Integrated Photonic Devices and Circuits

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Lecture – 11
Optical Waveguides: Theory and Design

Hi all, so here already into the theory of optical waveguide. So, I started a new chapter and the title of this chapter is optical waveguides theory and design. First of all, in this lecture today I will be discussing total internal reflection based Eigen mode solutions for slab waveguides. So, we have a clear picture about total internal reflections and also we have shown that total internal reflection in fact, it can provide 100% reflection at the interface.

When electromagnetic wave propagates from denser medium to rarer medium and the angle of incidence is greater than critical angle and that is almost lossless internal reflection happens and you can get very good optical wave guiding dielectric optical waveguide. So, I would like to discuss today slab waveguide or planner wave guide structure based on TIR total internal reflection to solve Eigen modes, different Eigen modes supposed to propagate in the planar waveguide structure.

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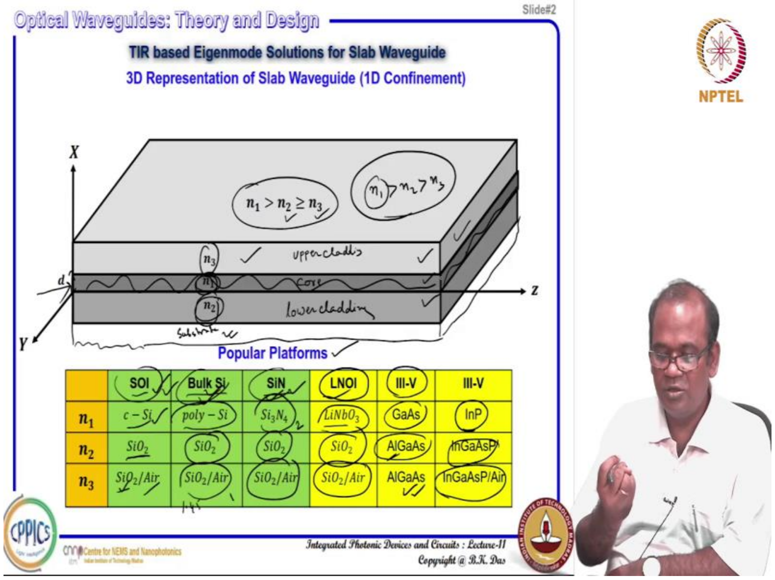

Optical Waveguides: Theory and Design

Slide#2

TIR based Eigenmode Solutions for Slab Waveguide

3D Representation of Slab Waveguide (1D Confinement)

Popular Platforms

	SOI	Bulk Si	SiN	LNOI	III-V	III-V
n_1	$c-Si$	$poly-Si$	Si_3N_4	$LiNbO_3$	GaAs	InP
n_2	SiO_2	SiO_2	SiO_2	SiO_2	AlGaAs	InGaAsP
n_3	SiO_2/Air	SiO_2/Air	SiO_2/Air	SiO_2/Air	AlGaAs	InGaAsP/Air

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NPTEL

So, here is an example of slab waveguide. So, you must know that this slab waveguide is normally not so useful for your photonic integrated circuit because for photonic integrated circuit you need a 2 dimensional wave guide 2 dimensional confinements. However, for starting to understand the optical wave guiding etcetera we will start with the planner waveguide structure and most of the time also when you fabricate photonic integrated circuits you need wafers or substrate and for photonic integrated circuit specifically 3 different layers are very important.

So, the bottom layer is called lower cladding. This is lower cladding and this is middle one is the core and this is actually upper cladding. So, this bottom cladding, core, upper cladding can be a suitable choice of material and suitable thickness and in the bottom you can have your substrate handling substrate that can be suitable thickness you can use that one that will be substrate typically that substrate is in the order of 1 millimeter thickness 1 millimeter sometimes 500 millimeter and so on.

So, there are popular platforms photonic integrated circuits and these platforms basically substrate or wafer we can commercially buy you can commercially there are manufacturers who can produce wafers for photonic integrated circuits. So, those wafers we know can be used useful for photonic integrated circuit are mostly few popular platforms here I have just shown one is first one is silicon on insulator and of course, bulk silicon and silicon nitride.

So, these are basically we call it this substrate not substrate entire wafer can be called substrate. Main thing is that it is basically we technological point of view we will call that as a SOI platform or SOI technology, bulk silicon or bulk silicon photonics technology or silicon nitride photonics technology. So, these are all basically silicon based these 3 green colour things silicon based and if it is a silicon on insulator.

You have a bottom substrate is of course, silicon. This so called core material, this core material is silicon and then lower cladding material is silicon dioxide sometimes it is called box buried oxide and upper cladding layer, this one can be silicon dioxide or here native silicon dioxide you can grow on silicon that same thing is good for silicon on insulator wafer. So, one can use that silicon dioxide obviously, you should keep in mind that for wave guiding purpose you need to maintain this condition.

That reproductive index of core material n 1 must be greater than n 2 and n 3 it can be possible that n 2 and n 3 can be equal in that case we will be calling that a symmetric slab waveguide. So, core region sandwiched between 2 identical lower refractive index material upper cladding and lower cladding that will be called symmetric slab waveguide otherwise, if it is simply just n 1 greater than n 2 something in this order, then we will be calling as asymmetric planer waveguides.

