Control Engineering
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Module - 12
Lecture – 01
Introduction to State Space Systems

Ramakrishna has already covered quite a bit of details in classical control. So, he has covered Laplace transform the transfer function modeling stability analysis in root locus bode so on and so forth. What I would like to do in these series of lectures is to go to the other design of controller; other option of control systems which we call as modern control systems and specifically in this lecture, I will be talking about state space, alright.

So, we will see what is state space control or the state space systems. We will see some modeling aspects of state space systems, I will present a few examples in this and then we will go ahead with other aspects as well. So, first of all, we have already seen what Professor Ramakrishna has already taught about the transfer function modeling approach and the question naturally arises; why do we want the state space approach, alright. So, before we go to the state space let us see why we need a different approach as compared to the Laplace transforms. So, on the slide, I have got 2 boxes over here, on the left I have got the good aspects of the transfer function approach, on the right I have got the drawbacks.

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So, a transfer function is actually very useful and it is actually one of the most dominant forms of modeling in the control industry. So, you go for process control aircraft any kind of systems ships so on and so forth. The dominant way of modeling a system and then controlling it is always from the viewpoint of a transfer function, alright. So, the basic concept of a transfer function is that it is some kind of a generalization of the Fourier transform, alright. So, once we see that within the Fourier transforms, we know that we are essentially trying to have a signal processing viewpoint of a system. So, you have a system and you give inputs or varying say frequency and the output is measured with respect to these inputs of varying frequency, right.

So, this is more a signal processing viewpoint and more specifically, this is called the input-output view and because we are looking at signals with frequencies a natural aspect that arises is a notion of bandwidth, alright and we know that bandwidth is very very important for control systems if your bandwidth of the system is fairly low, you cannot control the high speed dynamics, alright or you cannot analyze the high speed dynamics and vice versa if the bandwidth is too high you may start to bring in lots of noise and other aspects, alright. So, the bandwidth is of course, dependent on the specific task and the problem which you are looking at, but the bandwidth is again a very specific aspect of the signal processing viewpoint and it is a fundamental aspect in control systems.

The other nice aspect of control system of the transfer function approach is that we have these algebraic properties or transfer functions. So, I can take multiple transfer functions, I can put them in different second cascade them in parallel series I can take the feedback loops feed forward loop so on and so forth. So, the idea is that I can actually combine different transfer functions by using their algebraic properties, right. So, we have seen that you can just take 2 transfer functions in series and you can just combine them by the product of them or I can take a transfer function I can do a feedback with another transfer function you know so on and so forth and all these properties I am able to combine these different blocks primarily because of their algebraic nature, alright.

So, it allows us to do a lot of very useful things by combining different blocks from different subsystems and we can build more and more complex systems, right and of course, as I said before, this is very dominant in the industry you can take any advanced industry in the world today transfer function modeling is a single most important aspect when you come to the control of any system, alright. So, that is a good stuff.

Let us see; what are some of the drawbacks of all the transfer function approach the first and most important is they can be used only for linear systems, alright. So, we will go a little bit about what we mean by linearity and so on in the next lecture, but for now, I am assuming all of us have a fair idea of what is linearity. So, transfer function approaches cannot be easily used when we are looking at non-linear systems it can be done, but it is a little bit tricky.

Secondly the other important thing is it is usually limited to single input single output systems right. So, I have got a system like this say a transfer function over here I have got an input and I have got an output right. So, it is basically limited to these kind of systems now in general we see that a lot of systems are of the multi input multi output type. So, I have got different inputs and for each input in the combination of a particular process in the in the plant I have got different outputs right. So, y_1 y_2, so on till y_n all the way to G_n of S.

So, I have got inputs r_1 r_2. So, until r_n of S. Now we know from the lectures that Ramakrishna has already explained, we know how to handle this system, right, if we also know how to handle this system, we know how to handle every individual system. In this particular say highly complex system which is an interconnection of many different

systems, what we still have not seen is when 2 systems are interconnected, alright. So, I will actually a give a specific example and then it is going to become clear.

So, let us take I have got system one over here system 2 over here, right, we know how to analyze the stability properties we know how to compute performance criteria as all these kinds of things for each of these individual systems in you take any complex system a chemical process planned aircraft automated autonomous vehicles. So on and so forth; you never have single systems there are multiple subsystems and these subsystems are actually coupled to each other, alright. So, you may have the engine control system and then you have; say for example, the automatic flight control system or in cars you may have the abs system or advanced breaking systems which we have.

Now, these are coupled systems. So, when we say couple it means there is some kind of a relationship between these 2 systems right and these are very tricky to handle, when you have multiple such subsystems and the number of such subsystems in a in advanced vehicle or a chemical process plant can go to 100s, if not 1000s, right. So, for each one of this, we need to keep on say modeling first in this in the simple single input, single output approach, then we need to start to look at what are the coupling effects between these 2 systems and the coupling can actually be non-linear in many cases, right.

So, this becomes extremely cumbersome when you have multiple input multiple output systems. Now this does not mean that people are not doing this. So, in the aircraft industry; for example, you have extremely high order systems and there is really very complex amount of say dynamics and controls which is required and it is all done in the classical approach, right. So, you have beautiful transfer functions you have these couplings between the say each of the transfer function loops which are looking at and you can model all of this, the problem with the reason why we had to do this in the good old days was primarily because computing power was very limited, alright and we will see in a couple of slides why this change in the computing power allowed us to advance in the notion of state space and other modern control say techniques.

So, the reason was because we had limited computing, everyone preferred to look at individual loops design. The controllers for individual loops and then worry about the coupling between individual loops, alright and then design another controller to take care of the coupling those kind of things. So, this is; obviously, very tricky and its it can lead

to really messy designs and it is very difficult to really capture all the coupling effects. So on and so forth. So, that is another drawback.

The final drawback which is actually very important is that the internal behavior of a system is not known remember that we just said that a transfer function is typically an input output view of a system, right. So, you give a signal over here we you measure the output of the signal and that is all you are really concerned about. So, the transfer function then would be; just give me a minute. So, the transfer function then would be something like you know the ratio of the output to whatever is the input which is applied, right, what is missing is the internal dynamics of the system and it can be a very complex system, right, but we have absolutely no clue what is happening within the system, all we are concerned about in the transfer function approach is simply the response to a given input that is about it, alright.

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So, these are the drawbacks and let us see how the introduction of this new concept of a state space system solves many of these issues, alright, but first of all we need to see; what is the state space system. So, a state space system or a state space representation of a system is basically a mathematical model that connects the inputs the outputs and the internal variables of the system called the states, right, it connects all of these 3 different parameters by first order differential equations, alright. So, as we drew in the previous slide.

