LDPC and Polar codes in 5G standards
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Fixed Point Quantisation for SC Decoder and LPDC Decoder

Hello and welcome to this lecture we are going to talk about couple of things in this lecture, the 1st thing is fixed point implementation for the decoder I think I mentioned it briefly before I want to say a few more things systematically about it and 2nd thing we will see yes rate matching for the NDPC code in the 5G standard, so how that is done is like I said precisely described in the standard for a point out simple implementation for the rate matching that I will do the, so it can be modified later on. These are 2 things we will do in this lecture, so let us get started.

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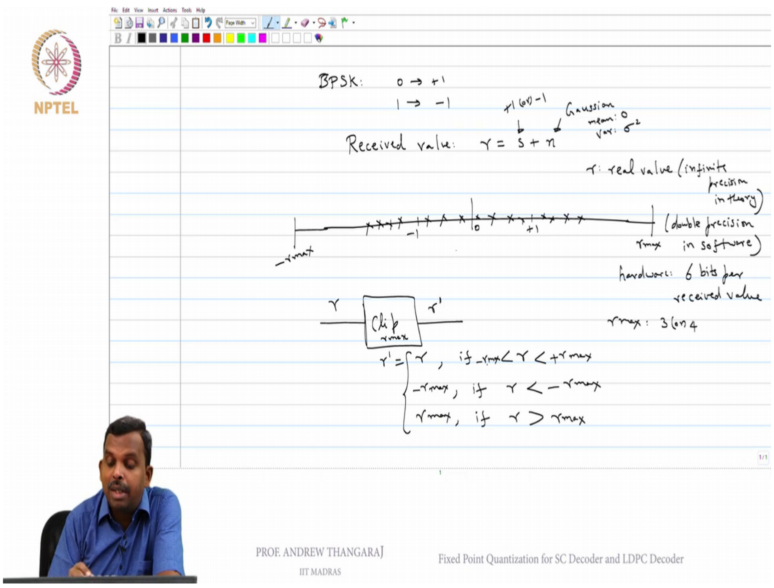

So if you remember the basic model that we have, we have BPSK which takes 0 to plus 1, 1 to minus 1 and then they have the received value after Additive White Gaussian Noise r is S plus some nice, right? So now this is plus 1 or minus 1 and this is Gaussian, Gaussian random available mean 0 and variance is going to be some Sigma Square that you will pick in the simulation based on the eb over and not that you want, so this is the picture so if you want to draw it on an a axis you can draw the value of r on a axis you have plus 1 here minus 1 here okay and the values are going to be around this minus 1 maybe depends on the Sigma Square, right? So you can have all sorts of values.

This is what you have okay, so generally as per this model this r is a real value, so what do I mean by real value you could get something like you know 1.002345 things like that, so the precision can be very high something like that at least in the model the way we write Matlab code, Matlab uses double precision numbers, so they have quite a few decimal points okay so maybe 32 bits, 64 bits like that okay so how… Implementations when you want to put your decoder on a chip or on a processor, on a board which has low power typically people who do not you such high levels of quantisation.

So he cannot use 64 bit perceived value and all that is really hard. In fact the number of pets they will allow depending on the area that you can have on the chip et cetera maybe just 5 bits or 4 bits per value, so this is technically theoretically infinite precision in theory. So in practice double precision say in software we need simulation probably using 32 bits, 64 bits these days okay, so that is what is double precision. So in hardware you probably want 5 bits per receive values, so you have to quantise and I will mention briefly how I am going to do the quantising this is how it is done typically, okay.

So when you quantise usually 1^{st} thing you do is set a maximum possible value, so you will clip the value that you considered to some regions, so for instance r equals s plus n, n is Gaussian, Gaussian can be plus infinity minus infinity can take a very large value sometimes occasionally does happen okay. So in practice what you do it is, 1^{st} you clip into some r max, r max minus R max okay, so what I will do is r will come in and I will clip and I will get some r prime which is clip, so what will this do?

So r prime equals r if R is between plus R max and minus R max and this will be minus R max if R is less than minus R max and it will be plus R max if R is greater than R max, so this is the clipping operation, so we go above r max or below minus R max you set it as r max, so a typical value for r max could be 3 or 4 or something like that in this case for the BPSK case that seems fine enough you do not need much more than that.

So notice I am doing 3 or 4 I am assuming the receive symbol is plus 1 or minus 1 plus noise, so there will be some gain in your receiver chain to make sure that that happens correctly okay, so R max and minus R max is typically for the decoding purpose you could take it as 3 or 4 it is not wrong okay, so once you decide on an R max, the values in between have to be represented by just 5 bits okay, so now let us say we will take 6 bits this is having some convenience.

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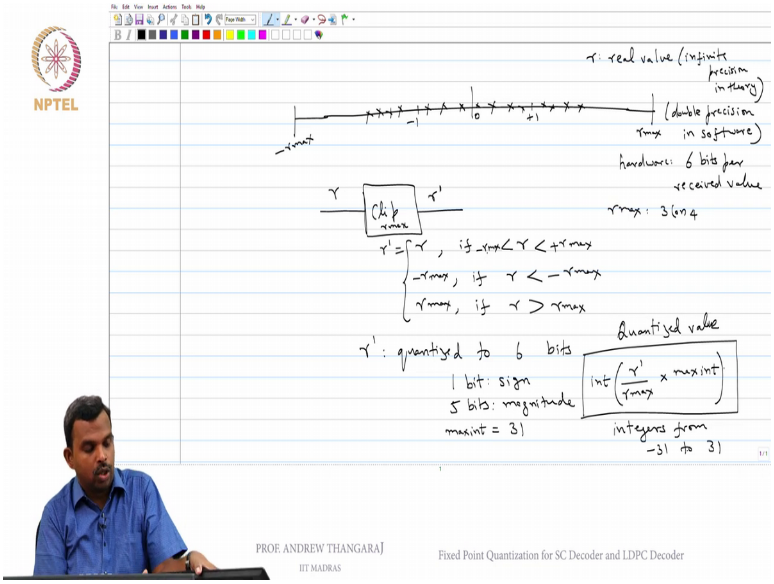

