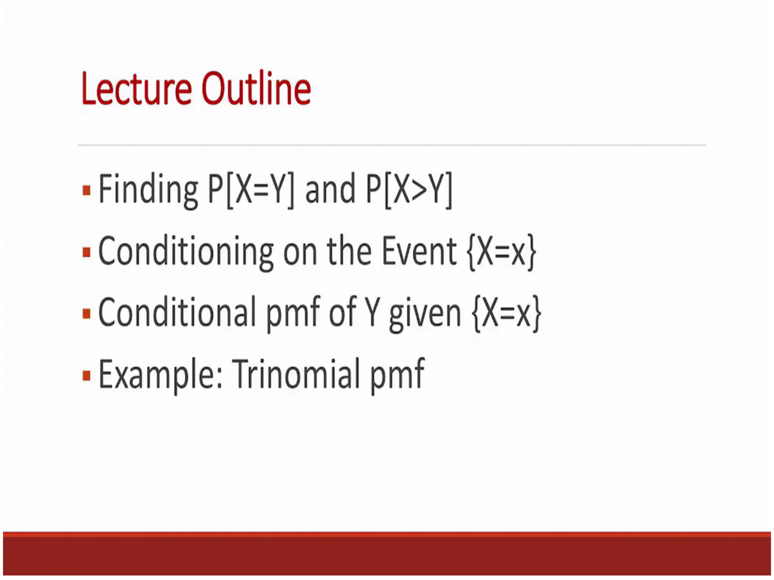
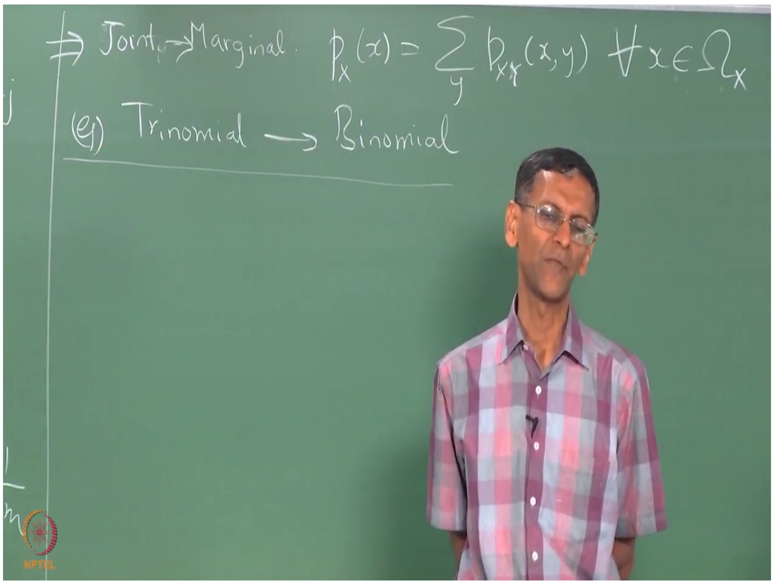
**Probability Foundations for Electrical Engineers**
**Prof. Aravind R**
**Department of Electrical Engineering**
**Indian Institute of Technology, Madras**

**Lecture - 13**
**Part 1**
# Manipulations of Joint PMF

(Refer Slide Time: 00:13)