So, as we drew in the in the previous slide safe example I have got the system I have got multiple inputs I have got multiple outputs right just write it as r 1, r 2, r 3, I have got 2 inputs, but it does not matter, I can have 10 inputs hundred outputs, yeah so on and so forth and then, there are multiple subsystems within this, right each of these subsystems is important because it shows how the input is mapped onto the output and we will see a couple of examples of this later. So, state space representation of this system what it does is to basically be able to take care of the inputs which I have all the inputs take care of all the outputs which we have and all the internal variables which are actually hidden if you look at it purely from an input output point of view, alright.

So, and of course, the relationship between these 3 guys are essentially just first order differential equations and we will see why it is first order differential equations in a couple of slides. So, a classical example which I gave before which I briefly mentioned before was the chemical processes. So, I can give a very short example of that one over here and there the idea was that let me yeah. So, there the idea is that let us say you have this chemical process plant there is a chemical process plant and you have multiple inputs, I have just drawn 3 inputs over here, but you can have more and you have a couple of outputs right although in a real chemical process plant you would have at least 22 different or 20 to 30 different inputs and 100s of measurements which are the outputs basically.

So, now what could be the input? So, one input could be the steam the volume of the steam which is coming in from the boilers right the other input could be the pressure which the compressors in the chemical process plant are applying to the particular liquid which is inside this inside the chemical process. So, this is because of the compressors right and then maybe we are having additional inputs like maintaining the temperature and so on, right. So, these are 3 different inputs which are being given to your plant and maybe we want to maintain the purity of a particular chemical that we are trying to manufacture safe example we want ninety nine point nine percent pure ethylene right for example, ethylene is something that is used to make plastics the plastics that you see over here and maybe you also want to measure the pressure and the temperature and the volume because we do not want these plants to go into a catastrophic failure and blow up right.

So, these are the measurements over here all these are the measurements and these are the inputs to the system and internally you have a lot of very complex dynamics going on right and the state space essentially manages to capture all these dynamics in one single framework and we will see a couple of examples of this later. So, that is one one very good thing if you have to do this with the transfer function approach as I mentioned before you need to model each path separately right then you to worry about how these things are coupled with each other and this becomes very complicated we will see in the state space approach why this is really very simple.

The second nice aspect of state space systems is that we can actually isolate individual subsystems. So, the key word is this one we can actually isolate individual subsystems and or say the components and we can actually say that these specific subsystems. So, to speak or either controllable or uncontrollable we can say that observable or unobservable of course, when we say components we actually mean states we do not mean literally the component itself, but the representation of the component, right. So, we are looking at a system where I can model inputs internal variables outputs I can even tell which specific internal variables the states are controllable uncontrollable observable unobservable. So on and so forth, alright.

So, we will see all this a little bit later and of course, the other important point is that we will see that state space is the linear version of state space systems is entirely dominated by linear algebra and there are excellent techniques and really good software tools for us to do analysis and design, alright. So, to recall to summarize this slide, the basic idea is that a state space representation of a system is not is a mathematical model which captures your inputs outputs through these internal variables called the states and all of this relationship are basically first order differential equations.

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So, let us look at the general procedure to obtain a state space system I will derive one or 2 examples a little bit later the basic idea is that we have seen in the transfer function approach whenever you give in a physical system we write down the differential equations which govern the dynamics right. So, for a simple mass spring damper, we have the second order dynamics for the RLC circuit, we have a second order dynamics so on and so forth, right and these dynamics are basically differential equations, right.

So, what we are going to actually do in state space modeling is to basically take the differential equation model of your physical system and then convert this nth order differential equation N could be 2 N could be 10 so on and so forth and we convert this Nth order differential equation which essentially determines the dynamics of your system into N first order differential equations, alright that is the basic approach to state space and so, how any states; we choose is basically one of the easiest ways to determine that is its basically the order of the differential equation which used to model the system with. So, if you have for example, an RLC circuit, it turns out the number of states we need are only 2 because the differential equation which governs the dynamics is a second order differential equation. There is of course, a different way to also decide how many states we need and the idea is essentially this for an electrical system.

We essentially say the numbers of states are basically equal to the number of energy storing elements. So, take a RLC circuit you have the inductor and the capacitor which

are the energy storing elements and so, you actually have 2 over here, right if you have 10 inductors and 10 capacitors you have a 20th order differential equation that is basically the story and then you have 20 states, alright.

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So, let us actually go to a simple example and we will start off with the ubiquitous mass spring damper system. So, we have as you can see here a mass m with the spring constant attached to this wall on the left. So, you have a mass m, it is connected to this wall over here by the spring constant k and then I have got a damper over here with the damping ratio of b and I have applied an input force towards the mass in this direction and so we are going to see what the dynamics are. Now we know by the basic force balance equation. So, the applied force the total amount of applied force is distributed among each of these 3 components.

So, by Newton's laws, we know the component of force along the mass is basically mass times acceleration where x is the displacement or the position of the mass. So, we have the component of force acting on the mass then we have a component of the force which is acting on the damping or rather the damping resists this component of the force and it is basically a function of the velocity that is d x by d t and of course, the damping ratio b and then the final component is this is a spring constant over here.

So, the component of force acting on the spring and the elastic force which it produces. So, these are 3 components now we know from standard the transfer function approaches which Ramkrishna has already taught the basic way to decide or to write down the transfer function of this one is to is to convert this into the Laplace domain, right. So, we apply the Laplace transform on either side. So, F goes to F of S, then you have m s square s square comes because of the d square by d t square, then you get the x office the displacement in the Laplace domain the damping ratio is a constant. So, it stays as it is and the velocity gets a s term again x of s finally, plus you get a k x of s, alright.

Now, if you remember the basic definition of a Laplace of a transfer function, it was nothing, but the ratio of the measured output to the applied input in this case the applied force, alright and there is nothing, but one over m s square plus b s plus k and then we know that when we correlate this with the standard expression for the characteristic equation, we know that we can derive properties of damping the natural frequency. We can talk about stability, we can talk about perform and so on and so forth; all the standard things, alright.

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Now, let us see how we do this in the state space approach. So, I will actually be doing this derivation. So, the final state space model is actually going to look like this and you can see that we actually have and you can see that we actually have these 2 variables over here x 1 dot x 2 dot these are actually the states of the system and we will see what

these other terms actually mean. So, let us see how we can actually say derive this. So, if we recall the basic derive the basic second order differential equation. So, we had m d square x by d t square plus b d x by d t plus k x, right.

Now, what I am going to do is to introduce a new set of variables. So, let us say a new choice of variables and remember what I said that because we have a second order differential equation, the number of states will actually be equal to 2. So, let us introduce 2 new variables, we just call them x 1 you can call them whichever you like, but x 1 is a good choice and x 2, alright and specifically I am going to assign x 1 is equal to x which is the basic displacement and x 2 to be the derivative of the displacement there is nothing, but the velocity.

