Control Engineering

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Module - 12
Lecture – 02
Linearization of state space dynamics

Hello and welcome to part 2 of state space modeling. So, today we will be talking about linearization of state space systems. So, in part 1 we saw that there are certain benefits of moving from the transfer function to the state space domain. Primarily we are dealing with non-linear systems we could deal with multi input multi output systems and so on. We saw that in the examples of the electric circuit where you had 2 sources a voltage source and the current source you had multiple inputs and multiple outputs.

So, today what we are going to see is that practically all the systems we consider in the world or for engineering are all are all completely non-linear right. And yet there is a good reason for us to not work directly with non-linear systems. We actually would like to work with the linearized version of non-linear systems. There are a couple of reasons for doing this I will I will explain that. So, we will see why we need the linearization procedure and I will explain one specific way of doing a linearization which is by using the Taylor series and the so called Jacobean matrix and we will end with end with 2 fun examples yeah.

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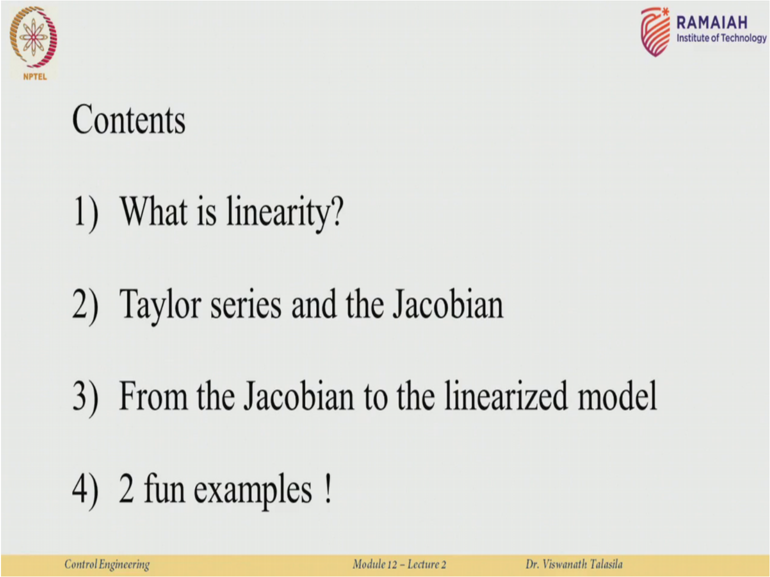

So, that is the basic structure. So, we will see what is linearity then we go to Taylor series and the Jacobean and then we will conclude good. So, our physical systems linear or non-linear they are all non-linear. There is no physical system which is completely linear, but before we even go ahead with seeing what do I mean by a physical system is non-linear and then how to handle this. It is important for us to understand the meaning of linear or non-linear. And the reason why I would like to stress a little bit more on this even though you have done this in electric circuits or what is called network analysis because most students are not comfortable with the distinction ok.

So, we remember it as a definition that they should be a superposition property, but it is not too clear what it actually means. So, let us see a very simple illustrative example of linear and non-linear.

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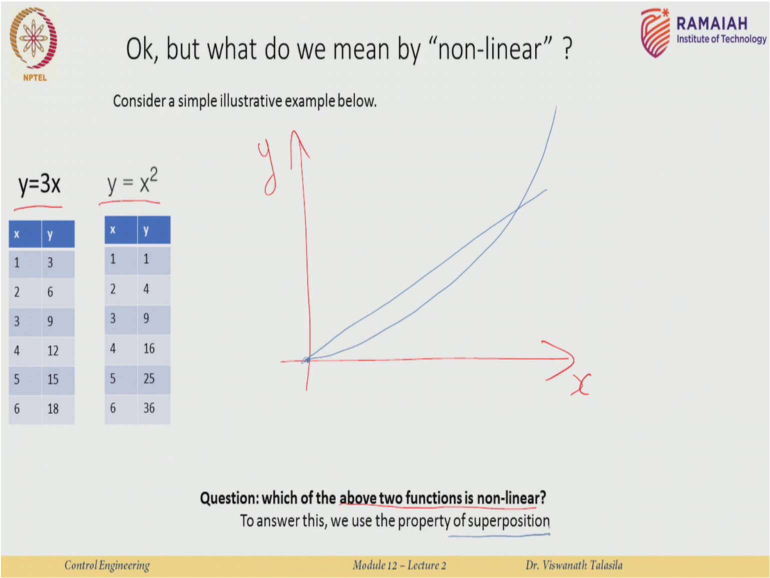

So, we consider 2 functions, the first one is y equal to 3 times x and the second one is y equal to x square. And the question I am trying to pose is which of the above 2 functions is not linear alright. So, if I just plot this particular function let us say I the x axis is of course, x and then I plot the output over here y. So, if I look at y equal to 3 times x the curve will actually be something like this is at the origin 0 ok.

So, when x is 0 you have y equal to 0 and then it goes linearly like this. If you look at y equal to x square it is a function like this, alright? And now the question is which of these 2 is linear or which one is non-linear alright. So, the way that we have studied this in electric circuit is or even in math, to answer this we actually invoke the property of superposition. I will explain super version in a slightly in a slightly different way.

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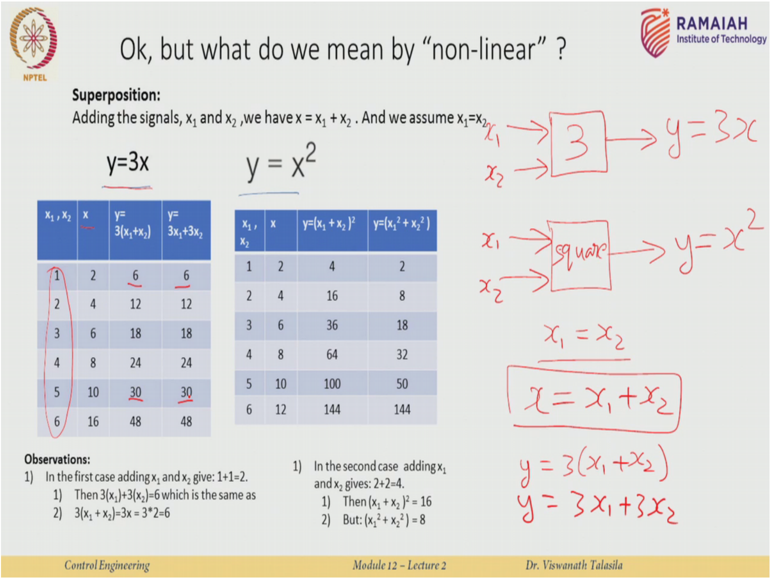

Alright So, let us see which of these 2 functions is non-linear. And the way we are going to do this is to assume that each of these systems each of these mathematical functions are actually systems alright. So, let us consider the first one, we have a system with the gain of 3. There is another system which is like this I give in inputs for both the systems.