So if you look at 6 bits 1 bit is going to show you sign so then after this r prime has to be quantised to say 6 bits okay 1 bit is sign the remaining 5 bits is magnitude okay, so that is one way to think about it so you can do a very quick and dirty operation to do this quantisation, it is pretty good in most cases, so you take the maximum integer that you want to allow it is just 31, right? So we have 5 bits you can go from minus 31 to 31 okay so those of you who know double, two's complement method of representation will tell me it is minus 32 to plus 31 but we just keep it as 31, so we will take minus 31 to plus 31 as the maximum integer and you have r max and you have this r prime as well and what I will do is for quantising simply 1st divided by r max okay and then I will multiply this with max int and then take the integer part of it okay.

So this is very convenient and simple way to do quantisation, so this is the quantise value okay. So hopefully this is clear to you okay so you take r prime the clipped value divided by R max multiplied by max int and take the integer value could take floor or seal or something like that it does not matter, so you can take the integer value okay. So this gives you integers, so what will happen once you do this is in the receiver you will have integer value, so we can see automatically one bit becomes sign 5 bit magnitude when you do this if r prime is negative this integers is going to be negative, so it will be minus 5 or plus 5 or things like that so I will show you when we write this in Matlab how the values look but this is how it works.

So you have your received value you clip it then you quantise you get integer values, so these are integers from minus 31 to plus 31 okay and you worked with this integer in your decoder okay. So you may be doing operations with this integer you may add them, if you add you have to make sure that you do not exceed too larger number, so all that you have to take care of, so I will stop with this as far as this course is concerned.

So we will do quantisation and we will work with quantised value, so now quantisation has some effect on the decoders these decoders we implement a suboptimal, so you have to be slightly careful when you quantise and we watch out for things that happen because of quantisation okay. So one of the effects is quantisation makes receive values equal, so for instance you may never get 2 receive values which are the same if you do not quantise but if you quantise you will get the same values.

So because of that there can be some sub optimality of course there will be sub optimality and you have to take care of those things in the decoder. Some things to watch out for but generally this will work, what works with real values will typically works with these kind of integer values as well, okay. So this is how the quantisation is done. So let me show you in Matlab how I have done these modifications to the decoder to put in this and work with integers okay.

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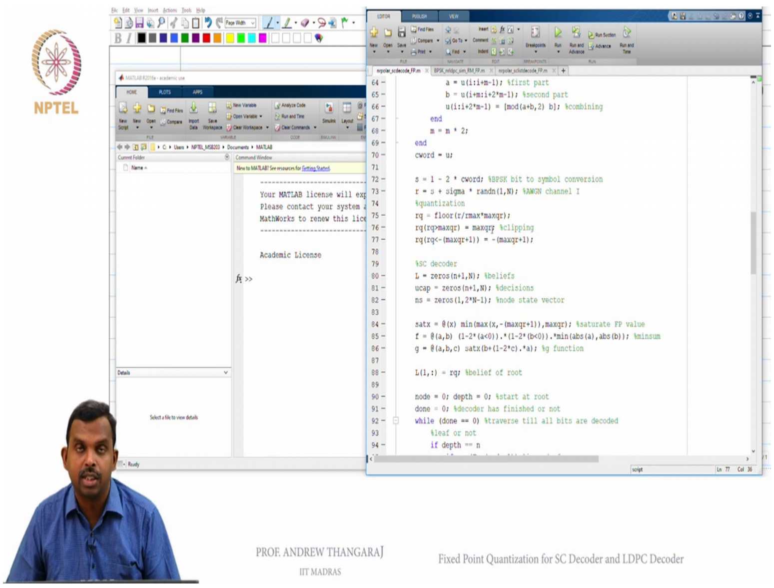

So the Matlab code for the successive cancellation decoder. For the successive cancellation decoder for polar codes, so previously we did not have the integer the fix point version of this code so I have saved it as, in our polar SED code FP and then I have put in the integer think, so you can see what I have done, I have done the R max as 4, you will have the code with you, you can try with 3 et cetera if you think that works okay and I have put Maxqr as 31 okay so this is the maximum integer received value.

The other things do not change nothing much changes everything else is same you decode and all that and now when you transmit you will do the BPSK just like before you will do the receive value just like before okay in this simulation this is how it works and then I will do the quantisation, you can do the quantisation in multiple ways I have done it in a slightly different way here you can see I have divided r by R max and then multiplied by max QR and then it takes the floor of it to get RQ and then…so I am not clipping 1st so I am doing the clipping a bit differently here, so you can do r by r max into max QR and then you do the clipping okay so this is the.

The clippings come here okay, so this is also fine so you do r by r max into max QR and then you clip and here can say I have clipped up to max QR on the positive side, on the negative side I have done minus max QR plus 1 okay so I am doing minus 32 to plus 31 which is actual two's complement representation of integers okay, so hopefully that is clear so you will see that this is equivalent. You can also do the other way you can clip r to 1st minus R max to R max and then this will not be needed you can just to the floor that is also fine.