Now, with these new choice of variables let us see how I can actually write down the dynamics and I can derive the; what I am still calling is a state space model which have not formally defined, but we will come to that later. So, let us see the relationship between x 1 and x 2 and we will see from there. So, first we see that d x 1 by d t is nothing, but x 2, right because if I take the derivative of x 1 I get d x by d t which is nothing, but x 2 over here, right and d x 2 by d t is nothing, but d square x by d t square, right.

Now, this d square x by d t square, we can rewrite it by using our second order differential equation and by dividing with this factor m everywhere. So, this will simply become F divided m because I am taking m onto the left hand side minus b by m d x by d t minus k by m x . So, I have just rewritten my d square x by d t square in this particular form, alright. Now what I am going to do if you look at the first equation over here you see that I have expressed x x 1 in terms of x 2 what I would like to do now is to express x 2 also in terms of x 1 and x 2 if possible.

So, let us see how to do that. So, I will actually do this here. So, we have this d square x by d t square and you can see that I have got this term F by m that is; obviously, constants or the independent input, they do not depend on the states, alright. So, that stays as it is and then I have got this b by m times d x by d t. Now if you look at what is this d x by d t its nothing, but x 2, right? So, I will just write this as b by m times x 2 x 2 is nothing, but d x by d t.

Similarly, the last term I have over here is k divided by m which is a constant it does not depend on the state. So, that will stay as it is and then I have the term x of t x is nothing, but x 1. So, this simply becomes x 1. So, this is my equation 2, this is my equation 1, if I combine equation 1 and 2; I get exactly the state space system and what you see is a d x by d t is related to the state x 2 whereas, d x 2 by d t is related to both x 1 and x 2, but of course, this is specific to this specific example; we have considered this need not be true in general; however, the first row in the 2 cross 2 states matrix will always be 0 and one and we will see the structure repeating even for a nth order state space system and the inputs are always treated separately.

So, you can think these are the input this is if you look carefully these are the actual dynamics of the system. So, here is where you have your accelerations velocities and so on and these are the states. Now let us say for this particular system, I also want to measure the output because without measurement you actually cannot do control analysis any such thing. So, let us say, I want to measure this as the output, now what are the 2 states which I have as we have seen the 2 states are x 1 and x 2 x 1 is nothing, but the position x 2 is the velocity now which you want to measure is entirely up to you depending on what problem you are you are looking at let us say I want to measure the position maybe I have got a say position sensor in one case in other case if you want to measure the velocity I may have an odometer or any or one of these things.

So, if you want to measure the position I call my measurement as y and remember what is accessible to me is x 1 and x 2, assume I can measure only the position or I want to measure only the position. So, I write this in the form of this, alright. So, this is my measurement equation, this is my measurement equation and this is my actual state space matrix as well as the inputs, alright. Now let us actually go ahead, we will come back to this in a slightly more complex example, we will also look at the basic theory of how to write a Nth order differential equation in form of in the form of state space systems, but the basic idea is always this you start off with you start off with the basic differential equation you make a new choice of variables and the number of such variables r is basically equal to the number of is equal to the order of a differential equation, alright. So, in this case I had second order differential equation.

So, I am taking 2 variables; these variable I have call the states, once you make a choice of these 2 states the idea is to relate the to the states with each other, alright and then you

essentially start to get these dynamics, alright and finally, of course, for as I said for analysis for control all these things we need to make specific measurements. So, then you introduce a measurement matrix as well and you get this structure.

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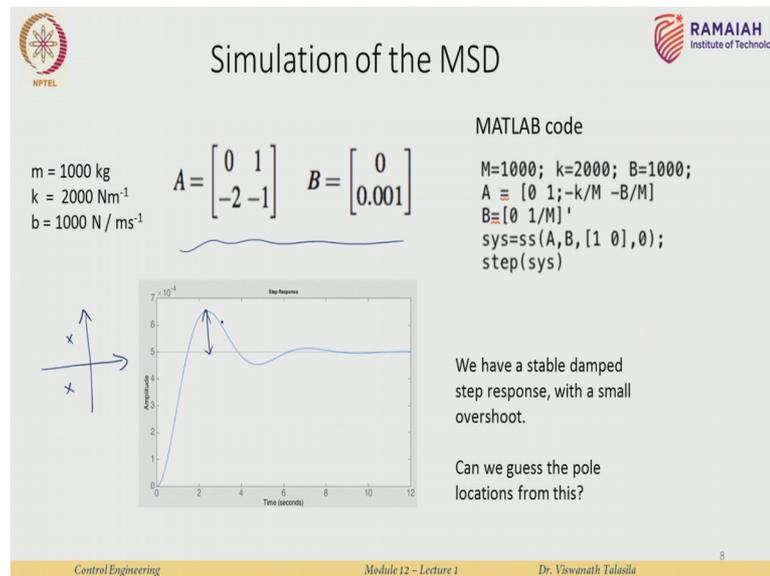
Now we will actually see the more formal version of this. So, so this was a mass spring damper; let us actually see a very simple simulation of this one. So, let us say, I take mass is equal to thousand kilos the spring constant equal to say 2000 Newton per meter and a damping ratio to be 1000.

Now, remember my A matrix remember this particular matrix over here 0 1 minus k by M minus B by M. So, it is actually called the A matrix and we will see why it is called the A matrix a little bit later. So, when I substitute the parameters k M and B in this matrix over here, essentially what you are going to get is this one; 0 1 minus 2 and minus 1 and then if you look at these inputs over here, I have A 1 divided by M for the moment let us assume F is equal to a unit step force.

So, F is equal to 1 and 1 over M which just becomes z 0 and 0.001. So, this is the A, this is the B and we will see what these things are later and if I do a simulation of this, I will run through the code a little bit later, we do a simulation of this, we see a step response of the following type and you essentially see that this is a damped stable response and with a very small overshoot over here and we have seen from the transfer function

modeling of a mass spring damper that when you have this kind of a response, we can sort of guess; what kind of poles that the corresponding transfer function would have. So, for this kind of response, it is very easy to see that the poles would be like this.

So, you have a complex pair of poles and it is in the left half plane, right which is why you have the stable and the damped response. Now of course, we are in this so called state space version where the notions of poles do not really hold, but we of course, want to relate how the system response comes with the notion of poles as well in the context of state space systems. So, one of the ways to do that is you have this matrix A over here. So, we see this is a square matrix and one of the natural questions that come in we see a square matrix is what are the Eigen values of A.

So, let us compute the Eigen values of A and it just follows the standard procedure you compute the determinant of s times i minus A and when you do this you see that the Eigen values are essentially this and for the same system if you look at the transfer function which is s square ma square plus b s plus k and you compute the poles of this transfer function, you will get exactly the same values as the Eigen value. So, you get minus 0.5 plus or minus j 1.3229, alright.