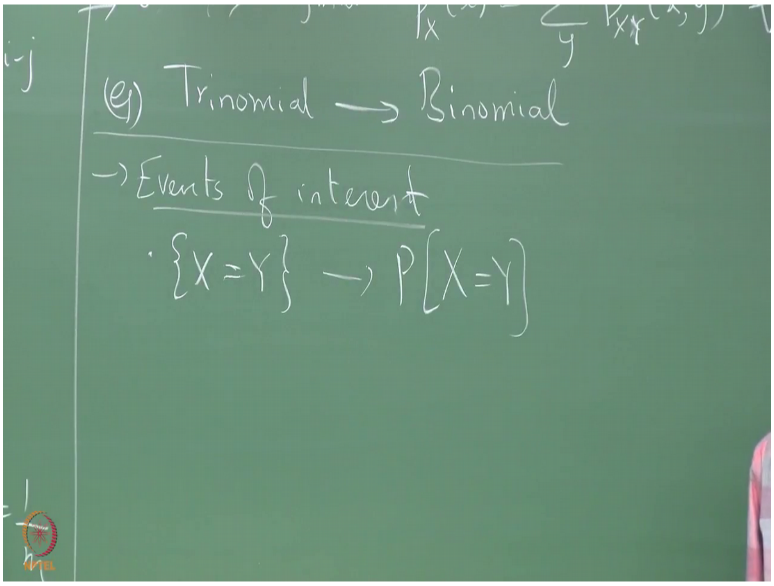



(Refer Slide Time: 00:23)



So, we move on with today's material. Now, that we have defined all these random variables x and y and so on; what is the events that we can look at some events of interest.

(Refer Slide Time: 00:30)

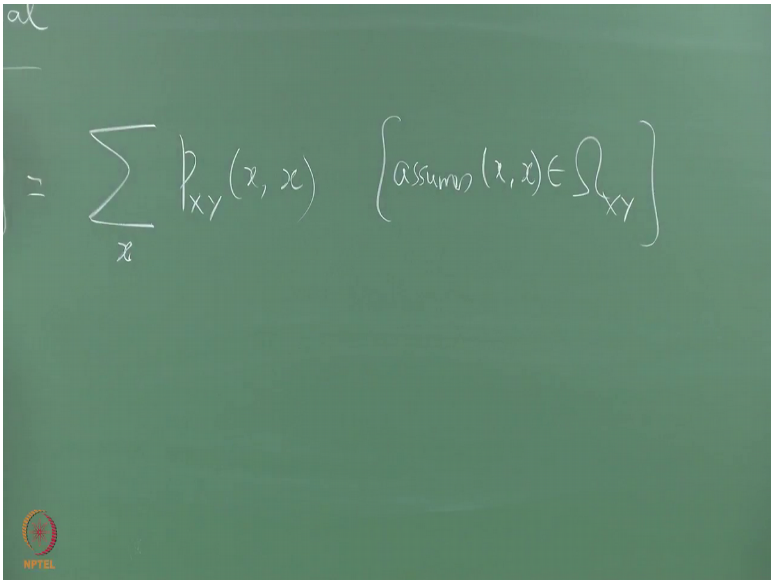



For example, you have the event x equal to y what is the meaning of x equals y why is why do we call that an event it is just like we call X equal to k an event in the in the single random variable case X equal to Y will become an event in the 2 random variable or in the multi random variable case.

So, this is this event happens whenever the case; for example, in this case trinomial example when the count of event one which occurs with the probability P i equals the count of event number 2, I call them 1 and 2 yesterday. So, I have to continue with that. So in the case of the trinomial coming back to put that on hold; so in general how do I calculate this this has a probability and we have already said that I mean for discrete case almost every event every such combination of points in this space is an event.

So, x equal to y is a valid event I mean it is a event for which we should be able to write out the probability how we do write this.

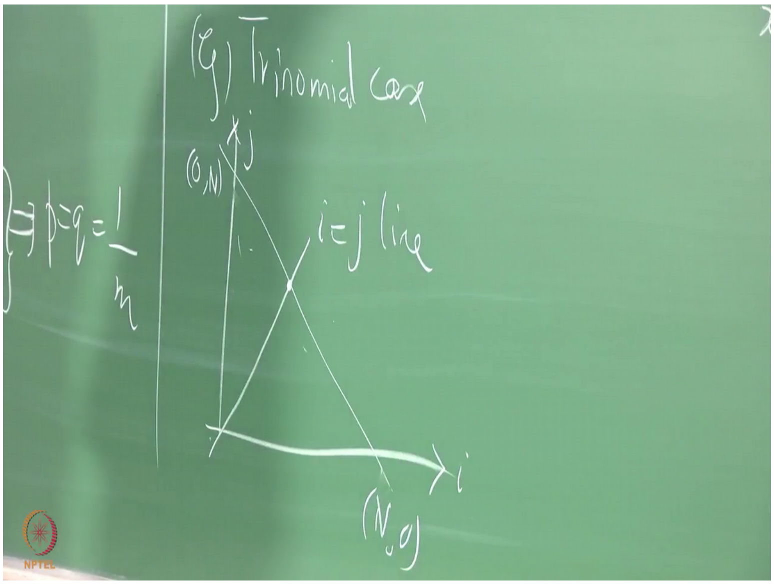
(Refer Slide Time: 02:03)