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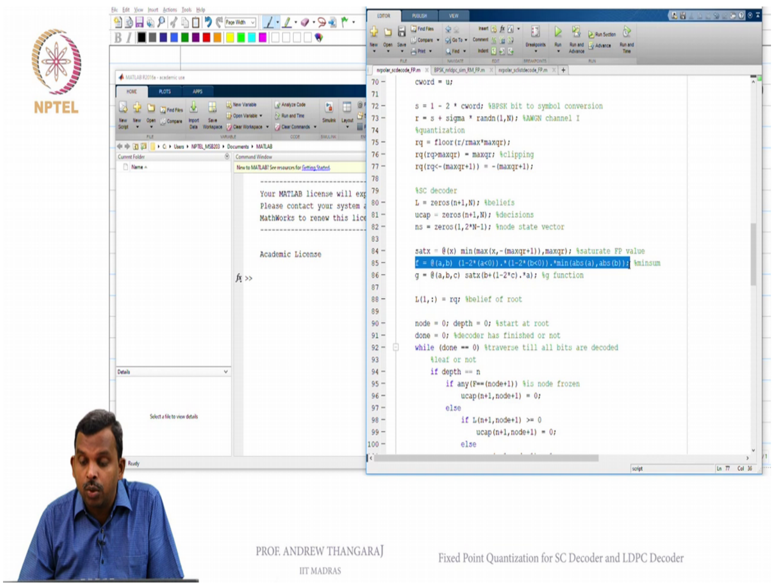

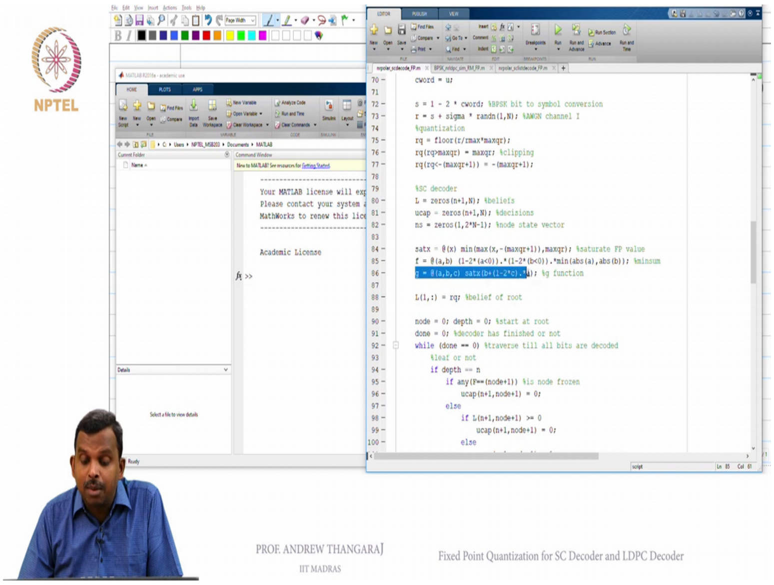
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cword = u;
s = 1 - 2 * cword; %BPSK bit to symbol conversion
r = s + sigma * randn(1,N); %AWGN channel I
%quantization
rq = floor(r/rmax*maxqr);
rq(rq>maxqr) = maxqr; %clipping
rq(rq<-(maxqr+1)) = -(maxqr+1);
%SC decoder
L = zeros(n+1,N); %beliefs
ucap = zeros(n+1,N); %decisions
ns = zeros(1,2*N-1); %node state vector
satx = @(x) min(max(x,-(maxqr+1)),maxqr); %saturate FP value
f = @(a,b) (1-2*(a<0)).*(1-2*(b<0)).*min(abs(a),abs(b)); %minsum
g = @(a,b,c) satx(b+(1-2*c).*a); %g function
L(1,:) = rq; %belief of root
node = 0; depth = 0; %start at root
done = 0; %decoder has finished or not
while (done == 0) %traverse till all bits are decoded
%leaf or not
if depth == n
if any(F==(node+1)) %is node frozen
ucap(n+1,node+1) = 0;
else
if L(n+1,node+1) >= 0
ucap(n+1,node+1) = 0;
else
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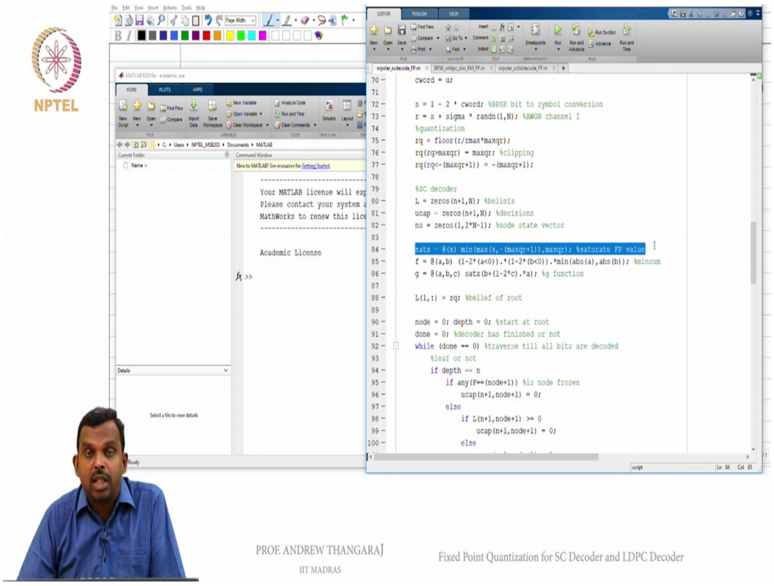
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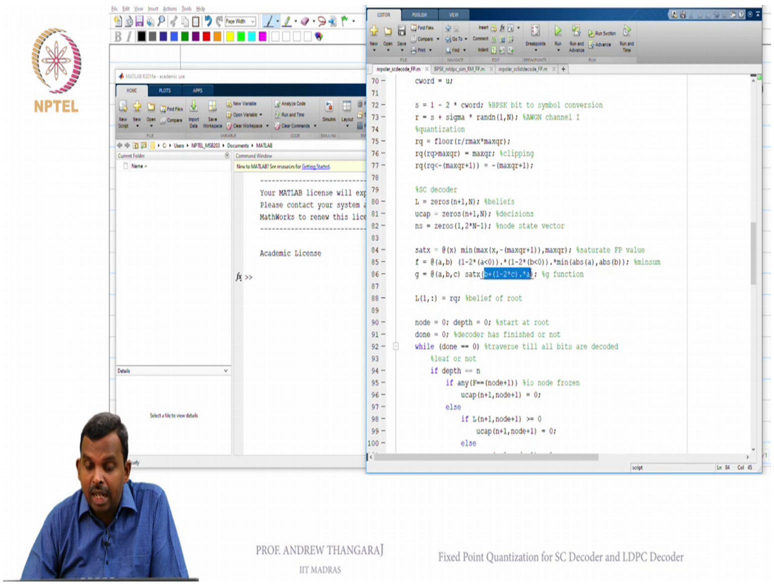
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So after this nothing much changes, couple of things to watch out for, so if you look at f and g operation so if you remember the polar decoder and we only do f and g between do not do anything else in terms of operations on the received values. The f operation is okay because it does not do anything to increase the value of what is coming in, so it just takes minimum of the 2 values, so it does not need any special handling.

