

Control Engineering
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Module – 02
Lecture – 03
Transfer Function Modelling
Block Diagram Representation

Hello everybody. In this module on module 2, lecture 3; we will talk about Transfer Function Modelling and what we also call as the Block Diagram Representation. And I shortly tell you what are these things mean and the advantages of the transfer function modelling and how the block diagram representation simplifies for us the analysis of systems.

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Motivation

- How to find the response of a system for a given input signal?
- E.g.

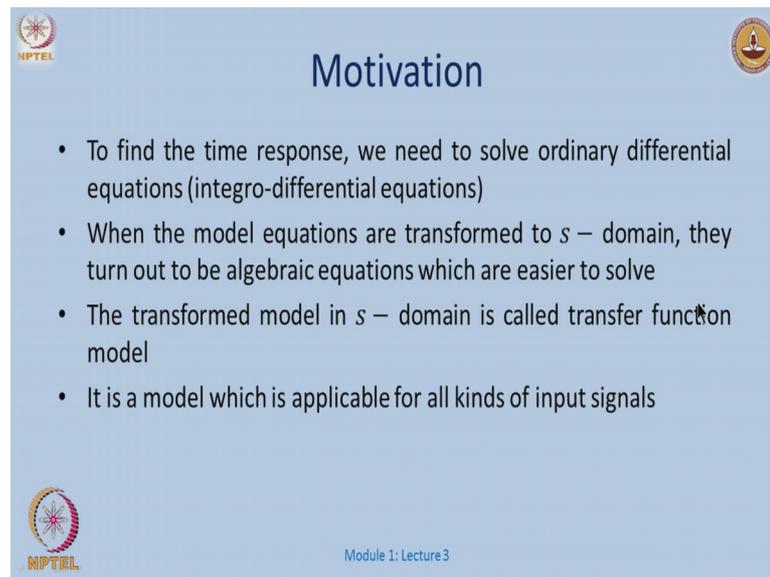
Find $V_0(t)$ for a given signal $V_i(t)$

$$V_i(t) = RI(t) + L \frac{dI}{dt} + \frac{1}{C} \int I dt$$
$$V_0(t) = \frac{1}{C} \int I dt$$

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So, let us start with an example again this we have seen earlier, let us say I thought with an RLC circuit and given a certain initial voltage; I would ask you to compute what could be; so, this is the input I asked you to compute, What is the output voltage to the capacitor? Or even what is I of t? So, what would I do; I would start writing down the appropriate voltage loss that V_i is R times I plus L ; dI by dt , plus the voltage across the capacitor, which is 1 over C integral $I dt$; this is the voltage.

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The slide is titled "Motivation" and features a light blue background. It includes the NPTEL logo in the top-left and bottom-left corners, and a circular logo in the top-right corner. The text is centered and consists of a list of four bullet points. At the bottom center, it says "Module 1: Lecture 3".

Motivation

- To find the time response, we need to solve ordinary differential equations (integro-differential equations)
- When the model equations are transformed to s – domain, they turn out to be algebraic equations which are easier to solve
- The transformed model in s – domain is called transfer function model
- It is a model which is applicable for all kinds of input signals

Module 1: Lecture 3

Now, the problem is find V_o for a given V_i , so what does this mean that you find that the time response or how V_o changes with a given V_i ; we need to solve certain set of differential equations; ordinary differential equations or even sometimes integro-differential equation; you see the little you know integral term over here. So, what you also earlier saw is that when I transform my differential equations into the Laplacian domain or the s domain the differential equations turn out to be simple algebraic equations which are then easier to solve you know linear equations are usually easier to solve than differential equations.

So, we transform the model into the s domain and then we would define what is called a transfer function and this is a signal which will relate a given input to output which will be computed again by what is called the transfer function of the system.

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The slide is titled "Transfer Function" and contains the following content:

- For an LTI system, transfer function is the ratio of the Laplace transform of the output to the Laplace transform of the input with the initial conditions being zero
- Mathematically, if $U(s)$ is the Laplace transform of the input function and $Y(s)$ is the Laplace transform of the output, the transfer function $G(s)$ is given by:

$$G(s) = \frac{Y(s)}{U(s)}$$

Handwritten notes on the slide include:

- A block diagram showing an input $u(t)$ and $U(s)$ entering a box labeled $G(s)$, with an output $y(t)$ and $Y(s)$ exiting. A red arrow labeled "ODE" points to the box.
- The equation $G(s) = \frac{Y(s)}{U(s)}$ written in red.
- The convolution equation $y(t) = g(t) * u(t)$ written in red.
- The equation $Y(s) = G(s) \cdot U(s)$ written in red.

Logos for NPTEL and a lamp are visible in the corners. The text "Module 1: Lecture 3" is at the bottom center.

And this is of course, model which is applicable for all kinds of input signals. So, what is the transfer function of a system; well the transfer function of an LTI or a Linear Time Invariant system, time invariant I can; let us recall the definition that the parameters of the system would not change with time for example, the mass or a spring constant or a resistance and inductance all these will be in variant of time.

So, for a linear time invariant system let me also recollect that what we will here focus on systems which are just single input, single output systems also called SISO systems. So, for these kinds of LTI SISO systems, the transfer function is the ratio of the Laplace transform of the output to the Laplace transform of the input assuming 0 initial conditions. So, mathematically if $U s$ is a Laplace transform of the input signal or the input function $Y s$ is the Laplace transform of the output then the transfer function G of S is G of S is Y of S divided by U of S .

So, what is how does is again look like, so I have input U of t and output Y of t and this is some system setting here some like a even say some say ordinary differential equation this is a system is represented by; so I transfer it into the Laplacian domain, it have a Y of s and the linear relation which describes, how a certain input results in a certain output is called the transfer function. So, given this transfer function I can simply say that given input signal U of s , the response of the system or the output is simply the multiplication of G of s and U of s , we call that in the time domain; we would actually have to do a

convolution here convolution. So, let say if Y of t is my output signal let me call G of t as a system and some convolution with the input.

So, now we get rid of these things, so this is G of t is replaced by G of s and this G of s is called the transfer function.

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Transfer Function as Impulse Response

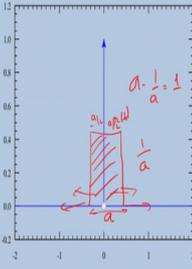


- Impulse signal ($\delta(t)$) is infinitesimally narrow and infinitely tall yet integrating to one
- It takes zero value everywhere except at $t = 0$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$\int_{-\infty}^{\infty} n(t) dt = 1$
 $\lim_{a \rightarrow 0} \int_{-a}^a \frac{1}{a} dt = 1$

- If input to the system is the unit impulse, then the output is called the impulse response i.e.,
 $u(t) = \delta(t) \Rightarrow U(s) = 1 \Rightarrow G(s) = Y(s)$
- That means transfer function is the Laplace transform of the impulse response of an LTI system when the initial conditions are set to zero



Impulse function $\delta(t)$

$Y(s) = G(s) \cdot U(s)$
 $= Y(s) = G(s)$



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So, simpler way or one of the ways of looking at transfer function is to look at the impulse response of a system which means the response of the system when the input is an impulse. So, earlier so what we had; we had Y of s was G of s times U of s and then if U of s is an impulse, we know that the Laplace transform of an impulse is just 1. So, I am just left with G of s, so let us keep it for the moment.

So, what is an impulse by definition is it is an infinitesimally narrow and you know and infinitely tall signal; you just you can you know in the speaking terms just says that it takes value of 0 everywhere; here its 0, here its 0 and I accept that t equal to 0; it is like an infinitely long signal such that the area is 1. So, how do we construct this impulse signal right if my sound a little superficial here, but let say I start with the little rectangle here and this rectangle has a width of a and a height 1 over a.

