Introduction to Time – Varying Electrical Networks
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Lecture No. 75
LPTV networks driven by modulated inputs

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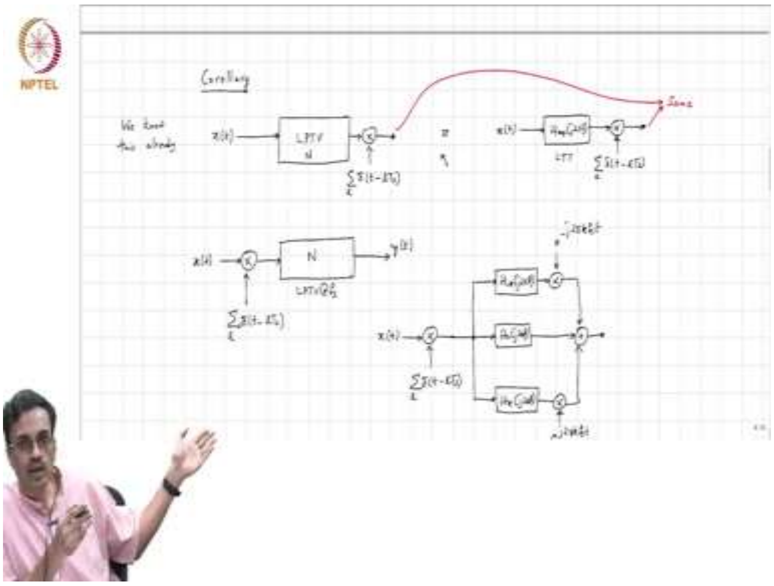

So, the next thing that I would like to talk about is a corollary of our principle that we have seen so far or I would send you all I would say. So, so far we have seen that if I have an input into an LPTV system and we sample the output at nTs. It does not matter whether it is nTs or nTs plus t naught. We know that we can always find an equivalent LTI system. We said that this is equivalent to have x of t driving an equivalent LTI filter, and if you sample the output.

In other words, if you sampling, I mean, one way to kind of think of sampling is you can think of this as a delta train, delta of t minus kt minus say lTs, and you get some output sequence here. So, what this is saying is that you will get the same output sequence if you take d minus lTs and so these two are the same.

Now, the next thing is, the next thing is what? Well, it turns out, as we will see going forward, that the opposite is also true. So, let us say you took an input x of t and modulated it with an impulse train. So, we know this already. And let us say it drives an LPTV system, at fs. Again, please note that the rate at which the LPTV system is varying, is exactly the same as the frequency of the modulating trip. Now, what we are going to see, and let us see, what let us call this y of t.

Now, the LPTV system can be represented by its as usual by its Zadeh expansion. So, you have h 0 of j 2 pi f, h sub minus k of j 2 pi f. So, this is e to the minus j 2 pi k fs times t, h sub k of j 2 pi f and you have e to the j 2 pi k fs times t and this is the equivalent of the system. And then it is now being driven by x of t and an impulse train delta of t minus LTS.

Now, there is, I mean, if you remember how we got the, we got this result. What did we say? Well, we were sampling. And when we sampled, we were sampling the outputs of the, we were sampling the sinusoids at k times Ts, and they all became 1. So in other words, if the multiplication and sampling happens side by side then it is easy to reduce the block diagram Like how we saw in the, in the first case, that is how we saw this result.

Here, unfortunately, it looks like, well, the multiplication the impulse train is here but the multiplication is happening on the other side. So if we want multiplication to happen on the left side, it turns out that it is pretty straightforward to do and I am just going to draw your attention to do that.

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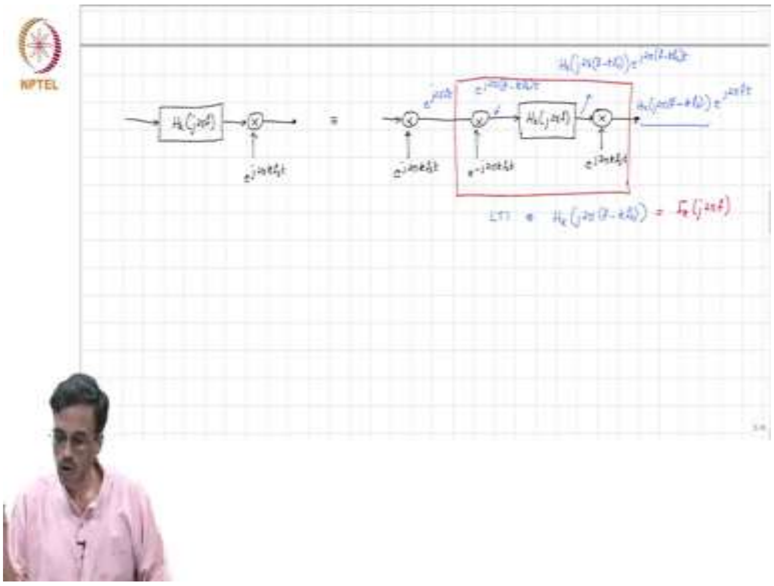