Let us in this particular case take only 2 inputs and I have an output over here. In the first case I am looking at y equal to 3 times x alright. In the second instance I am looking at an expression y equal to x square. And so, this is actually a squaring system over here. In both the cases what I am assuming in this specific example is that I have an input x 1. I have an input x 2 in the other case also I have x 1 and x 2. And for the sake of simplicity, but this is not required at all, but for the sake of simplicity let us just assume that x 1 is equal to x 2. And we are going to assume that this y equal to 3 x or y equal to x square are basically models of some physical system. And let us look at these 2 tables on the left over here. So, for y equal to 3 x and remember that we are assuming x 1 equal to x 2 in both the cases. So, y equal to 3 x I take x 1 and x 2. And I am going to define x is equal to x 1 plus x 2 alright.

So, this is the basic equation which I am going to use. So, when x is equal to x 1 plus x 2 and I consider x 1 and x 2 with these values x then becomes the sum of each of those right. So, one becomes 1 plus 1 2 2 is then 2 plus 2 4 and so on. Now let us look at the function y equal to 3 times x I can look at it in in 2 separate ways I can either look at it as y equals 3 times x 1 plus x 2 because remember x is equal to x 1 plus x 2 or I can treat it in this way y equal to 3 x 1 plus 3 x 2 alright. And let us see if both produce an identical answer. And we see in this particular case they actually produce exactly the same answer ok.

So, this is exactly what we mean by a linear function or a linear system. And I will explain intuitively what this means a little bit later. Now let us look at the second function alright.

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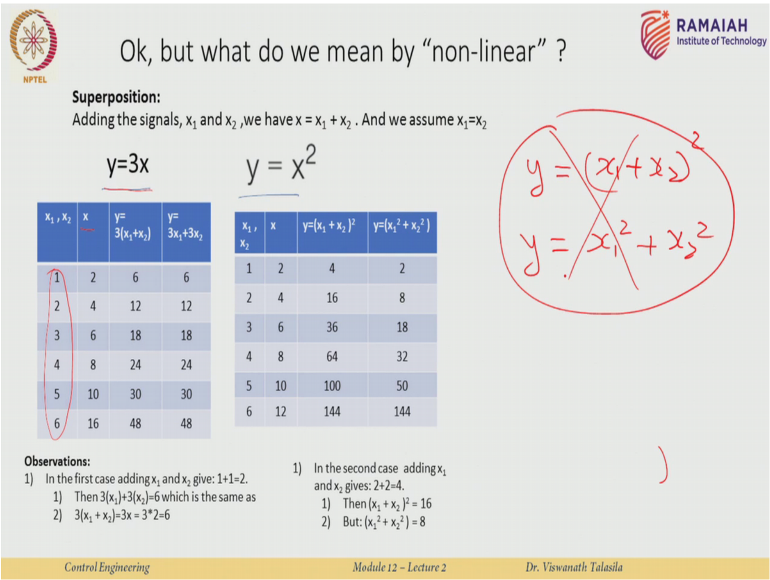

Which is y equal to x squared and again there are 2 ways of writing this y equal to x 1 plus x 2 whole squared. I can also assume the signals are being added in the following way and remember x 1 and x 2 are signals which are input to my system. We all know this is high school mad that the these 2 expressions are not equal to each other right. And we can actually observe this over here.

So, what is the interpretation of this? So, let us see what is the interpretation of this assuming that these 2 are models of 2 different physical systems. So, what it means is as follows. So, let us take the first one which is 3 x.

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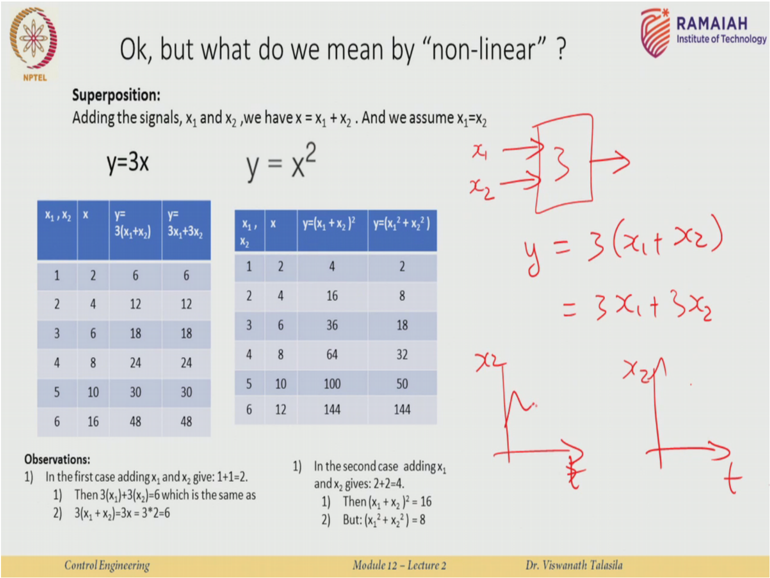

So, I have 2 inputs it is not necessary to just have 2 I can have 10, 100 whatever you like and this can be written in 2 ways right. And we have just seen this it is either 3 times x 1 plus x 2 or it can also be written as 3 x 1 plus 3 x 2.

So, what is the interpretation of this? If x 1 and x 2 are 2 signals right. As functions of time all I am saying is that I can add up the 2 signals x 1 and x 2 and I can pass them into this your system called 3 or I can pass each of these signals into the system 3 and add up the result later. How would that look like? That would look as follows.

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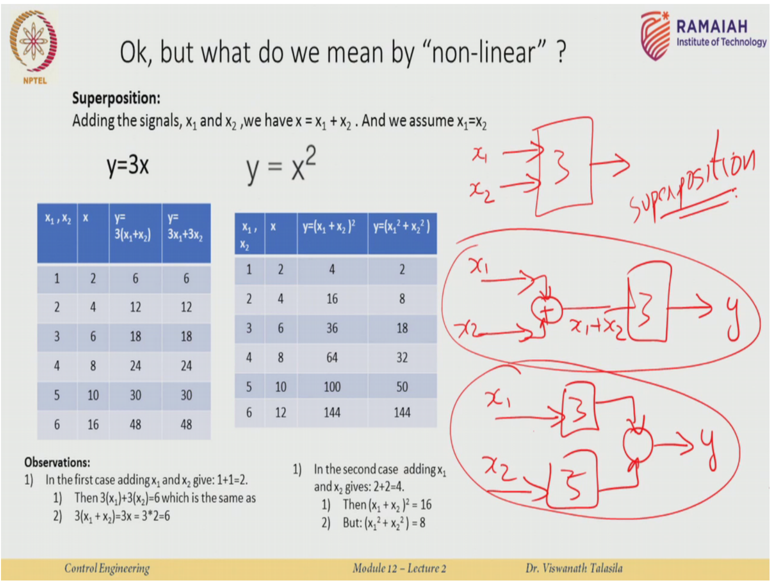

So, I can take x 1 I can take x 2 I can add these 2 up in a summation block. So, I get x 1 plus x 2 over here and remember both are signals and I can pass them into a block all 3 I get y. The other option is I can pass each of them into a block called 3 which is a model of the system. And I can add them up over here in order to get y. And both these systems are exactly the same. And if this is the case this is actually what we mean by superposition, actually a part of super version not the full thing, but is what we mean by superposition. I can pass my signals through the model first and then look at the output or I can pass each of the signals through the model and some of the response afterwards. And you are going to get exactly the same answer and we have seen this in linear electric circuit is where we apply the principle of superposition to actually compute system response to difference sources. What happens if it is non-linear well if it is non-linear we have just. So, we take the 2 signals again, x 1 x 2 and we pass so, we add them up ok.