So, now what is the area of this guy is 1; area is a times 1 over a is 1 and if I call this signal as say the rectangle r of t then I have minus infinity to infinity r of t would be 1; there is 0 everywhere else, it is a left of this point and even the right of this point and this

width is a ; from here till here. So, you can call this as a by 2 ; in the negative side an a by 2 , in the positive side.

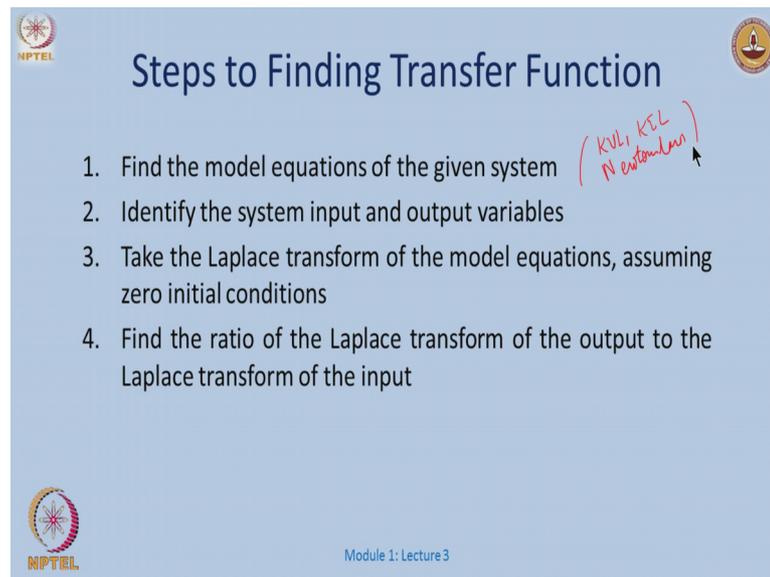
So, based on this how do I construct an impulse? So, let me take the limit of this signal as a goes to 0 ; nothing changes right is still the area is still a over 1 by a and the limit of this signal a ; if I take the area of the signal from the signal, it would simply 1 and therefore, I have this signal like this that minus infinity to infinity δ of t is 1 . So, if the input to the system is the unit impulse, then the output is called the impulse response and you see in that case; the output which I observe is equal to the transfer function; at least mathematically I can see here. So that means, that a transfer function is this transfer function is just the impulse response for an LTI system again, you look at the initial conditions to be 0 .

So let us do a little example of this, so I can take the case of simple pendulum, a point mass say everything is normalized to 1 and say if I just measure theta this way and I say just get give a impulse here; it may not be practically possible to give an impulse like this, but it might you can just look at it as a signal like this it is a very small a . So, what would you expect I just you know loosely speaking I just push it away so that so I gave it a little push.

So, it will just somehow follow its natural response, if it is a system which has no damping it will keep on oscillating all the time. If it is system which has damping, it will the oscillation spilt item; this is a natural response to the system. So, or even look at it in a different way say natural response of a human being, especially at 7 in the morning. So, the alarm clock is the impulse response, so perturbs you just; you know switch it off everything happens in the flush first time and then you go back to sleep and that is your natural transfer function; is the sleeping condition and that is the impulse response.

So, but if there is somebody trying to wake you up for a very long time then your response might be different, we will see how those response would be but the impulse is just the natural response of the system.

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The slide is titled "Steps to Finding Transfer Function" and contains a list of four steps. The first step is "Find the model equations of the given system", which has handwritten red notes "(KVL, KCL, Newton's laws)" and an arrow pointing to it. The other steps are: "Identify the system input and output variables", "Take the Laplace transform of the model equations, assuming zero initial conditions", and "Find the ratio of the Laplace transform of the output to the Laplace transform of the input". The slide also features NPTEL logos in the corners and "Module 1: Lecture 3" at the bottom.

1. Find the model equations of the given system (KVL, KCL, Newton's laws)
2. Identify the system input and output variables
3. Take the Laplace transform of the model equations, assuming zero initial conditions
4. Find the ratio of the Laplace transform of the output to the Laplace transform of the input

Now, the question is given a system; how do I find the transfer function, well the first step is to write down the equations. So, where do the equations come from? These again come from my voltage law KVL; KCL and even the Newton's laws all those things which we have been dealing till now.

So, first write down the model equations of a system; second identify the system input and output variables and we will see that you know the output the choice of output can be different or it is not always unique. Now, take the Laplace transform of the model equations which are essentially differential equations and assume 0 initial conditions and then find the ratio of the Laplace transform of the output to the Laplace transform of the input; I can always do this because it is a linear equation.

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Transfer Function : Example 1

1. **Model Equations:**

$$V_i(t) = RI(t) + L \frac{dI}{dt} + \frac{1}{C} \int I dt$$

$$V_0(t) = \frac{1}{C} \int I dt$$

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So, again let us come back to this example which we started with, so I have V_i as a input RL and C; the circuit elements generated certain current I of t and I want to find what is V_0 by V_i . So, this is easy to find out V_i is RI plus $L \frac{dI}{dt}$ plus $\frac{1}{C} \int I dt$ and this is my V_0 , this guy here.

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Transfer Function : Example 1

2. **Input and Output Variables:**

- Input: $V_i(t)$
- Output: $V_0(t)$

3. **Laplace Transform:** (assuming initial conditions to be zero)

$$V_i(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s)$$

$$V_0(s) = \frac{1}{sC}I(s)$$

4. **Transfer Function:**

$$G(s) = \frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{sC}I(s)}{\left(R + sL + \frac{1}{sC}\right)I(s)} = \frac{\frac{1}{sC}}{\left(R + sL + \frac{1}{sC}\right)} = \frac{1}{s^2LC + sRC + 1}$$

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without the $I(s)$ ($i(t)$) in the output
 $G(s) = \frac{I(s)}{V(s)} = \frac{1}{R + sL + \frac{1}{sC}}$

Now, what is the first thing is to identify the input and output variables; input is the voltage which I apply and output I identify as the voltage across the capacitor. So, my equation which was an integral differential equation, if I just take the Laplace transform

and use the appropriate properties that a differentiation becomes loosely speaking a multiplication with s , integration becomes a division by s and I end up with an equation like this.

So, now step 2; identify input, identify output, do the Laplace transform and the last step would be just take this ratio V_o by V_i . So, I could I always do this right V_o by V_i just take this guy, divided by V_i ; I have this 1 over s ; C ; I of s plus r plus s L plus 1 over C s and I just end up with this transfer function. Now I could also do it, you may ask a question what if I just write down here; what if I of s or even say I of t is the output; this is another choice of output, I will say instead of measuring the voltage here as the output, I just measure what is the current; I could always do that.

So, how does the Laplace transform look like; it is again we have kind of straight forward to compute from here. So, I say V of s is R plus s times L plus 1 over s C I of s . Now the transfer function G of s ; just to distinguish between this put a I here and there is an output here, how dimensionally measuring the current as the output is the ratio of the output I of s over V of s ; which from this expression can be simply computed this something like this; 1 over R plus s L plus 1 over s c . Again these are transfer functions for the same system, but here I am measuring V_o as the output and here I measuring the current as the output.

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Transfer Function : Example 2



- Find the transfer function of a system described by following equation: $\frac{d^3y}{dt^3} + 10\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + y = 10\frac{du}{dt} + u$ with zero initial conditions

➤ **Laplace Transform:**

$$Y(s)[s^3 + 10s^2 - 5s + 1] = U(s)[10s + 1]$$

➤ **Transfer Function:**

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10s + 1}{s^3 + 10s^2 - 5s + 1}$$



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Now, if I just start with some relation between input, so usually we will throughout this course denote U of t as the input signal and Y of t as the output signal. So, if I am given a differential equation in Y and U like this with again 0 initial conditions; how do I find the transfer function well again I just do the same steps, I take the Laplace transform of the left hand side, so Y of s . So, I am just differentiating with three times, it will be s cube differentiating twice will me give me an s square; differentiating 1s will give me a s and so on. The right hand side, I have $d U$ by $d t$ plus U , which will just transform to $10 s$ plus 1. So, once I have this; I can easily write down what is Y of s over U of s .