So, what we have right now is in the kth arm is h sub k of J 2 pi f, e to the j 2 pi k fs times t. And I can think of this as e to the j 2 pi k fs times t. I can multiply this by e to the minus j 2 pi k fs times t. And what should go there? What does this?

Student: H sub k of j 2 pi f.

Professor: So, this should simply be h sub k of j 2 pi f. And this is e to the j 2 pi k fs times t. You agree that both of them are equivalent. Now, what I want to do is draw your attention to what is there inside this box.

Student: T delta.

Professor: No, think carefully. So, let us say, we excite this by e to the j 2 pi ft, what is there?

Student: E for j 2 pi.

Professor: E to the J 2 pi.

Student: F minus.

Professor: F minus.

Student: K fs into t.

Professor: K fs times t. So, what is the signal there?

Student: H sub k j 2 pi.

Professor: H sub k j 2 pi times f minus k fs times e to the j 2 pi f minus k fs times t. It is an LTI system. So basically, whatever the input frequency is there it just gets multiplied by again. So, what comment can we make about output signal here?

Student: H sub k j 2.

Professor: It is h sub k of j 2 pi f minus. Let me just push the whole thing this side. H sub k minus k fs.

Student: T upon j 2 pi.

Professor: Times e to the j 2 pi ft. So, what conclusion? If you put in e to the j 2 pi ft the output is this. Does that depend on time? The gain imparted to a sinusoid at f by the red box does not depend on time. So, is it and the output is at the same frequency as the input. So, is it time invariant a time variant?

Student: Time invariant.

Professor: It is an LTI system. And what is the equivalent transfer function of the LTI system?

Student: H sub k j 2 pi.

Professor: It is simply h sub k of j 2 pi times f minus k fs. And all that this is doing, I mean, intuitively, this, h sub k of f minus k fs means that the response of that is moved from with from DC to a center at k times it moves towards the right.

Student: And on k fs.

Professor: And on k fs. Now why does that make sense? Let us see what the box is doing? It is taking an input frequency, pushing it to a. It is moving it to e to the minus k fs, that is going through the filter and then it is getting pushed back again. Pushed low and then pushed back up.

So therefore, you can write, so if you basically call this f sub k of j 2 pi f. So, by definition, therefore, I mean, instead of f sub k of j 2 pi f minus k fs I am just going to call that another filter, which is f sub k of j 2 pi f.

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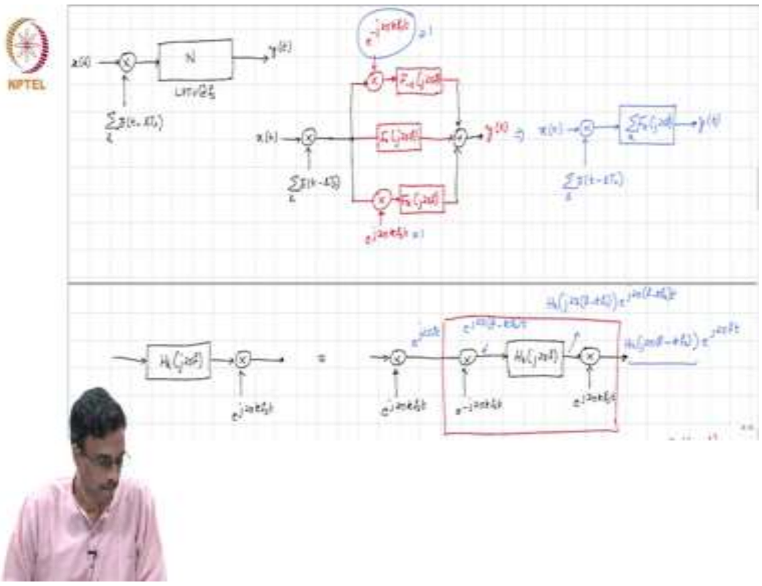

And therefore, I can rewrite. I can therefore, using this observation, I can rewrite this block diagram or manipulate this in this way. I will first multiply by e to the j 2 pi minus j 2 pi k, minus j 2 pi k fs times t this becomes, sorry, is this plus, yeah, so this is f sub minus k f j 2 pi f, this simply becomes f 0 of j 2 pi f which is basically the same as h 0 of j 2 pi f.