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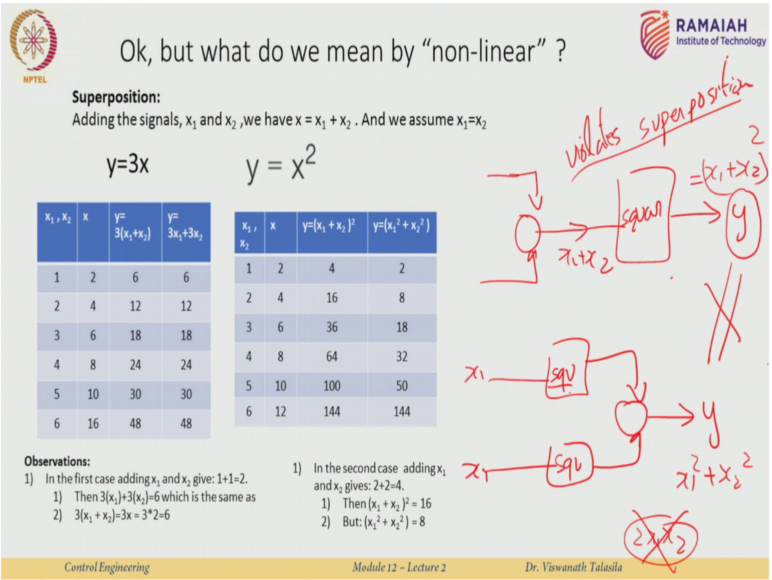

Let us so, we take 2 signals x 1 x 2 add them up you get x 1 plus x 2. And then you pass it through this block called the squaring block and you get your y alright. Now this is not equal to taking each signal squaring it up squaring it up and then adding them. This y over here is not equal to this y. And this basically violates it violates the principle of superposition. So, it violates the principle of superposition and we can see that in the first case y is nothing but x 1 plus x 2 whole square and the second case what is it is x 1 square plus x 2 square. And these 2 are not the same. There is a factor a residual factor of 2 x 1 x 2 which is left out right.

So, again if you violate superposition it essentially means that I cannot mix my signals pass it through the system and say that it is the same as passing each signal through the system individually and then mixing it up. That is not true for a non-linear system. So, good, now we know what is linear what is non-linear in a general sense we have understood superposition this applies to all physical example electrical mechanical and so on.

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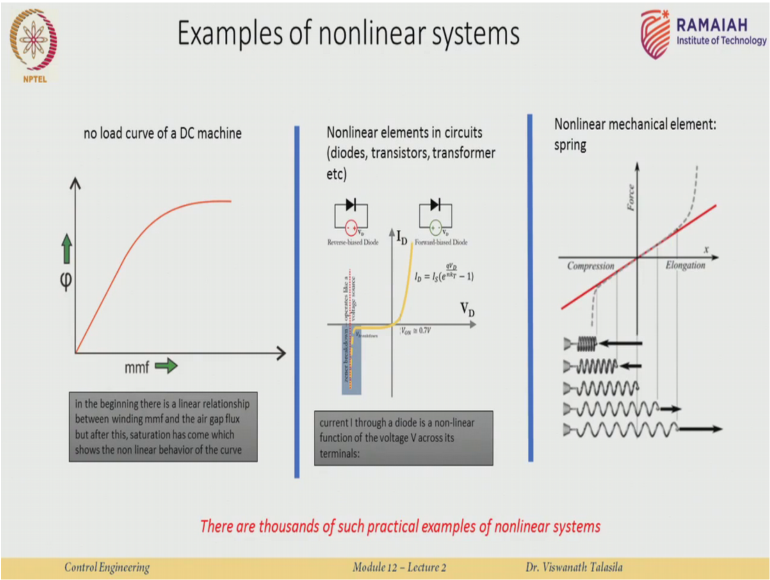

Examples of non-linear systems, I will just put 3 over here there are thousands of examples of non-linear systems.

So, there are your diode behavior it is actually one of the classic examples. So, you have this kind of a response over here right. So, in this region it is linear in this region it is absolutely not it changes completely right. Even if you look at a non-linear mechanical spring and there is no such thing as a linear mechanical spring all springs in life are really non-linear. So, if you look at the non-linear spring again you have this kind of an effect. So, all these are non-linear examples and there are thousands of such. So, while the claim is that all systems are actually non-linear what happens in practice many times we see systems which behave as if they are almost linear if he ignore certain very small effects. So, for example, I can have a spring which is like this which looks almost linear everywhere. And my range of operation of this particular spring depending on the application I am looking at maybe it is only in this range right. And I do not need to worry about what happens over here or over here because my operation actually happens in this particular range.

In this case it is enough for me to just use a linear model right. And I do not need to worry about the non-linear effects and this is actually what happens in practical engineering. As far as possible we try to use systems in their linear operating range alright. It is not always easy to do so, and we will see a procedure of how to linearize or non-linear system in the so called linear operating ranges alright. And that is essentially the topic of today's talk.

Now, why we want to do this is primarily because of this reason. The analysis and control of non-linear systems is very painful. It can get extremely mathematical the computational effort is very high and so on. On the counter if you look at the linear approaches you have excellent.

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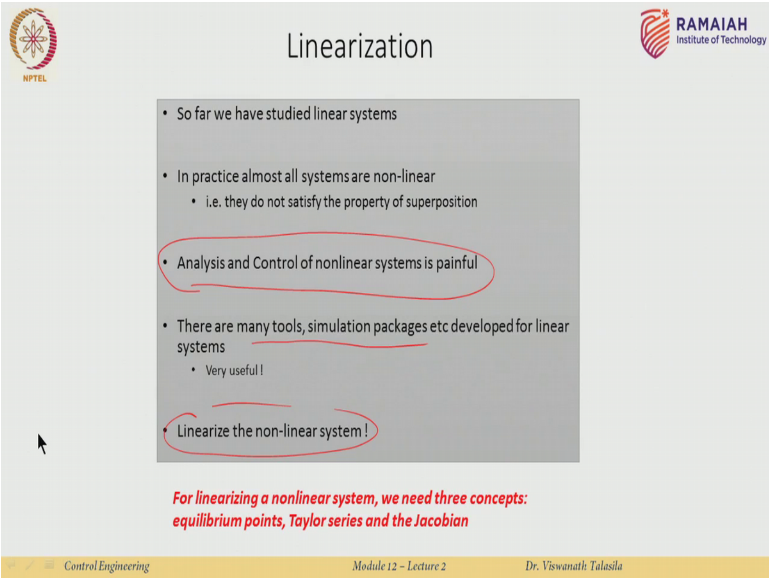

Simulation design tools you have simulation packages so on and so forth. Primarily relying on linear algebraic approaches alright. So, the only option we have because all systems are non-linear the only approach we have is to linearize the non-linear system. And we will explain how we are going to do that. We need 3 concepts the first of the concept of equilibrium point the second is the concept of the Taylor series and the related notion of the Jacobean. Using these 3 we will show you one specific way of linearizing a non-linear system.

