

Control Engineering
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Module – 11
Lecture – 03
Complementary Filter

Hello, I am going to teach you module 11, lecture 3 which is the complementary filter. The complementary filter is coming from an aspect where you use your high pass filter and the low pass filter as told by Doctor Vishwanath. So, I will be continuing on that on the implementation of the complementary filter. We have already seen the aspects of jitter of drift jitter in the accelerometer drift in the gyroscope. So, we see how we can implement the complementary filter to reduce the drift on the jitter.

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The slide is titled "Sensor Fusion" and features the NPTEL logo on the left and the RAMAIAH Institute of Technology logo on the right. The main content includes a list of bullet points and a block diagram. The bullet points are: "While measuring a particular variable, a single type of sensor may not be sufficient to meet all required performance specifications", "We combine several sensors into a measurement system that utilizes the best qualities of individual devices", "Inertial Measurement units (Accelerometer , gyroscope, magnetometer)", "GPS (Global positioning systems)", and "Camera". The block diagram shows three input boxes labeled "IMU", "GPS", and "Camera" on the left, each with an arrow pointing to a central box labeled "Sensor Fusion". An arrow then points from the "Sensor Fusion" box to an output box labeled "Angle". A red hand-drawn box highlights the "Sensor Fusion" box and its inputs. At the bottom of the slide, there is a footer with the text "Control Engineering", "Module 11 – Lecture 3", and "Dr. Viswanath Talasila".

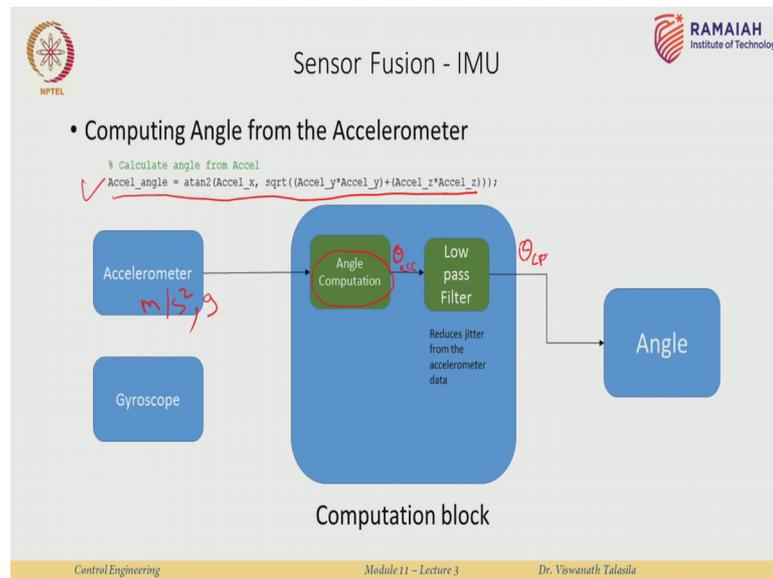
- While measuring a particular variable, a single type of sensor may not be sufficient to meet all required performance specifications
- We combine several sensors into a measurement system that utilizes the best qualities of individual devices
 - Inertial Measurement units (Accelerometer , gyroscope, magnetometer)
 - GPS (Global positioning systems)
 - Camera

Here in this presentation we are going to show the fusion of accelerometer and gyroscope

So, what is sensor fusion sensor fusion is a technique where we use multiple inputs refuse the data to get a particular variable out of it. So, for example, over here if you see that there is an IMU there is a GPS and there is a camera as Professor Vishwanath has already spoken to you about celestial navigation with dead reckoning that technique where you fuse the data from the dead reckoning sensors and the celestial navigation.

So, we use something some block call the sensor fusion block where we add these 2 sensors to get a particular output, here I am going to show you the fusion between the accelerometer and the gyroscope.

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Why do we need sense of fusion as we can see over here, the angle which is computed here can be used directly from the accelerometer by passing this angle information which uses the accelerometer values to calculate the angle. So, this is the formula where we calculate the angle from the accelerometer. It is $\theta = \arctan\left(\frac{a_x}{\sqrt{a_y^2 + a_z^2}}\right)$ of the Accel x, I will tell you the explanation θ and θ of Accel x divided by a square root of Accel y into Accel y plus square of Accel z. So, if we consider an accelerometer to be this duster, we have the x axis which is facing towards us the y axis which is facing parallelly to us and the z axis which is facing downwards from us. So, if this is the accelerometer that we are considering x is always the forward axis the y is the parallel axis and z is the perpendicular axis to the body of the accelerometer.

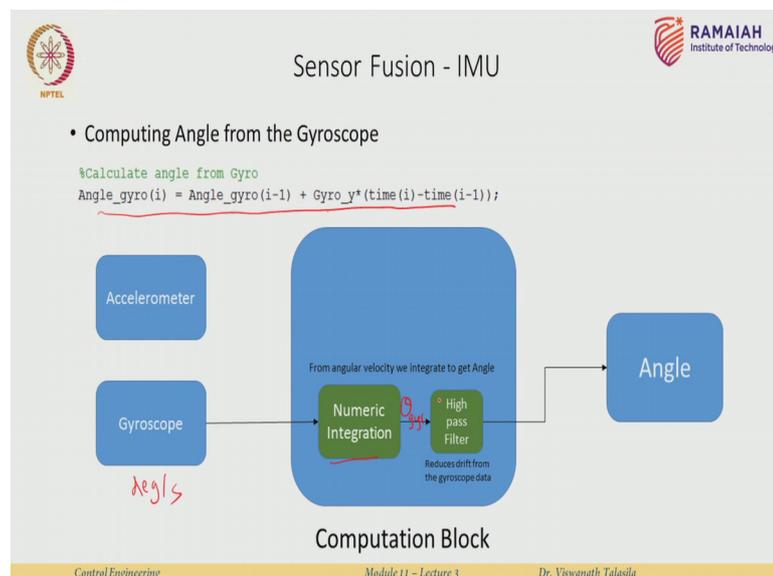
So, if we want to find out an angle for example, the peach angle which is supposed to be like that a movement which happens upwards in this directions we consider Accel x to be the forward direction axis the Accel y which being this axis and the z axis which is perpendicular to the axis of the maccelorometer. So, we get meters per second square and g values from the accelerometer g basically is the g force. So, if I consider an accelerometer to be stationary like this in air, the x axis which is the parallel to the

