Optical Engineering
Prof. Shanti Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 25
Gaussian beams introduction

Good morning. So, we finished one section of this course now the part on geometric optics. And, we are starting on the next part which is on Gaussian beams.

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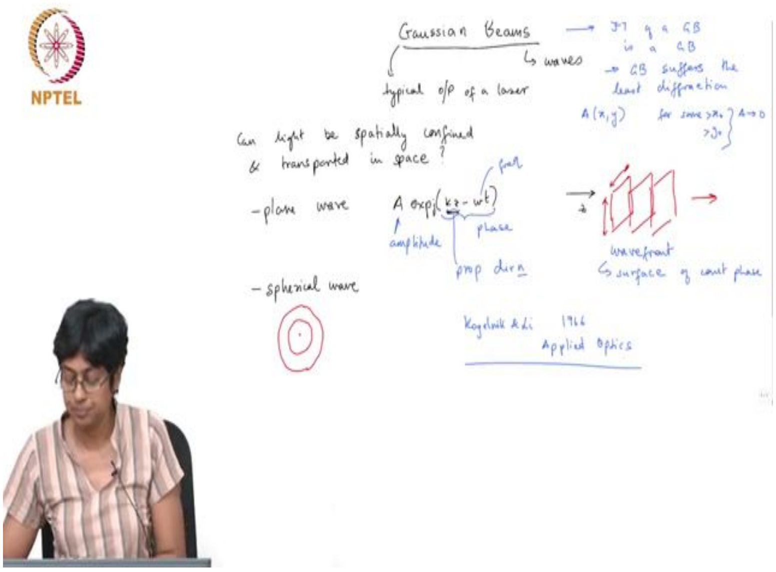

So, it is a big change, because up till now we have been considering that light travels in straight lines and we are considering rays, and the moment we go to Gaussian beams or any kind of solution to the wave equation. Then, you are no longer thinking of light as simply rays, but you are thinking of it in terms of waves.

The many reasons why we study light in this way and those reasons will become clear as we go along the course. One point to notice is that in many applications today, the source of light that we use is a laser, ok, that you would have heard laser diode if in many of our small systems will have laser diodes. Many larger systems will have different kinds of gasoline lasers or a variety of lasers and the typical output of a laser.

So, this is if I had to pick and say, what is the output of a laser? Laser can be the light coming out of the laser can be in a variety of modes, but the typical output of a laser is a Gaussian beam ok. So, that is one very crucial reason why we look at Gaussian beams ok. Now, I can approach this in another way I could ask the question: can light be spatially confined, and transported in space? And, again to answer this I am going to look at the different kinds of beams that exist.

Now, what do I mean by spatially confined; that means, as this beam propagates does it spread out and occupy all of space or does it travel. So, it is propagating. So, transported means it is propagating. As it propagates, does it propagate with a more specific direction or does it spread in space.

So, again we use lasers the most typical output is a Gaussian beam and when I am looking at beams, what is the usefulness of the beam? We are always thinking how can I manipulate light in some way? And, it most often is useful, if you can confine that beam as it travels.

Now, you would have heard of different kinds of beams, different kinds of waves. So, you might have in any electromagnetics course you would have definitely looked at what is called a plane wave right. I see no recognition on anyone's face ok. So, if I write the equation that represents a plane wave, I will have the amplitude of the wave and then I have a term that takes into account the phase of the wave right.

So, this is what shows it is complex right, I mean it is $k_z - \omega t$. What do these different terms mean, I am talking about the amplitude this entire term in the bracket is the phase, but this relates to the frequency or I can relate it to the wavelength of that wave and this to the propagation directions, that z is the propagation direction.

You could think of a spherical wave and in a sense these two kinds of waves are sort of completely opposite in nature right. Because, what is the property of a plane wave as it propagates, you can see from this equation, it is propagating in one direction ok. So, it is propagating in the z direction, but it is not a practical wave, I can never realize this wave in a lab when I say this wave I mean what is expressed by this equation. I can never realize this exactly, because of the amplitude. If you see, there is no specification on what is the relationship between the amplitude and the transverse coordinates x and y.

So, if you look at this expression it is actually saying this amplitude A it spreads out infinitely. So, if you might have seen pictures of a plane wave and they made you may have seen it drawn something like this right. So, these are the different planes of the plane wave, this is the direction z.

Now, when we they draw it like this, they are drawing this finite square shape of finite rectangular shape, but that is just because they want to show you something in the book that you are looking at in reality there is nothing in that expression that says, the amplitude spreads over this finite distance in the x and in the y direction. The way I have drawn it here there is a finite spread in the equation there is an infinite spread.

So, actually this mathematical equation though you might have seen it many times before may not have realized that it implies I am talking about a wave with infinite energy ok. What are these squares that we are drawing; these are what we call the wave fronts so for any wave when I discuss a wave when I talk about a wave propagating. I will use the wave fronts as one means of describing the propagation ok. And, what is a wave front a way front is nothing, but a surface of constant phase.

So, of course, the wave exists throughout the medium, but I am drawing at one position where the wave has and here it is a plane wave over this entire surface is a constant phase. And, then we draw the next wave front, where it has travelled an optical path length that gives it back the same phase it had over here ok. So, equivalent to a distance of lambda ok; the phase equi that is acquired after travelling a distance lambda.

