

**Introduction to Time-Varying Electrical Networks**  
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**Course Introduction and Motivation**

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Introduction to Time-Varying Electrical Networks

Received power =  $-100 \text{ dBm}$

$0 \text{ dBm} = 1 \text{ mW}$

$P \text{ in dBm} = 10 \log \left( \frac{P \text{ in watts}}{1 \text{ mW}} \right)$

$-100 = 10 \log \left( \frac{P \text{ in watts}}{1 \text{ mW}} \right) \Rightarrow 10^{-13} \text{ Watts} = 0.1 \text{ pW}$

Good morning everybody and welcome to the first lecture of the course Introduction to time-varying electrical networks. In this lecture we will look at the motivation for why the subject makes sense, for whom it makes sense; and what exactly we are going to be learning in this course. As a motivating example let us look at something that has changed the way we live our lives namely the mobile phone. So, how does this work? The key aspect of a mobile phone is the radio. Without the radio your phone would just be yet another toy, and let us see what the radio is actually doing.

It is picking out a very small received signal which is transmitted from base station, which can be pretty far away. And in the middle, there are trees, there are buildings and a whole lot of other landscape; and you are here clashing your phone. And the signal that is being transmitted from the base station on this tower here is reaching you potentially after taking many paths. For example, one path would be like this, one path could be like that and perhaps there is a direct path also. Because of the distance between you and the base station, the received signal power can be very very small.

Because a, the strength of the signals coming along these three paths is already very small and b, these paths could interfere in strange ways to result in signal cancellation locally. And as a consequence, the signal power that your phone is actually receiving is very very small to get some idea of what these numbers look like.

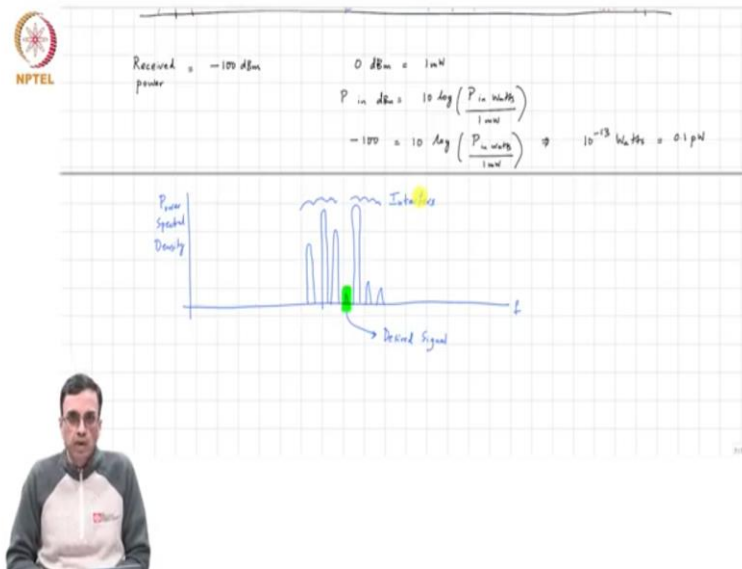
But a typical number would be minus 100 dBm in the sense that your phone should be able to respond to signals desired signals as small as minus 100 dBm. So, and what is 100 minus 100 dBm? One power is often referred to in dBm and 0 dBm basically means 1 milliwatt and P in general in dBm is  $10 \log$  the power in one in watts, divided by 1 milliwatt.

So, a minus 100 dBm signal is very small indeed and to see what that is that basically means that this minus 100 is  $10 \log$  power in watts, divided by 1 milliwatt. So, this basically means that we are looking at 10 power minus 13 watts of power, is the strength of the desired signal that your phone is actually receiving. And as you can see this is a tenth of a picowatt and this is very very small indeed. On the other hand, you are not the only person who is using your phone; you have a friend here who is also trying to talk to somebody, and his signal has to reach the base station.

So, he must be sending a fair amount of power typically several hundreds of milliwatts; so, that there is a hope of a small amount of that power reaching the base station. Therefore, the base station can make sense of what your friend is saying. Now, a big portion of that power is likely to hit your phone, simply because your friend is very close to you.