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So, in this particular case we are seeing that the poles of the system are exactly equal to the Eigen values of A. We will see this a little bit more formally why this is true, but this

is very interesting because it shows that this new modeling approach in the state space system at least with respect to said the dynamics of this kind exactly corresponds to what we see from the transfer function modeling approach. So, once we are done with this let us we have talked about states of a system, right. So, for the mass spring damper, we have said that the state one of the states is a position, other state is a velocity and we have done all these derivations over here, we have this model, we have a simulation as well so on and so forth, but we formally not actually defined what the state of a system is.

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So, the definition here is it is not a very polished definition, but this actually conveys a correct picture, a variable is called the state of a system because the future output of the system depends entirely on the on the current state value and whatever input we are going to give, alright. So, you have a system I modeled it by the so called states of the system and the output of the system the future output the system depends entirely on the current state and whatever input I give in the future.

So, I do not need to worry about what happened in the past, I will need to worry about the current state and in other words, it also means the future output in other words, it also means the future output depends on the past input only through the state and not through any other any other parameter of course, assuming you have done a good job at the modeling and you have not left out any variables or you do not have un-modeled

dynamics. So, as long as you take care of that the future output depends only on the past input through the state, alright.

So, if I put these 2 things together in effect this means that the state summarizes the effects of all the past inputs on your system on the future outputs. So, the state actually acts like a memory of the system. So, take for example, simple, so, cute where I have a capacitor and I am applying a voltage across this capacitor, right. So, I look at the voltage drop across the capacitor and I am interested in the dynamics. So, we know that the current through the capacitor is basically modeled like this right.

Now, what I am saying is if this VC which is a voltage drop across the capacitor if that is the so called state of the system, this particular variable I am saying that at time t equal to 0 or any specific you fix which can just say it to be equal to 0 at a time t equal to 0 if you have a certain value of the state at any time in the future VC of t will depend on whatever information you have in the past. So, the current value of the state and whatever input I am going to give afterwards. So, for example, at t equal to 1 as long as I know; what is the value of the state at time t equal to 0 and I know the kind of input, I am going to give at time t equal to 1, I can always calculate VC of T, I do not need to know what happened one over before I do not need I do not need to know what happened one day before as long as I have the total information of the state as of now, alright.

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So, that is essentially its loosely called as a memory of the system, alright now we have talked about what is a state we have done a little bit example with the mass spring damper we have seen that we have the so called input structure over here inputs over here, we have the measurements over here and we have the so, called state dynamics as I call them let us see this as a formal definition. So, we have seen the an example with the mass spring damper where we have looked at the all the inputs which are given to the system, we have looked at the measured outputs we have seen how to write down the basic equation for that one, we have looked at what I have called as a dynamics of the system, they have not really explained that. We will come to that and of course, I have talked about the states, now what is the general form of a state space model and specifically a linear state space model.

So, specifically a linear time invariant state space model and it is written in the following way. So, x dot equal to A x plus B u y is equal to c x plus d you remember all the variables are functions of time x and u and we are assuming that this is a time invariant system. So, the matrices a, b, c and d are actually constant for now in general, this need not be true. So, let us look at this particular equation the variable x is actually called as the state of the system and we had a definition of that is basically the memory, it captures what all has happened before the variable y is the measured output and the measure output is really your decision as well as it depends on the constraints of the system, there are some states which we can measure some states which we cannot measure and in general you may not even be able to measure states, alright.

But for this particular lecture, we will assume that y a are the measured states, alright and you over here is the input which we give to the system and you need not be a single input it can be multiple inputs we are looking at state space. So, multiple inputs are permitted now we have these parameters a, b, c and t a is basically the there are different words for this I can call it as a dynamics matrix or the state matrix or a much better word would be the interconnection matrix I will explain what this is. So, the interconnection matrix basically tells how different states of my system are actually connected with each other or related with each other or I can say coupled with each other. So, if you looked at the previous example the state x 1 which was a position is connected to the velocity through this parameter over here right .

So, x 1 is connected to x 2 through the parameter 1 and x 2 dot is connected to the states of the system through these 2 parameters, these are the interconnection parameters as we call them and they define the dynamics; they define how different states are connected to each other and so and interconnection structures are very important although we will not be dealing with that over here, but for us, integration structures here are basically the elements or the values which come up when we connect states, alright then you have the matrix B. Now before I go ahead, please note the dimensions of the matrices are very important.

So, let us say, we are looking at the Nth order state space system; Nth order state space system and remember that you get an nth order state space system typically from a nth order differential equation, alright. So, if you have a nth order state space system the matrix a will be of dimension n cross n, it cannot be more it is a square matrix of dimension n cross n now let and now let us look at the matrix B. So, B is a matrix as you can see from here B is a matrix which takes the inputs which are applied to the system and relates them to the state. So, it basically tells how my inputs are actually connected to my states in the case of the mass spring damper example which we took before the input the input matrix over here essentially says that whatever force have applied does not directly go to state number one because I have got A 0 in that entry over here, but this force directly goes to state number 2 which is the velocity by the way through the factor 1 over m. So, what b captures basically is how the inputs actually affect the states.

So, we actually call B as the input matrix, next we have this matrix C and C can be called as the output matrix, we have already seen in the case of the mass spring damper, we had an example of when we wanted to measure only the position. So, we had an output matrix like this where x 1 and x 2 is the state vector and this particular row over here was actually the C. So, so C is the output matrix. So, we are not going to be say dealing with the matrix D over here d is actually called a feed through matrix the feed forward matrix and we will not consider that over here.

So, let me just recall this slide. So, we have this the general form of the linear time invariant state space model A, B, C, D are constant matrices all the others are functions of time the a is the interconnection matrix, it tells how different states are coupled to each other b is the input matrix it tells how my inputs affect states which input affects which state and with what magnitude C is the output matrix it depends what I want to

measure and D is a feed forward matrix. So, is n cross n matrix the interconnection structure it is always equal to the number of states which we have b is the input matrix.

Let us assume we have m inputs m could be less than n or even equal to n. So, B will be your m cross B will actually be n cross m matrix and let us look at the output matrix C, let us say we are measuring p states in which case C will be a p cross n matrix, now in general; we could actually measure all the states if you have access to all the states in that case p will be equal to n. So, we have an n cross n measurement output matrix finally, we have the matrix read the feed forward you know matrix which is basically p cross m. So, these are the 4 matrices which we are going to consider we will be ignoring D for the rest of this lecture we are going to look at basically A, B and C, alright.

So, this is a general form of the linear time invariant sister. Now we have always said that we assuming we have an nth order say differential equation how do we go from that to the nth order state space system.

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And let us look at the general derivation over here. So, consider an nth order differential equation which actually models a linear time invariant system. So, we have the differential equation over here. So, u is the input y is the output in this particular case. So, we have these coefficients here which come about when you are modeling the system.