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Properties of Transfer Function

- ✓ Transfer function of a system is independent of the magnitude and nature of input
- ✓ Using the transfer function, the response can be studied for various inputs to understand the nature of the system
- ✓ Transfer function does not provide any information concerning the physical structure of the system i.e., two different physical systems can have the same transfer function

E.g. MSD system : $G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} = \frac{1}{s^2 + s + 1} \quad (M = B = K = 1)$

Series RLC circuit : $G(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 LC + sRC + 1} = \frac{1}{s^2 + s + 1} \quad (R = L = C = 1)$

Handwritten notes and diagrams on the slide include:
 - A block diagram showing input $U(s)$ entering a block $G(s)$ to produce output $Y(s)$.
 - The equation $Y(s) = G(s) \cdot U(s)$.
 - A diagram showing a block $G(s)$ with input $I(s)$ and output $V(s)$, with a note "units".
 - The equation $I(s) = V(s) / G(s)$.

So, given this transfer function what are its properties; well the first says that the transfer function of a system is independent of the magnitude and the nature of the input; which means it is essentially system level property, it is just based on what is the system at hand; not what is the input signal.

For example, if I take a system let us say any system with their transfer function G of s and I measure certain output with the certain input U of s . So, if this U of s is an say impulse, I will have a certain output it could be whatever; the G of s would be the same as when this could be a step input with the corresponding output, it does not really depend on the choice of the input, it is just a property of the system by itself.

The nature of the pendulum did not change, if the input is a ram or the input is a impulse or even a step; the transfer function with the one which transfers in a way given input

signal to an output signal is a property of the system itself. So, once I have G of s ; I can easily study what could be the transfer function for different kinds of inputs. For example, Y of s is G of s times U of s , so give this is fixed.

So, give me any signal U of s ; I can always study what is the response; if I want to do the time domain, I will just go find the inverse Laplace transform for which I know lots of tricks by now and so once I have the transfer function, I can study the response for various kinds of inputs to see how my system would behave. Well now, it is a transfer function unique, so give me a transfer function can I say what kind of a system is? can I say is it car, is it a bus, is it an aircraft or whatever. Well the answer is no, the transfer function does not provide any information concerning the physical structure of the system; which means that two different physical system can have the same transfer function.

So, let me just take example of the mass spring damper system, which we are studied in our earlier lectures. It had a transfer function which was like 1 over $M s^2$ plus $B s$ plus K ; if I just take a simple case of M equal to B equal to K equal to 1 , I have this transfer function and the RLC circuit which we was you in a previous slides, had a transfer function of this sort which for R , L and C being equal to 1 ; again translates back to this one.

So, these are same transfer functions for two different systems; therefore, if I just say well a 1 over s^2 plus s plus 1 , it could mean more than one system. Physically, I could realize with some mechanical system or even as an electrical system; however, what the transfer function does is; say if my input is the voltage and output I know is the current; let me write a in the Laplacian domain, then this G of s ; will have the appropriate units. So, it could be the impedance or the in reverse of the impedance and so on, it depends on how we look at the transfer say if I just write it in; so, V of s would be the output I of s , would be V of s over G of s . So, this would here it is a G of s here would capture the units of the inverse of the impedance similarly over here also.

So, the transfer function actually it captures the appropriate units to transfer the input signal to the output signal, if it is a force under velocity it will have an appropriate unit over here and so on.

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Transfer Function : General Form



• General form of transfer function of a system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}$$

$$= \frac{K' \prod (s - z_i)}{(s - p_1)(s - p_2) \dots (s - p_n)}$$

- n : Order of the system
- K : System gain or Gain factor – A proportional value that relates the magnitude of the input to that of the output signal at steady state
- z_1, z_2, \dots, z_m : Zeros of the system
- p_1, p_2, \dots, p_n : Poles of the system
- $n \geq m$ because the system becomes non-causal and is not physically realizable if $n < m$

Handwritten notes:

DC gain $G(0) = \frac{b_m}{a_n} = K$

FVT $\lim_{s \rightarrow 0} s \cdot G(s) = \frac{1}{s}$

DC gain $K = K' \frac{\prod (z_i)}{\prod (-p_i)}$

$\mathcal{L}^{-1} \left(\frac{s^2+1}{s+1} \right) \mathcal{L}^{-1} \left(\frac{s+1}{s+1} \right)$

nm cancel $\frac{s+1}{s+1}$

$G(s) = \frac{s^2+1}{s+1}$



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So, in general a transfer function would look of this form where Y of s could be any polynomial of order M in s, U of s the Laplace of the input could be written as any polynomial of order n which comes in the denominator. I could also factor that as some number K prime and tell you shortly what this is and do a series of what we call as zeros and what we call as poles. So, here I call n as the order of the system, M would be the number of zeros; n would be the number of poles.

So, there is something called also the DC gain here; which is essentially what happens when I just supply a DC input. So, this DC gain is just obtained by substituting s with 0 and in this case it would just become b_m by a_n . If you are wondering where this comes from, it is just an application of the final value theorem; to the output Y of s equal to G of s; U of s. So, when U of s is a step and I apply the final value theorem s going to 0 s times G of s what is the G of s for a step input is 1 over s.

So, this guys go away so what I am left with is G of 0, so this is you will referred to this as the DC gain of the system computed as b_m over a_n . So, these z's are called the zeros of the system; I shortly define them formally these are called the poles of the system and if I write my system in this form; my DC gain then would be G of 0, that would be let us call this s K. So, the K here would be this number K prime over a multiplied by the product of all the zeros with a minus over the product of all the poles.

So we will see short when this K prime is important to us, so one important thing to remember here is that the number of poles is always greater than or equal to the number of zeros because if the system has more number of zeros. So, if the system actually becomes non-causal, if you have number of zeros more than a number of poles. For example, if I have a system $s^2 + 1$ over $s + 1$ and this is a non-causal system and this is also not physically realizable by any known components.

So, and you could also look at it in a way that if I try to compute the inverse Laplace transform of this guy; $s^2 + 1$ over $s + 1$; it will not exist. Similarly, if I just take even a simpler one s and therefore, if in the future classes, when we are designing a controller its transfer function; if you know analytically say well I got a beautiful controller which again has a structure like this; this is incorrect.

This mean this never be physically realizable, so what we should be important in all through our problems is that the number of poles should always be greater than or equal to the number of zeros, if not the system becomes non-causal and it will never be physically realizable. So, we will keep this in mind and recollect this again when we do a control design and we will see how to actually overcome this problem.

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Poles and Zeros

▶ **Poles:**

- Roots of the denominator polynomial of the transfer function
- Values of s at which the transfer function becomes unbounded

$$\lim_{s \rightarrow p_i} G(s) = \infty$$

▶ **Zeros:**

- Roots of the numerator polynomial of the transfer function
- Values of s at which the transfer function vanishes

$$\lim_{s \rightarrow z_i} G(s) = 0$$

- Poles and zeros together with the system gain K characterise the input-output system dynamics

Handwritten notes:
 $\frac{SA}{(s+2)(s+3)} = 0$
 $P_i = -2, -3$ for
 $G(z) = 0$
 $z = -1$

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So, by definition what are poles well these are the roots of the denominator polynomial of the transfer function; how does this equation looks like just take this denominator polynomial and just equate this to 0; it will give me set of n solutions and those are the

poles of the system and what happens to the system; when s which is the value of poles when s is equal to sum p_i ; $G(s)$ close up to infinity at a very simple example could be something like this when I have $s + 1$ over $s + 2$; $s + 3$.