So, while this is interesting to look at, I might be able to arrive at some simplified version of this in a lab. This expression by itself represents a wave with infinite energy and is not something that is actually produced in the lab. However, I am interested in it, because it gives me the type of wave, which has one direction that it travels in and it travels consistently in that one direction.

The spherical wave and the and is completely opposite; it can start from a point and the wave fronts emerge as spheres around that point ok. So, when I draw it under a two dimensional surface it is going to look like circles, but the wave front is a sphere about the point and as you move away from the source which is the origin of this wave the spheres go on expanding in size ok.

So, there is no one direction this wave is traveling in it is traveling in fact, in all directions. So, too many of course, waves are very different behaviour, and we study them because some simplified versions of these are achievable in the lab and are used in actual experiments, but they are not exactly a description of what we are seeing and they definitely not a description of what is emerging from a laser.

So, we go back to the Gaussian beam and apart from the fact that this is a typical output of a laser; it also has some properties that make it relatively nice to study ok. So, it turns out that the Fourier transform of a Gaussian beam is going to be a Gaussian beam again. And, this is important when you are looking at propagation, because I can use the Fourier transform operation to understand, what happens to a wave as it propagates?.

So, this is a nice result, this is a nice fact. It turns out that of all the different beams the Gaussian beam suffers the least diffraction ok. So, in addition to it being the beam that we are typically going to get as an output it also has some nice properties that make it useful.

Now, we are thinking of the Gaussian beam as a solution, we know the output of a laser is a very directional beam all right. The beam emerging from a laser does not immediately start diverging right; it is a very directional beam. So, in that sense it is similar to the plane wave, where it differs greatly from the plane wave is the fact that the plane wave stretches out to infinity in the transverse direction right.

And, we know that the beam emerging from the laser has a finite width; that means, if I were to look at the amplitude of the wave in terms of x and y considering that it was traveling along the z direction. I know that at some large values of some value of x. So, for some x naught and for some y naught this a would tend to 0.

So, for larger than x naught and larger than y naught a would tend to 0; that means, the beam has a finite diameter, it does not spread out to infinity ok. So, we are not going to do a derivation of the Gaussian beam here. If you are interested, one of the earliest papers on Gaussian beams was by Kogelnik.

So, the authors were Kogelnik and Lee, it is I think a 1966 paper that appeared in the journal applied optics ok. Where they study this beam in more detail and give you mathematical derivation. So, if you are interested in more information, you can get it from there. And, see if we can arrive at something that seems to satisfy the Gaussian beam.

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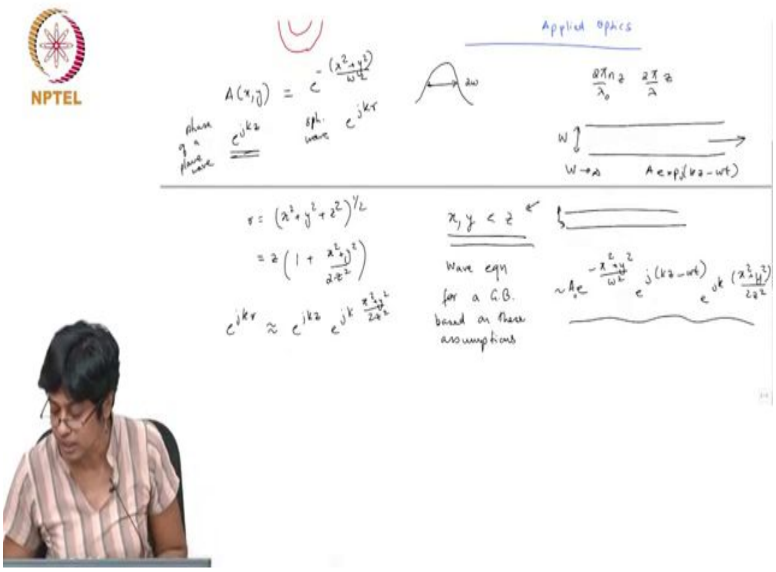

So, I know that, I want to consider a beam that has a finite spread in the x and y direction ok. So, I am going to guess that it has some relationship like this $A(x,y) = e^{-(\frac{x^2+y^2}{\omega})}$. So, if I consider the width the transverse spread of the Gaussian beam, then we consider this width as 2ω ok.

The plane wave in terms of phase had a jkz dominant right; this was a phase acquired as the wave travelled along the z axis in the z direction right. So, this was the phase. What is that k? The k is the wavenumber it is $\frac{2\pi}{\lambda}$ right and of course, it would be $\frac{2\pi}{\lambda}$, if this was in some medium all right.

So, that lambda you can consider I could consider it as it is $\frac{2\pi}{\lambda_0/\lambda_0}$ is the wavelength in air and if it was some other medium it would be this. So, I could write this as $\frac{2\pi}{\lambda}$ where that is the wavelength in that medium ok. So, this into z is an optical path length, sorry the nz is the optical path length and this into 2π is the phase that is acquired as the wave travels.

