Introduction to Time – Varying Electrical Networks

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Lecture 6.3

Applications of Inter-reciprocity: Analysis of Chopped Amplifiers (Continued)

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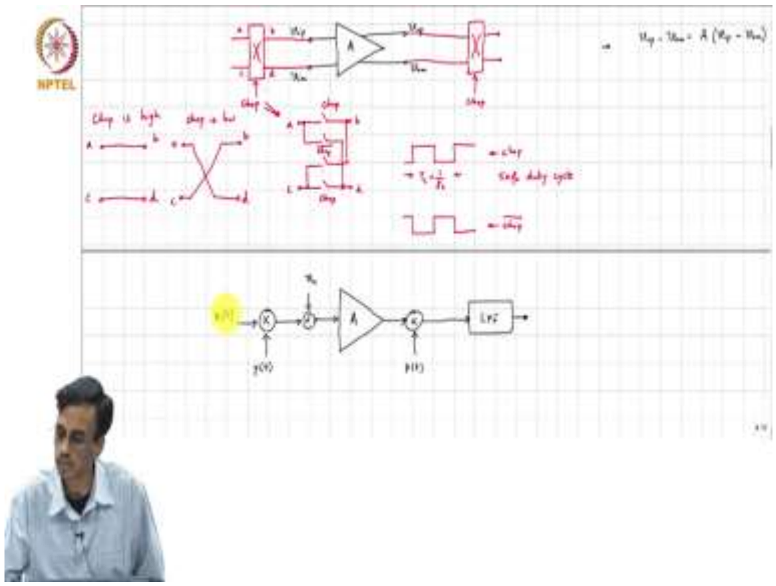

So, let us say, now a days all amplifiers are fully differential. In other words, what it means is that, if this v i p and this is v i m, this is v o p and v o m, then v o p minus v o m is simply A times v i p minus v o m. And those of you who have taken circuit classes will see why, probably already know why fully differential circuits are useful.

And the reason is that if there is a disturbance on both v i p and v i m which is very likely happen in a practical situation, then the amplifier, the amplifier is only sensitive, at least in principle, to the difference between v i p and v i m. So, anything that is common to both v i p and v i m is not amplified. So, in this scenario, we still have, the amplifier will still have offset and that causes all the, I mean the amplifier will still have input referred offset and flicker noise and that still causes problems.

So, however a fully differential operation makes it very easy for us to implement chopping with a slight difference in the chopping waveform and that is the following. You can, if you have, you can implement this block by... so, if I call this A, B, C and D and this is the chopped waveform and when chop is high and these are chop bar. So, chop is a 50 percent, so this is a chop signal, this is DC, this is 1 over f c, it has 50 percent duty cycle and this is chop bar and chop compliment.

At least in principle, that is also a 50 percent, you recycle square wave and so when chop is high, a is connected to d and c is connected to b. When chop is low or equivalently, chop bar is high, a is connected to d and c is connected to b. So, as you can see, this is equivalent to, if you want to draw a signal flow graph with this, you have an amplifier and the amplifier is amplifying the difference between the input and output of course. And the input here refers to v i p minus v i m and the output is v o p minus v o m. And there is of course, the amplifier flicker noise v n.

And what are we multiplying the input with? You are multiplying it with the chop waveform which is a square wave. Let me call this p of t. And how does p of t look? Well, it is a square wave with 50% duty cycle and it goes between 1 and minus 1. This is 0 and this is t c by 4, t c by 4, t c by 2, minus t c by 2. This is 1 and this is minus 1. So, this p of t.