So, here the poles are minus 2 and minus 3, so if s equal to minus 2 this value just goes to infinity. Similarly, zeros are the roots of the numerator polynomial said the numerator to be equal to 0 and what I then get are the zeros of the system. So, what happens when I evaluate the transfer function at zeros, so this will always be 0 just substitute; so in this case the zeros are at minus 1 and if I substitute minus 1 for s here, you might value of transfer function becomes 0; because this goes to 0. So, these poles and zeros together with the system gain K ; characterized the entire input, output response of the system.

So, what do I need or whatever I have learnt so far is I can; given a transfer function I can write my system as a set of poles, a set of zeros together with a possible system gain and these three will tell me the exact behaviour of a system given a certain input.

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Gain, Poles and Zeros : Example

- Find the system gain, poles and zeros of the system with following transfer function: $\frac{6s+12}{s^3+3s^2+7s+5}$
- $G(s) = \frac{6s+12}{s^3+3s^2+7s+5}$
- System gain: $K = \frac{12}{5}$
- Zeros: $s + 2 = 0 \Rightarrow s = -2 \Rightarrow z_1 = -2$
- Poles: $s^3 + 3s^2 + 7s + 5 = 0 \Rightarrow s = -1, -1 + 2j, -1 - 2j$
 $\Rightarrow p_1 = -1, p_2 = -1 + 2j, p_3 = -1 - 2j$

Note: Poles and zeros are purely real or appear in complex conjugates ($a \pm jb$) because all the coefficients of transfer function are real

Handwritten notes on the slide:
 $6s + 12 = 0$
 $s = -12/6 = -2$
 $-1 + 2j$
 $-1 - 2j$
 $a \pm jb$

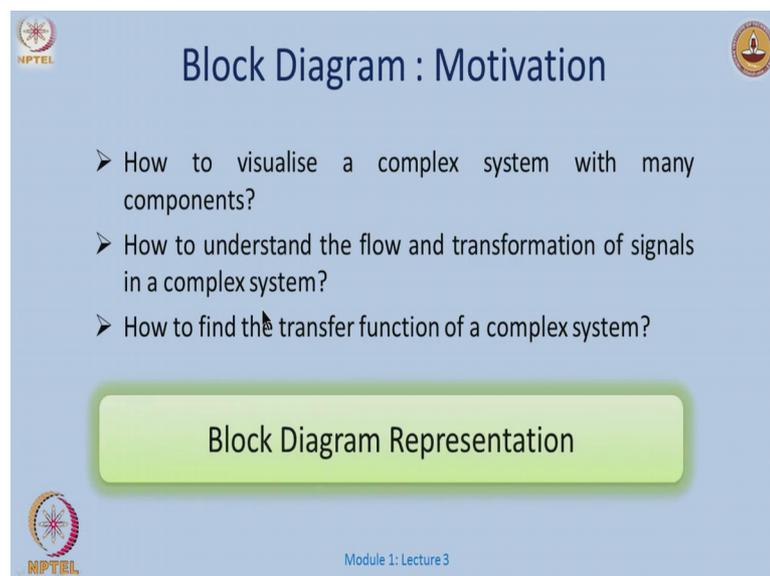
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So, let us see the system right given by $6s + 12$; $s^3 + 3s^2 + 7s + 5$ and I want to find; what are the system gain, what are the poles and the zeros. So, the DC gain just said s equal to 0 and I can easily compute this is the DC gain of the system is 12 over 5. What are the zeros? Set the numerator polynomial to 0; $6s + 12$ equal to 0; then this will give me s equal to minus 12 over 6 is minus 2.

So, the 0 would be a minus 2; so sorry this a little error here, so if I compute the zeros would turn out to be at minus 2. Similarly, I take the denominator polynomial equated to 0 and I get s equal to this three roots. So, at s equal to minus 1; minus 1 plus 2 j and minus 1, minus 2 j , so what do we observe from here; of course, the gain is will always be a real number, the poles and zeros they could either be a real numbers like a minus 2 here, minus 1 here and if they are complex, they will always be in conjugate pairs.

For example, there will never just exist a pole at minus 1 plus 2 j ; if minus 1 plus 2 j exist, there will always be a minus 1, minus 2 j or in general the complex poles will always be of this form; a plus minus j b and this is because all the coefficients of my transfer function are always a real values.

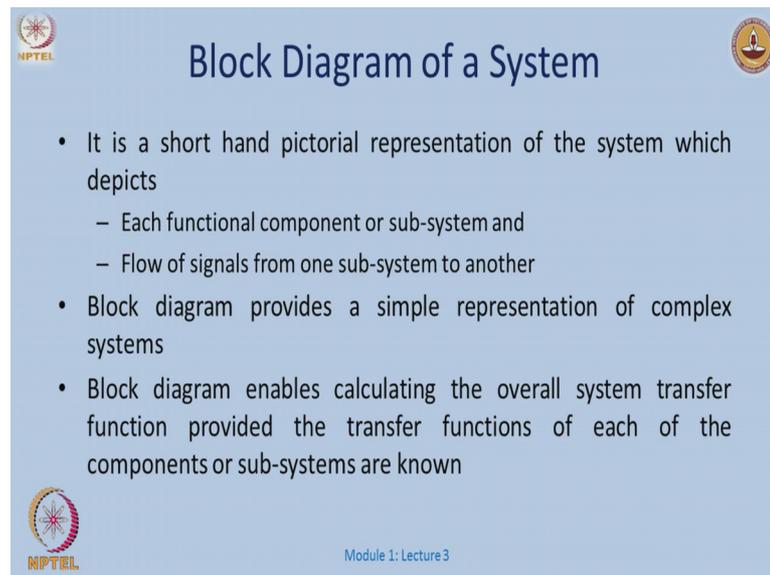
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The slide is titled "Block Diagram : Motivation" and features three bullet points: "How to visualise a complex system with many components?", "How to understand the flow and transformation of signals in a complex system?", and "How to find the transfer function of a complex system?". A central green button contains the text "Block Diagram Representation". The slide also includes the NPTEL logo in the top-left and bottom-left corners, and the text "Module 1: Lecture 3" in the bottom-right corner.

The second thing is; well if I have a system with different components; How do I visualize it? Is there a nicer way to represent that system and how to understand the flow transformation of signals in the complex system or given a series of interconnected systems; how do I find its overall transfer function? We will try to understand that phenomenon of it and this is what we will usually call as the block diagram representation.