On the other hand g needs some saturation, the reason is so what g does is adds to values, so if you take for instance plus 31 and plus 31 you add you get plus 62 okay but you are allowed only to go between minus 31 to 31 you want to quantise it back, so that is why I have written the saturation, it saturates he FP value and you can see that we have written it I have done max of x, minus okay so this will take care of the minus part, then after that it take the min of that max value, Max QR okay, so this will ensure that whatever value x is it does the clipping, so the saturation does the clipping between minus 32 and plus 31 okay it will not allow the value to go above that. If it goes above that on either side it will clip it to either plus 31 on the positive side minus 32 on the negative side, so this is what I have done here. Previously we did not have the satx on the g function so now we put the satx also.

So this few changes are enough to make sure your code will work with the integer side nothing more is needed here everything else is exactly the same and you can work okay. So this is hopefully clear so you have to make sure the receive values becomes an integer, the saturation effect is something you have to watch out for and if you are doing any operations in your decoder which increases the value you have to saturate again okay.

So sometimes when you saturate like this you have to be paying some attention for instance in the LDPC code there are 2 types of values one is the LLR the messages, the other is the LLR for the total LLR these 2 are saturated at different levels. They have to be saturated at different levels otherwise your decoder will fail okay so I will comment that next but this is something important. When we do operations and when you are expecting the value to grow you have to saturate okay.

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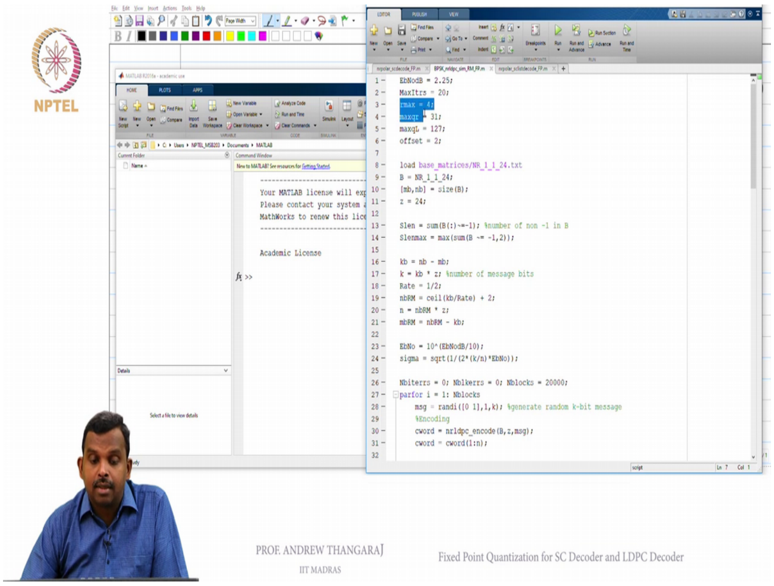

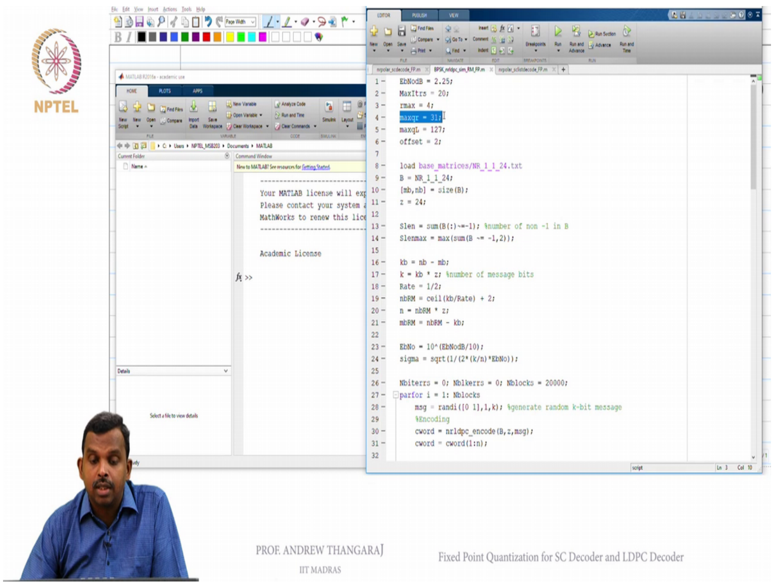

So now let me move on to the LDPC code and show you once again how this was done on the LDPC code. Once again you see here I have max…r max is 4 and then I have max QR as 31, so this is the quantisation for the receive values. I also have a max Q for L okay. In this L value I am saying is 127 I am allowing it to go 2 bits higher in magnitude than r okay so this is important because that L represents the total LLR which can be larger and then you have to subtract it from r to compute the extrinsic and all that, so you have to allow it to be large okay so that is very important.