So, my plane wave expression had this term. This is how the phase was changed in the plane wave expression? We are looking at a beam that is not exactly a plane wave; it is not exactly a spherical wave. How would this look, if I was looking at a spherical wave. If this is the phase of a plane wave ok. If I was looking at a spherical wave I would have jk_r right, where $r = x^2 + y^2 + z^2$.

Now, I know my Gaussian beam is not exactly a plane wave, it is definitely not a spherical wave. So, I am going to do a modification of this term. How is it different if we say that we are worried about or we want to take into account x and y in terms of the amplitude. So, we are considering a beam now that has a finite lateral spread, but that lateral spread is small and does not extend to infinity at all.

So, it is a good assumption to say that the variables x and y that we are going to consider for r wave I am going to be much less than the distance over which the wave is travelling ok. So, the beam is spread over a very finite lateral distance and it is very small. Those parameters are small compared to the distance travelled.

Now, if I do that, I can rewrite this as so my phase then I could consider it to be e^{ikr}, but in this case because of this assumption the r reduces to $e^{\pi jkz}$, that is the first term into $e^{ikx^2+jky^2+jkz^2}$ We know the way the Gaussian beam propagates and we are trying to arrive at we are making some intelligent guesses, and trying to arrive at what could the behaviour of the wave be, what could the expression be based on it is behaviour, based on what we know about the plane wave and the spherical wave?. So, if I write out then the wave equation for a Gaussian beam based on my assumptions right.

Since we have not done a formal solution of the wave equation, I cannot assume that what I write down based on these assumptions is exactly the wave equation for a Gaussian beam, but I know that it should have terms similar to these terms. And, so, if I write down what I know it should have a term. So, maybe there is some $A_0 e^{-(x^2+y^2)/\omega^2}$ right, and $e^{\pi jkx}$ minus I am taking the time into account, and $e^{ikx^2+jky^2+jkz^2}$

And, you can already see it is a wave more complicated than either the equation for a spherical wave or a plane wave right. And, that is one of the reasons, when you first solve the wave equation in electromagnetics and you study waves, you are generally just given these equations of the plane wave and the spherical wave, because that helps you understand how you can get different solutions to the wave equation? Yes.

Student: So, why do we extend those waves?

The extent of the wave, I am talking about a wave that so, his question is where did this arrive, why did this arise? I am talking about a wave that is if I was looking at a plane wave a plane wave, I could consider as traveling like this, but I have drawn it here with this finite extent, but actually according to the plane wave equation, this if I call this w I will say $w \to \infty$. The plane is infinitely large the equation for a plane wave is $e^{kz-\omega t}$. I have no x y component there is nothing limiting the amplitude a in terms of x and y.

So, the mathematical expression means this actually spreads across infinity right, but that is not a true solution, because if the wave actually spreads across infinity, it has infinite energy and I mean of course, we can never generate such a wave right. So, we are making the assumption that I am I now saying the way I draw I am drawing this. I am saying I am considering a wave of finite width, in other words the spread of the wave in this direction and in the orthogonal direction. So, in the transverse plane those dimensions are much smaller than the distance over which the wave is traveling and that is why I arrive at this.

So, if it is too detailed to work out the formal derivation of the Gaussian beam, it is not required for this class, but I want you to get the idea that and from your you all ought to have done course, some course in wave propagation or a electromagnetics before this. So, you would have arrived at an equation wave equation and you should have also studied the fact that there are many different solutions to the wave equation right.

The point that you need to understand here that is of importance here is many of those solutions you would have studied earlier, were mathematically ideal solutions. Then, they are not really what we generate in a lab; they are not really what emerges from a source like a laser.

Now, I am trying to use the knowledge that you had there to give you a sense of how we can consider the Gaussian beam, which is a real solution to the wave equation a practical solution to the wave equation. So, based on these assumptions, I modified or used or ideas of the plane wave and the spherical wave to arrive at this expression of it is, I would expect that the Gaussian beam expression is of this form ok.

So, now, what I want to do is I am going to give you the derived solution; I did not want to throw that equation at you because it has a lot of terms in it. And, if I just give you the Gaussian beam equation it says where did all of this come from. So, these few minutes should have given you some background as to what is the origin of these different terms ok.

We cannot explain every term of the actual Gaussian beam with this hand waving argument right, but at least some of the term should make more sense to you now. So, what I am going to do now is to write out the expression and this is their expression arrived at from a formal derivation or a formal solution to the wave equation taking Gaussian beam conditions into account.

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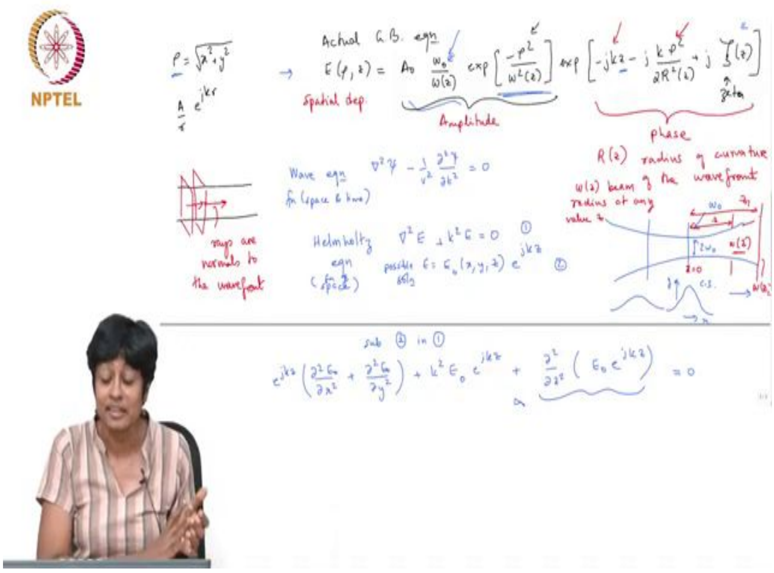