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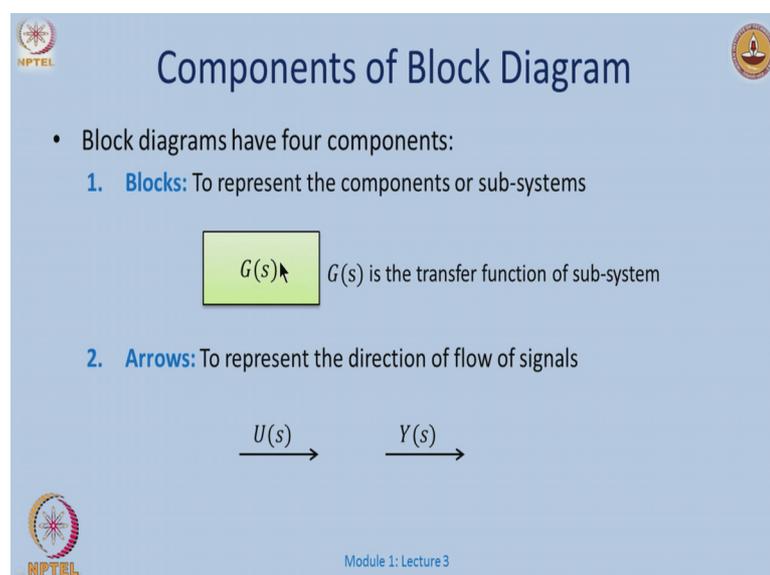
Block Diagram of a System

- It is a short hand pictorial representation of the system which depicts
 - Each functional component or sub-system and
 - Flow of signals from one sub-system to another
- Block diagram provides a simple representation of complex systems
- Block diagram enables calculating the overall system transfer function provided the transfer functions of each of the components or sub-systems are known

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What is a block diagram representation by definition? It is just a shorthand pictorial representation of a system which depicts each of the systems sub component, how signals flow within components or from one sub component to the other and this block diagram will give me easier representation of this kind of systems. There could be a multiple systems connected to each other and interacting with each other and we will see also how having this kind of representation, we will give us some easier methods to find the overall transfer function of the system; given that I know transfer function of individual components or the individual subsystems.

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Components of Block Diagram

- Block diagrams have four components:
 1. **Blocks:** To represent the components or sub-systems

$G(s)$ $G(s)$ is the transfer function of sub-system

2. **Arrows:** To represent the direction of flow of signals

$U(s) \rightarrow Y(s)$

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So, a various simple building blocks for block diagrams will be of four components once I will have $G(s)$; which is the transfer function, as we have defined so far; I will just have arrows to represent the direction in which the signals flow.

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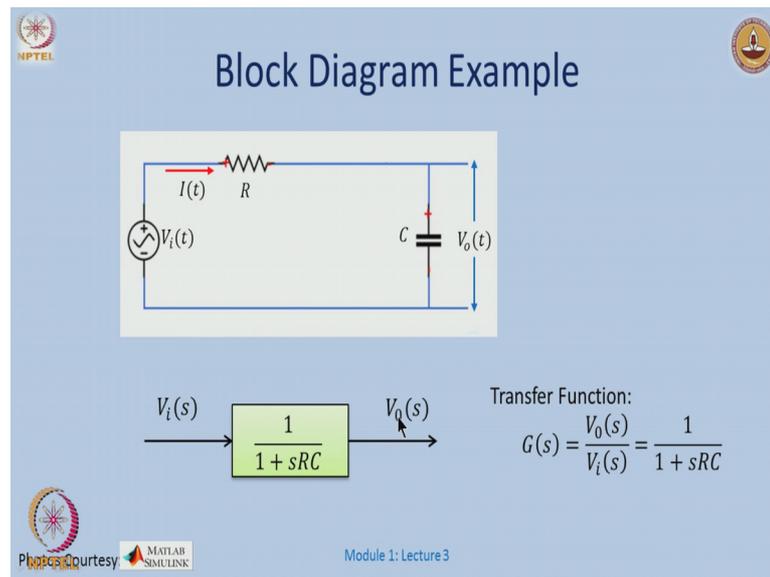
The slide is titled "Components of Block Diagram" and contains two numbered items:

- 3. Summing points:** To represent the summation of two or more signals. Below this text is a diagram of a summing junction, which is a circle with a plus sign (+) on the top and a minus sign (-) on the bottom. An arrow labeled $U_1(s)$ enters from the left, and an arrow labeled $U_2(s)$ enters from the bottom. An arrow labeled $Y(s) = U_1(s) - U_2(s)$ exits to the right.
- 4. Take-off points:** To represent the branching of a signal. Below this text is a diagram showing a horizontal line representing a signal path. A yellow dot on the line indicates a take-off point. An arrow labeled $U(s)$ continues to the right from the line. A vertical line goes down from the take-off point, and an arrow labeled $U(s)$ points to the left.

Logos for NPTEL and a university are visible in the corners. The text "Module 1: Lecture 3" is at the bottom right.

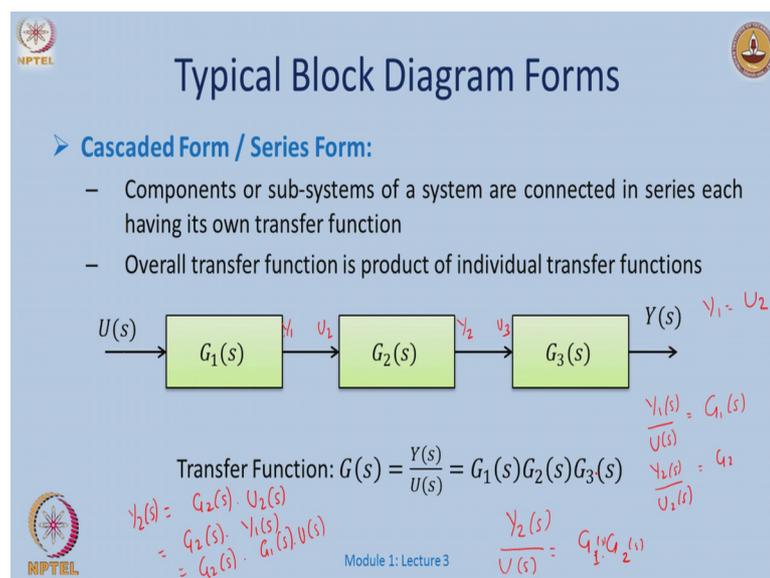
Then I will have summing points say I have a signal U_1 , a signal U_2 ; there is a plus and a minus here; the $Y(s)$ would be computed as U_1 minus $U_2(s)$; if there is a plus here it will be simply U_1 plus $U_2(s)$; take off points well this I was just like you are adding an additional Y or over here or something like this is a $U(s)$ here; if I just put a wire here and take the signal would still remain the same; since $U(s)$ here and the $U(s)$ also here, it is just represents the branching of a signal.

(Refer Slide Time: 28:23)



So, let us try to draw a block diagram of the simple circuit which we had started with; so, I have a V_i I have a R and a C over here; the L we just omit for simplicity. So, I can just say that V_i is my U ; V_o is my output signal and this have a very simple block diagram representation of this format. So, you will we can just compute this transfer function to be 1 over 1 plus s times RC ; this is very simple representation of the system.

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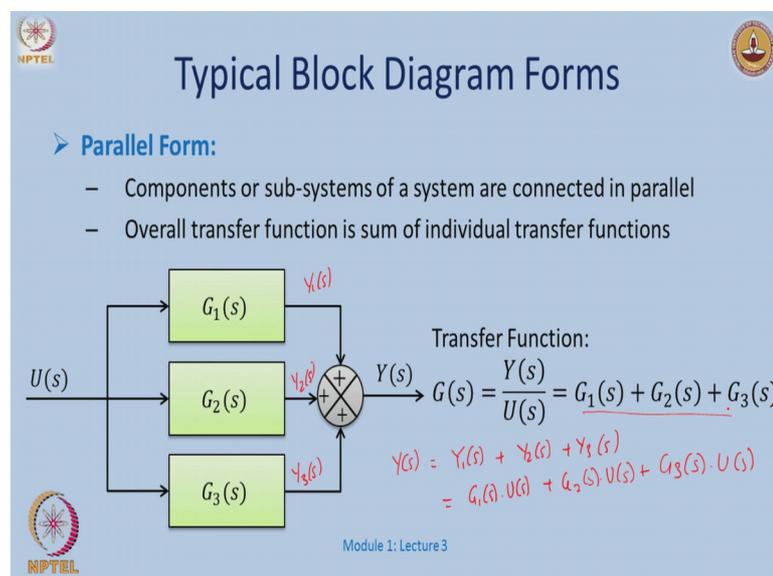
So, well what happens if I have systems which are in this form; this is U of s which generates a certain Y of s via G_1 ; this output serves as an input to G_2 ; G_2 will have a

certain output which will serve as an input to G_3 and so on. In this case the transfer function is simply this one Y of s over U of s is G_1 times, G_2 times G_3 . Let us see this actually is a straight forward to compute, so let me call this as Y_1 .