Remember, the kth arm that h sub whatever is just simply moving that h sub k to. Now this becomes e to the j 2 pi s plus k fs times t and f sub k of j 2 pi f, and this is y of t. Now, well when you multiply this so now, you can see why this makes sense. If you are now you are cascading two multipliers, so multiplication by two numbers is basically you can multiply the two numbers together and you have a single multiplier, when you multiply these two together what happens?

Student: It is like 1.

Professor: That all becomes 1. So, this becomes 1 all these become 1. So therefore, is equivalent to taking x of t sigma or l delta of t minus LTS. And what do I get here? All these become 1, so that is simply sum over f sub k of j 2 pi f and this is y of t. Does that make sense?

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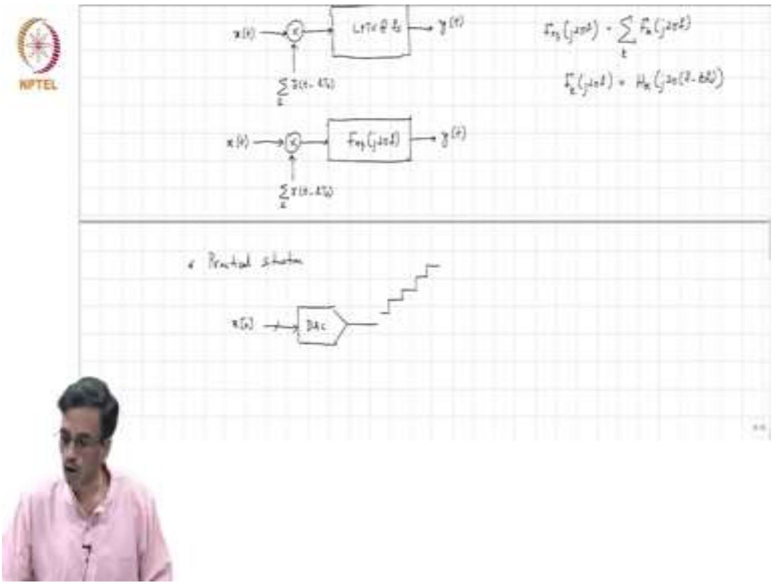

So, what is the order the moral of the story? Well, if you have x of t and you have an impulse train and you excite it, you excite an LPTV system how and you get some output, you can think of this as the output of the same impulse train exciting an equivalent LTI system and you will get the same waveform in in both cases.

So, f equivalent of j 2 pi f is simply the sum of f sub k of j 2 pi f where f sub k of j 2 pi f the f filters are simply the harmonic transfer functions j 2 pi f minus k fs. So, this is a dual of the other situation where there we multiply the output by an impulse train and look at the output signal. Now, it is I mean, you can think of that as this is simply turning the other one, the other input to output. Does it make sense?

So, the obvious question now is so, why is this even relevant? And a practical example of why this may help is. So, in practice, I mean, whenever you want to convert from digital to analog so you have a discrete time sequence x of an exciting a DAC. The output of the DAC basically will typically do will have a staircase like wave output here.

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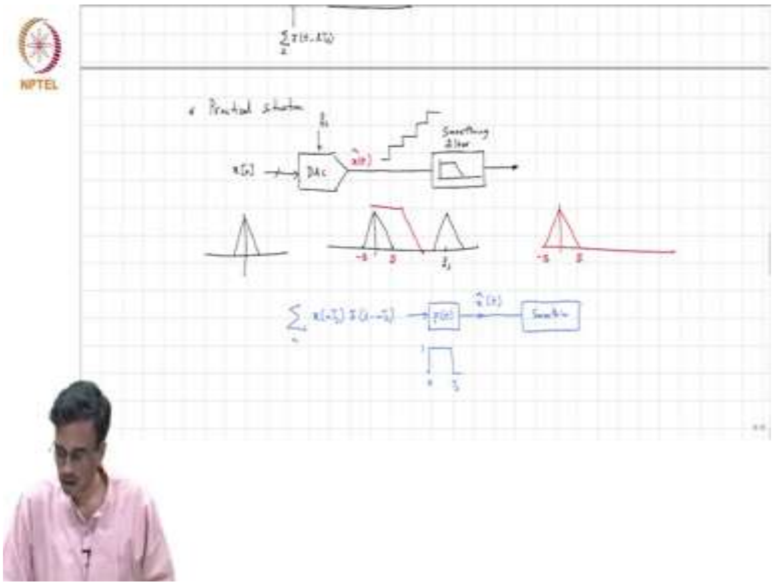

