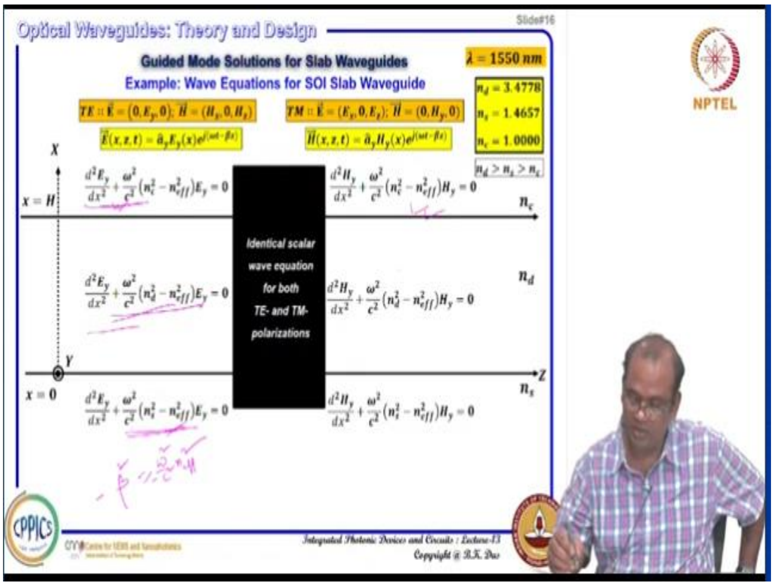

We can do them independently that is why let us go ahead a little bit both the TE, TM polarization is same and if I just consider that this thing this beta = beta square is there that I put down here I can write a little bit simplification the left hand side I can write like this right hand side I can right like this left hand side TE polarization E y component this is our H y component, but 2 different polarization TE polarization one mode TM polarization another type of solutions we can consider.

So, this thing beta square you see omega / c n effective we are considered like a beta so, instead of beta square I will be writing n effective square and minus beta square minus beta 2. So, that is the reason this is coming along with this omega square / c square epsilon r n d square was there this term up to here it will be there and then beta will be writing simply for x TE polarization and TM polarization identically equation if you see straightforward.

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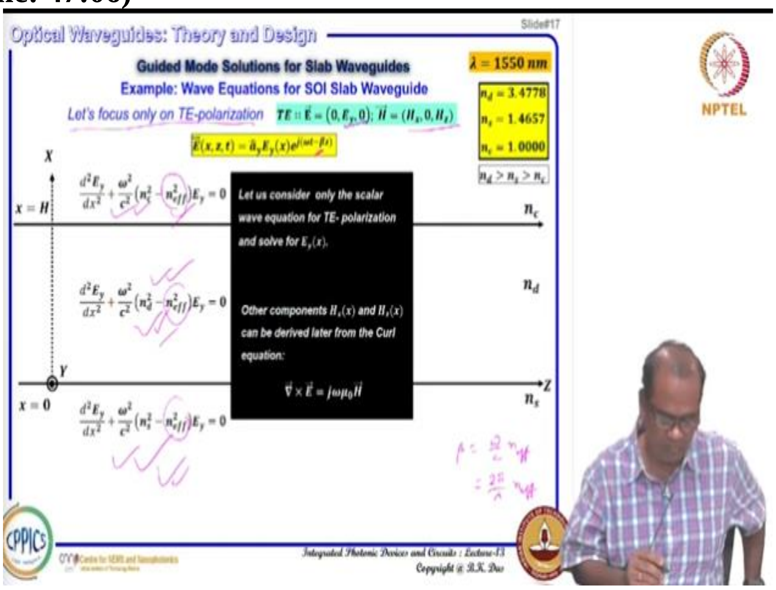

Now let us let us focus only one TE polarization that means, we have E y component and you have H field is Hz plane just only TE polarization and this is the scalar equation for the cladding layer scalar equation for E y component in the core region device layer is scalar equation for the sustained region. So, straight forward directly I am coming from the Maxwell's wave equations vector wave equation now, it is converted to the scalar homogeneous median 3 different independent wave guiding cover region and co region and substrate region.