Now let me call this as U_2 , let me call this as Y_2 and U_3 and so on. So, for G_1 , Y_1 s over U of s will give me G_1 of s , similarly for G_2 ; this will be the ratio of Y_2 of s over U_2 of s . So, there are; so Y_1 is U_2 , Y_2 is U_3 and so on. So, Y_2 of s is G_2 of s times U_2 of s , now what I know is this is G_2 of s times Y_1 of s . Now what is this Y_1 of s , Y_1 of s is G_1 of s times U of s ; so therefore, if I were to compute what is Y_2 over U of s that will simply be G_1 times G_2 again of s and s here.

Similarly, I could even do the third cascade, so I will get the overall Y over divided by the overall U it is p G_1 times, G_2 times, G_3 . So, here is a what I call as the series connection; series inter connection in which case the transfer functions just to multiple.

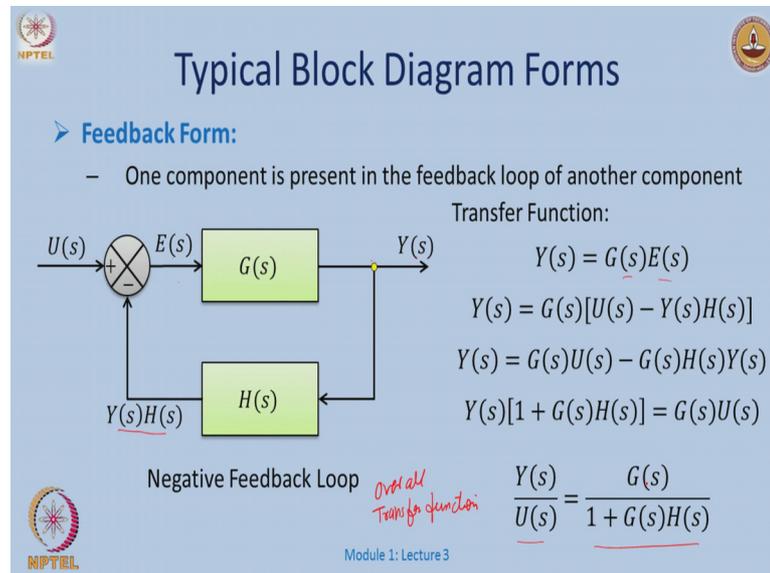
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Similarly, I can also have components inter connected in parallel and then what would be the overall Y of s over U of s . So, let us say that this U_1 via G_1 generates an output of Y_1 of s ; here I have Y_2 of s and here Y_3 of s ; by the definition of the summing block Y of s would be Y_1 of s plus Y_2 of s plus Y_3 of s ; what is Y_1 ? Y_1 comes via G_1 and U_1 . So, I will have G_1 of s times U of s plus G_2 of s times U of s plus G_3 of s times U of s

and resulting in this expression that the entire G of s is Y over U is just a summation of these three; G_1 plus G_2 plus G_3 .

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I could also have things connected in what I call as a negative feedback loop, so I have a signal here it goes via G , it is and this Y is fed back via H of s again over here. So, how does the transfer function loop in this case, so in the feedback form, so I want to find out what is Y of s over U of s . So, let us start from here where does this Y of s come from; Y comes via G and E is written as Y is G of s times E of s ; this is nice. Where does E of s come from? E comes as U minus Y of s times H of s ; this is little summing block here with a plus and a minus. So, E is plus of this signal minus of this signal, so this I write here U of s minus Y of s H of s .

And I just do a little simplification, so I just say Y of s is G times U minus G ; H and Y , I eliminate or I just get this guy to this side and I just get that Y of s by U of s is G over 1 plus G times H and this is as we call as the negative feedback loop and we will loop and we will use this with extensively throughout this course, so it might help you just to memorize this thing as a formula that if I have a G here, H here, Y over U or the overall transfer function this I could even called as the overall transfer function between Y and U ; given this individual transfer functions here, the forward transfer function block has G of s , the feedback one has H of s and this is the overall transfer function from Y to U as G of s over 1 plus G times H .

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Loading Effect on Transfer Function

- **Assumption:** While deriving transfer function, there is no loading i.e., no power is drawn at the output of the system
- This assumption must be satisfied even while deriving transfer functions for each component in a block diagram
- If one component is acting as a load on another component:
 - Transfer function of each component cannot be determined separately
 - Transfer function of both components combined should be determined
 - Both components are put in the same block in the block diagram representation

Module 1: Lecture 3

Handwritten notes: $G_1 \rightarrow G_2$
 $G_1 \cdot G_2 ?$

Now, is this always true if I say that you know I have G_1 , I have a G_2 that the overall thing is always G_1 times G_2 and when I do this; I just everything is in s . So, I just sometime omit this, but this is assumed that this is all in the Laplacian domain is this always true, so let us first talk of with an example, it is in this slide.

(Refer Slide Time: 34:35)

Loading Effect : Example

RC Circuits in cascade

Handwritten notes: $\left(\frac{1}{1+sRc}\right) \cdot \left(\frac{1}{1+sRc}\right)$
Is this the overall transfer function?

Photos Courtesy: MATLAB SIMULINK

Module 1: Lecture 3

So, I could possibly you know if I just be a little lazy, I could just look at this system as you know I have this one RC circuit connected to another RC circuit, How did the transfer function of this RC circuit look like; let us go back few slides. So, this look like

something like this $\frac{1}{1 + sRC}$, so the transfer function of the first guy would be $\frac{1}{1 + sRC}$, second guy would be the same $\frac{1}{1 + sRC}$.

Now, the question to be asked is; is this the overall transfer function; let us see if that is the answer.

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Loading Effect : Example

RC Circuits in cascade

► Model Equations in Laplace form:

$$V_i(s) = RI_1(s) + \frac{1}{sC}(I_1(s) - I_2(s)) \quad (1)$$

$$V_0(s) = RI_2(s) + \frac{1}{sC}(I_2(s) - I_1(s)) \quad (2)$$

$$V_0(s) = -\frac{1}{sC}I_2(s) \quad (3)$$

Photos Courtesy: MATLAB SIMULINK

Module 1: Lecture 3

Let me just derive these transfer functions starting from the four steps which I had listed out earlier, I just write down the individual equations with the K V L here, a K V L here and then just compute V output by V input. The first loop says V_i of s is RI_1 plus $\frac{1}{sC}(I_1 - I_2)$, similarly I could write down the expression for V_o and V_o explicitly written down is in this form.

(Refer Slide Time: 36:15)

Transfer function is obtained by elimination $I_1(s)$ and $I_2(s)$ from Eqs.1, 2, 3

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{R^2C^2s^2 + 3RCs + 1}$$

Module 1: Lecture 3

Now, I do all the math and the transfer function obtained by just eliminating I_1 and I_2 would be simply like this.