To should keep in mind that why do we need n 1 higher because we want to compile light in this region in this region light will be confined and it can propagate along Z direction or if you want it can propagate also X direction, Y direction. X direction it will not be allowed it will be confined because of this this type of arrangement. So, if you are launching light the slide can propagate in the either Y direction or Z direction clearly.

However, as a homogeneous medium this is like infinitely extended Y, Z plane, but along the X direction there is a in homogeneity is there. There is a core region where the fractal index is higher along the X direction n 1 region and top cladding or upper cladding and lower cladding. So, this is just 1 example I have just given silicon normally for silicon on insulator the silicon crystal refractive index is in the order of 3.47 silicon dioxide is relatively less 1.45 and then again top cladding also silicon dioxide or air.

So, if it is bottom is silicon dioxide top is silicon dioxide then it is a symmetric slab waveguide for SOI and if it is bottom is n 2 is equal to SiO 2 and upper is air then it will be called as asymmetric. Similarly, we can have bulk silicon where this core material is poly silicon. So, that is these things are chosen depending on the available technology CMOS compatible technology because all these SOI, bulk silicon they are chosen.

So, that you can utilize CMOS compatible fabrication process that we have discussed earlier and again similar like SOI the bottom cladding lower cladding is silicon oxide, native oxide you can easily fabricate it you can form and then you can upper cladding that is n 3 that will be SiO 2 or air you can keep it just air. Air refractive index is 1 and silicon dioxide is 1.45 if you are operating at 1.55 micrometer.

Similarly, there is another platform well known platform silicon nitride. So, their core material, the silicon n 1 material refractive index that is silicon nitride refractive index in the order of 2 at 1550 nanometers and then bottom cladding can be silicon dioxide and top cladding silicon dioxide or air. Symmetric and asymmetric slab waveguide you can make. So, I must mention that these 3 things these 3 material platforms it is written in the fabrication process almost standardized.

And foundry semiconductor foundries also adopting you can fabricate CMOS compatible fabrication process like for photonic integrated circuits CMOS and silicon is correlated. So, this is adopted accordingly. However, there are other interesting materials also material platform people started using because of different other types of advantages. You will know that silicon for photonic integrated circuit purpose it is not suitable for laser.

Laser diode integration and it may not be the best material for high speed modulator. So, that is why people already explore and also going forward with other platforms like lithium niobate on insulator lithium niobate you know we have discussed earlier lithium niobate is a very good material platform for high speed modulator. So, the technology already developed to burnt lithium niobate crystal with silicon dioxide on silicon.

So, it can have your core material lithium niobate thin film and then lower cladding can be a silicon dioxide and upper cladding against silicon dioxide or air. So, this is also one platform people use and this is for particularly for modulator purposes and some other nonlinear photonic application purpose because lithium niobate is also known for nonlinear, very good nonlinear optic material and then 3 5 compound semiconductor.

So, 3 5 compound semiconductors like gallium arsenide and gallium arsenide can be core n 1 material and upper and lower cladding can be AlGaAs and aluminum gallium arsenide because, if you are just making alloy with aluminum gallium arsenide and aluminum some fraction of gallium is replaced by aluminum atoms then the refractive index reduces. So, that can be considered as a lower cladding.

This is also lower cladding can be upper cladding or lower cladding, you can just have a lower refractive index. Similarly, this gallium arsenide basically used for laser diode photo detector all those type of thing it is a direct bandgap semiconductor. It is useful however, gallium arsenide laser you fabricate typically the emission wavelength not satisfying your communication band 1550 nanometer rather it is around 1 micrometer less than 1 micrometer.

So, that is why another platform is used also that is called Indium phosphide platform that can be used as a core and you can have Indium gallium arsenide phosphide another compound alloy you can grow technologies are available that type of wafers are available as a lower cladding n 2 and air or another layer of Indium gallium arsenide phosphide can be upper layer. So, these are the; platforms available and if you use suitable technology available to you.

Then you can make different types of photonic structure that can be active devices that can be passive devices that can be programmable devices and some devices some platform is more important more interesting, because it can also integrate electronics for example SOI and bulk silicon, silicon on insulator and bulk silicon you can co-integrate electronics as well as photonics. However, lithium niobate on insulator platform you cannot co integrate your electronics chip will be separate. 3 5 semiconductors you can co-integrate electronics also.

But you know CMOS electronics you cannot integrate some high power electronics all those different types of technology is possible. But normally people use these 3 5 semiconductors for separate active opto-electronic devices like lasers and photo diodes. Nevertheless, I would like to discuss that whatever this platform you get and always you are getting these 3 layers in your hand already before you can design your waveguide structure and you can fabricate your waveguide structural photonic integrated circuits.

You need to know what is the wafer you are going to use and what is the refractive index in the core region and what is the refractive index in the upper cladding what would be the refractive index of the lower cladding and what is their dimensions and specifically the dimensions of core layer is very important because they are only you want to confine your light. So, these things known to you.

So, based on that, you can actually go for fabricating photonic integrated circuits. So, as you see as you get your wafer it is just simply a slab waveguide because you have upper cladding, lower cladding and core. So, let us try to understand how light is confined at least 1 direction as you get your wafer. As you buy your wafer from manufacturer you can do sometimes customized wafers also. You are already getting a slab waveguide or planner waveguide structure.

So, I need to understand first, how this planar for a given thickness. How this planner waveguide structure can be analyzed and understood how light is confined in 1 direction and I can understand the modal picture just simply by total internet reflection method whatever we have learned so far using that thing, you will be able to understand the propagation in 1D waveguide.