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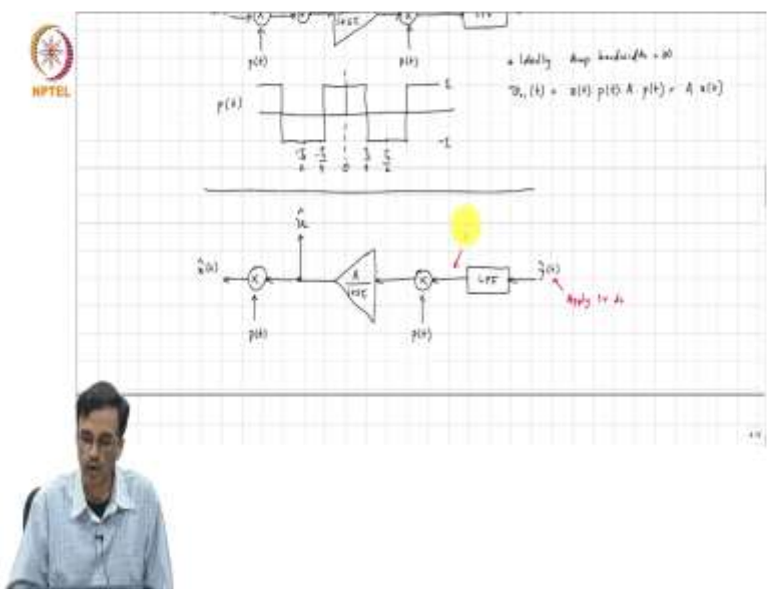

And if the amplifier has got infinite bandwidth and v n is 0, what do you expect to see? Well, x of t, I mean ideally, if our bandwidth is infinity and y of t, let us call this v o 1 of t, which is the output voltage just before the low pass filter, is simply x of t multiplied by p of t multiplied by A by p of t again, which is nothing but x of t times A times p square of t, which is, p square of t as you can see, is 1. So, this is A times x of t.

So, when compared to using a cosin modulating signal, we can see that we have a gain which is A rather than A by 2. And what comment can we make about v n? Let us say v n is a DC offset, then at the output of the amplifier, it is amplified offsets, then it is getting multiplied by a 1 minus 1 tone. It on average will be 0 because the low pass filter is going to remove this.

So, basically, under ideal circumstances, it works. Now, the question is what happens, of course we know that in the real world, no amplifier is going to have infinite bandwidth, so the question is okay, let us see what happens when you have an amplifier with a bandwidth whose, which is got a bandwidth of one over 2 pi tau. So, the transfer function of the amplifier is A, we assume it is a single pole model with a bandwidth and that bandwidth corresponds to time constant of tau.

Now, the question is what happens to y of t. In other words, we would like to understand what all inputs can lead to an output at DC. So, again we go back to our, so this is the original signal flow graph. Now, it is time to draw the adjoint. So, this is x hat of t, this is p of t, this is v n hat, this is A by 1 plus s tau. This is p of t again. This is the low pass filter, this is y hat of t. So, again what do we need to do? We apply DC here, what do you get here? 1 volt, now that gets multiplied, let us call this, at point A, what signal will you get?

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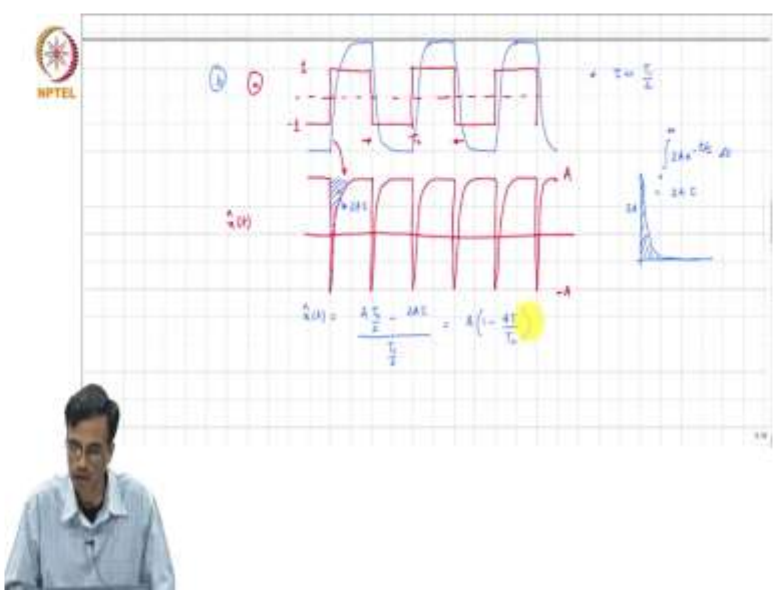