So, only difference is that everywhere I have used n effective n effective n effective because everywhere any field is existing per mode the beta value is same for everywhere do not forget that this beta is nothing but omega / c n effective, that means 2pi / lambda times n effective propagation constant. So, it is propagating as a mode or whatever. So, that n effective I have just considered in terms of beta that is the consideration we have considered though it is unknown we need to solve that.

So, keep in mind n effective what type of n effective solutions I will be getting for example, in the ray optics model, what we got we tried to solve theta and they in from theta we try to solve your n effective and then beta so on.

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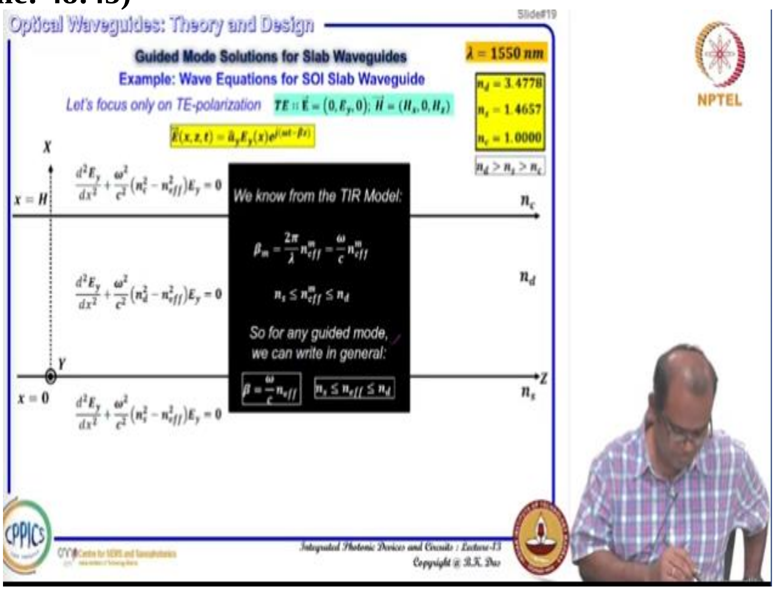

Now so we know from the total internal reflection model here what is that the beta m = 2pi / lambda n effective. So, this n effective m equal to we consider their m effective m equal to say n d sin theta m we do not want to consider this theta m we are just without theta m were trying to get in solutions. So, what is that when we go by c n effective so I can say that n effective is less than n d this is the consideration I have already the earlier discussion.

So, for any guided mode, we can write in general beta = omega / c n effective that is what I am keep on talking n effective must be less than n c, same thing I am repeating just so that things are consistent.

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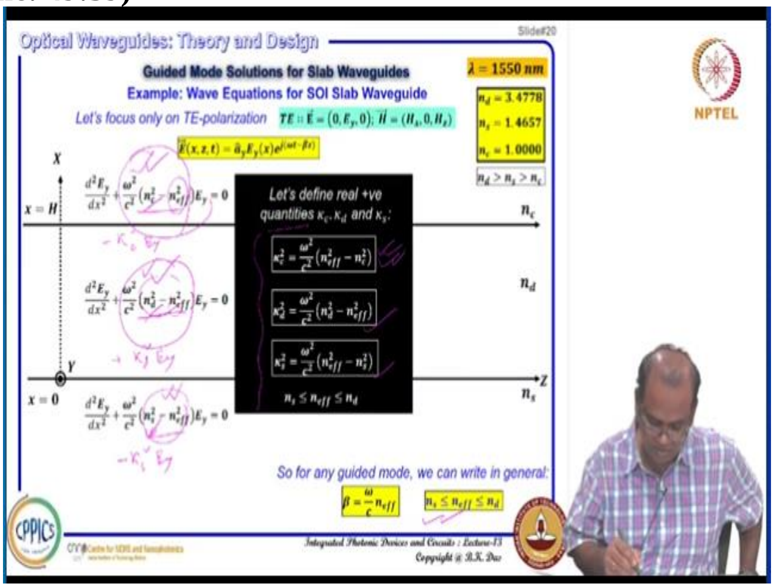