Now, what we are going to do here is as we have done in the mass spring damper case where we had the second order differential equation and we introduced 2 new variables x 1 and x 2 what we are going to do here is to introduce n new variables because we have an nth order differential equation we would expect to have n states. So, the way you would do that is fairly straightforward. So, let us say I have got a I introduced a new variable called x 1 x 2 all the way to x n, alright and again we have n because the order the differential equation is n, alright.

So, x 1 I will relate it to the first variable over here y, alright x 2 I am actually going to call it as d y by d t and so on; now note that yeah. So, so I have x 1 is equal to y x 2 is equal to d y by d t and x 3 in a similar way x 3 is basically d square y by d t squared I can go on all the way to n and basically what I would get is d n minus 1 y by d t n minus 1. So, there is exactly the same as a mass spring damper example where we had written x 1 was equal to x and x 2 was equal to d x by d t exactly the same as that except I have got n states over here.

So, I have got the slightly longer looking expression, but is exactly the same, alright. Now as we did in the case of the mass spring damper, we will now look at the derivatives of these states and we will see how these states are actually coupled with each other. So, if I take d x one by d t we see that its nothing, but d y by d t and d y by d t as you can see is nothing, but x 2. So, I get x 2 over here, similarly, d x 2 by d t is nothing, but d square y by d t square and d square y by d t square is nothing, but x 3 x 3 I can go all the way here.

So, I can go d x n by d t will be equal to well this would depend on all the states which we have over here, alright. So, let us see how to do that. So, d x n by d t is nothing, but if you look at this particular expression over here is it is nothing, but d and y by d t n, alright and how do we derive this particular thing we take a look at our basic differential equation over here where I have d n y by d t n the nth order derivative of y and we rewrite this expression and the way you rewrite that as you have done for the mastering example before would simply be u of t and take all these terms over here on to the right hand side.

So, that becomes u of t minus a n minus 1 then what do I do with this expression over here I replace it with x n, alright this particular term is nothing, but x n. So, it becomes

minus a n minus 1 x n minus in a similar way minus a n minus 2 x n minus 1 and you can go on like this and then you have this expression over here a one d y by d t on I take it on the right hand side. So, I get minus a 1 and d y by d t is nothing, but x 2 minus a one x 2 and then finally, this expression a naught y. So, take it on the right hand side, I get a naught x 1, alright.

So, basically what I have done is exactly the same as a mass spring damper where I have written all the states in terms of the other states. So, we essentially have now the coupling between the states the interconnection matrix and when I put all of this in a matrix form we want to know how that looks like in the case of the mass spring damper if you remember we had x 1 dot x 2 dot it was nothing, but 0 1 what does that k by m I think yeah minus k by m and minus b by m, alright times x 1 x 2.

Now, we are doing exactly the same thing except that we want to see how this looks on the n order nth order system and it looks exactly like this.

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So, you see that these first set of rows the n minus 1 rows which I am I am actually showing you over here the n minus 1 rows are basically all the velocities which are related to the previous one. So, x 1 dot would nothing would be nothing, but x 2 x 2 dot would be nothing, but x 3 so on and so forth same as the mass spring damper case, we

have done the final expression the d x n by d t it basically comes from this from this guy over here.

So, this expression I have rewritten as a last row of this matrix and of course, we have an independent variable here the u of t it is not a state variable it is an independent one. So, we take that outside. So, this is basically my b u of t this is my A x of t and of course, this is x dot of t. So, x dot is equal to A x plus B u there is a standard expression for a nth order differential equation. So, this is for a nth order system and of course, as before we want to measure the states now I can again decide which particular state I want. So, I am just calling x bar as a state vector which is nothing, but x 1 x 2 all the way to x n and in this particular case, again I am assuring the first state I could also measure the third state for example, in which case I would have a matrix like this.

So, if I measure the third state, I will just have a matrix like this x bar or I can even have a combination of measurements. So, I can measure state 2 and maybe state 3 and 4. So, I would have something like this that is really up to you how you can measure depends on the sensors you have the access to these states so on and so forth. So, this is the basic state space model of an nth order differential equation and I have just shown you how we can derive this one and the logic for this basically follows the same process which we did for the simple to cross to mass spring damper nothing more and nothing less, alright.

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Now, let us work out a simple example. So, we have this. So, we have already done the second order state space model for a spring damper we have seen the nth-order derivation for to deliver entered a state space system let us look at this simple third order differential equation and see how to did how to derive the state space model, alright. So, what we are going to do is to do what we have just done for the previous case, since it is a third order differential equation, we are going to define 3 new variables x 1 x 2 and x 3, alright and as we have done before we relate the first variable to y the second variable to d y by d t and the third one to d square y by d t square and a third one to d square y by d t square, alright.

Let me write this here and the first thing which we always try to do is to find out the relationship between these states essentially to compute the interconnection matrix. So, for that we take the derivative of the first state and we find that it is is nothing, but d y by d t and d y by d t is basically x 2. So, the derivative of the first state is x 2 the derivative of the second state. So, x 2 dot is nothing, but d square y by d t square and d square y by d t square is x 3.

Finally we have x 3 dot which is d cube y by d t cube and d t by d cube y by d t cube we can actually derive from this expression, alright. So, what we do. So, take all these terms onto the right hand side and then divide with the factor 2 that is going to give d cube y by d t cube and so, let us actually do that. So, we get d cube y by d t cube is remember I am dividing all the terms with the factor of 2.

So, it is 10 divided by 2 that is 5 u of t this term is minus 4 divided by 2 d square y by d t square. So, minus 2 d square y by d t square and then a 6 d y by d t becomes minus 6 d y by d t and finally, eight y becomes 4 y and what we do now is to replace each of these ys or the d y by d t with appropriate state. So, we know that y as we have defined here is x 1 d y by d t is nothing, but x 2 and d square y by d t square is x 3.

So, this equation is nothing, but 5 u of t minus 2 x 3 minus 3 x 2 minus 4 that is a final the last row of the state space model and that is exactly what we get over here and the first 2 rows in this case as we have seen will always be 0 1 0 0 0 1 and now we have the inputs in the final matrix over here in this form and of course, again for no particular reason I am considering that I measuring the first state that need not be true I can

measure the second state or I can measure a combination of states any such thing that is really up to you.

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So, let us take another example we will use a parallel RLC circuit and we will try to derive the state space model. So, actually I have shown the final derivation over here. So, let us see what the actual circuit looks like. So, let us suppose we have a current source I s and we have connected this parallel with the resistor branch and inductor branch and a capacitor branch and let us say we measure the voltage across the capacitor and call it VC or V out as you like. So, this is C, this is L and its off and I will further denote the current through the resistor being as I R through the inductor being I L and through the capacitor being I C and now what we want to do is to write down the state space model for this one and you can see that I have already considered the 2 states as being the current through the inductor and the voltage across the capacitor and again the reason for that is because this particular RLC circuit has 2 energy storing elements.