So now that we know that this is what we are going to do linearize the non-linear system.

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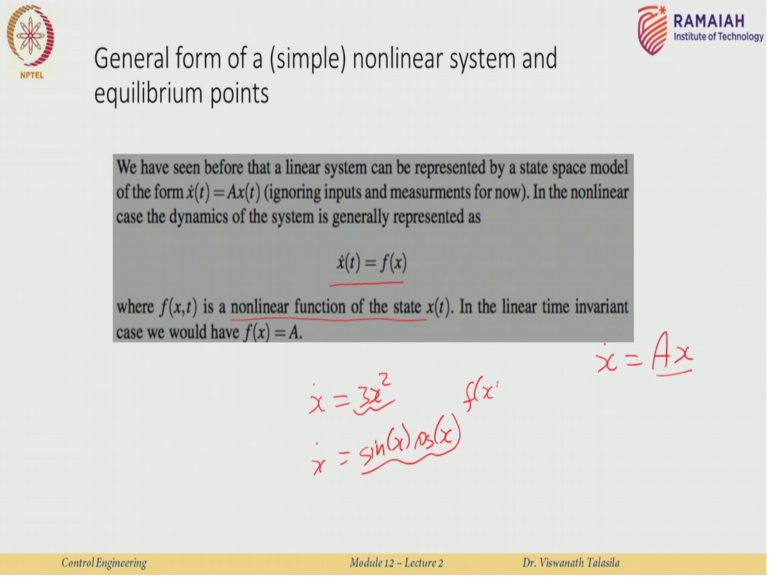

So, first let us look at the general expression for the non-linear system. So, if you recall linear state space system was x dot equal to ax plus bu y equal to cx plus d u, let us ignore the inputs and outputs for the minute. So, what we will have is basically a system of the type x dot equal to a times x alright. And notice that a is independent of x that is the crucial thing a may be a function of time a may be constants whatever it is, but it is independent of x. In the non-linear system you have the classical way of denoting a non-linear system dynamics as x dot equal to f of x. And f of x comma t is specifically a non-linear function of the states x of t. Of course, in the linear case f of x will just be equal to a.

Now, I can write my dynamics as simple example would be say 3 x square alright. Or I could write x dot equal to sin x times cosine of x, these are all examples of non-linear system and we essentially are just calling these as f of x. So, that is a general notation for a non-linear system.

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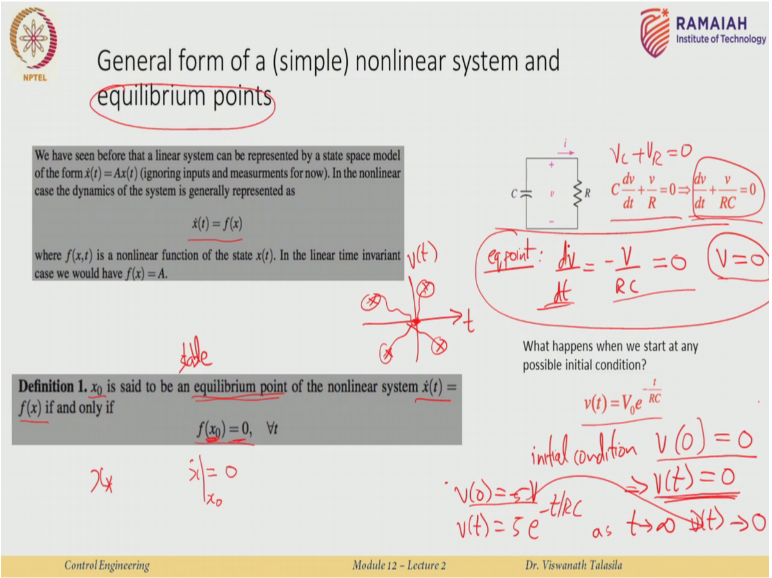

Now, it is very important to understand the concept of an equilibrium point. And the definition of an equilibrium point is as follows. So, given a non-linear system x dot equal to f of x as we have seen before a point or x or you can call it as x naught. Some people also use x star as another notation. A point x naught is said to be an equilibrium point of this non-linear system x dot equal to fx, if and only if at that particular value of x naught the dynamics is equal to 0.

So, this basically means that x dot is equal to 0 when evaluated at x naught. So, that is the notion of an equilibrium point. It basically says at that specific point x in x naught in this particular case your dynamics are 0. Let us look at one simple example of the so called scalar case remember that when you talk of states you have state vectors. So, in general we would talk about x as actually being a vector of states right. For this particular slide let us ignore that. What we are going to really look here is just a single state alright.

So, let us look at this example of the RC circuit. And we can write based on the kirchhoffs voltage law. This relationship over here, where the voltage across a capacitor plus the voltage drop across the resistor it basically sums up to 0 alright. So, when I rewrite this a expression I get dv by dt plus v by RC equal to 0. Now what is the equilibrium point for this particular system? That is a question. And the and the procedure to compute the equilibrium point is really straightforward, we know that the definition says the derivative must go to 0 at that particular point ok.

Now, what we are going to do is to write dv by dt equals minus v by RC alright. To compute the equilibrium point I need to set this to 0. Now this is equal to 0 only when v is equal to 0 right. Because no one will take r or c equal to 0 and construct a circuit like that. So, R and C will be non 0 values and this equation holds true only when the voltage across the capacitor is equal to 0 alright. So, this is an equilibrium point now it is actually an interesting equilibrium point and we will come to one of this notion a little bit later in a more complex case.

So, first of all remember we computed and said that when the voltage is equal to 0 for this model of the system dv by dt plus v by RC is equal to 0, this is an equilibrium point because the dynamics is actually equal to 0. Now the solution of this simple first order differential equation is basically this one. So, v naught e power minus t by RC where RC is your time consumed basically. So now, the question is we know that when v is equal to 0 alright when you start the circuit simulation with the initial condition, such that v at time 0 is equal to 0 alright. In this case for all future time this answer will be 0 alright. It will it will never be a non 0 value it will always it always be 0.

Now, what happens when we started a different initial condition say for example, v of 0 equal to it could be 5 volts alright. We see here that v of t will be equal to 5 a power minus t by RC and for any reasonable say combination of R and C as t tends to infinity actually it does not even need that much time, but as t tends to a sufficiently large value v of t will tend to 0. So, what this means is that when my initial condition is 0 my the dynamics of the system will always stay at 0. It never goes to any other finite value. When I started any other initial condition my dynamics will always converge to 0 alright. So, if I actually draw x and t as a function of time actually v sorry about that.