So, now let us define to solve I need to solve this equation, this equation and this equation all are identical equation only thing is that here n c square, n d square, n s square these 3 region differences like that, but we know that this n effective is n between n d and n s this is the case. So, if I just compare this thing, this thing with a just compare n effective with respect to n c, n effective with respect to n d, n effective with respect to n s you know that n effective this one is greater than n s n effective is greater than n s.

And they are n effective is less than n d, n effective is greater than n c. So, that means, once this is greater than this one that means this value is negative, but this is less than n d. So, this is positive, but this is negative. So, what we consider we define positive real quantities kappa c, kappa d, kappa s. What is kappa c you know that n effective greater than n c we define like this, just I try to express this thing, particularly all this thing n effective is the unknown thing.

But I am defining with this expression, this is also positive because n effective greater than n effective less than n d, n effective greater than n s. So, if I just use this expression here, that means, instead of these one, I will be writing minus kappa c square E y. And here I will be writing plus kappa d square E y and here I will be writing minus kappa s square E y. So, if I define this thing where omega c and n c n s n d are known only n effective common in all the expression that is unknown we want to solve that.

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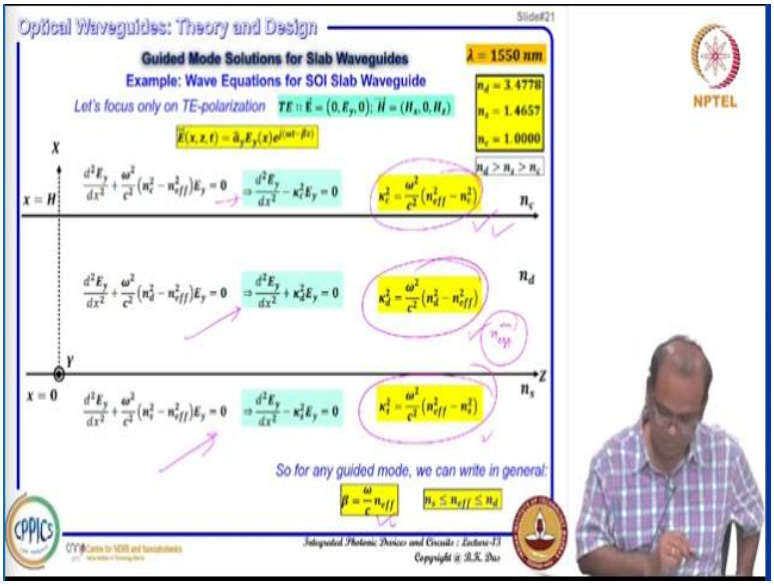

So, this is that with this definition, we will just define this expression will come like this, this expression will come like this, this expression will come like this and everywhere I just defined same thing again whatever I discussed this on and of course, beta = omega / c n effective by the way, we are not considering n effective m in this case, because I do not know how many modes will be there, if they n effective will be discretized or they will be continuous, it is not because ray optics model we saw that n effective will be discretized.

Because theta is discretized but in this case, we know that some confinement will be there along x direction and some propagation or a vector space velocity everything will be there in the z direction that concept we will be following however, what I see that this thing is propagating along z direction but n effective and n d relationship depending on that thing these are actually described and n effective instead of n effective m I just considered n effective m normally n effective whenever I am getting solution that time we will see whether that will be discretized or not.