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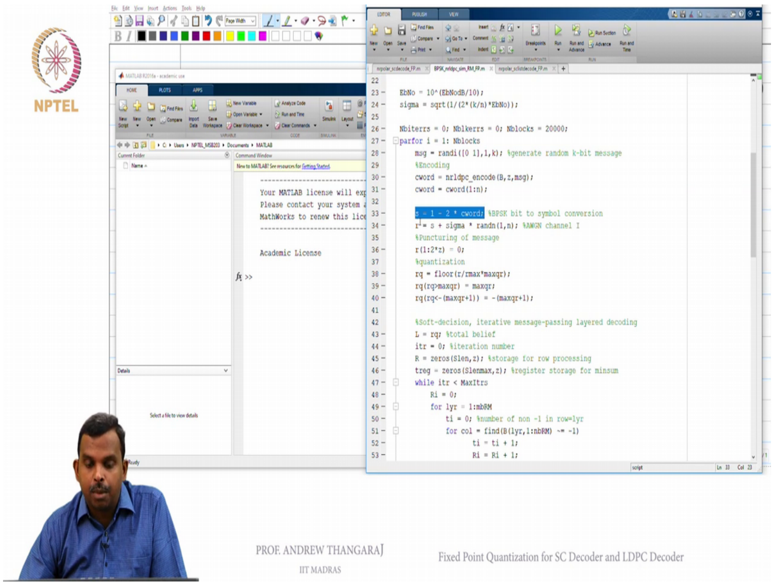

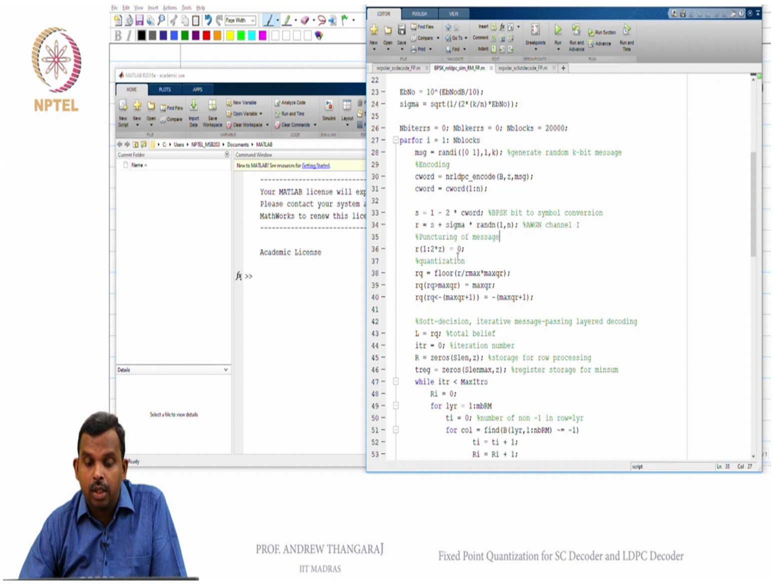

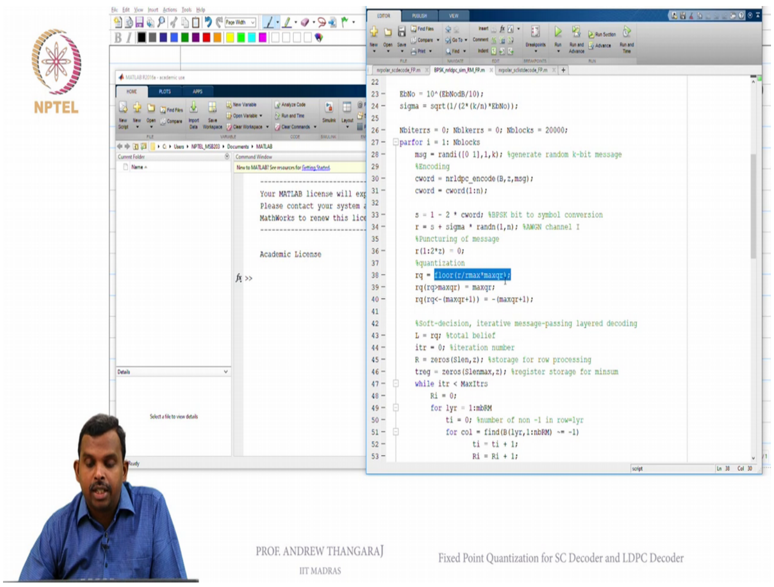

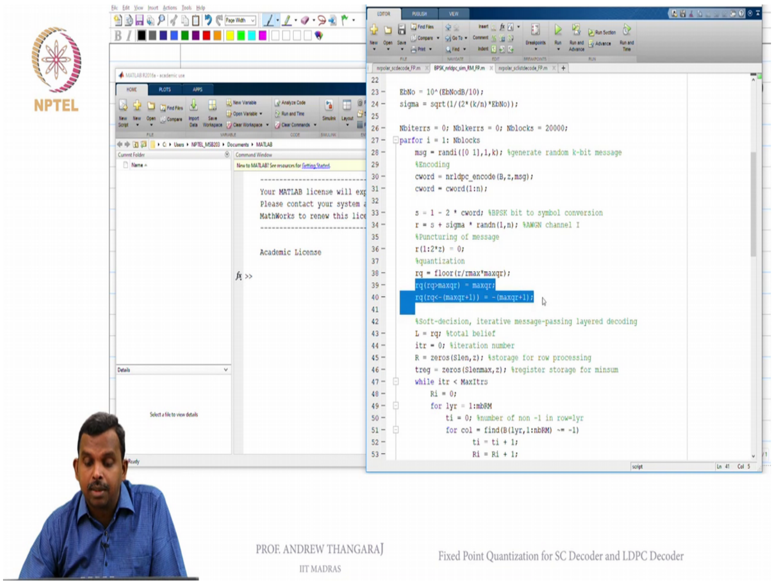

And then other than that what I do is largely the same I will compute the BPSK modulation then find the receive values of K and then of course this is puncturing and then after that I am doing exactly the same thing okay. So I do the computation of the floor, right? Convert it into integer and then you do the clipping okay so this clips okay so that is fine I have done the clipping okay.