So, the actual Gaussian beam equation and it is a slightly complicated equation. So, I want to get it written. Let us say,

I say, it is E which is a function of r and z ok. And, actually let me not say r, because I used r earlier as this. And, I want this variable I will call it ρ is now just $x^2 + y^2$, it is just a function of the transverse coordinates ok.

$$E(\rho, z) = A_0 \frac{w_o}{W(z)} \exp \frac{-\rho^2}{w^2(z)} \exp \left[-jkz - j\frac{k\rho^2}{R^2(z)} + j\zeta R(z) \right]$$

So, there is some amplitude A_0 and then I will explain what these different terms are $-\rho 2$ $-\rho$ minus rho squared by omega squared z exponential minus j k z minus j k rho squared by capital R squared of z, this is 2 plus j, and this is zeta ok. I am not drawing that very well zeta of z ok; this is the actual expression for a Gaussian beam.

Now, some parts of it you it with any wave equation you must recognize the main parts of the wave equation, the expression for the wave right. What are the main parts, you have a set of terms that relate to the amplitude of the wave right.

And, you have an exponential and everything in this bracket relates to the phase of the wave. You can see it is way more complicated than either the expression for a spherical wave or a plane wave right. In fact, I did not write out the expression for a spherical wave, but that would be $jkrA/R$, because as the wave propagates in the radial direction at any point its amplitude is dropped right. So, the power is constant ok.

So, while you may not understand each of these individual terms in this Gaussian beam expression, you can still see as any expressions for a wave should have for any wave, there must be an amplitude term and there must be a face term and we can see that here. You should also be able now to see some similarity between the expression we arrived at taking into account some assumptions that modified the plane and spherical waves right. And, we know what those assumptions were based on, given that what we know about the Gaussian beam is a beam of finite width that does not spread in the x y direction.

We arrived at some expressions like this right. And, you can see that mirrored over here remember $\rho^2 = x^2 + y^2$. So, it is a function of the transverse coordinates that I have this term relating to this part we have not written out the time dependence. So, I have written only the spatial dependence here ok. We can always study only the spatial dependence, because you are $e^{-j\omega t}$ is going to repeat everywhere.

So, you can always just study spatial dependence. And, finally, if you actually need the full wave at any point of time at any instant of time, you will take into account that $e^{-j\omega\theta}$. So, we have written only the spatial dependence of the Gaussian beam here. So, that is why you do not see the term $-j\omega t$, I can write that does not give me any information about the spatial extent of the wave.

So, this has been taken into account, this has been taken into account here right. So, I have expressions, which take into account some of the phase varying terms in my Gaussian beam, I do have some extra terms, I have this term and this term ok.

And, those of course, you would understand, if we did the formal derivation. What is important for you now is to have a better sense of these terms in this Gaussian beam, to get an idea that now we are talking about or discussing a beam that it travels more or less in one direction the z direction. But, it does have a finite spread indicated by this and that spread is taken into account in the amplitude right. This is coming in the amplitude, which means that the amplitude does not spread out to infinity right. And, as we go along the other terms will become clearer.

What, why are there other terms? Because, if I write down expressions for any wave, what is each term in that expression doing? It gives us the information of how the wave changes as it propagates right. And, this idea that propagation itself does something to the wave should not be new to you, because even in ray optics we considered propagation through free space as an operator, as an optical operation right.

So, if you did nothing and you let your ray travel through free space or an homogeneous medium, it changed some parameter of your ray. And, that is exactly the same with any wave, any beam, as it travels the very act of traveling changes the beam. And, that is why we classify beams into plane waves, spherical waves, Gaussian beams; there are a multitude of beams.

You might have in your life up till now heard of plane waves and spherical waves, but you have Gaussian beams, you have Hermite Gaussian beams, you have Legare Gaussian beams, you have Inchi beams, you have Methow beams and that is just a fraction of the beams.

Now, what does each of these names mean? It means you are talking about a class of beams, or a set of beams, which have a specific behaviour as they travel, the way they change as they travel has some specific property. So, the Gaussian beam needs this complicated expression, in order for us to calculate or to understand that, if I know all the parameters of the beam at one plane, what does the beam look like at some other plane? Right.

So, I may have measured the beam parameters at one plane and I allow this beam now to travel, I may need to know, what are the parameters at another plane? They were even in your optical design system; if you do not know the parameters, if you do not know how the phase of the beam has changed, you cannot design a lens that will give you focus.

Because, the lens assumes that it will focus a beam given it has a certain input phase on it. If you are missing calculating the input phase and you are saying I am going to use this phase to change the beam in this way. If your input phase is wrong the optical element is not carrying out the desired operation right. So, I need to be able to calculate, what is the phase of a beam after it has travelled some distance?