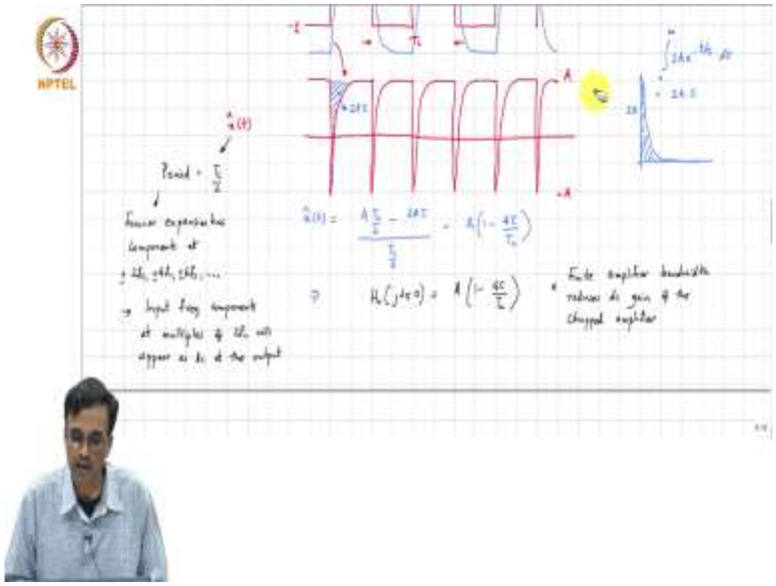

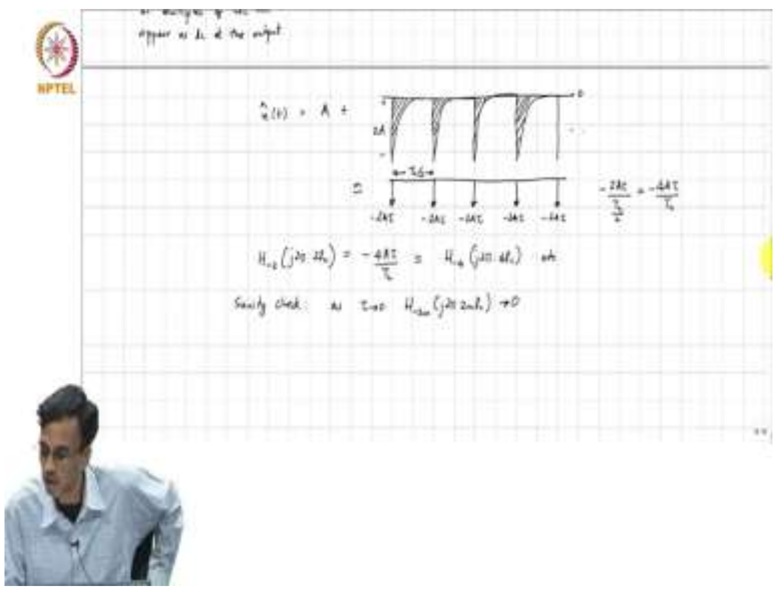

You will get p of t as, and by the way, this should have been p of minus t but since p of t is an even function, it is the same thing. So, what do we do now? What do you get at A is basically, a signal like this. So, this is 1, this is minus 1.

Now, what comment can you make about the signal at b? A square wave is passed through an amplifier with a DC gain of A and a bandwidth and a time constant corresponding to tau. So, what do you think you will expect to see at the output? Let us assume that, I mean, remember what is this duration? This duration is the chopping period DC. So, what comment can you make about, let me draw that in blue over the same picture here. What comment can you make? How will it look like?