So, in the spectral domain, the staircase like output waveform, so, if x of n has a spectrum like this, this will have images around and let us say this DAC is sampling at fs. So, if you did not do anything, what you will have is you will have images at multiples of fs. So, what you need to do is to put in a smoothing filter. And the job of this smoothing filter is to remove that high frequency images.

So, a very common example of this is whenever you are trying to modulate an RF signal, I mean a baseband signal to RF, you will convert the baseband digital signal using a DAC. The output of the DAC cannot directly be modulated up. Because if that happens, then all these images will also appear it had RF, and that becomes a problem for the for your neighbor who is trying to receive a very weak signal. So, you must adequately suppress those images before you can go into a mixer or modulators, so that that job is done by a smoothing filter.

Now, this is the output of the DAC. So here basically what you end up getting at the output is clean spectrum like this. So, if this is b and this is minus b, at least ideally, you are supposed to see, no images. Of course, in practice, there will be some finite rejection. Now, the output of the DAC can be so let us call this x of t or let me call this x hat of t. X hat of t can be thought of as you take a sequence x of n. I mean you can then you multiply or rather.

So, you think of it as taking a signal x of n ts delta of t minus n ts sum over n. This convolved with that is the impulse sequence, and every DAC has got a pulse shape, so this goes into a filter, pulse shape is p of t, typically this p of t is non-return to 0 pulse. So, this is goes from 0 to t s and this is the x hat of t and that is the output signal of the DAC. Now, this goes into a I mean, this has to go into a smoothing filter. This is, but this sequence is a model edit sequence. It is basically equivalent into what we see here.

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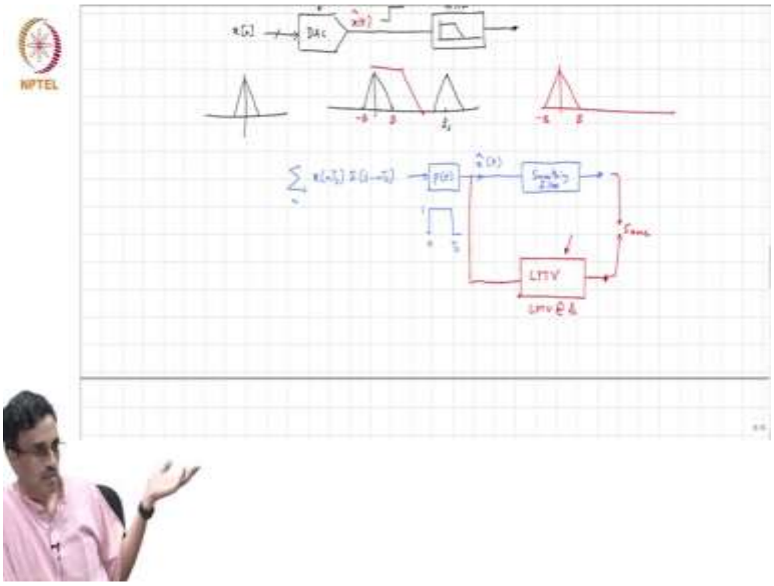

So, what this is saying. This is a corollary to what we have seen earlier where we sample the output of a of an LPTV system. Now, what we have just seen is telling is that well, there is no need to. I mean, in principle, if this smoothing filter is replaced by an LPTV system then also, there is I mean, at fs so you can find an equal and LPTV system whose response I mean, where these two can be the.

Student: Same.

Professor: Same. There we were trying to replace.

Student: LPTV system.

Professor: An LPTV system with an LTI system, here we are doing the opposite. You can potentially use an LPTV system to smoothen the output of the DAC. And the hope is that, because you have an LPTV system, you will be able to what do you call you will be able to perhaps make a filter, which is, which has a much sharper roll off at high frequencies than is possible with an LTI filter.

Remember, an LTI system is simply a special case of an LPTV system, where only that h naught is there and none of those other filters are there. So, there is obviously a lot more degrees of freedom here than they are in an LTI system. So, these can potentially be used to advantage.