Alternately I could look at and say, I know all the properties of the beam here. If the beam were traveling through free space or through a medium homogeneous medium, I can use this expression and say this is what the beam would look like at this plane. Now, if it does not look like that, I know what aberration has been caused by this propagation.

And, I can correct for that aberration, because I should have got a beam with this I did not. And, so, I can add a negative of the effects that made the beam deviate from what it should? I can correct that. And, some form of this is used when people are studying stars and the light that is traveling from a distant star comes through our atmosphere.

Now, the distant star is the perfect point source as far as we are concerned it is. So, far away we can consider it a point source. I should get a nice perfect focus assuming my optics of my telescope is perfect, but it does not happen, because the beam gets distorted through the atmosphere. But, since I know I should get a star spot I can correct for the distortion and in fact, this measure of the distortion that it is seen right.

So, there are so many things you could use this to say I expect a beam to look like this, it does not. So, this is a measure of the distortion that tells me something about the medium that it travelled through. Maybe, I use this as a sensor. So, there are many many reasons why you want to be able to calculate just maybe that you want to be able to study propagation or for all these other reasons, but you always need to have an expression for the beam and that expression is going to tell you exactly what happens to the beam as it propagates ok. So, for the Gaussian beam, if we look at the spatial variation, what I get is this expression.

Now, just to give you some background. So, that is not really that I pulled this equation out of a hat. The equation that was solved the original equation that was solved to arrive at; this is actually the wave equation ok. The wave equation takes of course, into account both the spatial and time variation of a wave. So, it is the del squared operator let us say my wave is psi minus 1 over v squared del dou squared psi up to t squared is 0 right. This is the original wave equation that we are solving.

But, this equation, of course, is a function of space and time and in a lot of cases we can neglect the time part, when you are just studying the wave. Finally, of course, if you want the full description of the wave you need to include the time part, but if you are looking only at the spatial variation this equation will reduce to what is called the Helmholtz equation, which is only an expression of and I put it in terms of electric field right now.

So, this is just as it is a function of space. So, I have kind of gone a step before this solution ok. Again trying to justify why the solution is acceptable?. Now, if I did not know anything, if I did not make any of those assumptions 1 point that I could consider was the solution to this one possible solution is to say, I have an amplitude and I have a phase ok. So, here I have not made any assumptions. Now, if I substitute this, so let us call this equation 1 and this equation 2.

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

$$\nabla^2 E + k^2 E = 0$$

$$e^{ikz}\left(\frac{\partial^2 E_0}{\partial x^2} + \frac{\partial^2 E_0}{\partial y^2}\right) + k^2 E_0 e^{ikz} + \frac{\partial^2}{\partial x^2}\left(E_0 e^{ikz}\right) = 0$$

$$\frac{\partial}{\partial z}\left[\frac{\partial E_0}{\partial z} e^{ikz} + jkE_0 e^{ikz}\right]$$

$$\frac{\partial^2 E_0}{\partial z^2} e^{ikz} + jkE_0 \frac{\partial e^{ikz}}{\partial z} + (jk)^2 E_0 e^{ikz} + jk\frac{\partial E_o}{\partial z} e^{ikz}$$

$$\frac{\partial^2 E_0}{\partial x^2} + \frac{\partial^2 E_0}{\partial y^2} + 2jk\frac{\partial E_z}{\partial z} + \frac{\partial^2 E_0}{\partial z^2} = 0$$

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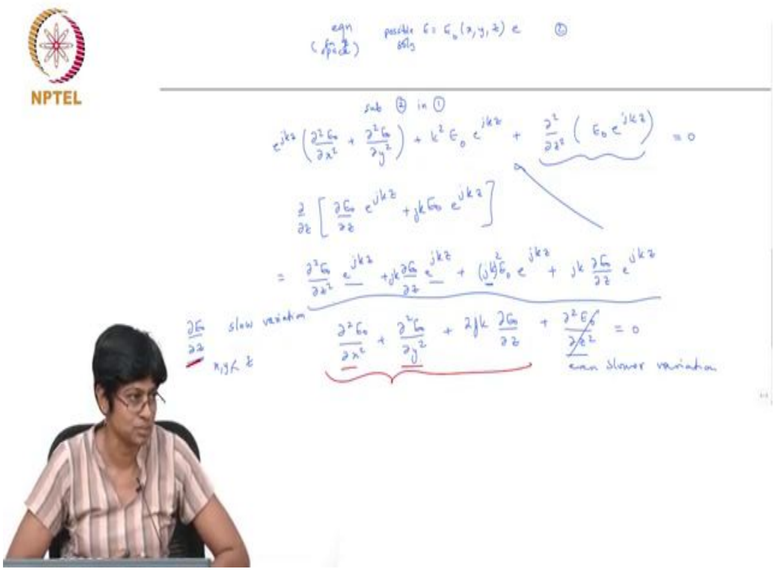

If, I substitute equation 2 in equation 1 I am going to get. So, this is E, I am going to get dou squared E naught by dou x squared is dou squared E naught by dou y squared. Of course, there is a e power j k z there, there is this k squared E there, which is I am not writing out x y z of E naught, but everywhere please note E naught is a function of x y z ok. And, I am going to have dou squared by dou z squared of E naught E power j k z right, this is equal to 0.