Then later on we will be moving for more rigorous fabrication theory Maxwell's equation to get more clear picture for the moment just to get a modal picture of wave guiding mode, how light is confined that picture will be easier to understand using total internal reflection methods.

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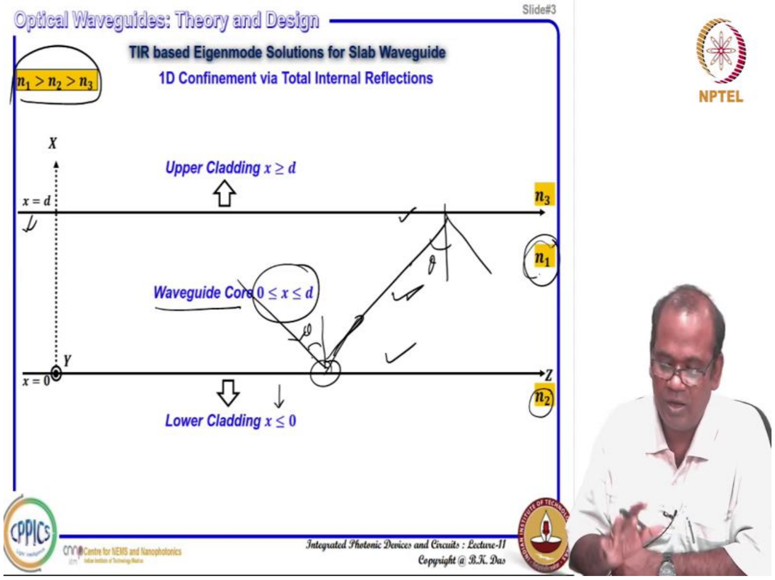

Optical Waveguides: Theory and Design

Slide#3

TIR based Eigenmode Solutions for Slab Waveguide

1D Confinement via Total Internal Reflections

$n_1 > n_2 > n_3$

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Let us move on. So, this is the picture I just replicated this wafer 3 layer system this is your n 1 core region stretched between this region x less than d 0 that means thicknesses d that is called as a waveguide core and your lower cladding refractive index n 2 that is this region that means x less than 0 interfaces always your YZ plane at x = 0 and another interface also YZ another YZ plane at x = d.

So, this one and we are considering this white plane is infinitely extended for example for the moment as I mentioned earlier that in this type of situation you can think of that light is field to propagate along Y direction as well as Z direction only it will be confined along the X direction because of the total internal reflection here and here, why total internal reflection will take place because we have chosen this condition refractive index of the this region if you see, n 1 greater than n 3 and n 1 greater than n 2.

So, any wave in this region if you are launching here, if you somehow get a angle which is greater than critical angle it will get total internal reflection will take place. Similarly, something comes here, so, if this theta is greater than critical angle in this interface, then it will be totally deflected. So, this is the picture we are just considering d and we will try to understand how 1 dimensional confinement is possible and what will be the mode solution.

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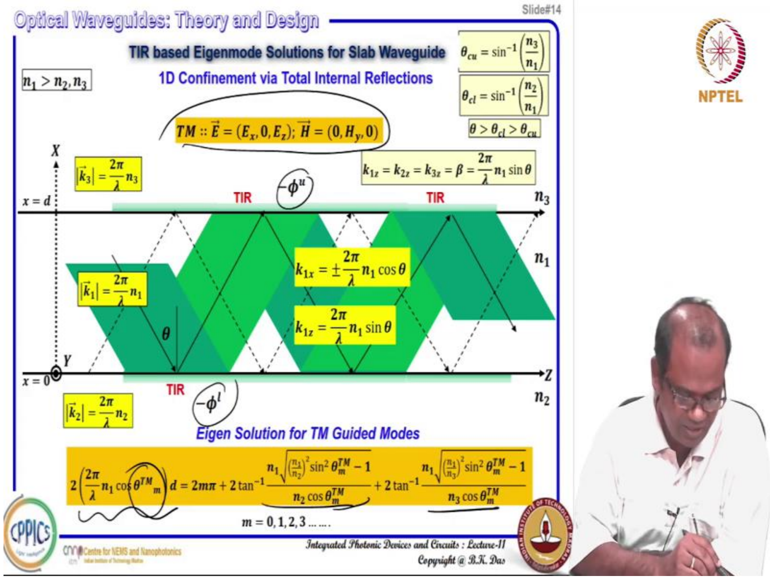

Optical Waveguides: Theory and Design

Slide#14

TIR based Eigenmode Solutions for Slab Waveguide

1D Confinement via Total Internal Reflections

$n_1 > n_2, n_3$

$\theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$

$\theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\theta > \theta_{cl} > \theta_{cu}$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$|\vec{k}_3| = \frac{2\pi}{\lambda} n_3$

$k_{1z} = k_{2z} = k_{3z} = \beta = \frac{2\pi}{\lambda} n_1 \sin\theta$

$|\vec{k}_1| = \frac{2\pi}{\lambda} n_1$

$k_{1x} = \pm\frac{2\pi}{\lambda} n_1 \cos\theta$

$k_{1z} = \frac{2\pi}{\lambda} n_1 \sin\theta$

$|\vec{k}_2| = \frac{2\pi}{\lambda} n_2$

Eigen Solution for TM Guided Modes

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta^{TM}{}_m\right) d = 2m\pi + 2\tan^{-1}\frac{n_1\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m^{TM} - 1}}{n_2 \cos\theta_m^{TM}} + 2\tan^{-1}\frac{n_1\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m^{TM} - 1}}{n_3 \cos\theta_m^{TM}}$$

$m = 0, 1, 2, 3 \ldots\ldots$

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So, this is a picture we have shown that a beam parallel beam is coming as we have already discussed during total internal reflection understanding. So, rays coming planar wave and going in this direction and total internal reflection can happen this theta you can consider such that is greater than critical angle lower critical angle lower interface critical angle for the lower interface and also it can be greater than critical angle for the upper interface you know critical angle you can define.