So, basically what you do is you write; you look at the probabilities of these points x comma x right assuming that x comma this of course, assumes that x comma x is in the is a valid point in omega x if there is no point x comma x and omega x y then the probability of x equal to y is what if I cannot like; for example, in the drawing of the 2 balls right with the like with num different numbers x you can never get x equal to y.

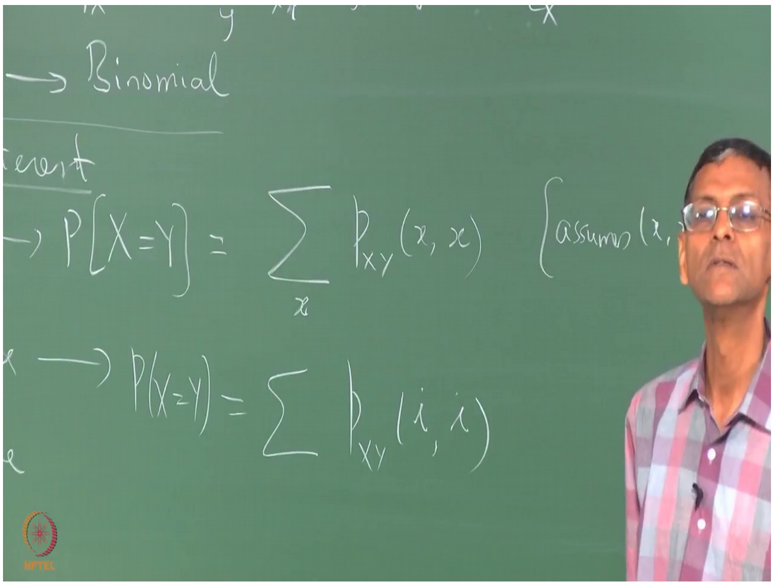
So, in that case that probability will be 0, but in the trinomial case this is not going to be 0. So, in general what you do you; you add up all these probabilities right with the equal arguments x and y. So, this assumes some you have a at least one point in x; x in omega x y otherwise the probability is to be really 0. So, when I say sum over x I am talking appropriate values of x you have to look at the situation the specific case and see how what the elemental summation is going to be.

(Refer Slide Time: 03:20)




So, in the trinomial case what happens? So, here we can draw this picture all right this is the point 0 comma n, here I am plotting the points i and j and here I am plotting the points n; n comma 0 know the points here; so x. So, this is the line i equals j and we will assume without much loss of generality that that what n is even all right why do I need. So, that it makes my job little easier. So, this point I am going to assume belongs to the grid what is this point n by 2 comma n by 2. So, this is i equal to j line.

(Refer Slide Time: 04:10)