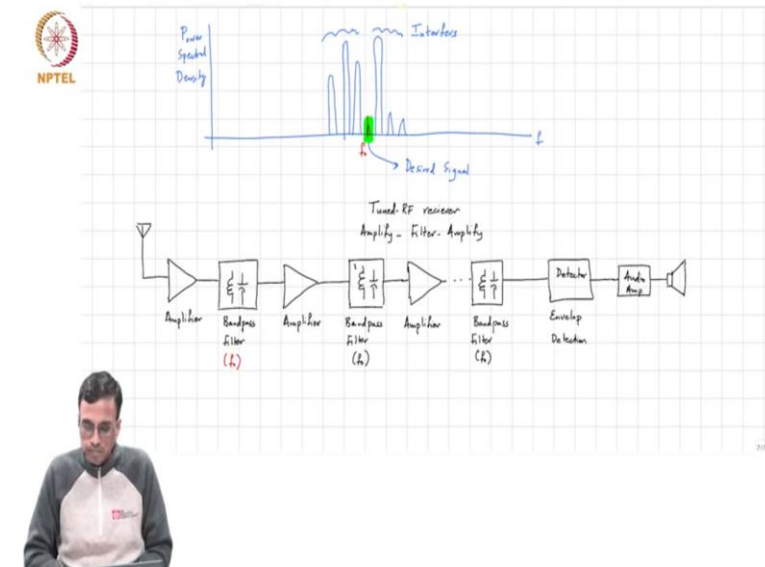
So, your phone not only has to receive a very very small signal; it also has to fish out the small signal of the information on the small signal in the presence of very very large signals from other sources. Your friend's phone is simply an example. It could be a microwave oven running close by; it could be any other source of RF radiation. So, to cut a long story short what your cellular phone is doing.

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Is looking at this crowded RF spectrum where the whole bunch of signals. This is frequency and this is power spectral density and you have many unwanted signals; and your job as the receiver is to pick out this small information bearing signals, which is our desired signal from among all these interferes. And remember that the desired information bearing signal is modulated onto on RF carrier; simply because propagation at these frequencies can be done efficiently. So, radio waves propagated at certain frequencies efficiently; and therefore, the desired signal the information bearing signal is modulated onto on RF carrier transmitted. And at the receiver our job is to pick out these small signal and decode the information bearing signal.

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How does a modern phone do this? Well before we get to the modern phone; let us step back in time and see how radio receivers were built you know almost 100 years ago. So, the received signal which was received by an antenna would be very very small; and as you have just seen and to be able to figure out what received signal looks like. The electronics that follows the antenna must be able to see a sufficiently large signal. So, what is done or what was done was to have an amplifier up front.

Well, you could make the amplifier gain very very large and therefore be able to figure out what was being transmitted. Unfortunately, the fact that you have so many other interfering signals means that if the gain of the amplifier is too large; then the output of the amplifier would saturate. So, to prevent this you would have an amplifier with whatever gain was feasible before saturation on only an effect sets in; and filter, the output typically done using tuned lc circuits. And so, this is basically a bandpass filter and if you are interested in receiving a signal at a frequency  $f_{naught}$ ; the bandpass filter would have to be tuned to a frequency  $f_{naught}$ .

Now, those of you have taken a basic circuits class, know that it is very difficult to achieve a very narrow band lc network; simply because the inductors and capacitors are lossy. So, the quality factor of the lc network that you can build is often limited, and since the information bearing signal is so narrow band compared to the carrier frequency. If you had to filter out all the interferers in one go; it turns out that the quality factor of the filter components that you need to

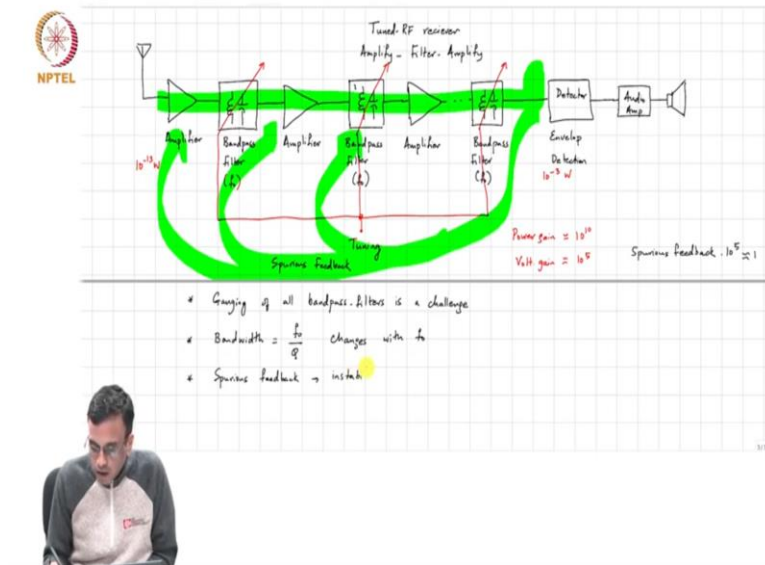
build your bandpass filter is so high has to be practically infeasible. So, what people did was to have a cheesy error somewhat gentle bandpass filter first.