For example, upper interface here in this interface critical angle would be sine inverse n 3 / n1 is the critical angle for the upper interface and theta cl is the critical angle that is sine inverse n 2 / n1 that is in a lower interface if your this angle is greater than both lower cladding critical angle and upper cladding critical angle. So, I can say total internal reflection in the lower interface as well as upper interface lower upper interface lower interface all these total internal reflections and I can see.

And you can think of the another way it is another completely opposite direction it is this direction you are considering another angle you can consider also theta TIR hitting here. So, here also it can happen TIR, TIR and then TIR, TIR. So, in this way you can get some way total internal reflection and both interfaces and certain angle entries should have should actually satisfy a certain angle so that this is satisfied.

Next is you see, this is the material medium where refractive index is n 1. So, corresponding anyway vector that is a vector constant I would say with propagation constant I would say that is we are already discussed earlier that is 2 pi / lambda n 1 lambda is the incoming whatever the light is coming electromagnetic wave is coming that has a lambda wavelength in free space and n 1 is the refractive index that is a dielectric material of course and even though we are considering this a semiconductor platform 3 layer platform.

But we should choose these lambda is such that, that lambda is transparent, that photon energy should be less than the band gap and so that it can see like a dielectric material medium. So, this is the proportion constant in core region, this is the propagation constant in the lower cladding region, just 2 pi / lambda n 2 and this is the propagation constant in the upper cladding region good.

So, next thing is that suppose this is your theta, now, if I try to get what is that this is suppose k 1, k 1 vector I am considering, so, what is the x component k 1x. k 1x is k 1 times cos theta, so, 2 pi / lambda cos theta since you have another reflection coming here. So, it is also can be considered as a k 1 here this vector we can define then it will be also k 1x will be there and here also k 1x is there.

So, in that case I have in this total internal reflection both interfaces if it is happening k 1x it can have + 2 pi / lambda n 1 cos theta that is the x component and since it is opposite direction also 1 component is there that is +, - and if I just try to see the longitudinal component k 1z 2 pi / lambda n 1 sine theta. So, this is the longitudinal component in the core region I know that what is that transverse wave vector component and what is the longitudinal wave vector component.

You can think of identical situation for another ray coming in opposite direction here this also you can consider this can be also similar theta. So, here also you can see k 1x positive and here you can see k 1x negative and here also decomposition you will be getting k 1z. So, longitudinal component is fixed for all but you have counter propagating if you are just independently thinking that the electromagnetic wave having a propagation constant k 1x component then it is a counter propagating wave is there in that direction.

So far good then if I just try to find out what is the k 3x. k 3 is this one. k 3x earlier we have discussed that in case of total internal reflection we can think some kind of imaginary transmitted to a vector which one having longitudinal component k tz which will be continuous like this one to here I will be considering here this k 3z k 3z will be equal to k 1z because tangential component will be same, but normal component will have an imaginary.

So, during total internal reflection and normal component will have an imaginary value and that value we have derived earlier also if theta is the angle of incidence here we are considering then n 1 / n 3 that is the in this interface n 3 n 1 refractive index contrast and sine square theta -1 and since this is a positive value for theta greater than theta c theta cu. So, we can say that j 2 pi / lambda k 3. So, 2 pi / lambda is the normal a vector in the free space and in n theta and this one we just write kappa 3 j times 2 pi / lambda kappa 3.

So, that is the imaginary part. So, we know that that is how the feel will be decaying in the X direction because of the x component is imaginary that will be attenuating. So, any wave vector component is real that will be actually like a traveling wave it will be generating that we have discussed earlier. So, similarly, we have k 2x in this direction if it is I am just defining k 2x that should be also imaginary 2 pi / lambda kappa 2.

So, that means in this region also it will be decaying anywhere it is total internal reflection happening that will be decaying. So, I consider that some field will be there in the top side and upper side because of this condition for the total internal reflection. Now, this total this thing because it is decaying it is actually normally we call it as Evanescent field. So, that field is existing but that field is not carrying energy along the X direction.

X direction it is decaying that is actually for our photonic integrated circuit. So, those who are studying waveguide theory etcetera. Normally this type of decaying period is they call like Evanescent field. The field is there but energy is not flowing in the X direction but that field will carry energy. Whatever field is there in the Z direction because we know that here K 3z will be there. Here we here we k 2z because it is medium 2 and here it is k 3z component will be there and k 1z is there they are identical.

That is why k 1z can carry energy in this direction k 2z also care in the cladding region also carrying energy in this region and k 3z also will be carrying energy in this direction. So, that means the field exist in the second media, but it is not like a propagating along its direction not propagating along Y direction, but they can it can contribute whatever field is existing there, that fraction of the field also contributing energy propagation in the Z direction.