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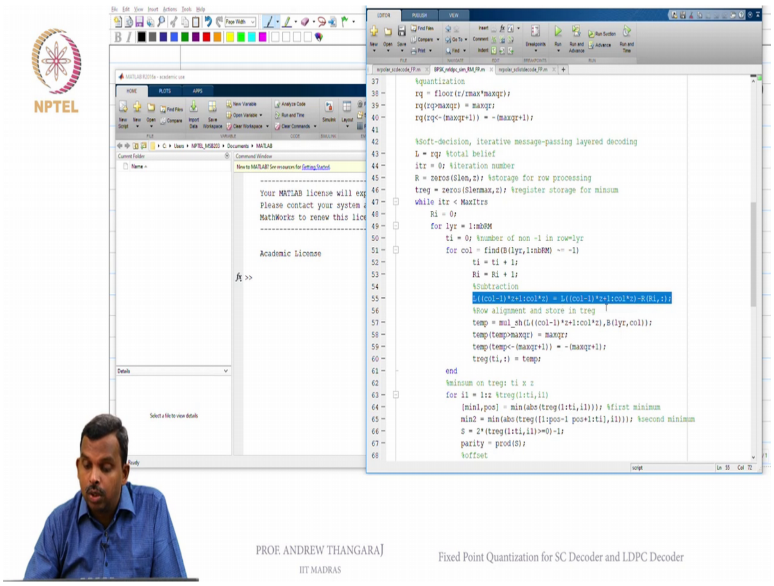

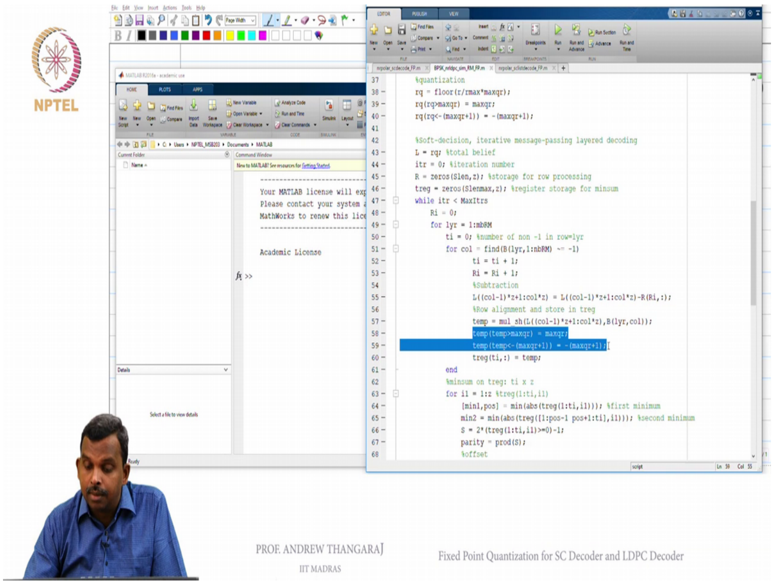

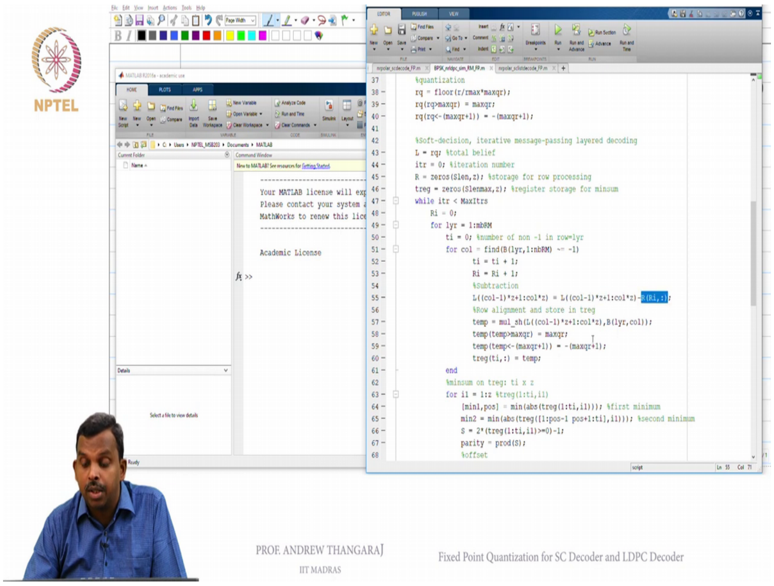

So now in the decoder so this is inside the LDPC decoder you remember the LDPC decoder you do row operations, right? In the layer decoding row after row after row you work with it and then you subtract, you first subtract the total LLR minus what is already in the row then you do the min some on the row and then you add that value back to L total okay, so here is the subtraction, subtraction happens here the capital R is what is in the storage for the row processing okay.

So you do the subtraction and then you to some row alignment and after that notice here I do this okay so anytime you do an operation like this you have to also make sure that you quantise again okay so because this L has going from minus 127 to 127 this is only minus 31 to 31, so this can go above 31 so you quantise again make sure that it is quantise. So this make sure that your bit with the… Number of bits you are allocating for the storage remains 5 okay so that is very important 5 or 6 in this case okay.