No, no, you take a square wave, pass it through a single stage amplifier with a DC gain of A, it will basically do this and if tau is much smaller than T c by 2 or t c equivalently will do this. Then, do this and so on. So, the assumption is tau is much smaller than T c by 2. Now, what comment can we make about x hat of t, the signal at x hat of t?

The blue signal here is going to be multiplied by the red signal here. So, what comment can we make? So, what will happen to, when? So, basically you will see something like that. This red part is the same as this blue part. Then in the second half cycle, what happens? You multiply it by minus 1, so basically what you will see is something like this, then again you will see something like this and so on.

So, what is the peak amplitude here? It is going to be A, this is minus A. And what do you think you can, so the first thing that you observe is that, what comment can we make about H 0, I mean what do you think is the area of this part? It is the same, that hatched area is basically something is an exponential that goes from, with a height of 2 A and a time constant of tau. So, the area of this hatched part which is the same as the area of that hatched part there, is nothing but 2 A times tau, which is nothing but 2 A, e to the minus t by tau d t, integral 0 to infinity which is nothing but 2 A times tau.

So, what comment can you make about, so basically, the average value of this is the same as the average value over half a cycle of t c which is basically A times t c by 2 minus 2 A tau divided by t c by 2, which is nothing but A times 1 minus, no think carefully, 4 tau by t c. So, the average value of x hat of t is A times 1 minus 4 tau by t c. So, what is the conclusion? What does this mean?

Yes of course, if time constant is 0, you will get A which you expected but what is the conclusion? If we analyze the adjoint network, we applied DC at the output port and found that the DC value of the input port is A times 1 minus 4 tau by t c. So, what does this mean as far as the original signal flow graph is concerned? In the original signal flow graph, H 0 of j 2 pi 0 is A times 1 minus 4 tau by t c. So, what conclusion, so finite amplifier bandwidth reduces DC gain of the chopped amplifier. Alright? Okay?

The next thing, what is the period of the waveform x hat of t? The period of this waveform is t c by 2. So, when you expand this in a Fourier series, what all components will you get? What all frequencies will you see? You will see, so Fourier expansion is as components at 2 f c, 4 f c, 8 f c, 6 f c etcetera. And so, what comment can you make about so what? How does this?

So, what does this mean? This means that inputs frequency components at multiples of 2 f c will appear at DC at the output. So, this basically means that there is down conversion of input signal components at all even multiples of the chopping frequency. Now, the question is what happens to the, what is the gain from an input, for instance, at 2 f c to the output at DC? You understand?

So, we would like to, we know that input components at multiples of 2 f c will fold or will get translated to DC. Now, the question to ask is, what is the gain from an input at 2 f c to output at DC? So, how do you think we can find that out? Yes, Fourier expansion of this periodic waveform. Now, we do not do the Fourier expansion of the Fourier waveform but let us get an idea if by, if we can get a good approximation by basically recognizing that this output is simply 2 A, just A minus, so x hat of t is simply A plus a waveform which kind of does... and so on.

This is 0 and this peak is, this is 2 A. And the time constant is tau. So, this can be approximated as, you can think of this is as an impulse with an area of 2 A. So, this is a periodic impulse trained with an area of 2 A. And this is nothing but DC by 2. And if you have a periodic impulse train, you know that all the Fourier, an impulse train in frequency domain is also an impulse train in the time domain and all the Fourier co-efficients have the same value.

So, if you know the DC component of the impulse train, the strength of all the harmonics is also the same thing. So, what is the DC component of this value? It is nothing but 2 A tau divided by tc by 2 which is 4 A into tau by t c. So, all the Fourier co-efficients have the same magnitude and that is because you have approximated this narrow exponential pulse by an impulse function.