Then it would basically kind of tame the interferers a little bit; then you get into another stage of amplification which is now feasible, because the interferers are lower in strength. Follow this up by yet another bandpass filter also centered at  $f_{\text{naught}}$ , which would attenuate the interferers even more and also result in the sharpening of the bandpass characteristic. And do this several times over before the interferers are sufficiently attenuated, and the signal the desired signal is sufficiently amplified; so that you could go into a detector which typically was an envelop detector.

This would give out just we sense the envelop transmitted RF signal which is desired, and before this would be a baseband signal or the information bearing signal; which would then go into an audio amplifier which would go and drive a speaker. So, this is now what is called the tuned RF receiver and the basic idea is very straight forward. The scheme is to amplify filter amplify and you keep doing this until you knock off interferers, and you amplify the desired signal.

Now, why is it called tuned that is because we have a whole bunch of bandpass filters which need to be tuned to your desired channel. And why is it a tuned RF receiver? Well, the whole signal chain all the way from here to here is all operating at the carrier frequency or the RF frequency. Now, let us see what the problems of this radio receiver are, so that we can better motivate the need for what we are going to be studying in this course.

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First, of course when people were buying the radio you wanted to be able to listen to multiple channels; obviously, not all at the same time. So, you would need to have a tuning knob so that you could tune to different frequencies if desired. So, in other words you would need to move or select the desired channel, by moving the center frequencies of these bandpass filters all at the same time. Because it does not make sense for one filter to move and the other to stay where it was; you want to move all the bandpass filters center frequencies together in unison, and this is called ganging.

So, if you have represented in a picture, you would have a knob that would go and tune the center frequencies of all the bandpass filters in unison. So, of course there is a practical problem with this; you can while you can attempt to tune all the center frequencies in the same direction by the same amount. In practice manufacturing tolerances will mean that you turn the knob and all the frequencies do not really move by exactly the same. As a consequence, individual users had to go and adjust their radio receivers, so that this happened; so, basically that was one problem.

So, ganging of all bandpass filters is a challenge; especially when you consider manufacturing variations of all these components; the next problem is that selectivity. Remember that the bandwidth of an LC network of a second order LC network is dependent on the quality factor of the components. Now, when you see assuming that the quality factor remains largely unchanged

over the RF band; what happens when you tune your radio receiver to a different channel is the following. The center frequency changes; but the quality factor of the bandpass filter remains the same.

And therefore, the bandwidth of the filter or the bandpass filter, which is basically the center frequency divided by the quality factor changes, as you tune your center frequency. So, if you tuned your bandpass filter from say 500 kilohertz to 1500 kilohertz; then the bandpass filters bandwidth would also change. This is not desirable because all stations are mandated to have a fixed bandwidth. Perhaps the next drawback was the following and this is perhaps the most damning of the problems of the true tuned RF receiver; and that is that of feedback to see why spurious feedback is such a big problem.

Think about the receiver; the receiver is of the order of let us say 10 power minus 13 watts. But this voltage before it goes and hits the audio amplifier whose job is to mainly drive the speaker, is perhaps of the order of milliwatts. So, the net power gain from the input to the output is about 10 power 10; which means a voltage gain of about 10 power 5. So, we have in effect from here all the way up to here a high order amplifier with a large gain of the order of 10 power 5. Now, even if there is a small amount of spurious feedback from the output to the input; so, this is all what I will term a spurious feedback.

This can happen in multiple ways perhaps it happens to the supply, perhaps is just coupling from the output to the input; so, this is spurious feedback. And remember that a feedback system a high order feedback system can easily get unstable; it is conditionally stable and to kind of estimate how much spurious feedback one can tolerate. Well, it is we just look at how much the gain the loop gain must be before instability section. So, the spurious feedback times gain which is 10 power 5 must be approximately 1. So, the amount of spurious feedback one can tolerate before you have potential instability is of the order of 10 power minus 5; which is about 100 dB.