So, if I take v as a function of time if you start over here at the origin initial condition is equal to 0, you will never have dynamics going anywhere else it will always stay there. You can start anywhere else over here or this can be an initial condition this can be an initial condition this whatever it be, you will always converge to this value 0. And it turns out that this is a specific case of what is called a stable equilibrium point and more specifically a globally stable equilibrium point.

Now, we will not go too much into these definitions, but when you look at design analysis of control system these concepts become very clear. So, let me summarize this slide, basically saying that a particular point of your solution we are looking at v over here, one of those values of v is said to be an equilibrium point if the dynamics is always equal to 0 at that particular point. And we also have a simple procedure to calculate what that equilibrium point is which is basically you equate the derivative to 0 and you solve for that f of x and that is what you get ok.

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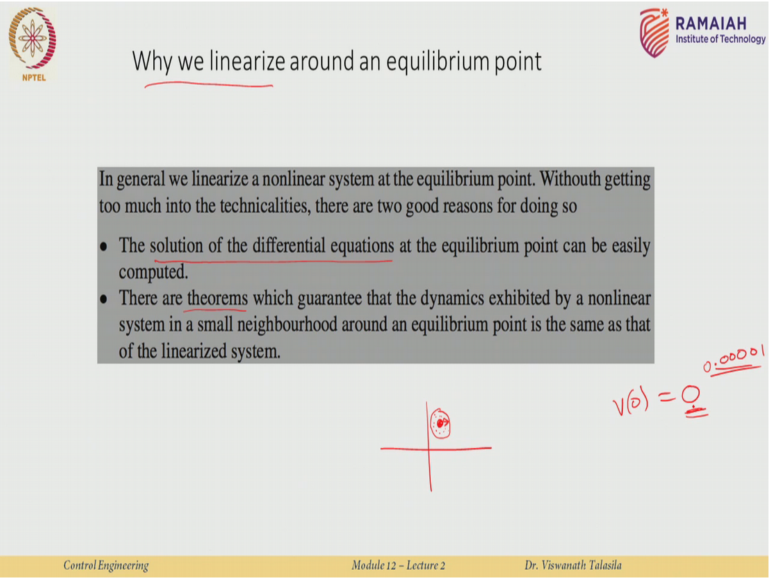

So, this is basically what I said in the previous slide, that if you start off your dynamics let us say if you start a dynamics at x naught where x naught is your equilibrium point the dynamics will always remain at that particular value for all future time t. It also means that if I start at any other initial condition and accidentally or through design or whatever the system manages to reach that particular state from that time onwards the system will be essentially stuck at the state forever alright. So, that is a basic notion of an equilibrium point. And of course, there are various kinds of equilibria there are saddle point centers all kinds of things we will encounter one or 2 of them in this particular lecture otherwise it is it is actually a fascinating study ok.

Now, we have been talking about equilibrium points we have been talking about linearization. Now there is a natural question which comes, why do we want to linearize about an equilibrium point why not somewhere else. The answer to that is fairly one is a very practical reason. And it is basically that it is very easy to compute the solution of differential equations at the equilibrium point. All you need to do as we have seen in the previous slide just equate the derivative to 0 and just solve it. Generally it is fairly straightforward as opposed to computing the general solution of the differential equation itself which can be very complex.

The second and more relevant reason more important reason is that we have theorems which actually guarantee that if you linearize a system about an in the neighborhood of this equilibrium point right. So, if you linearize the system at this equilibrium point we are guaranteed that the behavior of the system at this equilibrium point is very similar to the behavior of the system anywhere else around the equilibrium point in a small neighborhood right. So, in the previous case we had the RC example. And we saw that the value of 0 is an equilibrium point you start anywhere close to 0 right. Say 0 point 0 0 1 or whatever it be in a very small region around that equilibrium point, your behavior will be very similar to what it is at the equilibrium point fine.

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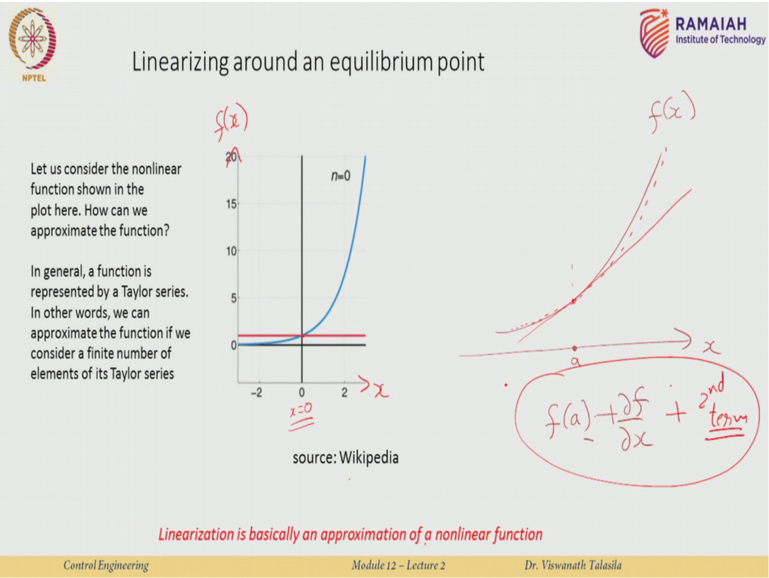

So, that is basically why we look at linearizing around an equilibrium point. So, let us actually first look at a slightly graphical sketch of how we would approach the problem of linearization, and in the next slide we will actually see the so called Taylor series and a more formal way of doing the linearization alright. Let us say that we have this parabolic like function in the blue line which we are seeing over here. And we want to actually fit another function to this one alright. And or rather we want to approximate this function and this various ways of approximating such functions. So, in the first case I can simply have that at a ok.

So, let me call this as x and let me call this as f of x. X is an independent variable and f of x is a function. So, as x varies we have a change in f of x also. Let us take a particular value of x say that we take x equal to 0, at this point. And then we compute what is this f of 0 we get some value let us say it looks to be about say almost one. Now this is one approximation to this curve right. So, you have this curve and one of one particular approximation to this curve says that I have I know this particular value I do not know the rest of the curve, but I know that one particular value well that is that is a good start.

Now, what can we do once we know this one particular value, well we can actually compute the slope right. This looks like a nice curve and one of the one of the nice things we can do is just compute the slope over here alright. The slope. So, if this function is f of x the slope over here would simply be partial f with respect to x alright. Now this is another approximation. So, the way I would actually write this is let us take this particular value in x we will call it as a. So, x is equal to a. So, at the first instant I got the value of f when x is equal to a we will call that as f of a right.

Now, to that I am going to add the slope over here which is a straight line right. So, I will add plus partial f by partial x. And essentially I get this kind of in this is an approximation. So, all I am saying is given my nice parabola I now have an approximation to the parabola which is a straight line clearly this is not true right. I mean you still have all this deviation over here which needs to be taken care of. So, what we could do is now to fit this in a slightly different way. So, you had your parabola again and you computed this point you computed a straight line why do not we actually try to fit a second order say polynomial r on this particular point right.