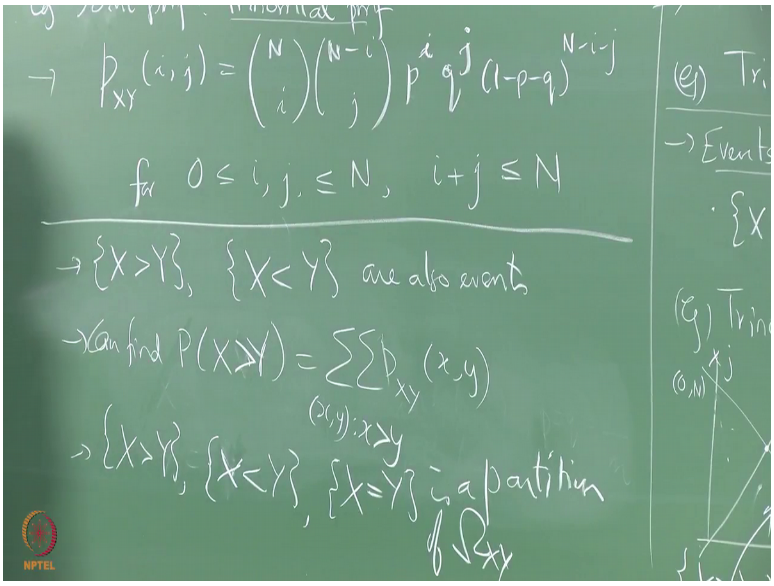


So, what happens here how do I write this; I do not have the key i, let me not rewrite you know take that potential again I will just write it as P x y i comma i, you can all anybody can substitute j for i out; there I am not going to do it for you write it all over again, but what we are interested in this are the limits of the summation what are the limits of the summation what are the limits of the summation one why one why cannot; why can they both take the value 0, they can both take the value 0 also know it is it is possible that x does not occur or y also does not occur remember i said x and y are not a par sorry event 1 and 2 are not a partition right it is not like you must get each one of the one of the 2. So, we go from i equal to 0. What is the maximum value?

Student: N by 2.

N by 2 for n event; so having understood this case x equal to y. Remember whenever you make statements about random variables they in general they are events that is what I want you to remember take away from all this discussion why should you look only at x equal to y why not look at x greater than y or x less than y, they are also events let me do here go from here the events of interest.

(Refer Slide Time: 06:00)




So, you have X greater than Y; X less than Y are also events and that is why we write them with the curly brace the curly braced notation is universally used to indicate events. So, going coming to this diagram which is not all that well drawn, but when I rely with it right the 45 degree line is a little skew, but it is. So, which portion of this diagram corresponds to the event that X is larger than Y and which portion corresponds to x smaller than y that is exactly like going back to school right there is no only thing that only thing you have to remember when you looking at the probability at these events you have to add all these probabilities.

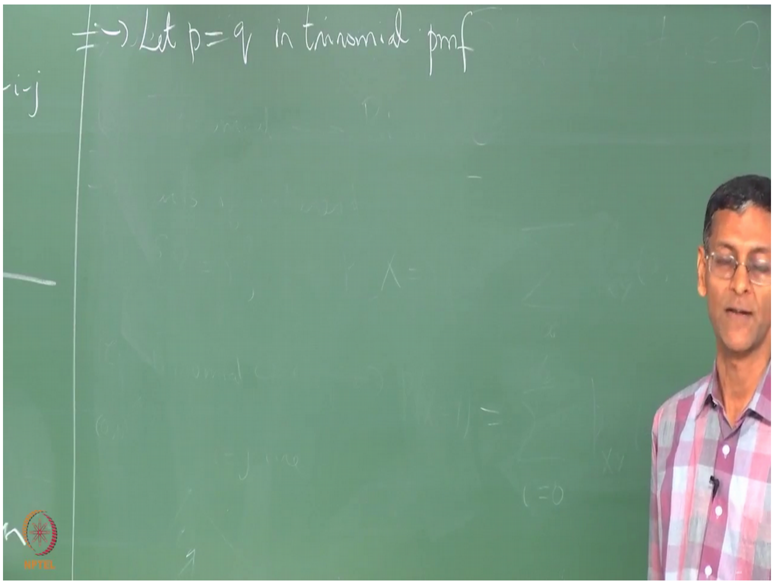
So, these probabilities we will give you what prob; what event this is x greater than y. So, this is and likewise this will be X less than Y and X is equal to y does not fall into either of these 2 camps it is distinct in the in the discrete case. So, so can find P X greater than Y by what you can say well P X Y, I have now in general I have a double summation I have to say P X Y; X Y such that. So, X Y such that X is small, X is greater than Y strictly greater than Y.

So, you have to add over all points in omega x y that give you the non-zero probabilities and all those points where the X coordinate is bigger than the Y coordinate. So, this in general is a messy summation it cannot be done easily especially in the trinomial case there is no nice close form solution even here there is no nice close form solution there also, but that is the need the sum does not exists; I think I have mentioned it in some earlier lecture existence simply means that the sum is finite and here; obviously, all these probabilities have to exist it is just that you cannot write a close form for it in terms of P and q or and n. So, you have to start getting used to a fact right that.