So, I can just concentrate on this part for the moment and that is going to be nothing, but dou by dou z of the first derivative of these. So, I will have dou E naught by dou z E power j k z plus E naught j k E power j k z. And, if I take this derivative again, I will have dou squared E naught by dou z squared correct.

So, if I now write this part along with this equation, what will happen is I have a j k squared here. So, I have a negative term. So, finally, and everywhere every term has e power j k z. So, that is going to cancel throughout. And, finally, what you will be left with is dou squared E naught by dou x squared plus dou squared E naught by dou y squared plus 2 j k dou E naught by dou z. And, you will have one more term and that is the term dou squared E naught by dou z squared equal to 0.

And, again I could go back to my variations for this wave and say well we are seeing dou E naught by dou z is a slow variation. So, again I am taking into account the fact that the variables x and y are much less than z. So, the variation of the amplitude with respect to z is a slower variation, than the variation of the amplitude with respect to the lateral concern. What does that mean? It means as the beam propagates, the changes in the amplitude in the lateral extent are far more dramatic, then the changes in the amplitude as it propagates along the z axis.

Yes of course, there are changes as it propagates along the z axis. Think of a wave that you are bringing to focus right. At the plane that the wave leaves the lens to the plane where the focus has occurred of course, the amplitude has changed, but the amplitude has changed greatly in the lateral region much more that variation from exiting the lens to the focal plane slowly happens right. So, you this is a slow variation. Compared to dou squared by dou x squared and dou squared by dou y squared and that is why we say this is an even slower variation and you can neglect it.

So, the equation that is actually solved to arrive at the Gaussian beam expression that I had written above, the equation that is actually solved is this equations, derived from the spatial Helmholtz equation and you are taking into account the fact that you say, this variation dou E naught by dou z is a much slower variation than these ok. So, I am building all the groundwork for the Gaussian beam derivation, but we are not going to do the Gaussian beam derivation ok. But, I did not want to just give you that Gaussian beam expression, because it is quite daunting to see all those terms and not having a sense or idea what those terms.

So, this expression I did not explain all the different terms there right. So, we recognize some of those terms now that we know what they are doing to the wave? But, we did not talk about what is omega of z, what is capital of R of z, what is zeta of z?. So, let us do that now.

So, R capital R of z relates to the radius of curvature of the wave front ok. What we mean again, if I think of a plane wave I would draw it like this and I would say it is wave front can be considered to be planes. And, fact what are the rays we have been talking about all this time the rays are nothing, but the normals to these wave fronts right. So, the rays are normals to the wave front.

So, in a plane wave we had these plane surfaces spread out to infinity and they repeat every time the wave, we can look at them every time the wave has travelled 2 pi a phase of 2 pi ok. What are the wave fronts of a spherical wave?

Student: (Refer Time: 44:06).

What is a wave front of a spherical wave?

Student: (Refer Time: 44:14).

It is the spheres. The spheres around the origin are the wave fronts of a spherical wave. What are the wave fronts of a Gaussian beam? Right. So, the Gaussian beam is a really interesting beam in some places it has a very plane wave like nature. In some other places when I say places, what do I mean as it propagates? At some places during it is propagation it is going to appear like a plane wave. And other places it is very similar to a spherical, it is similar to these waves right.

How do I draw the wave fronts of a spherical of a Gaussian beam well, they are depend I it depends greatly on where I am drawing them. But, if you look at any textbook, what they will draw for you if they are showing you a Gaussian beam propagating is they will draw something like this, this is a side view. So, this is the z direction.

So, the overall the general direction of the wave is one direction that is it is plane wave nature. And, yet if I were to look at the cross section, it is not that it spreads out to infinity. The cross section anywhere at any plane would be Gaussian in nature that is why it is called a

Gaussian beam. That means, the amplitude of the wave on the intensity of the wave dies down as you go out now I am drawing a cross section, this is a cross section. So, this is in the y direction, this is in the x direction.

So, as you go side wards the amplitude dies down clearly showing this is a wave with finite energy does not spread out to infinity. And, any cross section I take is going to have this Gaussian nature, only when the wave has spread out a little more this beam or the peak is happening at a lower place and the spread is more, but it is Gaussian anywhere ok.

And, to talk about the wave front I have to; I have to be specific about where along it is propagation. So, there is a place where you have the minimum beam waste, that waste is called omega naught. So, this is 2 omega naught and that is the omega naught that you are seeing here. This is the minimum beam size for this Gaussian beam.

Omega of z is the parameter that tells you, what is the beam radius at any value z because you can see a plane wave I assume it has the same radius or the same diameter as it propagates right. But, a Gaussian beam you can see from the way I have drawn it comes down to a narrow point and then it is diverging again right. So, it has a certain omega of z omega is the relating to the radius ok. So, if your diameter would be 2 omega of z, but if I go to another value of z it has a different value of omega right and z is measured from this beam waste this is always considered the e origin this is z equal to 0.

So, this is the distance z for which you have this omega of z and then this would be z 1 and you would have omega of z 1 and so on. I did not say there is no variation; it is a slow variation ok.