So, at least for the first several harmonics transfer functions, you basically, this value will be accurate. So, because this is negative, so this must be all must be negative and this must be, all the co-efficients must be minus 4 A tau by t c. So, what comment can you make therefore of H minus 2 of j 2 pi times 2 f c. It is nothing but minus 4 A times tau over DC. And which is also approximately equal to H minus 4 of j 2 pi times 4 f c etcetera.

Now, what is sanity check? Of course, sanity check is DC, we already know, but what comment can you make about the H minus 2 of j pi 2 f c? Is there a sanity check that we can do for that?

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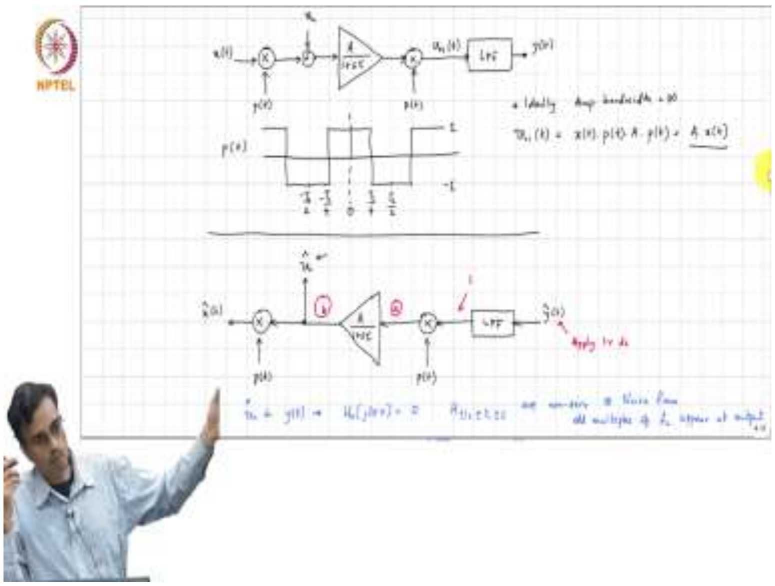

As tau becomes 0, what do you expect? So, as tau becomes 0, remember that as tau becomes 0, the output is simply A times x of t because p square of t is 1 and you can see that from this diagram too, as tau becomes 0, this waveform will simply becomes a constant A, there will be none of that exponential tail.

So, as tau tends to 0, the H sub minus 2 m of j 2 pi m 2 pi into 2. 2 m f c becomes 0, tends to 0. The next thing is what comment can we make about the hormonic transfer functions to, from the noise source to the output and for that, what do we need to do? We have to look at the waveform at B. The waveform at B is shown in blue, that is this waveform here. And that is basically, a square wave.

So, what comment can we make about, as far as v n hat v n to y of t is concerned in the original signal flow graph, what comment can we make about H 0 of j 2 pi 0? That will simply be the DC value of this periodic waveform and that is z. And that we knew already. If you apply DC there, it is going to get chopped to plus and minus p of t and that is going to be completely eliminated by the low pass filter.

And of course, there are higher order harmonics and what is the fundamental frequency of this waveform at B? That is simply a period of f c, a period of t which basically means the Fourier series has got components at multiples of f c, however as you can see, since the duty cycle of the square wave is 50 percent, it will only have odd harmonics.

So, H plus minus 1, plus minus 3, plus minus 5 etcetera are non-zero and what does this mean? Noise from odd multiples of f c will appear at tau. This is an example of the use of adjoint techniques where, you can also try and do the, the adjoint makes you see very clearly what is going on. In principle, you could, I mean finally when you have a circuit you can always analyze it.

So, you could, in principle go on putting an input tone at f c and then put it at 2 f c and then, arrive at the same result. But this is messy and you have to do it once for the offset, I mean once for v n and once for x of t and that too for each frequency. This, you know, applying the adjoint technique basically saves a significant amount of labor. So, this is a very useful trick.