So, in other words a signal that is at the output of the amplifier, here must have a spurious feedback back to the input of less than or much less than 10 power minus 5; and that is extremely difficult to achieve in practice. So, consequently it was not uncommon for radios to oscillate and so designers had to spend a lot of time trying to ensure that the gains of these spurious feedback paths were minimized by design; and so that was yet another.

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\* Gauging of all bandpass filters is a challenge  
\* Bandwidth =  $\frac{f_c}{Q}$  changes with  $f_c$   
\* Spurious feedback leading to instability

Low Frequency

\* Filter bandwidth and  $f_c$  are decoupled  
\* Channel bandwidth independent of  $f_c$   
\* Gain & filtering at baseband  
\* Spurious feedback paths are broken

Direct Conversion Receiver

Let us now take a look at what has happened over the last 100 years; this is the block diagram of a modern radio receiver. We have an amplifier for the same reason that we have one and the tuned RF receiver. But, after the amplification we go through a process of demodulation; where the received signal is multiplied by sine and cosine waves, which are generated by a local oscillator. Now, as you seen from your basic signals and system classes; multiplication by a sine wave is equivalent to demodulation. So, the information bearing signal is now at baseband; so, this is the baseband signal; and the desired baseband signal is filtered.

This which is now done using a low pass filter and you have a baseband amplifier in each path; which eventually drives which goes into DSP. So, the DSP of course uses complicated algorithms to go and figure out what the input signal is. So, but as far as the RF receiver is concerned, you can see that most of the filtering and amplification happens at baseband frequencies. Consequently, these filters can be implemented using active RC techniques and do not need inductors. Further, it is much easier to get gain at baseband frequencies than at RF frequencies; so, these amplifiers in the baseband are very power efficient.

But, perhaps most importantly channel selection is done by selecting the frequency of the local oscillator. So, moving the frequency of the local oscillator which is generated at the receiver; essentially moves the channel that the receiver is receiving, and consequently the bandwidth of the low pass filter are decoupled. The center frequency of the local oscillator is decoupled from



the bandwidth of the low pass filter. As a consequence, the channel bandwidth remains independent of  $f_c$ . Gain and filtering performed at baseband, where they are much easier to accomplish.

Next, remember we finally need the same gain of about 10 power 5 down the signal chip. However, there is a big difference between this receiver and the tuned RF receiver; and that is how parasitic feedback is addressed. Now, this signal here at the output of the receiver is at baseband; so it is a low frequency. So, let us say a part of this somehow couples on to the antenna. Low frequency output couples through the amplifier which will reject low frequencies; because this amplifier is typically a bandpass amplifier. And whatever remains is modulated onto a carrier at  $f_c$  and therefore is going to be filtered by the lowpass filter.

So, as you can see the process of modulation or multiplication or heterodyning as it is often called; as you can see the process of modulation or heterodyning breaks the problem of spurious feedback, simply because the feedback is at low frequency. Whereas the process of demodulation translates this feedback signal to a much higher frequency, which is eliminated by the filter. This is happening because the feedback loop has translation inside it; so let us now see what happens to spurious feedback in this receiver. Now, the output of this receiver just like in the tuned RF case has to be large gain multiplied by the input; except that the output frequency is very different from the input frequency.

Now, suppose a small amount of the output frequency leaks to the antenna. Remember as we discussed before it is very difficult to get high isolation between any two nodes on a circuit; so, now even if the large output signal at baseband leaks to the antenna. It is first of all rejected by this bandpass amplifier which is right at the antenna. Further whatever remains is going to be multiplied by local oscillator frequency and is going to be modulated to RF; which is then eliminated by the lowpass filter. So, essentially the process of modulation has broken the feedback spurious feedback loop.

Simply because, the gain is at a very different frequency than the signal received by the end; so, this turns out also to be a clincher for the receiver that uses heterodyne, so spurious feedback. So, to summarize therefore the modern day receiver which of which an example we discussed above is, this is what is called a direct conversion receiver. The direct conversion receiver which is an

example of a modern radio receiver addresses all the problems of the tuned RFSC. First, the filter bandwidth and the channel frequency are completely decoupled. Most of the gain is achieved at baseband, where it is easy to achieve.

Most of the filtering is done at baseband where it can be done in a compact and power efficient manner. And finally, the problem of spurious feedback is eliminated because the gain occurs at baseband; and the process of modulation/demodulation essentially breaks this feedback. Now, as you probably realized the key operation that enables all these nice things to happen is indeed the process of modulation or demodulation. Without heterodyning or modulation or multiplication all these are equivalent terms are used interchangeably. The magic of the modern day radio receiver would simply not be possible.