One is the inductor one is a capacitor hence you will have 2 states and not more than that. So, how are we going to write the equations for this one? So, first what we are going to do is to focus on VC and then on I L. So, the way that we would do it is if you look at the current through the inductor we know that the voltage drop across the inductor is given by the constitutive relationship which is basically L d I L by d t is nothing, but V the voltage drop across that one and specifically which we I can write it as

VC itself because the voltage drop across the capacitor over here is the same as the voltage drop across the inductor which is in parallel with the capacitor. So, this is your equation number one when I take the term L on to the right hand side.

So, that is one over L VC. So, this is actually the first state equation to write the second state equation we will employ Kirchhoff's current law at this particular node by observing that the expression for the current through the capacitor is nothing, but the total current which is coming in minus the current which is going through the through the resistance branch minus the current which is going through the inductance patch. Now we see that this we have I L in this expression.

So, I do not want to touch I L because we want the interconnection matrix between the voltage and across the capacitor and the current through the inductor to contain both terms what I will do is to replace the other terms if possible yes of course, we will stay as it is because it is an independent source for I R that is the current flowing through this resistance branch we know that it is nothing, but if you consider the voltage drop across this to again be VC because all these branches are in parallel.

So, I R is nothing, but VC divided by r I L of course, stays as it is what about I c the current through the capacitor as you know is given by c d V c by d t. So, this is equation number 2 and if you plug these 2 equations into a matrix form let me erase this one for one minute. So, if you plug these 2 equations in a matrix form you are actually going to get this expression over here. So, let me show how we can do that. So, d I L by d t that is the derivative of the first state d V C by d t derivative of the second state and remand and please see here this term C the capacitance I will remove it from here and divide it on the right hand side everywhere.

So, that is one over C I S 1 over C times R and 1 over C times I L and now we write down the a matrix and then the state vector the state vector here is nothing, but just I L and VC and I just have to fill in the elements of the matrix. So, d I L by d t is only a function of VC with the 1ne over L and the other 2 and for d V C by d t, you simply just fill these 2 terms over here. So, this is your a matrix. It is the same as this expression over here, this is A, this is your B matrix. This is the input, alright.

So, this is how the final state space model will look like let us say that you want to make a measurement of both the current flowing through the inductor as well as the voltage drop across a capacitor in that case your y equal to C x will have the following form will be 1 and you will have I L and VC assuming you want to make both measurements, alright.

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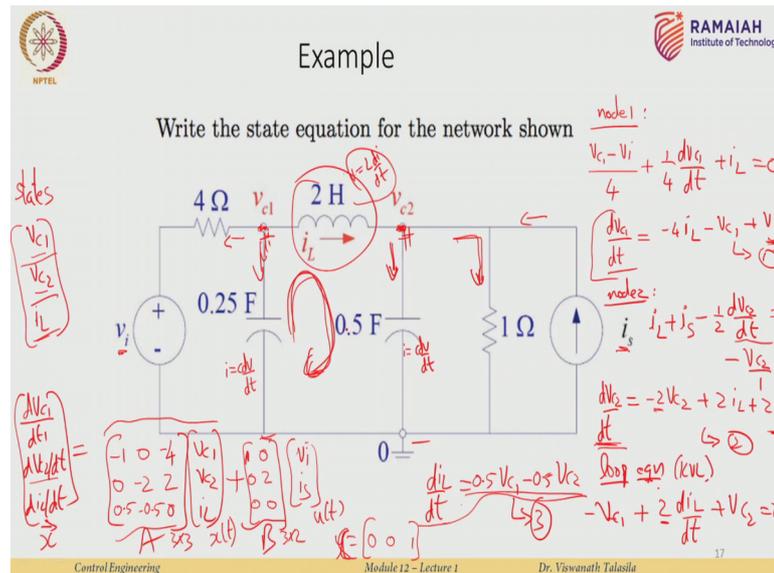
So, let us look at the next example which is again electric circuit. So, we have 2 sources V input and I s and we have got basically 2 resistors 2 one inductor and 2 capacitors right. So, let us see what is a state equation of this one and we are going to solve this again in the same way as a previous example we see that we have 3 energy storing elements there are 2 capacitors and one inductor, alright and the voltage through capacitor one is written as V C 1 the voltage to the second capacitor is VC 2 those are the 2 states for the capacitors and the current through the inductor is written as I L.

So, the states for this states for this particular system will be VC 1 VC 2 and I L and so let us actually try to write down the dynamics of each of these variables the d time derivative of this one. So, let us start off and we can analyze this circuit in 2 ways in 2 steps actually. So, the first step is to basically write down the nodal analysis at these 2 particular nodes. So, basically we are going to write the k C L at these 2 and in the second step we are going to apply a k V L in this particular loop, alright.

So, first let us look at node one and we will write the kickoff current law at node one and the way, we can do this is we are going to assume all the currents coming in are positive all the currents leaving that particular node are negative. So, they sum up to 0 that is a basic idea. So, at node one, we have by Kirchhoff's current law. So, VC 1 minus V I divided by 4 that is the current which is over here and then we have the current which is leaving this particular branch here and that current is going to be given as V C 1.

So, this current through this I am going to write as by using this constitutive relationship I equal to C d V by d t. So, it is basically going to be C is 0.25 that is 1 over 4 d V C 1 by d t that is a current in this particular branch, right and a current in this particular branch which is going out is this I L this is equal to 0. So, if I rewrite this expression, I will actually get d V C V N by d t is nothing, but minus 4 I L that is a term on the right hand side minus VC 1 plus V input, alright. So, this is equation number 1. Now this is the k C L applied at node one if I apply k C L at node 2.

I am actually good at this particular node I have the current I L which is coming in and I have this current which is going out there is a current in this particular branch going out and this I s also coming in. So, I have I L plus I S the 2 incoming currents I need to know your minus the 2 outgoing currents. So, the first one will be again C d V C by d t, again I apply this constitutive relationship over here. So, I will basically get 0.5 which is one over 2 that is my C d V C 2 by d t that is a current in this branch over here and the current in this branch is nothing, but the current in this in this branch is simply [vocalized-noise] C 2 divided by 1, alright.

So, if I now rewrite this in terms of my d V C by d t. So, I get d V C 2 by d t is basically I multiply with 2 everywhere, if you take all the terms on the right hand side and then multiply with the negative sign. So, you would actually get minus 2 V C 2 plus 2 I L plus 2 is right. So, this would be my equation number 2. Now if I look at the loop equation that is this particular loop over here with the ground at the negative potential these 2 as being positive defined over here.