Let us say that we are able to fit something like this. This is another approximation. So, what I am going to do now is to add a second order term over here. And when I add up all these terms together the final fit to this particular curve will be something like this, almost what we want. So, this is a very good approximation of this particular parabola. Now in general you may not have just parabolas you may have say fairly high order curves.

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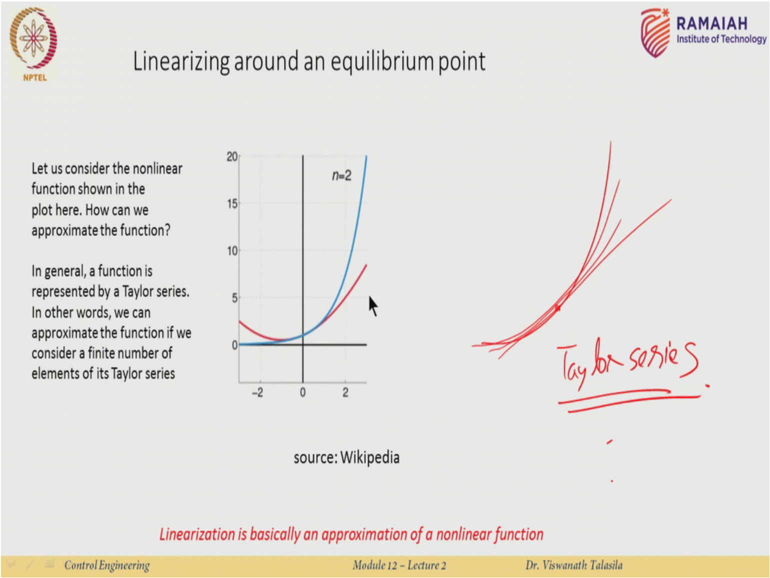

In which case I may need to take first is my constant value, then I need to take a straight line I may need to take a second order polynomial you need to take a third order polynomial so on and so forth. All these curves which you are trying to fit to this particular polynomial are basically approximating this polynomial right.

So, this actually goes by the name of the Taylor series. And the basic idea of the slide is that linearization is basically an approximation of a non-linear function. How do how we do it formally is by the Taylor series expansion. And so, we have already seen what this thing is right. So, if I go back to the previous slide you can see the animation over here it is a second order it is a third order fourth order and so on right. I can show this again. So, there you go first order second order third order and so on right. So, these are different fits to the particular function.

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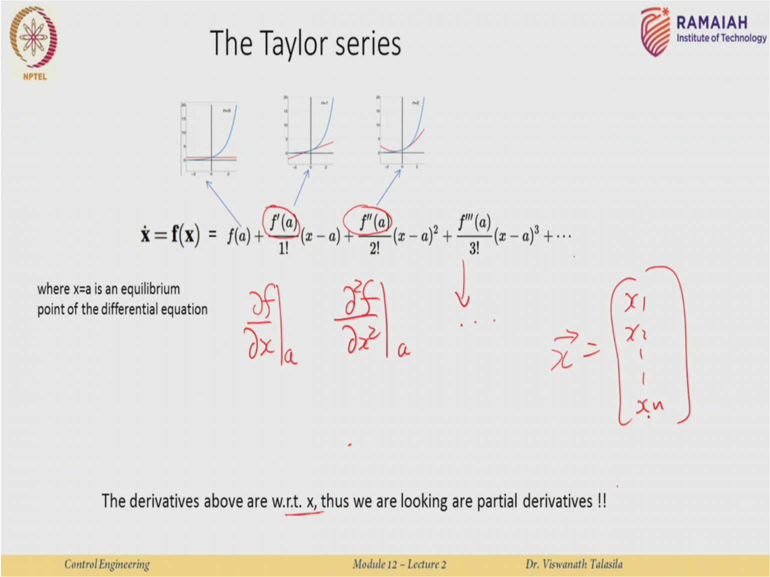

And the way we compute those fits to the function is by using the notions of derivatives we have already seen the first order the first order derivative where we took the slope of the line, that is that is a slope of the line over here right. Slope of the curve sorry.

So, if we add up all these terms of the Taylor series essentially you are actually going to have a perfect fit to your parabola alright. And that is basically approximating the non-linear function. And of course, where we approximate is important we have already seen that it is better to approximate at the equilibrium point because it is easy to compute the solutions and other reasons right. So, we always try to approximate at the equilibrium point. So, if you look at these terms over here the f dash a f double dash a and so on, these are nothing but the partial derivatives with respect to the independent variable in this case we have only one independent variable, evaluated at the equilibrium point right. F double dash will be the second order partial derivative evaluated at the equilibrium point and so on right.

So, all the derivatives we are considering are with respect to the independent variable x in this case one, but in general it will be a multi dimensional independent variable right. Because remember that we have the state vector which was x 1 x 2 So on till x n. So, let us see how that works out.

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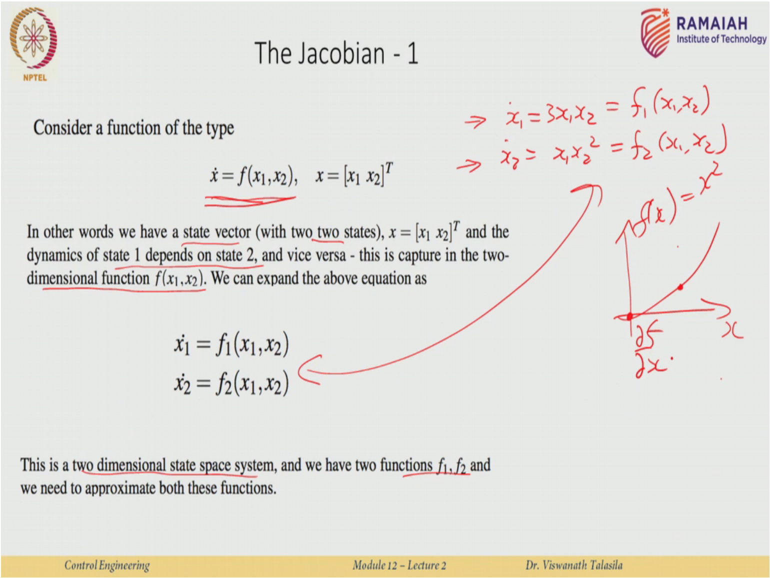

So, in the previous case we had a function of the type for example, it could have been x dot equal to 3 x square write a scalar dynamics which you can call. Now what we are going to look at is functions of 2 variables in the next slide we will go to n variables and see how we can generalize this.

So, we look at functions of 2 variables this can actually look as follows. So, x dot equal to f of x 1 comma x 2 right. And the way I can write this down as one specific example it could be something like 3 x 1 times x 2 x 2 dot equal to x 1 x 2 square something like that. Now this would be my f 1 of x 1 comma x 2 this would be nothing but f 2 of x 1 comma x 2 and that is basically this expression x expanded in 2 equations ok.