So, that region also very much part of the waveguide that even is field region, including the core region, you have some extended Evanescent field that even Evanescent field also propagating with a longitudinal wave vector k 3z here and here will be k 2z and here will be k 1z and as you know, tangential component is always continuous tangential component of the y vector k 1z, Z direction tangential in the interface.

So, they are continuous. So, you can consider that power energy will be carried by the Evanescent field in the Z direction propagation direction, I am not considering the field propagation along Y direction, because the Y direction and just not considering at all anything happening and just ignoring at the moment. Now, I just mentioned this k 1z what I discussed in the previous slide equal to k 2z longitudinal Y vector that is tangential component in the medium 1 in the core region longitudinal component tangential component in medium 2 here.

And tangential component and medium 3 they must be equal and we give another symbol we call it as a beta propagation constant we can said the effective propagation constant along the propagation direction that will be actually nothing but k 1z whatever the k 1 value is there and just written there, whatever k 1z that will be k 2z that will be k 3z. So, we write like this fine. So, these are the things we need to understand more rigorously in future.

Now, we know that when total internal reflection takes place, there is some kind of phase change happens because, that reflection coefficient become complex. So, when the flex reflection coefficient become complex like this a + jb gamma t or gamma tm we consider like that, if it is complex like this a - jb / a + jb that is will be 1 into e to the power - j phi something like that total internal reflection will take place, but it causes some kind of phase that phase I am just writing down here just to give you a clear picture what happens.

So, now Eigen mode solution; what is that? This is very important is no doubt what I have written here you have the k 1x and that has a + k 1x and - k 1x you have forward propagating wave and backward propagating wave along X direction. So, if you just concentrate on that, we make the k 1x and that somehow clear some kind of standing wave pattern along the X direction then, that that can be considered as a mode.

How that is possible we know any standing wave suppose you are considering a cavity simple fabricate cavity or a rigid rotor cavity if any wave propagates here and this direction and comes back and it comes back with 2 phi phase shift round tip travel if it contributes 2 phi phase shift or integer multiple of 2 phi phase shift these then suppose this a propagation constant propagating wave propagating in this direction and coming back this direction. So, if it is L, so round tip phase is propagation constant, it is a beta into z.

So, propagation constant k and round trip is L 2 times 2kL, if this 2kL is becoming 2m pi then we can consider that is actually somehow creating self-consistent and resonance standing wave is generated. So, similarly, I can say that this k 1x when going first reflection here, this is going that means x component reaching up to this point, taking energy to this point and then it is flipping back. Then it will come back again. So, it will when it is coming back to this point that means k 1x actually completing 1 round trip.

So, from this point to this point k 1x actually helping phase completion along the x direction is 1 round trip total round trip. So, if I just consider k 1x times 2 d, d is forward direction and reverse direction. So, that is the round trip phase, but, from here to here reaching, it is undergoing, if we consider this point A it is undergoing total internal reflection and another total internal reflection here.

So, that means, when this point to this point this is A and this point A and B point to consider if coming A to B if you want A and B exactly identical that means, the k 1x whatever round trip phase is contributing along with that you have to consider whatever phase change is happening because of the total internal reflection and that is actually stopping the wave propagation along X direction that is upper interface we are considering - phi u and lower interface we are considering - phi l.

So, for example, for 1 round trip you have this is the phase for k 1x and additional this phi l and phi u lower interface and upper interface. So, since there are - sign we have seen that actually you remember that that was actually 2 tan inverse n 1 square root n 1 / n 2 square - sine square theta, so, on. That tan inverse minus sign is there you could use that minus sign. So, whenever you will be inserting this phi value that time will be considering plus sign that exact value will be putting.

So, this is the value this is the equation we can use for more solution round trip phase shift 2k 1x d - phi l - phi u lower interface upper interface and this round trip is 2 phi times m integer then that will be a solution for modes where m can be considered as 0 1 2 3 and so on. So, this is called Eigen solution for guided modes. So, written down here again solution for guided modes where if you have just consider T polarization you know T polarization where in this configuration different stream electric field will be oscillating only along Y direction.

Only Y component is there and magnetic field will be in the XZ plane. That is one type of polarization we call it T polarization and I just we normally distinguish T and T m just to take advantage of boundary conditions and without compromising any violation to satisfy Maxwell's equation and wave propagation. So, if I consider these 3 components that is good enough to carry energy and another 3 set of components called t m that also good enough to carry independently energy.

So, we can treat them separately if all the 6 components are there. So, we can take this set of components and treat them because it actually it satisfies one type of boundary condition because E y is the only electric field that is tangential to the interface. So, for E y we know that is the phi TE in the lower interface the lower interface means you have n 1 refractive index and n 2 refractive index that comes in here tan inverse this one n 2 n 1 / n 2 n 1.

And if you are considering upper interface whatever the phase is changing for TE polarization n 2 will be replaced by n 3 because here interface is n 3 upper interfaces. So, if I insert this one here and this one here, then that solution if I am getting at tall some solution that will be corresponding to TE polarization because this phase change in the interface whatever phase change occurring because the total internal reflection that has been considered for T polarization.

So, now and just after substituting how it look like so, 2 times k 1 cos theta I have just written cos theta TE m you will be understanding why we are putting that, so, 2 pi / lambda n 1 cos theta m times d. So, that is what you wanted to get that. 2k 1x d. So, 2 times k 1x equal to 2 pi / lambda n 1 cos theta that is what we have written. TE I have put because I have used to the phase change phi u and phi l for TE polarization and m.