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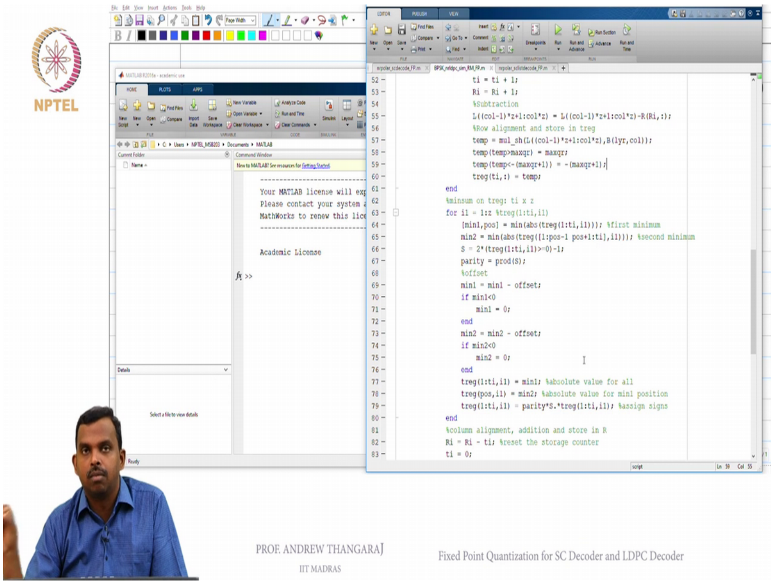

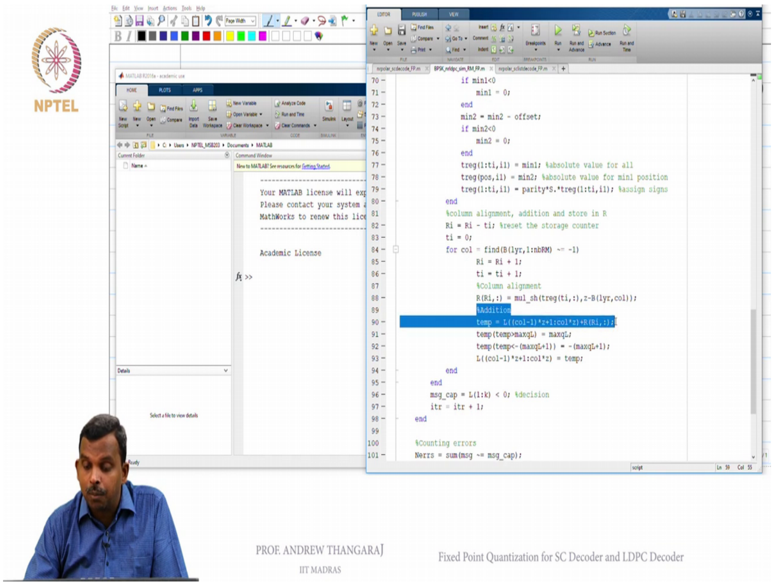

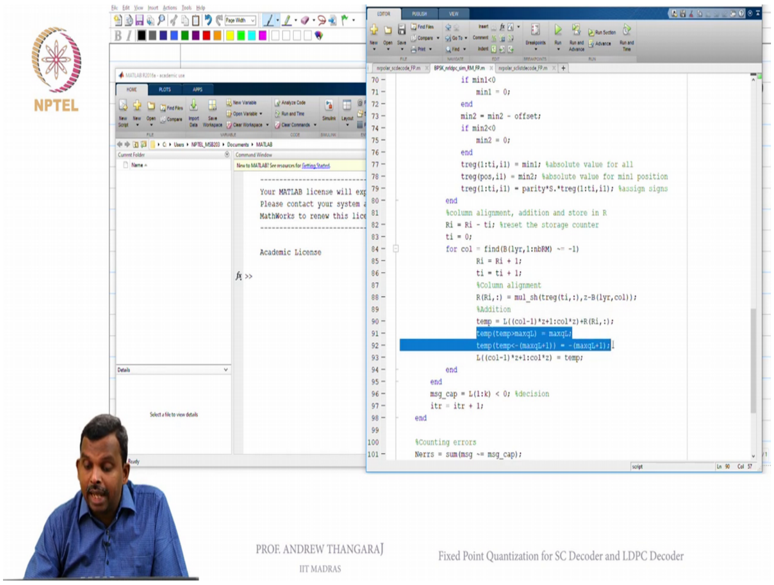

And then you do the min some operation here, the min some does not require any checks for increase in value because you are not doing any addition, so that is okay and then you have addition here again okay. So notice what happens here, so once you have the min some value you have to add, so this is the addition part and when I add this temp ultimately becomes L, okay so to quantise again but the quantisation is to Max QL now okay so I am going from minus 128 to plus 127 okay, so I do that here to make sure I do not overflow here and that is fine, so these of only changes other than that there is nothing else to change everything else work as such okay.

So converting your code to fix point is little bit easy to start with but when you look at the operation inside making sure that the bit with are valid for the different types of number you have inside your decoder can be a little troubling. In the polar codes it is very easy and simple you only have to saturate the g operation. In the LDPC code you have 2 different values one is this r storage that you use for the row processing, the other is the L value which is the total LLR total belief and as you keep processing the L value has a larger quantisation than the are value, so you have to take care of that and keep saturating. Whenever you do addition or subtraction you have to keep saturating in a different way, so that is something important once you do that both of these work if you want I can run these for you.

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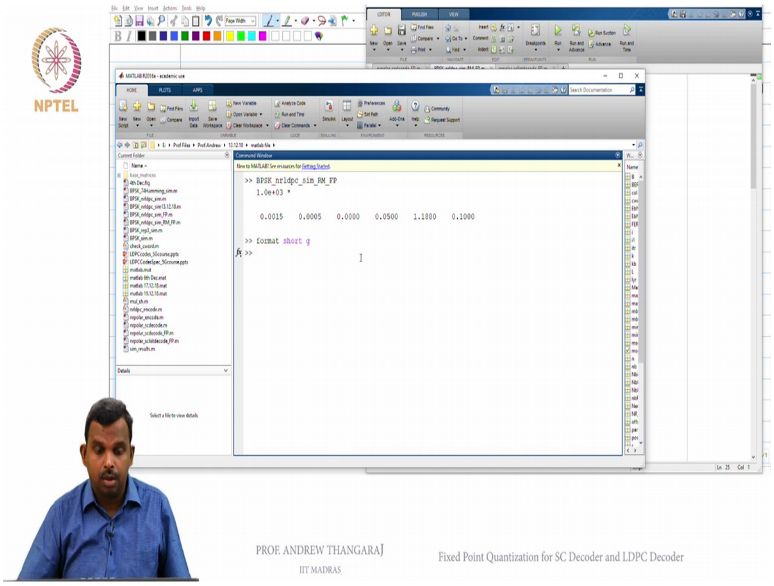