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Block diagram: Input  $x(t)$  enters a multiplier block with gain  $A \cos(2\pi f_c t)$ . The output is  $A \cos(2\pi f_c t) x(t)$ .

Equations:

$$x(t) \rightarrow A \cos(2\pi f_c t) x(t) \quad \text{--- (1)}$$

$$y(t) \rightarrow A \cos(2\pi f_c t) y(t) \quad \text{--- (2)}$$

$$\alpha x(t) + \beta y(t) \rightarrow A \cos(2\pi f_c t) [\alpha x(t) + \beta y(t)]$$

$$= \alpha \times \text{(1)} + \beta \times \text{(2)}$$

Superposition is satisfied  
→ Linear

Time-varying

$$x(t-t_d) \rightarrow A \cos(2\pi f_c t) x(t-t_d)$$

$$\neq A \cos(2\pi f_c(t-t_d)) x(t-t_d)$$

Let us take a closer look at multiplication as a system; we have an input which is processed by a multiplier, multiplied by  $A \cos 2 \pi f_c t$ . And the output is the product of the input with this cosine. If the input is  $x$  of  $t$ , the output is  $A \cos 2 \pi f_c t$  times  $x$  of  $t$ . If the input is  $y$  of  $t$ ; the output is  $A \cos 2 \pi f_c t$  times  $y$  of  $t$ . If the input therefore is  $\alpha$  times  $x$  of  $t$  plus  $\beta$  times  $y$  of  $t$ ; the output as you can see is  $A \cos 2 \pi f_c t$  times  $x$  of  $t$  times  $\alpha$ , plus  $y$  of  $t$  times  $\beta$ ; which is simply  $\alpha$  times the output of equation 1 plus  $\beta$  times the output of equation 2.

So, evidently multiplication obeys the law of superposition; so this is evidently a linear system. Specifically if you put in an input of 0; the output is 0. Now, let see what happens if you delay

the input by some time  $t_d$ . The output is  $A \cos 2\pi f_c t \times x(t - t_d)$ ; and why is this true. Remember all we are doing this changing delaying the input, this does not change; so, the output is simply  $A \cos 2\pi f_c t$  times the delayed input. And as you can see this is definitely not the same as taking the output that we had earlier, and delaying it by  $t$ . So, what I meant to say was if you delay the input, the output is not a delayed version of what we had before. So, this basically means multiplication or modulation or heterodyning is an example of an operation that is time varying (36:23).

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Multiplication - mixing, heterodyning, modulation

- Linear
- Time-varying

Analogy: Time & amplitude is continuous

Digital: Time & amplitude are discrete

DSP

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- Linear
- Time-varying

Analogy: Time & amplitude is continuous

Digital: Time & amplitude are discrete

DSP

Interface electronics

Analog input → Amplifier → Filter → Quantizer → Digital Signal

Analog-to-Digital conversion

So, to summarize therefore multiplication or mixing or heterodyning or modulation all mean the same thing; and they are all at least ideally they are linear operations, but that time varying operations. And as we have just seen the key that makes the modern radio receiver work is this process of modulation. So, that should convince you of the importance of studying circuits that are linear and at time varying. Yet another case in point is a modern signal processing chain. Remember that signals that we are interested in are in the natural world like images, voice, RF whatever.

These signals are all continuous in time and amplitude; but eventually need to be processed digitally in order for information to be stored efficiently and transmitted. So, we need to convert these these analog signals into digital form, and this is basically a digital signal is one where time and amplitude are discrete; whereas real world signals the time and amplitude continues. So, you need some electronics between the real world and the virtual world, which is able to translate continuous time continuous amplitude signals into signals that discrete in time and amplitude.

And this is what is called the interface electronics; also called the signal conditioning electronics. And how does this work? We have a combination of amplifiers. Some filtering we have sampling which converts continuous time discrete time followed by quantization; so that the sampled signal is converted into a signal with discrete levels. So, this gives you a digital signal or a signal that can be represented digitally; because the number of levels is discrete and it only occurs at clock edges. So, this process is called analog to digital conversion.