So, that is basically what we are saying if you have a state vector with 2 states and the dynamics of state one depends on state 2 and vice versa this is captured in the 2 dimensional function f of x 1 comma x 2. And that is exactly what we have done over here. So, this is a 2 dimensional state space system and we have 2 functions f 1 comma f 2 remember in the previous case when we had the dynamics like this x and f of x right. Where this parabolic thing (Refer Time: 32:22) x square (Refer Time: 32:24) so, when you had this kind of an equation we actually considered at the equilibrium point in this case equilibrium point is over here. So, we considered at the equilibrium point the derivative of f with respect to x right.

So, basically we are trying to understand how the dynamics of the system changes with respect to the independent variable. In this case we had only one independent variable. In our example in the current example we now have 2 independent variables right. So, if we try to look at it visually it will actually look something like this.

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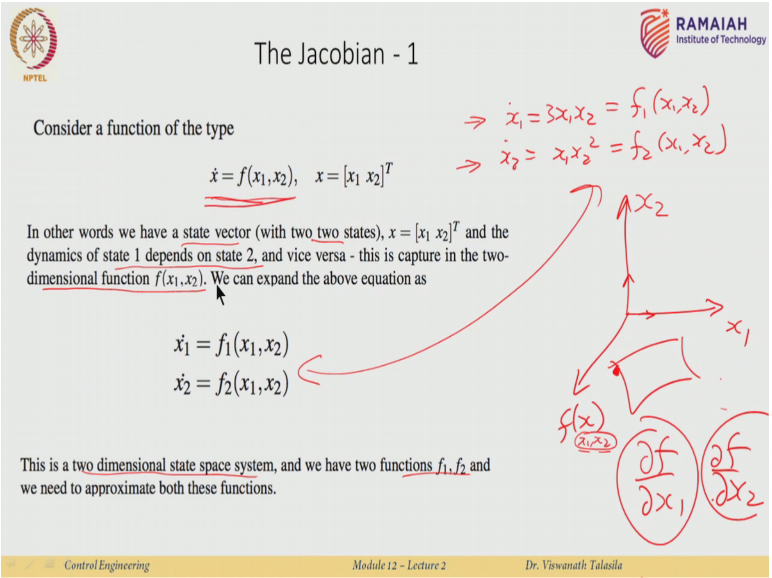

So, I have got An x 1 one of the independent variables the other independent variable is x 2 right. Then you have your function itself f of x. So, for a change in x 1 and a change in x 2 I will have a particular value of f of x right.

So, typically this would be a surface or something like that right. So now, because f of x which is basically x 1 and x 2, because f changes with respect to x 1 and x 2 I need to compute 2 derivatives. So, it one derivative will be respect to x 1 the change of the dynamics in the x 1 direction the other derivative will be the change of dynamics in the x 2 direction. Now we need 2 derivatives unlike in the scalar case way we just use a single derivative alright ok.

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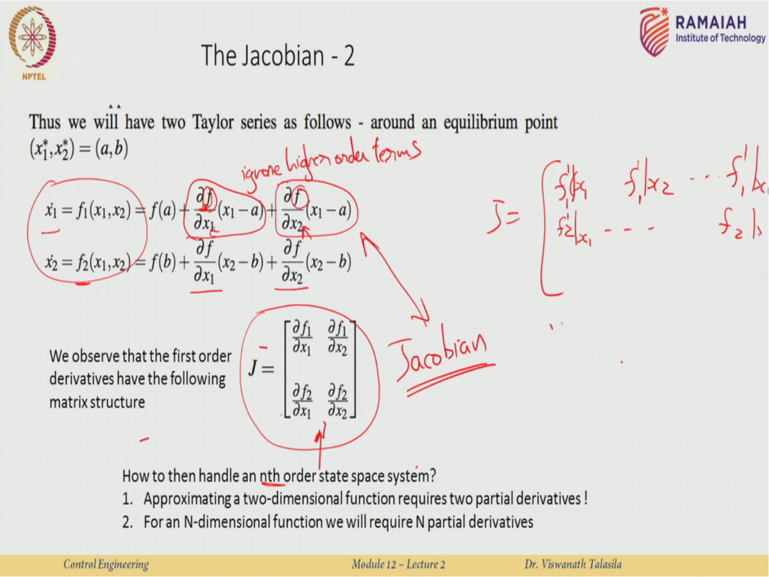

So, how will that look like? Well, if you just write the Taylor series and you ignore the higher derivative terms, it can call it the higher order terms ok.

So, we take only the first order terms over here for the first equation x 1 dot or the first state in our case we actually have these 2 expressions over here right. The derivative with respect to x 1 the derivative with respect to x 2 these are again I am repeating this simply capturing the change or the variation of your dynamics your system in the x 1 direction. This one is capturing the variation of the dynamics in the x 2 direction, which is why we always have 2 derivatives.

Similarly, f 2 also varies as a function of x 1 and x 2. So, again you have these 2 derivatives. If we put these derivatives in a matrix form you have this kind of a structure over here. It is a square matrix again and essentially youre going to just take all the partial derivatives and then put it over here. Now this is a special kind of a matrix called the Jacobean, and the Jacobean basically is derived from the Taylor series right. The first order terms of the Taylor series we are ignoring the higher order terms, because in general first order terms are usually good enough approximations to the non-linear functions in general not always. Now we know how to handle second order dynamics, how would we handle nth order state space system well based on what we have just seen for a second order state space system if we compute the derivative with respect to x 1, and the derivative with respect x 2, for an nth order you will just keep on going until you compute the derivative respect to x n alright.

Now, note that when I say f it is not just f it is f 1 and so on I will have f 2. So, the derivative of f 2 with respect to x 1. So, until derivative of f 2 with respect to x n and so on. Put all of this in a matrix form and this will essentially generalize to This kind of a structure.

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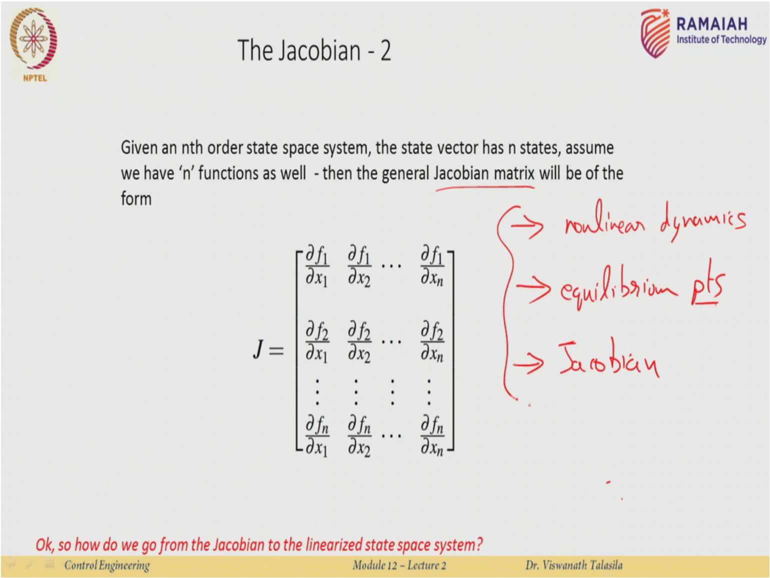

So, this is your the classical the Jacobean matrix. And this is where we stop one part of the lecture and what we have done. So, far is the following.