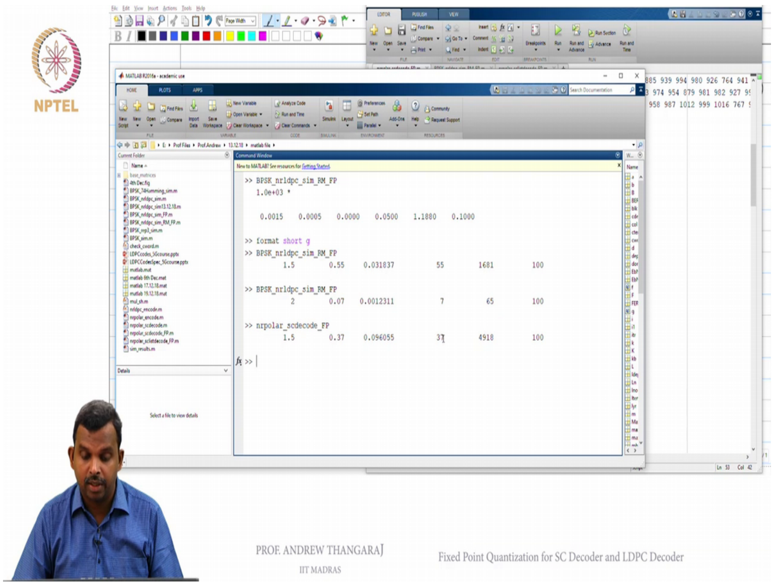

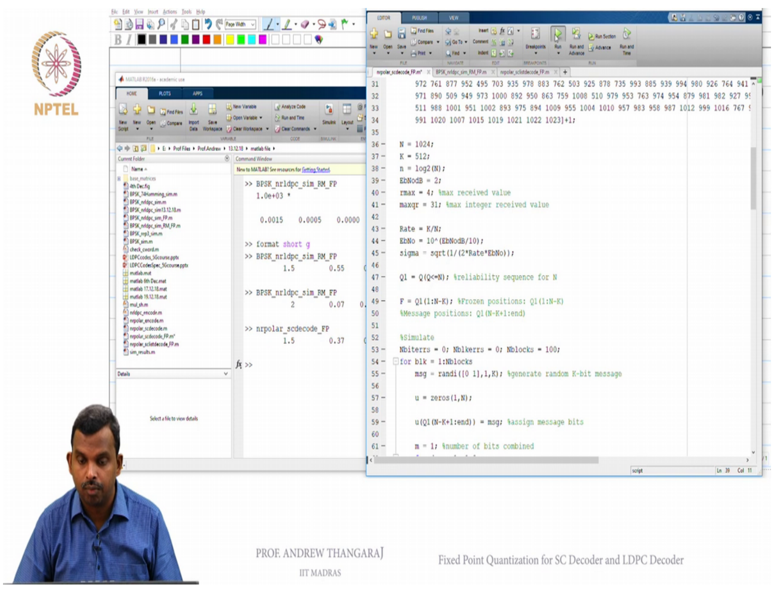

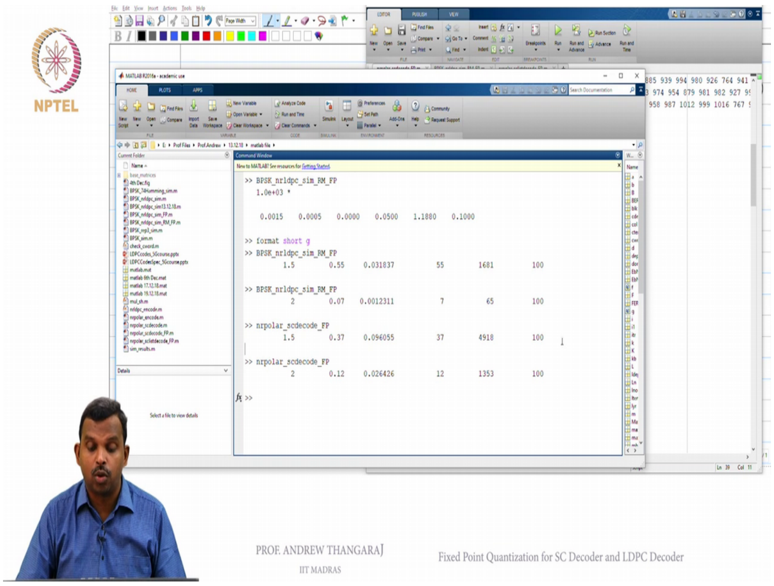

Okay so let me show you how this decoder runs I have set it up so that EbNodB is 1.5, DB maximum iteration is 10 okay and then I am running it for 100 blocks okay. Let us run this okay so it is finished running and gave you some answers may be you cannot see that very clearly so maybe I should run it once again, I will run it once again just to show you how these numbers look it is not too bad we got 5 errors out of 100.

So it shows 55 errors out of 100, so it shows that it works and gave you some decoding, so if you want you can go to 2dB to increase that and we will get 20 hopefully. Yes so it had only 7 out of 100 in error when you went to 2dB okay. So that is the decoder, it works in simple way, so hopefully the fixed point part of it is clear you can also run the polar decoder this is, let us also run it at 1.5 I think again 100 blocks okay so let us run this once again to be sure that we get something reasonable okay it worked reasonably 7 out of 100 and if you go to 2 you will get you get some performance 12 out of 100 was the error so this is how the fix point is done okay.