Student: So, how can we do this?

So, we have not neglected we did not neglect dou E naught by dou z that is retained in the expression we are saying that itself is slow compared to the variation with respect to x and y right that is slow, but we retained dou E naught by dou z in our expression, we said it is slow and therefore, the second derivative is even slower and that we neglect ok.

Now, in all of these derivations, even when we did ray optics. We you would have seen you talked about third order aberrations, fifth order aberrations, or when we arrived at the expression for a lens, we as we did a we took the so, in your derivation for the refraction at a curved surface you had a cosine term. And, we neglected all the higher order and we said let us assume that it is one right.

I can always do the derivation repeat the derivation taking the first the next term into account as well. And, that is how you arrive at your 3rd order, 5th order 7th order? So, even here this is a general expression for a Gaussian beam, where we say the second derivative dou square d naught by dou z squared it is very small compared to the variation with respect to x and y, that it is varying no one is saying it is not varying.

But, it is so; small compared to the transverse variation it is negligible. In certain applications where the dimensions are such that the either you are not it is not acceptable to make these assumptions, you need to be so, accurate or the variations are not a negligible, you would go back to the derivation and say I will not neglect that term I will keep that term.

Why do we do neglect it? Because in most cases, we have neglected it; that means, is an error factor in our solution, but that the error introduced is probably smaller than the errors were getting from other aberrations you do not see that error. And, in any application where it is going to be so sensitive you would go back and say let us include all the terms.

Today's class was just an introduction to Gaussian beams ok. As, I said if you are interested in the details of it please do go refer to that paper that I mentioned or there are textbooks which will have the derivation, but the idea for us again is I want to shift your focus now from rays to waves.

So, that we can now start looking at because not every system can be analysed using rays, if the wave nature it not be neglected and in many cases it cannot be. Then, I just even to design my optical systems I must be able to take this into account ok. And, so, we are just shifting our focus so, that we are able to take the wave nature into account, we look at the Gaussian beam in more detail in tomorrow's class.

Good morning. So, we have moved on to the next topic Gaussian beams and that is what we spent yesterday's class on we did not do a formal derivation of the equation, but we looked at how the Gaussian beam is different from a plane wave and it is spherical wave two beams that we are familiar with. And, using some logic based on how we expect the beam to behave or what kind of beam we are looking for? We explained the expression for a Gaussian beam which has been derived elsewhere.

What we were doing at the end of the class was to look at the different parameters in this beam a little closer.

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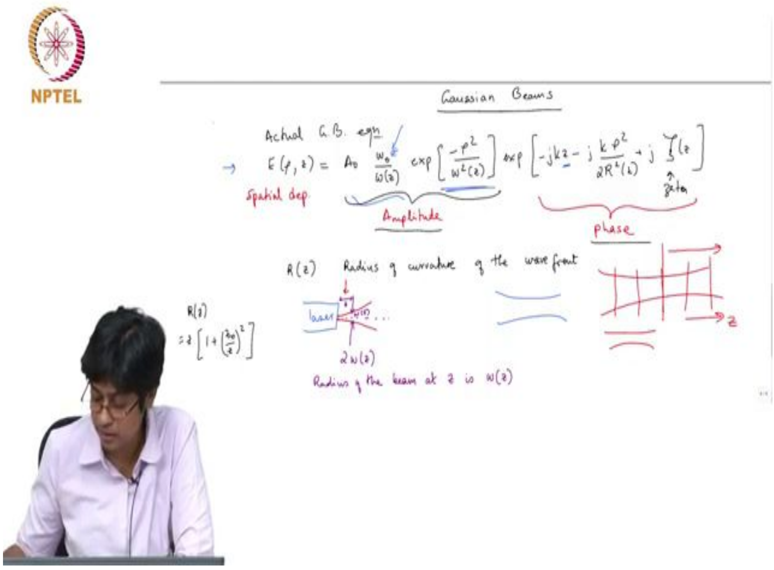

So, this equation has written here very clearly demarcates the part of the beam, which is the amplitude relates to the amplitude and the part which relates to the phase. But, within this you can see there are different terms. So, we looked yesterday at R of z, which we call the radius of curvature of the wave front. And, to explain this I had said imagine you were looking at a Gaussian beam propagating along the z direction.

If, you were to take the cross section at any plane, it would have a different radius of curvature ok, which is quite different from the plane wave or the spherical wave. This plane wave at any plane, it has a planar surface. So, it is wave front is a planar surface the set it spherical wave at any plane it is wave front is a sphere. So, the radius of that sphere is going to be different, but you have a sphere right. Whereas, with the Gaussian beam and we will look at this in more detail, the nature of that wave front is changing as the beam propagates.

Now, there was some questions about why I drew the beam like this? So, just to clarify if the beam were coming out of a source so, let us say I had a laser. And, the beam emerging from this laser is a Gaussian beam, what I would expect from this source is not as I have drawn here with a increased radius or diameter and then it decreases and then it increases again. But, from the source the source would have the minimum radius and it would be diverging from there ok.

So, in other words what I am drawing in this figure is basically from this point onwards ok. Now, why is that it is because a Gaussian beam is generated it is one of the default modes generated by most laser cavities, but as long as the light is within the laser cavity, it is guided by the walls of the cavity. So, it is not diverging there, but the moment the light leaves the laser, it starts to diverge. So, it is minimum radius, it is minimum size is going to be at the point it exists the laser leaves the cavity ok.