So, we have a non-linear system or we have the dynamics of the non-linear system. We have then computed the equilibrium points we have defined it and just said how we can compute the equilibrium points of the dynamic system. And finally, we have seen that to approximate the non-linear function we actually use the Taylor series. And as far as possible we typically usually the first order derivatives of the Taylor series right. So, we use a Taylor series the first order derivatives and then we compute the Jacobean ok.

So, this is what we have done. So, far now the question is how do we go from the Jacobean to the linearized state space system, how do we get the linearized state space system once we have the Jacobean. And by the way I will be deriving one or 2 examples for the Jacobean. So, this will become clear in a couple of slides.

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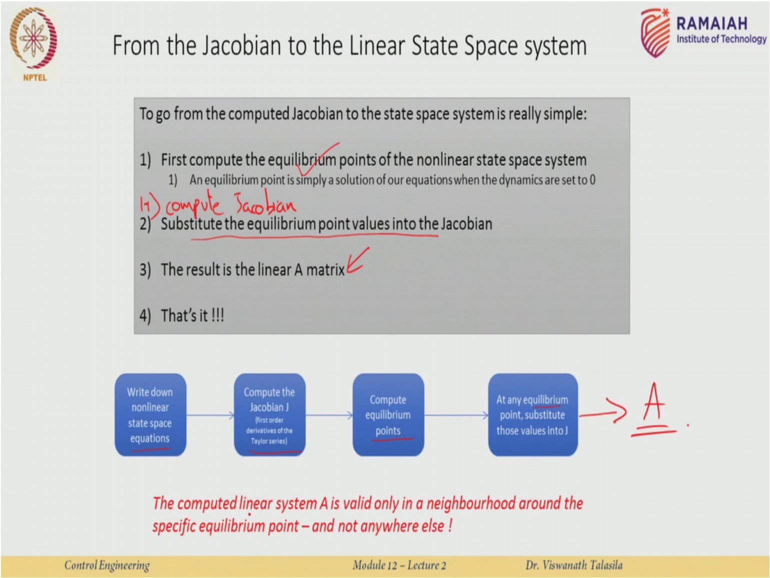

So, how we Go from the Jacobean to the linearized state space system is as follows. So, again all of this is at a high level when you look at the example it becomes much more clear. So, first we have computed the equilibrium point which we have which you talked about. And then we compute the Jacobean. So, I would actually include another point over here I would say compute the Jacobean right.

So, first we compute the equilibrium points of the system then we write down the Jacobean the matrix basically. And the equilibrium points which we have we substitute the equilibrium points into the j into the Jacobean and we will see an example of how to do that. When you substitute the equilibrium points into the Jacobean what you are going to get is a linear a matrix. And that is it is as straightforward as that. So, the procedure is you write down the non-linear state space equations compute the Jacobean compute the equilibrium points substitute the equilibrium points into the Jacobean and this gives you the a matrix. A point which we will come back to maybe in the next slide which is very important is that the computed linear a matrix or the system matrix a is valid only at the equilibrium point and in a small neighborhood around the equilibrium point it is not valid anywhere else. We will see that very shortly ok.

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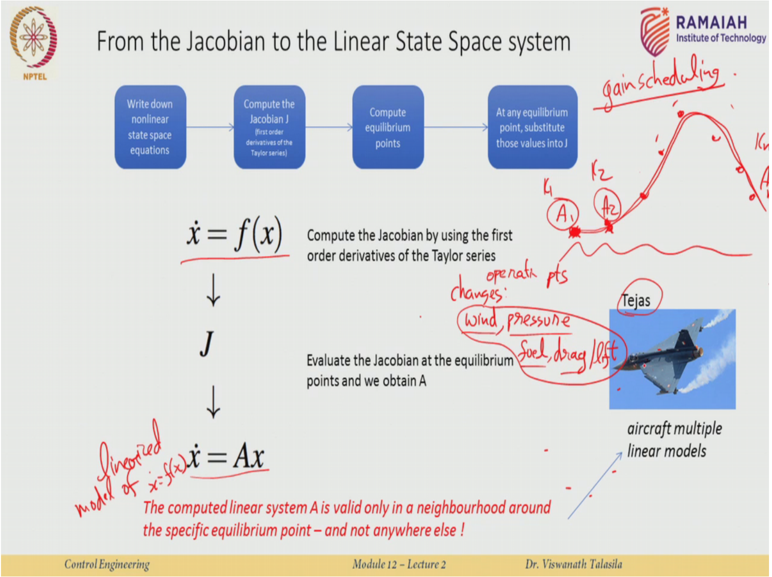

So If you rewrite this a little bit this. So, we have the basic non-linear dynamics then we compute the equilibrium point the Jacobean and you get x dot equal to ax and this is the linearized model of x dot equal to f of x alright. So, there we go now yeah. So, one example is if you take the tejas of an indian light combat aircraft or for any aircraft for the matter of fact and aircraft the general non-linear dynamics are very complex right. And you really can not design simple controllers for these really non-linear dynamics. So, what engineers have done over the past fifty years or So. They have design what are called as linear models at each operating point of the aircraft flight path say for example, just one particular example if we. So, this is the terrain earth and the aircrafts trajectory basically takes it along this along this path ok.

So, we have our wonderful tejas which is actually flying around this path. Now at each and every point on the flight path multiple changes happen. So, what are these changes let us actually write some of them down. So, your wind your conditions change atmospheric conditions change, the pressure conditions change of course, when wind changes even turbulence and other factors also change. So, your pressure changes that these are the external parameters the internal parameters are that the fuel is reduced a little bit your plane maybe having some extra drag because of the kind of wind conditions which are affecting it and so on right. Each of these changes every single change actually changes the model of the aircraft dramatically.

So, these are actually called operating points right. So, as we go from one operating point to another operating point the model of the system is actually different. So, what people have actually done is to take each of these operating points and linearize the system around this operating point. So, to get a matrix take another operating point and then linearize it over there and so on and so forth. So now, you have a bunch of system matrices a one to a n along the flight path of the aircraft for aircraft. For each of these system matrices each of these a matrices they actually design a controller. So, corresponding to each matrix you now have a controller right. Now you can call this k one I can call this k 2. So, on until kn and this is in flight control literature this is actually called as gain scheduling, without getting deviated from our topic. The idea of this example is that this is a highly non-linear problem right this flight path which encounters. So, many changes in wind pressure fuel drag lift so on and so forth is a very highly non-linear System.

And the only way we can handle it at each operating point of this flight path we actually linearize the system right. So, then you get this set of linear models and for each model we actually design controllers. So, this is how it is classically done in aircraft industry and chemical process plans and so on.