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Observe:

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{R^2C^2s^2 + 3RCs + 1} \neq \frac{1}{(1 + RCs)(1 + RCs)}$$

- Overall transfer function is not equal to product of transfer functions of two RC circuits in cascade
- This is because the assumption of no loading fails when individual transfer functions are derived
- Here, the second RC circuit draws energy from the first one and hence its individual transfer function is not valid when in cascade

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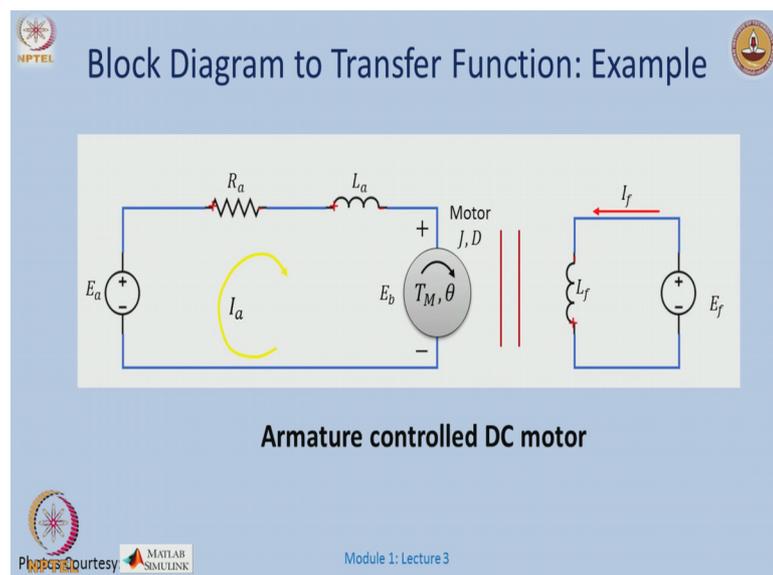
And this guy is not equal to the product of the individual transfer functions, the overall transfer function of this system is not equal to the product of the transfer function of the two individual RLC circuits even though the c (Refer Time: 36:38) is cascade; What is going wrong here? So, there is a catch here is that whenever I want to do an inter

connection like this, the assumption is that while deriving the transfer function; there is no loading that there is no power drawn at the output of the system.

So, this assumption must be satisfied all the time while deriving transfer function in this form. So, if one component is acting as a load on the other component; the transfer function you know it just cannot be determined individually. So, we should actually look at both components together, they should look at in combination. So, what is happening here if you see this circuit, the overall transfer function is not equal to the product of the individual transfer functions. This is because again the assumption of no loading fails; how does it fail? The second RC circuit, it draws energy from this component here; this one, it draws energy from this is where my output is measured for the caret from here till here.

So, this circuit draws energy from the first one and hence its individual transfer function is not valid when it is in cascade. So, this is what we have to be careful when we apply the formula of two transfer functions in cascade, the overall transfer function being the product of it. We will possibly see little more examples on this; for the moment we will just conclude by saying that this is not always true. Of course, and there are ways to make this which we will postpone to a little later, but before that let us try to do what I would call as a block diagram representation of a transfer function.

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A very simple example; I take DC motor which is also called as separately excited DC motor, the field is supplied by an external source; I have the armature certain armature resistance, a certain armature inductance and this we know produces a back emf, this back emf is also in a torque; which produces a certain displacement theta; if I consider J and D to be the moment of inertia and frictional component of the motor. So, in these sub-systems; what are my components? So, if I denote R a as the armature resistance, armature inductance the current of the armature.

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Block Diagram to Transfer Function: Example



➤ **Variables and Constants in the model:**

- R_a = resistance of armature (Ω)
- L_a = inductance of armature (H)
- I_a = armature current (A)
- I_f = field current (A)
- E_a = voltage applied to armature (V)
- E_b = back emf (V)
- T_M = torque developed by motor (Nm)
- θ = angular displace of motor shaft (rad)
- J = moment of inertia of motor and load referred to motor shaft ($kg - m^2$)
- D = friction coefficient of motor and load referred to motor shaft ($\frac{Nm}{rad-s}$)

K_T : Motor torque constant
 K_b : Back emf constant



$E_a - E_b = I_a (R_a + sL_a)$
 $E_a = I_a (R_a + sL_a) + E_b$

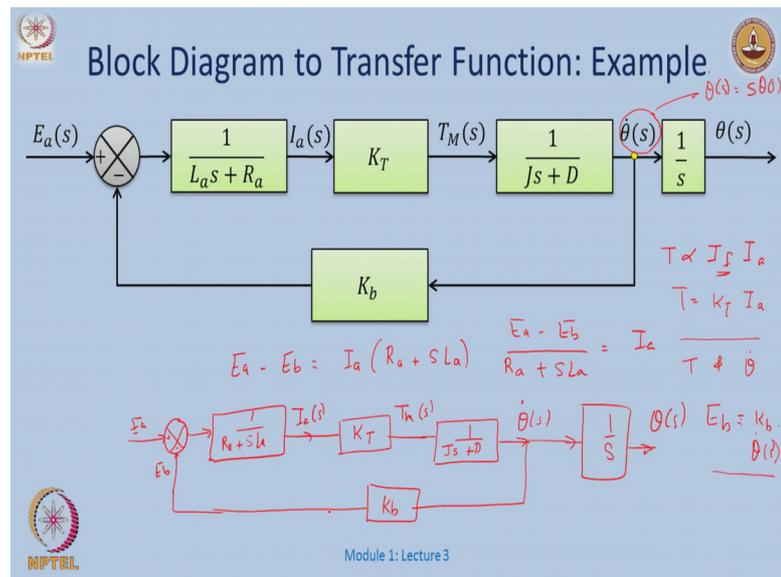


Module 1: Lecture 3

So, if I just write down the equations for this; the first one, so I have E a as the applied voltage this is R a; this is L a and this will have a certain back emf. So, the first of my equations if I were to write down would be E a equal to and if I just call this I a; from this notations here this is I a; R a I write everything in the Laplacian domain R a plus s; L a plus E b or E a minus E b is I a; R a plus s I a.

So, what happens next? Once this back emf is generated, I will have a certain torque resulting in a displacement theta and of course, taking care of the moment of inertia of the motor or possibly even the load and D would be the combined frictional component of the motor and the load. So, there are two things here; one is the back emf constant and the other one is the motor torque constant.

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Let us first look at how this block diagram is generated; the first thing in the previous slide I had this thing that $E_a - E_b$ is the armature current $R_a + s L_a$. So, if I were to write this as a block, so the first block would say that E_a generates a certain armature current here. So, as this E_a as the input possibly or which results in a certain armature current.

So, how will that look like, so it is only try to achieve you know this bigger block diagram here. So, I start by saying E_a ; there is a negative sign here, so $E_a - E_b$. So, $E_a - E_b$ if I would write this in a better way $E_a - E_b$ divided by $R_a + s L_a$ gives me I_a . So, $E_a - E_b$ goes via a block $1 / (R_a + s L_a)$ to give me I_a ; everything is again in the Laplacian domain in this direction.

So, what do we know in a motor that the torque is proportional to the field current and the armature current. The basics of a DC motor teach us this way, when here my field is constant the separately excited machine. So, if you go see this picture here this is always constant E_f is constant. So, the torque as a result of this would have a constant effect therefore, what is variable here is just the armature current as a load varies the armature current would vary and therefore, the torque I can re-write as T is some torque constant times the armature current.

So, I will have K_T and the T_m here ok, now what does this torque do; well this torque, so we had seen this relation for rotational motion between torque and the angular

velocity and what does it go through? This torque goes through the J ; the moment of inertia of the motor and the load and the friction. So, the torque is related to the (Refer Time: 43:47) T_m ; goes via this thing via $J \dot{\theta}$ plus.