Something can be a perfectly healthy number without being expressed in closed form, but; what is the partition? Now I have a partition of the of the X Y space what from sitting here what is that partition you might right there is three way partitions sitting right in front of me what is that three way partition these three events X equal to Y, X greater than Y l X less than Y is a partition the sum of those three probabilities must equal one and they are exclusive only one can happen at any time; it is less than Y; X equal to Y is a partition; partition of omega X Y. Now we can use certain symmetry arguments here especially for X greater than Y and X less than Y. This symmetry argument may not work. So, easily for X equal Y.

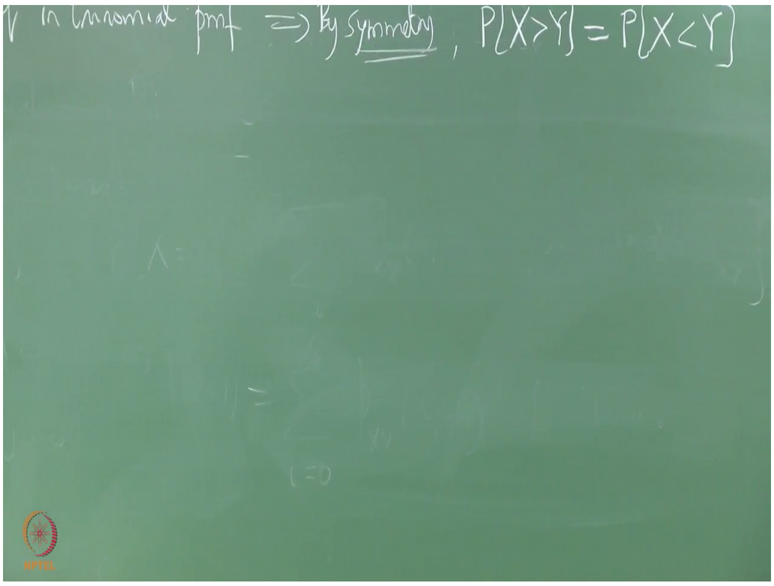
But it certainly in the sense that supposing I tell you that p must equal q or p equals q not must equal p equals q for p equals q what can you say about x greater the events x greater than y and x equal x less than y.

(Refer Slide Time: 10:13)




The trinomial pmf like joint pmf of course, and this omitting over joint, but let P equal to q how can you say about x greater than y and x less than y it must be those probabilities must be equal right without even any calculation why do you why can you say out with some great assure degree of assurance because you know in a probabilistic sense you know either favoring event number one which for which x is the count nor for every event 2 for which y is the count.

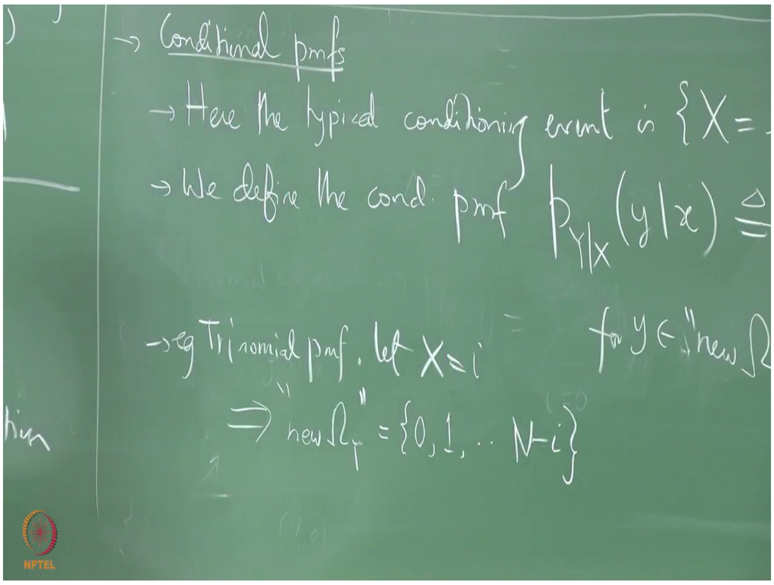
(Refer Slide Time: 11:00)



So, by symmetry; this symmetry argument come in very handy later on probability of x greater than y equals probability of x less than y of course, this tells you nothing about probability of x equal to y and. So, this tells you nothing about each of the individual numbers just. So, happens that this 2 are equal, and if you know this and if you know probability of x equal to y it becomes difficult to find all three of them.