So, the picture that I drew yesterday this is just a general picture of how a Gaussian beam propagates and this is going to become clearer as we go along. Remember if we are talking about the light coming out of a source, it is coming out of a laser it is coming out at it is minimum size and then because there is nothing to guide it anymore the divergence starts.

The point of importance is unlike a spherical wave, even though there is divergence it is not diverging in every direction as a spherical wave. It is not like a plane wave which theoretically has 0 divergence and continues along one direction with the same size everywhere ok. There is some divergence, but it lies somewhere between these two. And, in fact, we are going to see not only does it lie somewhere between these two these values of the radius as well as the divergence depend on where along the path of travel you are looking at the beam ok.

Now, I have been using the word radius. So, you have to be very very careful because the R of z that I have written down here capital R of z, this is the radius of curvature of the wave front, which is different from the diameter of the beam ok. So, I could say so in fact, the radius here and let us say this was the optical axis, it is symmetric about the optical axis, then I would say this is omega of z, where z is this distance z is always measured from the minimum size and the diameter therefore, is 2 omega of z. So, the radius of the beam at z. So, radius of the beam is omega of z ok.

We had the same idea when we were doing geometric optics and we talked about the size of your lens the diameter or the radius of your lens, opposed to the radius of curvature of the lens right. So, do not get confused with these two parameters both very important, but completely different. So, R of z actually has this expression ok. We will come back to this right now just want to give you these expressions ok.

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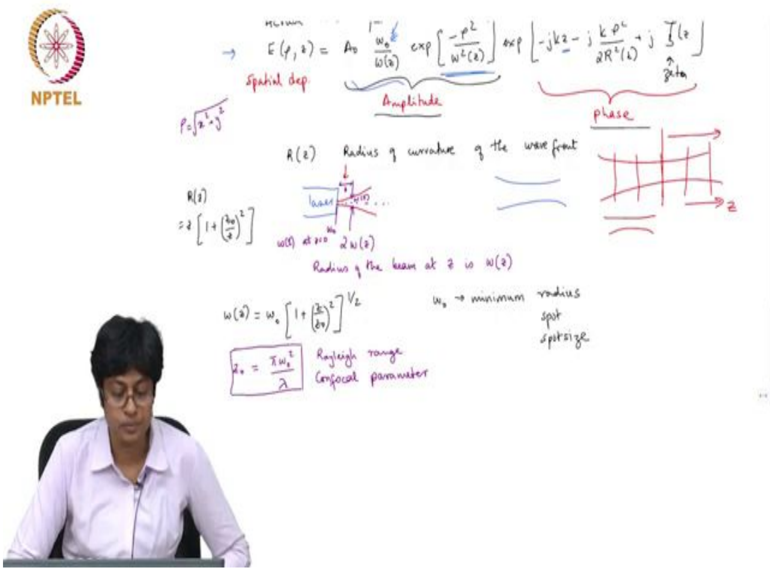

We also have omega of z and now we already know this is the radius of the beam, but I can write it in terms of the minimum spot ok. So, what is omega naught? Omega naught is the minimum spot size. So, I will say minimum radius, but typically for a Gaussian beam we do not say minimum radius, we will say minimum spot or minimum spot size ok. So, in the example where I gave of the light emerging from a laser the minimum spot is this value here or other words the value it is omega of z at z equal to 0 ok.

So, what this expression for omega of z is telling us as the beam propagates it has this minimum value, as the beam propagates there is a change. And, that change is dependent upon z, how far you have travelled away from the minimum spot? And, it is also depend upon this parameter z naught, which is given by this relationship ok. Now, if you, if we had done a formal derivation, in the course of the derivation these terms would have been developed, I am just giving you the expressions now ok.

z naught is often called the Rayleigh range of the Gaussian beam or it is sometimes called the confocal parameter. And, in today's class these parameters are the significance of these parameters, ought to become clearer ok. So, one point should be becoming clearer to you, when we talked about the plane wave?.

In order to describe the plane wave, I used an amplitude and a direction. With the spherical wave I would need an amplitude and the origin of that point source from which the wave originates. But with the Gaussian beam, I do need an amplitude, I do need a direction, but I also need this omega naught or z naught because omega naught and z naught are related right.

So, I need the wavelength as well right. So, the Gaussian beam in order to describe it fully or to understand it fully, there are few more parameters involved ok. And, that is typically why when you are introduced to the concept of beams. Beams, that are solutions to the wave equation, you are normally introduced to the mathematically simple ones, although they are practically impossible to generate right.

But, there is very simple the expression that describes them is simple and with that simple expression, you get the idea that as the wave propagates it is phase is changing from the expression you can get information about the phase the direction and so on ok.

Those same concepts are being applied here, but the way the Gaussian beam or rather the change in the Gaussian beam as it propagates or the effect of propagation on a Gaussian beam is very very different from that of the plane wave and the spherical wave ok. So, I think we have covered all the parameters in this expression remember rho was the parameter just taking the transverse coordinates into account ok. And, of course, we will come back to zeta as well.