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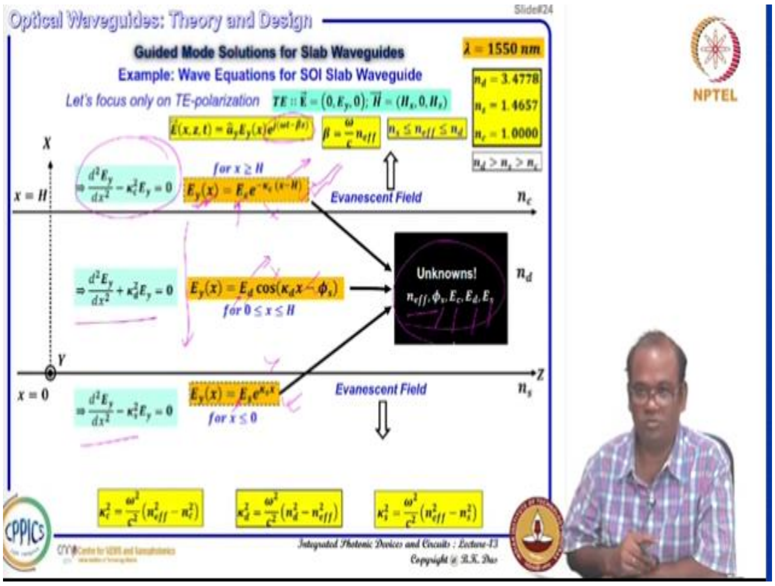

Now you see this is the equation we are considering in the previous slide if you just check this one kappa c is positive, if I try to solve this one, what would you solution it is like exponential decaying solutions it will be there and what will be the solution this is plus this will be oscillating and we have just solution like a oscillating cosine function or sin function anything we can consider and only thing is that you introduce one some kind of phase kappa dx - phi s.

This phi s again we are introducing additional constant we need to solve that of course, and some amplitude you see here I am considering similarly, this one because this side k c case both are positive, but this is coming negative and this is positive. So, only central solution in this region that will be oscillatory that means, like a standing wave and here because it will be kappa d will be plus minus so on solutions will be there just standard first order second order differential equation you solve this. And this is for x = 0 we are considering also.

Because I will not consider amplifying here also plus minus kappa s plus minus kappa c will be there, but plus sign will not be considered here because plus and means it is actually exponentially increasing in the cladding region field cannot be increasing without amplification etcetera, but here also it is decaying but x is sensitive x less than 0. So, negative sign will come it is actually decaying only the standing wave solutions. So, from the wave vector equation, we just come out with this type of solutions.

Obviously, I am just carrying this thing because that will be useful for further discussion and then these are the solution for 3 regions solutions, all these 3 things if they are part of this mode E y x and here or E y x is that is part of this mode all of them so trouble with this phase upon that means the phase velocity of these 3 solutions phase velocity and that will be always seeing the longitudinal direction beta propagation constant which is I discussed earlier that is actually k 1 z in ray optics model that beta is coming. So, that will be propagating with this phase velocity all of them should be multiplied by the space factor that should be kept in mind.

Now in this equation, what are unknown things we have introduced n effective, phi s, E c, E d, E s these are the unknowns we need to solve them. So, that is actually the next target we have to solve because you see this is a solutions here, which is actually 2 part x greater than H this region because this region differential equation, you can consider homogeneous medium and differential equation this one. And from here to here homogeneous media with a differential equation like this.

Homogeneous media with a differential equation like this and all of their solution individually in each homogeneous medium, you find and since, they are I am trying to solve the mode, which is actually propagating along z direction and these solutions are associated to that particular mode. That means I have to assume that all these field distribution here and here that pattern that distribution as a whole that will be traveling in a forward direction with this phase factor.

And because of the differential equation whatever constants are there we need to know their relationship. And at least we should maintain because it is E y we have to check that E y, this tangential component, E y component if I use this equation and this equation exactly if I put x = H they must be same. So, from there, I can try to find out what is the relationship between E c and E d and try to find out all these unknown values. With this I stop today for this lecture.