I have written because this theta solutions are will be getting for m = 0 and m = 1 and m = 2 and so on. So, this is the left hand side and right hand side we have 2 m phi and then phi u and phi l, I have taken right hand side. So, whatever the value here, if I take phi u and pie l this side that will be 2 m phi + phi l + phi u. If it is TE polarization I will be writing TE and TE this expression I will be writing.

So, that is what we have written of course, you can just think of canceling this 2 2 2 then it will be getting a very nice equation. So, left hand side theta T, right hand side also theta T is there and you can solve because for a given m if you are putting m = 0 you will be able to solve theta TE 0 that is a one solution for theta TE that means, your theta TE solution is now discretized because m is discretized right hand side.

You are getting solution you are putting m = 0 you will be getting one solution for theta TE you are putting m = 1 you will be getting another solution for theta TE. So, this is very important to conclude this actually ensures that any theta greater than theta cl and theta cu greater than the critical angle in the lower interface and upper interface is ensuring total internal reflection but that not ensuring whether it is a guided mode or not .

That guided mode insurance it is coming through this equation because round trip phase accumulation because of the transverse component of the m vector that is what satisfied some standing wave or resonance condition. So, that condition is giving this one and where from we find that the theta value is now discretized. So, all theta value will not give you a standing wave pattern along the X direction.

So, all theta value you would not see any confinement along the X direction only a discrete solution for m = 0 m = 1 m = 2 m = 3 using this equation you can get a solution. So, now, if I just try to get TM polarization TM polarization you know this is other way. So, your electric magnetic field is having only y component H y, but electric field is in the Evanescent plane. So, other things, so, in that case also you can get a similar solution only thing is that this phi l and phi u you have to consider for TE and TM. So, earlier you use TE equation now, we have to use for TM.

So, the solutions come; similar thing, but you will be getting different solution for theta TM because the phase equation is different and the interface because of the total internal reflection on whatever phase it is acquiring that will be different for TM polarization. For that purpose you will get different solutions different set of solution for theta TM polarization if components are like this. So, you will get 2 set of solution one for TE solution and other for TM solutions and they are equations slightly different.

So, different type of theta values you will be getting and m actually again it is defined for 0 1 2 3 4 for TM polarization you can put there m also another integer because just to distinguish TE, TM good. So, now, we know that well theta TM, which particular theta value are satisfying to create a standing wave along x direction and that will be consider as a mode. So, what is next to them.

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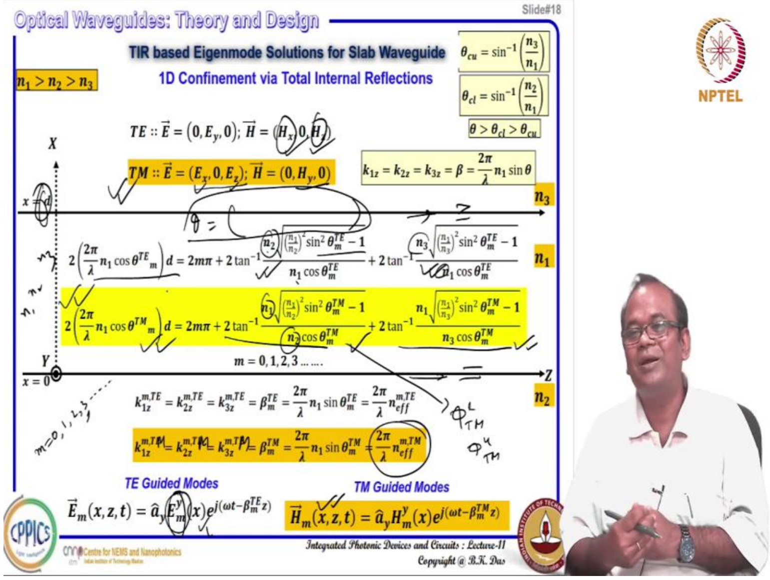

Optical Waveguides: Theory and Design

Slide#18

TIR based Eigenmode Solutions for Slab Waveguide

1D Confinement via Total Internal Reflections

$n_1 > n_2 > n_3$

$\theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$

$\theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\theta > \theta_{cl} > \theta_{cu}$

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z);\ \vec{H} = (0, H_y, 0)$

$k_{1z} = k_{2z} = k_{3z} = \beta = \frac{2\pi}{\lambda} n_1 \sin\theta$

$\theta = ($

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta^{TE}{}_m\right) d = 2m\pi + 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m^{TE} - 1}}{n_1 \cos\theta_m^{TE}} + 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m^{TE} - 1}}{n_1 \cos\theta_m^{TE}}$$

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta^{TM}{}_m\right) d = 2m\pi + 2\tan^{-1}\frac{n_1\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m^{TM} - 1}}{n_2 \cos\theta_m^{TM}} + 2\tan^{-1}\frac{n_1\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m^{TM} - 1}}{n_3 \cos\theta_m^{TM}}$$

$m = 0, 1, 2, 3 \ldots\ldots$

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$$k_{1z}^{m,TE} = k_{2z}^{m,TE} = k_{3z}^{m,TE} = \beta_m^{TE} = \frac{2\pi}{\lambda} n_1 \sin\theta_m^{TE} = \frac{2\pi}{\lambda} n_{eff}^{m,TE}$$

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ϕ_{TM}^{L}

ϕ_{TM}^{u}

TE Guided Modes

$\vec{E}_m(x,z,t) = \hat{a}_y E_m^y(x) e^{j(\omega t - \beta_m^{TE} z)}$

TM Guided Modes

$\vec{H}_m(x,z,t) = \hat{a}_y H_m^y(x) e^{j(\omega t - \beta_m^{TM} z)}$

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So, now, I have written that thing, this is the Eigen mode solution for T polarization Eigen mode solution and I have written that k 1z, k 2z, k 3z equal to T polarization that will be equal to this one beta propagation constant. This is the thing next, you see k 1z m for TE polarization k 2z m for TE polarization that means field components are like this and k 3z that will be called as a beta mTE.