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Analog-to-Digital Conversion

Input  $\rightarrow$  Gain & Filter  $\rightarrow$   $\sum_k \delta(t-t_k)$   $\rightarrow$  Quantizer  $\rightarrow$  Output

Sampling is a time-varying operation

- \* Linear time-varying electrical networks

Background - Signals & Systems  
- Electrical Networks

Topics

- \* Refresher on basic circuit analysis  
- write circuit equations

Virtually every signal chained today follows this paradigm, where you have an analog to digital converter; which eventually interfaces to the DSP. And the DSP then plays around with this digital bit stream in whichever way it works. Now, as you can see if you plot the signal chain block diagram; you have gain and filtering. And you have sampling which is basically mathematically equivalent to multiplying the input by a Dirac impulse stream; and followed by quantization. Quantization introduces error which often is considered as an additive error and this is the order of the signals.

So, the input is amplified and filtered, sampled which in mathematical terms can be represented as multiplication by a discrete an impulse stream of Dirac impulses, followed by a (correction) with quantization. Now, in practice the quantization error is often designed to be so small as to be negligible as far as the output is concerned. So, ADC for all practical purposes can be thought of as essentially converting a discrete a continuous time signal to a discrete time, through the process of sampling. As you can see here sampling is the same as multiplication; but it is now being multiplied by a different signal.

Earlier we had a cosine, now we have an impulse stream; and just like a demodulator or modulator. Sampling is also a time varying operation. Even how important a to d converters are in modern day electronics. It makes sense to learn more about the time varying operations that go on inside an analog digital converter. So, this course basically introduces students to the

important area of linear time varying electrical networks. And what exactly are we going to be learning in this course and what background should one have. I am assuming that you had a basic course on signals and systems, and you understand the fourier transform and laplace transform well.

I am assuming that you have also taken a basic course on electrical networks at the undergraduate level. So, you must be familiar with Kirchoff's voltage and current laws and how to solve networks and so on; and with this you should be able to follow this course. Now, what topics will be will be we covering? We will first refresh our memories on basic circuit analysis. And what this means is that we need to be able to write circuit equations systematically.

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- \* Linear time-varying electrical networks

Background - Signals & Systems

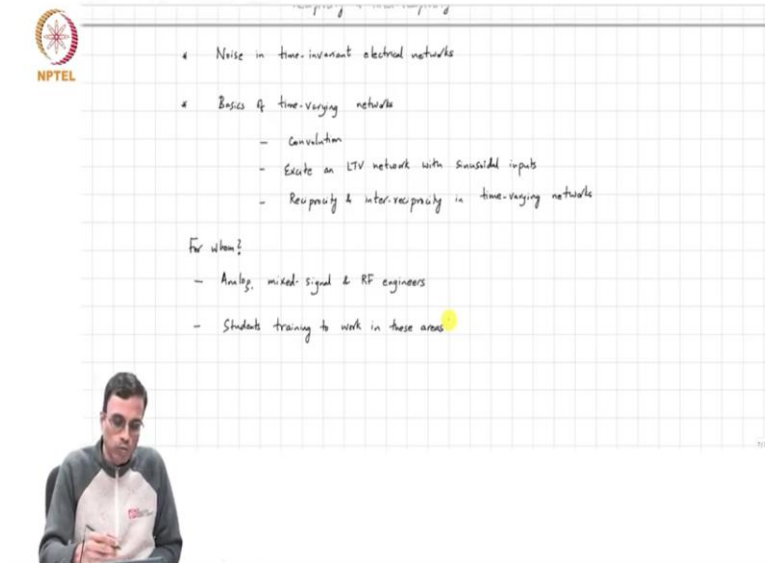
- Electrical Networks

Topics

- \* Refresher on basic circuit analysis
  - write circuit equations systematically
  - Tellegen's theorem
  - Reciprocity & inter-reciprocity

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- \* Noise in time-invariant electrical networks



This will form the base for the material on time varying networks later in the course; but this also serve as a refresher for those of us who have done this long ago and forgot. We will refresh our memories with regard to Tellegen's theorem and concepts of reciprocity and inter reciprocity; which are very important later on when we study time varying networks. Then we will also cover noise in time invariant electrical networks.

We will then move on to basics of time varying networks; so, we will re-look at everything we have learnt with respect to time invariant networks. Now, with the lens of time varying systems, so we will look at convolution; we will see what happens when you excite an LTV network with sinusoidal inputs.

Then we will look at reciprocity and inter reciprocity in time varying networks. And we will see many applications of such analysis, as and when we come across come across then. So, this is going to be the outline of this course and who is this course targeted towards. Well, it is targeted towards analog, mix signal and RF engineers as well as students training to work in these areas. With that I will stop; I hope to interact more with you during the course. Thank you.