So, by applying k V L, I would actually get V C 1 it is minus V C 1 because of the polarity plus the voltage drop across the inductor which were the constitutive relation which I have written over here V equal to L d I by d t. So, it becomes minus V C 1 plus R 2 d I L by d t plus a voltage drop across this capacitor, right. So, plus V C 2 is equal to 0,

if we rewrite this in terms of the derivative expressions the states basically. So, I get d L by d t remember I need to divide with 2 I am going to take it on the other side.

So, it becomes 0.5 V C 1 minus 0.5 V C 2. So, I had 3 states which I have defined. I now have the 3 first order differential equations for each of these states. So, I simply put them together, I am not I talking about measurement, we will see the output, we will see that later. So, when I put all of this together we get d V C 1 by d t d V C 2 by d t and d I L by d t. So, this is the state vector x dot, this will be related by the A matrix times the states itself. So, we see 1, we see 2 and I L and now this relationship is basically the ones which we have derived over here.

So, let us look at equation number 1 on the right hand side and we just plug in the terms which you have computed. So, it is minus 4 I L. So, I get A minus 4 and A minus V C 1. So, minus 1 V C 2 is not there. So, at 0, we will see what to do about the V I a little bit later. Now for d V C 2 by d t, we see that we have a minus 2 V C 2 and we have a plus 2 I L and we see one does not come over there, it is A 0 finally, for d I L by d t, it depends entirely on V C 1 and V C 2 and its basically 0.5 and minus 0.5, this is 0.

Now, we still have to plug in the inputs because we have a V I and is over here. So, what we are going to do with that is we have 2 inputs V I and is and what we would need is a 3 cross 2 matrix over here because this is A 3 cross 3 this will be A 3 cross 2 matrix, alright and we see that d V C by d t from equation L, it depends on V I. So, I have A 1 over here, it does not depend on this. So, this is 0 d V. So, d V C 2 divided by d t that is equation number 2 that depends on is and you have the constant being as 2.

So, this is 2, this one 0 I L does not depend on d s I L does not depend on I s as you can see over here. So, this is all 0s. So, this is the basic state space model for this electrical network. This is the A matrix. This is the B matrix. This is u of t and this is x of t, right and the measurements again are up to you; you may want to measure the current flowing through this inductor in which case my measurement matrix will be 0 0 1, my C matrix will be 0 0 1 or you may want to measure the current across in other 2 capacitors. So, the C matrix will change appropriate, alright.

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So, the entire derivation; the final state space matrixes are actually given over here.

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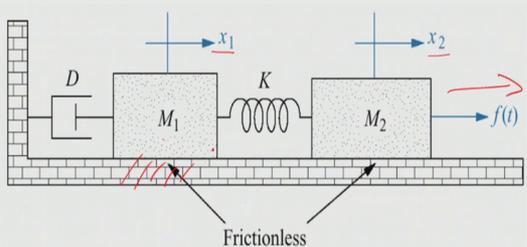
Let us look at one more problem which is on a couple mass spring damper. So, we have 2 masses and they are coupled by the spring, with the spring constant K mass 2 is pulled in this direction by a force f and mass 1 is connected to the wall with this damping with a damper D, alright and mass 1; it has a displacement x 1, mass 2 has a displacement x 2 and we are just going to assume that there is no friction between the mass and the ground over here, alright.

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So, there is no friction between these 2, but you can easily incorporate that that is not a problem. So, if you look at the; so, the basic way that we always write the differential equations or the dynamics for a mass spring damper is always look at the force balance equations, alright. So, the applied force will have to be distributed across the individual components which are there. So, if you look at mass 2 first. So, I am applying a force F, it resolves into 2 components one is an acceleration component on mass 1.

So, that is mass times acceleration over here and then the spring which is coupled between mass 2 and mass 1 and you have already seen in the previous lectures; how to write these dynamics. So, it is basically K of x 2 times x 1 that is the displacement between the 2 things between 2 masses. So, this is the second order differential equation for mass 2. In a similar way, we write the equation for mass 1, note that the force F; it does not directly act on mass 1, it acts actually through mass 2. So, term on the right hand side is 0 and because we have these internal forces for mass one experiences an acceleration which is d square x 1 by d t square and then you have the damping which acts in terms of its velocity d x 1 by d t and k is again couple between m 1 and m 2. So, we have K x 1 minus x 2, alright.

As we did before, we are going to define the basic we want to write down the state space model of the system. So, the first thing we need to do is to identify the states, right. So, the way that we are going to do as we did for the original mass spring damper system.

So, we identify x 1 is equal to basically the position x 1 itself. So, x 1 of t is equal to x 1 itself then we have a second state which I can call as x 2, but I can also call it as V 1, alright, it is the velocity. So, I call it as d x 1 by d t, these 2 states are for mass M 1.

Similarly, I have another state x 2 which is the displacement of mass 2. So, I am calling it x 2 itself and sorry this is x 3 x 2 and then I have a state x 4 which I am going to call as basically V 2 the velocity of mass 2. So, this is nothing, but d x 2 by d t.

Now, this notation is a little bit confusing. So, what we are going to do is to simplify this a little bit we will remove this x 2 over here. So, we have x 1 and V 1 as the states or the first mass then we will call this as x as x 2 and V 2, right otherwise everything else stays exactly the same. So, these are the states for mass M 2. Now all you need to do since we know these already by the relation d x 1 by d t is nothing, but V 1. So, one state is taken care of the second state d x 2 by d t is also taken care of by this expression and what we are left with is to actually to compute the time derivatives of these 2 states and then you will get the interconnection matrix.

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So, if you go through the steps, you will actually see that d V 1 by d t and d V 2 by d t are essentially derived by you using these expressions over here, right. So, we saw that V 1 is if you take V 1 equal to d x 1 by d t and now I am looking at d V 1 by d t. So, it is nothing, but d square over here and I can rewrite this expression as we have done before

and then derive this these terms over here or d V 1 by d t. In a similar way, we can do you can derive the expressions for d V 2, alright.

So, we get these terms over here, put everything together in exactly the same way as we have done before, remember that the states where x 1 V 1 x 2 V 2 and then you get this a matrix over here and the b matrix, alright exactly the same way as we have done before.

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So, this example for the coupled mass spring damper; so, we have seen you know in a previous example for the simple mass spring damper that the Eigen values of the A matrix are exactly the same as the poles of the corresponding transfer function, right. Now let us see if this property holds in general.

But before we do that; we need to actually establish the relationship between the state space and the transfer functions or how? So, given a transfer function model; how do you compute a state space model or vice versa given a state space model, how do you compute the transfer function model, right. So, this is a very simple derivation to show that. So, remember our basic equation was x dot equal to A x plus B u and Y is equal to C x plus D U, right, there is a general form of the basic state space system.