So, instead of beta because it is polarization sensitive now, you are just considering which polarization depending on that which Eigen mode solutions you are solving because you have different for TE polarization you have this Eigen mode solution and for TM to be different. So, beta TM you know this is n 1 sine theta, you are solving theta m TE here. So I can find a theta m theta 0 for TE polarization theta 1 TE polarization theta 2 for TE polarization. So, that will be m = 0 this will be for m = 1 this will be for m = 2.

So, I am just putting a value here and corresponding theta TE value I can solve and using that theta TE value I can find out beta I can find out beta 0 TE polarization beta 0 discrete values of beta propagation constant also. Beta 1 for TE, beta 2 for TE, beta 3 for TE and so on, I will be getting. So that means longitudinal component I will be also getting discretized. So, one thing is that the k 1x first it is discretized depending on, this is the Eigen mode solution.

This is basically the k 1x times d 2 times roundtrip that k 1x is discretized because theta TM solutions you will be getting different because of the m and once k 1x is discretized you can say that k 1x square + k 1z square that should be equal to k 1 square because k 1 is equal to you have a x you can define like a k 1x + a y a z k 1z say in the XY plane XZ plane you are considering the k vector. So, I can have a propagation constant also I can get and whenever I am writing you know 2 pi / lambda by a beta expression it is like that 2 pi / lambda n 1 sine theta m.

Normally, if it is a homogeneous medium of n 1 the propagation constant would have been 2 pi / lambda into n 1, but it is in the waveguide you have to multiply longitudinal direction the propagation constant with sine theta T and whatever solution comes you have to consider this one. So, instead of n 1 you are using these one n 1time sine theta solution. What about the solutions you are getting these sine theta solutions I will be calling as a effective index n 1 time sine theta that is not n 1 exactly what it is n 1 sine theta effective index.

So, since theta t I said that I have a set up theta TE 0, theta TE 1 then theta TE 2 then theta TE these are the solutions I am getting corresponding to m = 0, m = 1, m = 2, m = 3. So, that means, all these discrete values if I put I will be getting any effective also discretized. So, those mods also will have an effective 0 TE polarization and effective. So, first m is actually corresponding to theta solution 1 TE polarization n effective 2 TE polarization.

So, you will be getting effective index of the different order of guided mode. So, m actually this m entry actually defining individual propagating modes by giving the discrete solution of theta T theta m you are getting. So, once you get discrete solution of theta m then you get a discrete solution for k 1x and this good solution for k 1z. k 1x actually confining discrete nature of the actually as a specific standing wave pattern corresponding to one discretized k 1x will give one particular standing wave pattern along X direction.

And corresponding k 1z will be that pattern how it will be propagating along the Z direction that will define. So, every n entry will give you set of k 1x and k 1z, k 1x is giving a pattern along X direction field distribution and k 1z how that field will propagate along Z direction that is that will be giving. What will be the phase velocity of that pattern because that will be coming in your phase from that is the propagation constant that is a real by the way.

So, real part of the propagation constant that means, it will oscillatory in this Z direction it will be oscillating and moving and X direction because it is counter propagating standing wave giving you mode. So that is the case and beyond core region upper cladding and lower cladding, because you have the imaginary propagation constant particular direction, so, that will be attenuating.

So, I can say that standing wave pattern I will be creating here, x = 0 to d this region, some kind of oscillatory standing waves, nodes antinodes will be coming. So, when only one antinode will be coming that then it will be called fundamental mode 2 will be coming there is a first order mode and so on a different type of mode and each of them will have individual longitudinal component along Z direction, that will be giving you phase constant or phase velocity.

This is all about for TE polarization I am discussing. So, now, the solutions for this T polarization T guided modes, I can say that because you are getting a standing wave in the X direction, so, I can say that if more solution, I am not considering y variable m x zT because Y direction you can consider this in finitely extended it is constant along Y direction. So, electric field is only a function of xz and time.

So, if it is I will I would like to know, because Z direction it is propagating and X direction you are getting some kind of standing wave pattern some kind of resonance because of the k 1x. So, you see that electric field distribution will have some kind of x distribution you will see some kind of function. So, you see these types of functions we could predict by considering a plane wave getting total internal reflection back and forth.

Upper interface and lower interface and then here you can see that you particular mode say m for m = 0 you will get one particular distribution m = 1 you will be getting another particular distribution along the X direction m = 2 another particular distribution along X direction and this y is nothing but your y component I mean to say y component for TE polarization will be getting and this pattern this y component means vector is along Y direction.