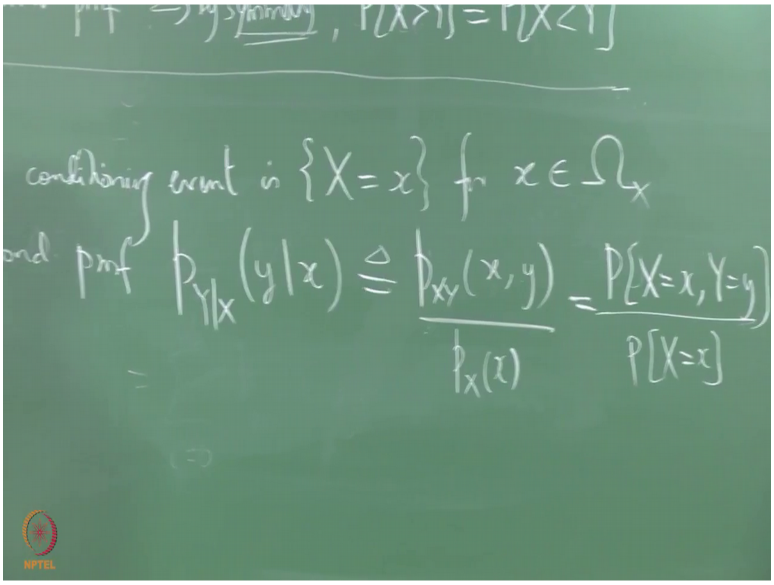
So, you should always be aware try to exploit to maintain as possible; now let us see one more type of pmf conditional pmf.

(Refer Slide Time: 12:04)




In the one random variable case we also looked at conditional pmfs how did we de what kind of conditional pmfs did we considered over there we considered some simple conditioning like x greater than x naught and so on. Here also we can do the same thing, but the type of conditioning which is more interesting here is you observe for example, let x take a certain value.

(Refer Slide Time: 12:51)




So, here the conditioning events are conditioning event is let say X equal to x which is let say in event of nonzero probability if for some x in omega x; in other words observed that you have done the experiment and you have observed that capital X has taken some value what does what does it tell you about why. So, you define here when I say here I am talking of this situation revolving 2 random variables this is why this is means what we take or is of course, y equal to y is also equally right typical.

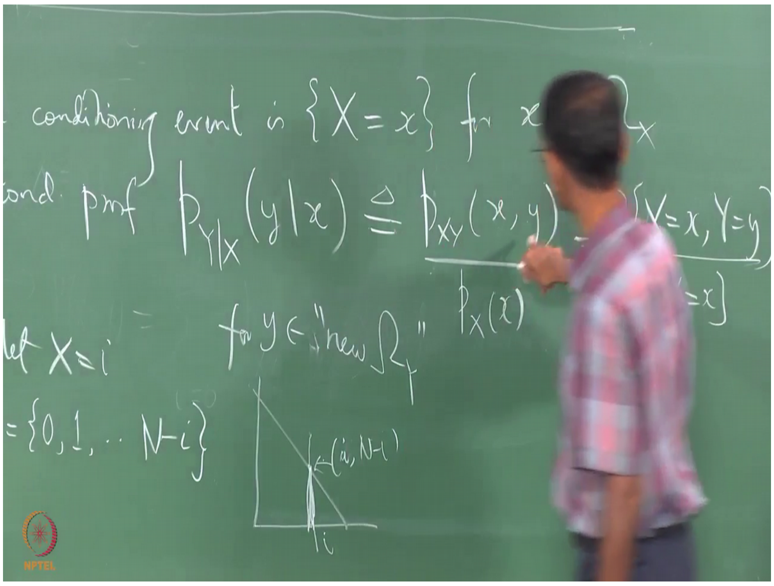
So, we define the conditional pmf P of y given x this is the notation we are going to use y vertical line x has a single subscript argument y given x like this. So, this x takes this value small x and this is the argument of the pmf. So, this is defined as this joint probability which comes from the joint pmf, remember here y is the argument. So, you are looking for probability may be before this I would not erase anything, but let me also let me write this like this and also write this a conditional; I mean probability like this may be I should have written this first and then this, but does not matter right let me move away. So, that you can all see that the expression.