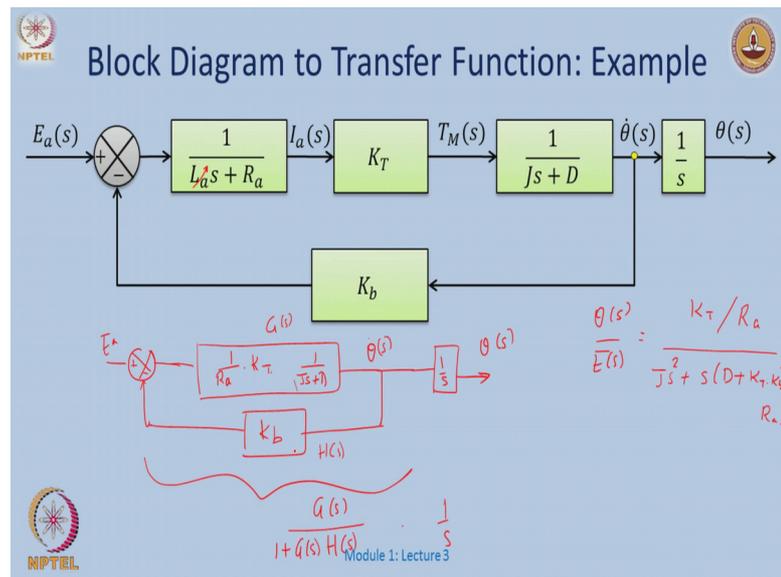
So, the relation between the torque and the angular velocity is goes through this, it generates $\dot{\theta}$ and this $\dot{\theta}$; I can integrate this is an integral component would look like $1/s$ and I get a θ . Now, how to close this loop, so I have that the back emf is proportional to the speed of the motor. So, here we define K_b as the back emf constant, so if you recollect the basics of DC motors, I will have $K_b \dot{\theta}$. So, this $\dot{\theta}$ of s via K_b completes my entire loop here. So, let us go through the process, so with an arrow in this direction. So, my voltage input voltage first generates an armature current.

Student: (Refer Time: 45:06).

I was about to say that, so there is a question of what is this $\dot{\theta}$ of E he said we will eliminate differential equations, but then there is a $\dot{\theta}$ of s . So, it is just an abuse of notation $\dot{\theta}$ of s actually means $s \theta$. So, I am just using the notation here a little bit, but this actually means a this one, so that will that will again keep that in mind. So, I just wrote this just to make a little obvious that we are dealing with the angular velocity and we are used to this differentiation.

So, $\dot{\theta}$ of s essentially would mean $s \theta$ times θ of s , so again back to this. So, we start with input voltage which generates a certain armature current and this armature current results in a torque; torque results in a certain angular velocity and this angular velocity, I can integrate to get the displacement and I also know that the to close the loop that E_b is related to $\dot{\theta}$ via this expression and then I have this entire close loop system.

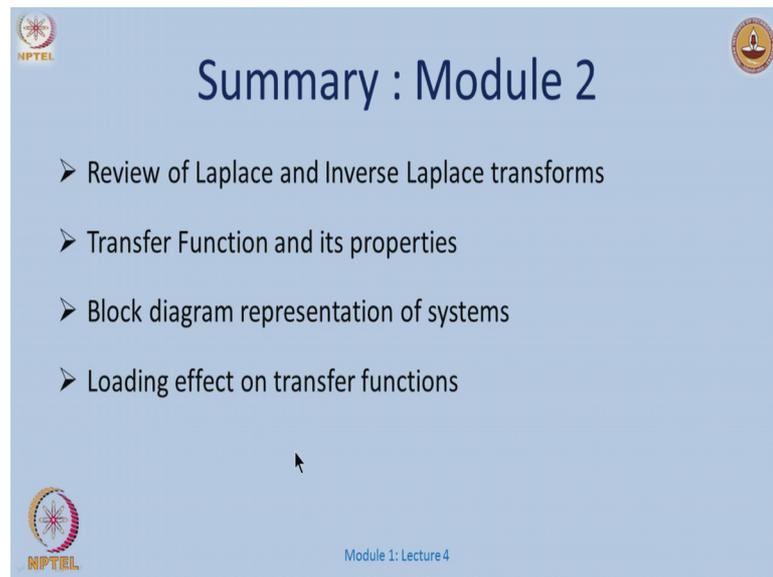
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So, I can rewrite; how do I get the transfer function for this, so let us for simplicity say I just ignore L for the moment. So, I will have E_a ; a negative feedback loop and so I will have a single block here; always would simply be multiplied. So, I have a 1 over R_a , I have K_T , I have 1 over $J s + D$; I have a θ dot of s then 1 over s, I have a K_b here; this guy was from here till here and this. So now, how do I write the overall transfer function as the ratio of the output displacement to s?

So, I will just recall what properties I learnt about the block diagram earlier as, so here I have G of s; H of s is G plus 1 over G times H . So, I go back to my block here, so let me call this G of s; I call this H of s. So, my overall transfer block here would look like G of s over $1 + G$ of s; H of s and then this would be multiplied by 1 over s and I can just write down the final expression that this will look like straightforward computation which can be done very easily K_T over R_a over $J s^2 + s D + K_T$ times K_b entire thing multiplied by R_a ; this is we can just do this calculations from here to here.

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The slide is titled "Summary : Module 2" and lists four topics: Review of Laplace and Inverse Laplace transforms, Transfer Function and its properties, Block diagram representation of systems, and Loading effect on transfer functions. It includes NPTEL logos and a lamp icon, with "Module 1: Lecture 4" at the bottom.

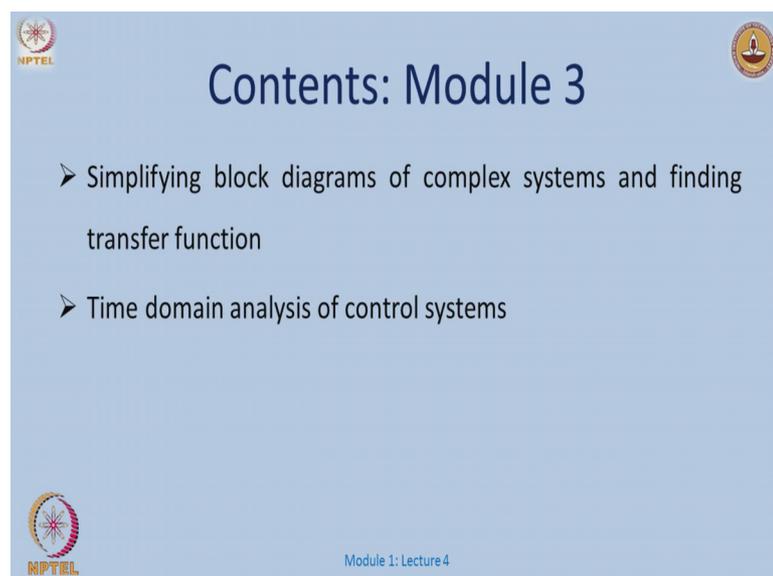
Summary : Module 2

- Review of Laplace and Inverse Laplace transforms
- Transfer Function and its properties
- Block diagram representation of systems
- Loading effect on transfer functions

Module 1: Lecture 4

So, what have we done so far; well we had defined transfer functions and its properties we had seen how defining a transfer function helps in block diagram representation of complex systems and also saw some special cases where two systems in cascade do not direct their Laplace transforms, do not directly multiply or the transfer functions do not directly multiply in case there is loading effect on the transfer functions.

(Refer Slide Time: 49:00)



The slide is titled "Contents: Module 3" and lists two topics: Simplifying block diagrams of complex systems and finding transfer function, and Time domain analysis of control systems. It includes NPTEL logos and a lamp icon, with "Module 1: Lecture 4" at the bottom.

Contents: Module 3

- Simplifying block diagrams of complex systems and finding transfer function
- Time domain analysis of control systems

Module 1: Lecture 4

So, what we will next look at is, if I have a system which has lots of summing points lots of you know negative feedbacks; how do I simplify those block diagram of a complex

system and then to find the overall transfer function. We will also then see how we can actually use this transfer function to see how my systems would behave to different kinds of inputs.

Thank you.