Now, when I convert this into the Laplace domain; so, d x by d t d by d t is basically the s term in the Laplace domain. So, you get s X of s and then you get A x of s, remember that A is a constant matrix. So, it just stays as it is as is for B, C and D. So, this is the first

term of the Laplace expression, right that is what I have written over here. Now you take this term on to the left hand side and you subtract and you are going to get these terms. So, s I minus A times x of s is equal to B times U of s, right, it is confusing, I can just write this down. So, s X s minus A of x of s is equal to B u of s take the x of s term common out and all you are going to get is s I minus 8 equals B times U of s.

Now, so, this is we can call this as equation 1. Now we have a second equation Y is equal to C X plus D U, right, this in the Laplace domain. So, this is the Laplace equivalent of Y equal to C X plus D U that is this over here. Now what we are going to do is to take this X of s which we have computed over here. X of s is equal to s R minus A inverse B times U, right. So, I can write this one implies x of s is nothing, but s I minus A inverse B U of s, right. So, I take this x of s and I substitute this, there is a small correction to be made in the slide, this is not U of x, they should be U of s, alright.

Let us continue. So, we have that x of s is equal to this expression, we substitute this back into this term over here and then we derive this one, alright. So, Y is equal to C x of s is nothing, but s I minus A inverse B times U that is these terms over here plus D times U that stays as it is. Now we know that the transfer function is defined as G of s is equal to the output by the input. So, take the U terms common out of these 2 terms, alright and then and then divide with U of s and you are going to get this. So, this is the basic relationship between the transfer function and the state space model in this particular case, we are assuming A, B, C and D matrices are given to you and the goal is to see how we can go and derive a transfer function and this is the basic expression for that.

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So, let us actually derive a; solve a very simple problem I will not go through the details the all the details are there in the slide. So, we have x dot equal to A X plus B U and then Y equal to C X. So, this is the A matrix, this is the B matrix and this is the C matrix and the objective here is to actually compute the transfer function given the state space model and we know that it is given by this expression which we have just derived before, right. So, all we need to do is to plug in all these terms and so, for s I minus.

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So, let me write down the expression again here equal to C s I minus A inverse B plus D, alright. So, we need to first compute s I minus A and compute the inverse of that. So, s I minus A is basically this over here. So, that is basically s I is basically this s times I minus the A matrix was just this one, alright. So, when you actually subtract this, we actually get this s I minus A, then you compute the inverse of that. So, inverse of s I minus A is basically the joint of s I minus A divided by the determinant of s I minus A, if you remember the basic expression of inverse.

Let us take our matrix, let me just give a different notation I will call it as matrix M. So, the matrix M is actually computed by taking the adjoint of the matrix divided by the determinant of the matrix, alright. So, that is exactly what we have done here I joined basically means these 2 terms are swapped and the sign of these 2 terms are reversed that is exactly what is happened and we compute the determinant here, alright. So, this is s I minus A inverse. Now to s I minus A inverse; we need to multiply with we need to pre multiply with C and post multiply with B, alright.

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So, when you do that. So, this is the expression over here. So, you pre multiply with C and post multiply with B and when you do that remember in our particular example, we had D over here is equal to 0. So, the D term does not exist. So, this does not matter the D term here really does not matter for us, we can ignore that and what we get is C s I minus A inverse times B, you do all the multiplication and you are going to get this

transfer function, right. So, this is a simple example to show given a state space model how to derive the transfer function.

Now, it is important to remember that you may have made a mistake in the derivation; the simple way to double check is to compute the poles of this transfer function right. So, there is a quadratic equation. So, you just use minus B plus or minus square root of B squared minus 4 A C by 2 A and you compute the poles in this particular case, I think it will be minus 2 and minus 3, these are the poles and to double check that these poles are actually correct and that we have obtained these poles from the correct state space expression, what you can actually do is to compute the Eigen values of the matrix A and I will explain why that is true we already saw a simple example at the mass spring damper that this was exactly the same. So, if you compute the Eigen values of the matrix A, it turns out you will also get minus 2 and minus 3, alright.

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Now, let us see why this is actually true. So, the Eigen values of A R and the relationship to the poles of the transfer function and it turns out as you can conclude in this last line the roots of the characteristic equation of a transfer function which are the poles are exactly the same as the Eigen values of the a matrix of the equivalent state space model. So, let us the derivation is fairly simple. So, we have our expression which is over here. So, y equal to see s I minus A inverse B U plus D U of s, alright. Now let me just rewrite this remembering that the inverse of a matrix is the adjoint by the determinant. So, that is

what I have written over here. So, this becomes C adjoint of s I minus A divided by the determinant s I minus A, alright, you multiply with B U and add the D you over here.

So, this entire thing is equal to Y of s. Now just do a simplification over here. So, this becomes C adjoint s I minus A B U plus this term now goes on to the right hand side. S I minus A determinant D U whole divided by s I minus A, alright. Now you see in the numerator you again have this U term as constant in terms of course A function of s. So, take U constant or take U as a common factor. So, and then if you divide by U on the left hand side you would actually get y of s by u of s on the left hand side and that is equal to this expression over here, alright.

So, this was basically Y of s by U of s when I take U as a common factor in the numerator and then and I bring it onto the left hand side. Now this transfer function you see is equal to this one with the denominator being s I minus A the determinant of s I minus A; how do you compute the poles of a transfer function? We basically take a look at the denominator and we actually look at the roots of the denominator right we see the roots or a demon of the denominator are basically the determinant of s I minus A. It is exactly the same, alright.

So, computing the Eigen values of s I minus A is exactly the same as computing the poles of your transfer function. So, that is why in the previous example, we were actually able to prove; I mean show numerically that the Eigen the poles of this transfer function which are minus 2 and minus 3 exactly equal to the Eigen values which is also A minus 2 and minus 3 and this is true in general.

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So, what we have learnt in this lecture was that we introduced the concept of a state and a state space system and we saw why the state space systems are very important. They can actually model non-linear systems. They can take care of multi input, multi output system, so on and so forth which it is a transfer function approach could not easily do or could not do in some cases.

We essentially introduced the notion of the state we saw the basic form of a linear time invariant state space system which was the x dot equal to a x plus B U and y equal to C X plus D U right and then we did a few examples for the state space modeling and then we derive the relationship between the state space and a transfer function crucially, we proved that the poles of a transfer function are exactly the same as the Eigen values of the A matrix of the state space system.

So, this establishes that the dynamics we are looking at in the state space and the dynamics we looking at to the transfer function only for linear systems are exactly the same. So, what I am going to do in the next lecture is to look at since we talking since we are saying that state space can handle linear or even non-linear systems, we look at what are the basic differences between linear and non-linear systems. We will look at why it is important to linearize or non-linear system and there is really good reasons for doing. So, because handling non-linear systems is extremely difficult and we will actually look at the technique of the Taylor series to linearize a non-linear system and finally, we will

come conclude with 2 really nice examples one is a predator prey I will write the non-linear dynamics and show how we can linearize them and finally, we look at the Van Der Vaal oscillator.

Thank you.