So, why do I call it a pmf; I am going to allow a small y to vary while keeping the small x fixed. So, and what is y going to vary over of course, minimally you will want to vary your all value all the points in omega y, but it turns out in in for example, they say they say in this case is like trinomial you will get a you may get a smaller range you may not get the full range if you know that x equal to x y may not in many cases it may not have the entire freedom to vary all the way that it did in the absence of that very information.

So, y will have. So, for some I will say new omega y which depends this new omega y this depends on the value of x. So, in the trinomial case we will do this let us say x equal to i. In other words what have we observe we have observed that the event one has occurred i times this is not right a typical algebraic statement of saying let the variable x take the value i, this is an event remember again from one last time I am saying it when I say let x equal to i means the random variable x took the value i, then what is the range of the and obviously, this i has to be in y it has to be in omega x.

So, it has to be between 0 and n in the trinomic case it cannot be outside that. So, given that x is equal to y what is this new omega y and the new omega y is what this in general depend on i 0 to n minus i, set of integers for the maybe I should have drawn that picture and kept it, but I can draw it again.

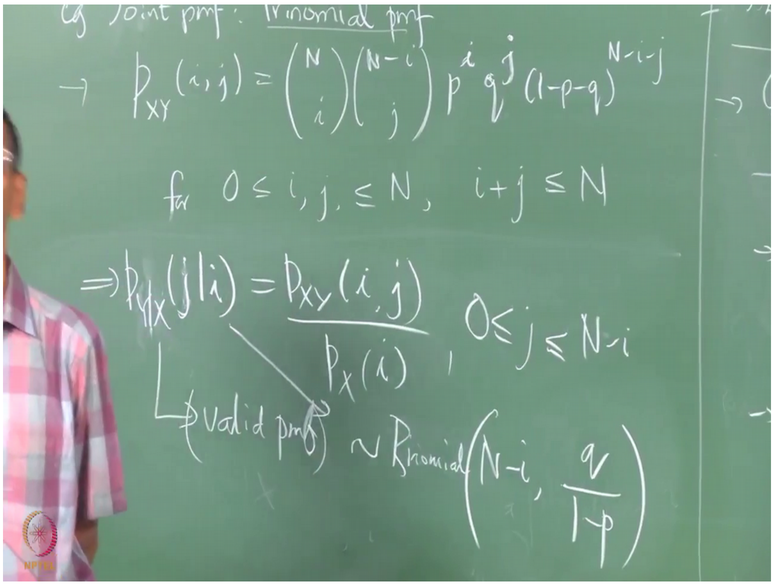
(Refer Slide Time: 17:11)




If I say x is i, this particular i; that means, I only have this much freedom for y and so here I have the point i comma n minus I do not have the full freedom of allowing y to vary from 0 to noise, incidentally this new omega y is my notation and you would not find it in books, but I have to call it something.

So, I am calling like that right I mean my terminology right, but it is crucially important to freeze this before you proceed any further because these are the values that you are going to allow for this y. So, what is it there is a reason for keeping all this right because we will need this the explanation the nice thing about doing it on the board is something like this can stay is it not; so no flipping back and forth between slides.

(Refer Slide Time: 18:39)




P x given y of what did we call it sorry y given x right P y give x of j given i. So, I am going to still continue to use j here.

So, this is P x y of i j divided by P what is the denominator P x of i right this is binomial we have already worked it out this is. Therefore, the way to understand this this conditional pmf is it is joint divide by marginal joint by marginal if you do the calculation what do you think you will get this has a nice close form expression, because there is no summation nothing involved you are taking this expression and dividing by you are taking a trinomial expression with the same i j exactly put here right the understanding is j varies from 0 to what is what is the variation of j 0 to this is the new omega y now n minus i n minus i.