And this entire pattern it will be propagating with a phase velocity omega by beta TE m z. So, web phase velocity will be fifth Ph power m a math mode that will be omega / beta n TE we can say if it is TE polarization we can say phase velocity. So, normally, since m = 0 you will get one solution m = 2 another solution 3 another solution. So, each mode will travel in a different phase velocity that is very interesting, because in the waveguide, you can get a different set of solutions.

But all of them will travel with a different phase velocity different phase constant and different distribution pattern also. So, this is all about mode. So, when you see a certain specific mode along certain direction X direction and that pattern is actually unique and it maintains along the propagation direction as it propagates along the Z direction with a phase velocity that is omega by beta phase velocity.

So, this beta m is nothing but whatever you are getting this longitudinal components, because k 1x is contributing to standing wave k 1z actually contributing to the energy propagation along Z direction. So, this is how the both the picture we can understand at least just simple total internal reflection ray optics picture just resonance condition standing wave and components propagation constant component etcetera we are starting from the just planar, plane with total internal reflection.

And the interface we could somehow interact is I have not considered any compromise so far to or to solve this type of different type of mode solutions just this arrival this this type of distribution, we have to just we know that what will be the distribution, but exact distribution you know that this will be like a standing wave here you to recreate it and outside it will be exponentially decaying because the imaginary part.

So, because E y component is tangential the field strength in this lower interface and upper above the interface and below the interface there should be continuous. So, you can use the boundary conditions and to find out the entire profile solutions I hope it is clear. Next is same thing I can think of for TM polarization. TM polarization again as I mentioned earlier, the electric field will be in the Z plane and magnetic field will be normal direction.

So, that is the tangential component electric field magnetic field. So, for that purpose we have another set of solution, again more solution because you are entering phi u phi l TM or whatever I have considered phi l subscript or superscript or whatever and phi u TM. We have to present 2 tan inverse 2 tan inverse all the value will be there. So, this function will be slightly different here it is n 2 here it will be n 1 and n 2 here.

So, this n 2 and n 1 will be interchanged proper this is you are considering lower interface phase change and this is upper. Upper means n 3 coming n 3 and here numerator denominator intelligence for TM and TM and that is the difference just to keep in mind. But you can I think I have shown how to derive how to starting from final equations when theta E greater than theta c what would be the phase difference phase change occurring at the interface that you will be able to derive using that equations already everything is should be TM we should write TM.

So, in that case you will be getting longitudinal direction all the solutions effective index solutions you will be getting and similar to that you will be getting mode field solutions. But here we have written instead of TE polarization I have just consider electric field but here we are considering magnetic field because the magnetic field is the tangential. So, all the boundary conditions in the lower interface and upper interface you can use magnetic field.

Because magnetic field or electric field in the boundary interface there will be continuous tangential component will be continuous, so, only component H y for TM polarization that must be continuous across the interface. So, you can use that boundary condition to make some continuity here and there and there and they are getting clear field distribution picture, but thing is that you said keep in mind we are actually finding the only one component for TE polarization. What about other 2 components?

How to find the other 2 components? You go back to Maxwell's curl equation from that curl equation once you know you just insert electric field only electrical component known to you have this type of solution for example, then you insert there then you get other components easily. So, similarly, per magnetic TM polarization, you have the solution for H y, y component is there. So, you can use curl equation one of the curl equations you use and then you can find other components E x and E z.

So, all the components you will be able to solve for both separately for TE polarization and for TM polarization and you can get whether, what sort of how many modes you can get. For example, next thing we will be discussing that I know that m can be 0 1 2 3 and so on. So, how many more solutions how many solutions I would be able to get because the integer means you can start from 0 to infinity integers.

So, you will be able to get infinite number of solutions. But all these infinite number of solutions will not satisfy confinement alone X direction. So, you have to ensure the confinement along is direction. So, all the solutions you have to check whether that is giving you confinement along the X direction or not if it is not giving confinement along X direction that is not a guided mode.

You have to ensure the confinement along X direction and up to the value suppose up to 3 you are getting solutions which is giving well different confinement along X direction then we can say that there are 3 modes can be guided in this waveguide if it is m = 5 solutions are giving confinement along X direction, then we can say that 5 modes are supported by the waveguide and beyond that, it will be even if you launch light at certain angle solutions you will be getting but that will be leaked.

That will not be confined that will be shown how the angle must be less than the critical angle in the interface. That is why it is not confined. So, I have been now discussing how to get a solutions of the number of modes for a given d value thickness for a given n 1, n 2, n 3 and lambda I would like to know how many more it can support and how to define d. So that I can say that it will be supporting only one mode or I can design d values to allow TIR 5 modes 5 TIR modes. For example, 5 modes to be guided.

So I need to find out what is the value of d require for a certain operating wavelength lambda. So, those types of things, I will be just giving some examples like silicon on insulator platform maybe and we will try to find out the values how to solve those values. It is not so easy to be that you can directly get a simple analytical solution from here directly theta TM because it is something like this equation if you look left hand side also theta TE is there.

Right hand side also theta value is there, theta is the variable. So, you cannot get something like theta very easily theta something like that right hand side, if you would get an analytical you just insert the value and you can calculate easily what is that theta value and this type of equation is called transcendental equation easily you cannot get some kind of analytical expression for theta. So, we need some kind of graphical method to solve this.

These 2 equations and try to understand how to estimate the number of modes and their field distributions and their propagation constant or effective indices or phase velocities for a given wavelength? If you change the wavelength situation will be different. So, those thing we will be discussing the next lecture. Thank you very much.