So, you vary j from 0 to n minus i, what will this come out to be turns out this is a valid pmf right this conditional pmf is a valid pmf note that it is always valid if even before going any further how do how do I know that it is a valid pmf if I add all these quantities I should get one and that is that should be pretty obvious, because if I add all these points keeping x fixed and varying y what do I get? I get back this the denominator right. So, conditional pmfs are always valid pmfs. So, in this case note that it is a valid it is right it is a valid pmf once you considered it in totality all the where all points j what is the pmf that you get what do you think it turns out it is going to be binomial with what parameters.

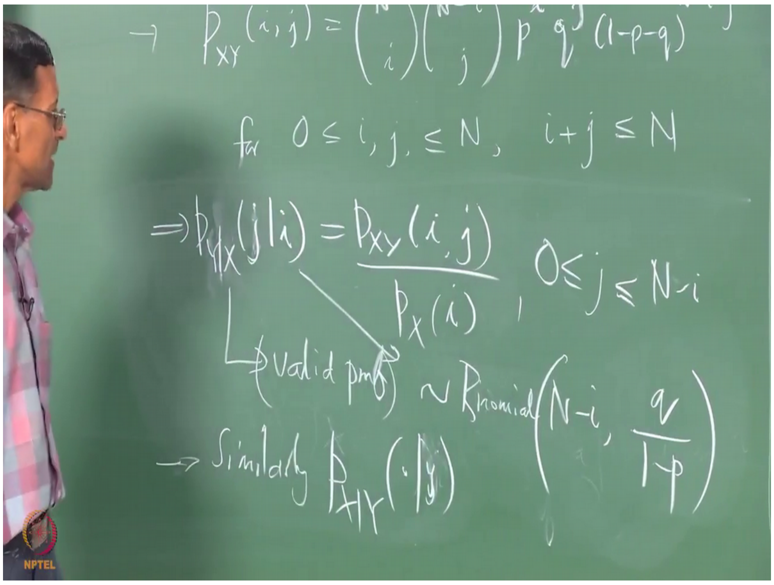
So, this is bino this guy is binomial of course, n minus i right is going to be that parameter right it is going to be it is going to go from 0 to n minus i. So, therefore, that n is n minus i and the probability parameter it turns out this P divided by 1 minus sorry q q divided by 1 minus P not P it is q divided by 1 minus P please do the division and work this out for yourself, because these kind of manipulations; I do not have time for and this going to give the result and ask you to verify it for yourself all you have to do is take this divide by this which is n choose i P power i 1 minus P to the power of n minus i and it will neatly work out to be this please do it for yourself right and check you can do it later on at home; I am not going to no spend time on this here.

So, just to point out how this trinomial example is. So, useful in containing all this concepts that is why we take it up in some detail. Of course, the concept is far more general and is that is what is very important. And in general these conditional pmfs or conditional pmfs will keep coming up a lot in probability, because it tells it say it says right given that for example, the height is.

Something what is the distribution of the weight if the height and the weight are the joint distribution that you are looking at I thinks. So, that is what right. So, it is crucially important concept to understand because we are not just interested in nearly describing joint events and joint probabilities we are also interested in observing something and asking questions of inference right the whole idea of statistical inference is based on observation ok.

So, clearly I mean you can also condition on you do the reverse conditioning can also find I mean add that here similarly.

(Refer Slide Time: 23:17)




Find P of x given y there is no reason why you must only condition on x you can equally well condition on y as situation might be and. So, you can define this dot given some j if want to call it or small y in general this again will be a joint divided by marginal I am I do not want to spend more time on it right. So, I think it should be fairly obvious how to calculate this.

So, when you look at this is a pmf you put a dot here and you look at the collection of probabilities the countable collection with what all the possible values of this dot that is why I do not want to put x and in the case of this trinomial what are you going to get you are going to get once again binomial this will become n minus j right if you take if you said y equal to j and then what will happen to this it will become p by 1 minus q and note that this q by 1 minus p or p by 1 minus q has to be smaller than 1, why.

Student: (Refer Time: 24:24) p plus.

Because, p plus q is smaller than 1, this must be smaller than 1.