ground and the y axis which is parallel to the ground does not measure any acceleration, but the z axis which is measuring the gravity is measuring 9.8 meter per second square. So, we call that to be g and we normalize the value of 9.8 meter per second square into a g value.

So, you have get plus or minus 1 g depending on the orientation of the sensor. So, the accelerometer value that we get as g is passed through the angle computation block. So, the angle computation in this block when we get the g value perform the certain function which is mentioned over here this angle computation is then passed. So, we get a value of theta accelerometer. So, this theta accelerometer is passed through the low pass filter which is removing all the high frequency noise as we know that the accelerometer has a lot of high frequency noise, we can remove it by designing customize low pass filter for this particular function where this low pass filter reduces the jitter from the accelerometer data. So, the value we get over here is theta low pass. So, this is the angle that we get from the accelerometer.

Similarly, we can do the same from the gyroscope. So, from the gyroscope we can compute the angle.

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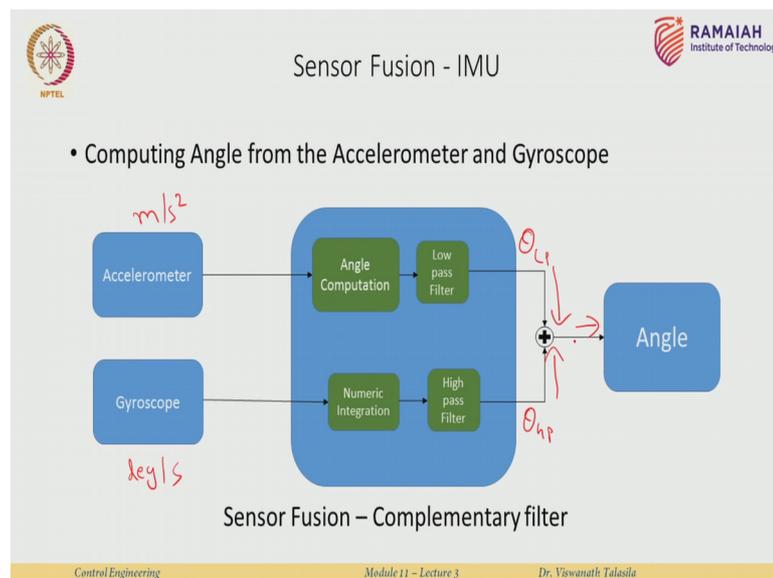


The gyroscope basically gives you value in degrees per second. So, which is your angular velocity this value from the gyroscope which is degrees per second if passed through the numerical integration which is shown as a function over here this is the

MATLAB function where we pass the gyroscope in through the numerical integration by gyro into time i minus time i minus 1 and added with the previous angle of the gyro. So, this is the function of a numerical integration. So, so after we compute the numerical integration of the gyroscope value we get an angle which is theta gyroscope.

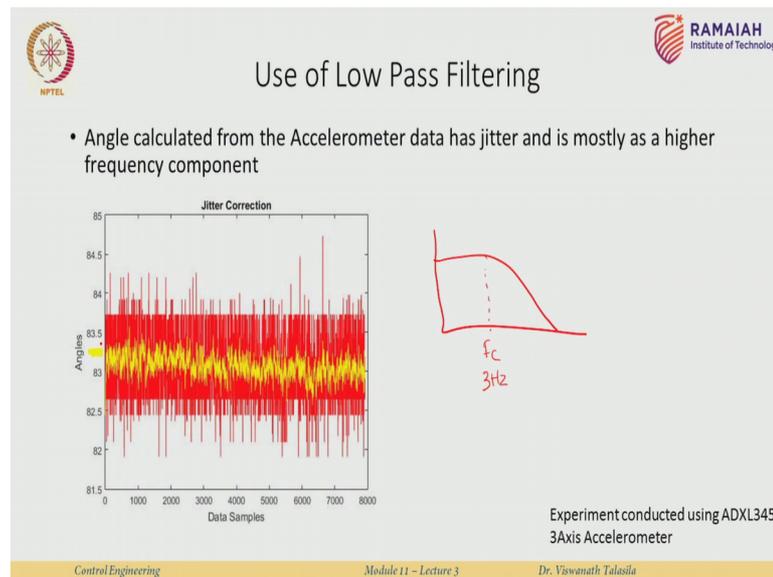
So, as Professor Vishwanath has already mentioned that the gyroscope values have after integration give a lot of drift. So, the drift is compensated by passing this angle through the high pass filter. So, the high pass filter basically cuts of low frequency noises which is the drifts and gives you a value theta high passed value which is the again the angle. So, now that we know that we can compute the angles from the accelerometer and the gyroscope, we tried to fuse these 2 sensors which is your accelerometer and your gyroscope to compute the angle for better performance.

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So, now we can see here that the accelerometer giving values in meters per second and gyroscope giving values in degrees per second the accelerometer is passed through the angle computation and the low pass filter the gyroscope is passed through the numerical integration and the high pass filter both of these which is the theta low pass and the theta high pass are shown previously is then added and then we get the angle.

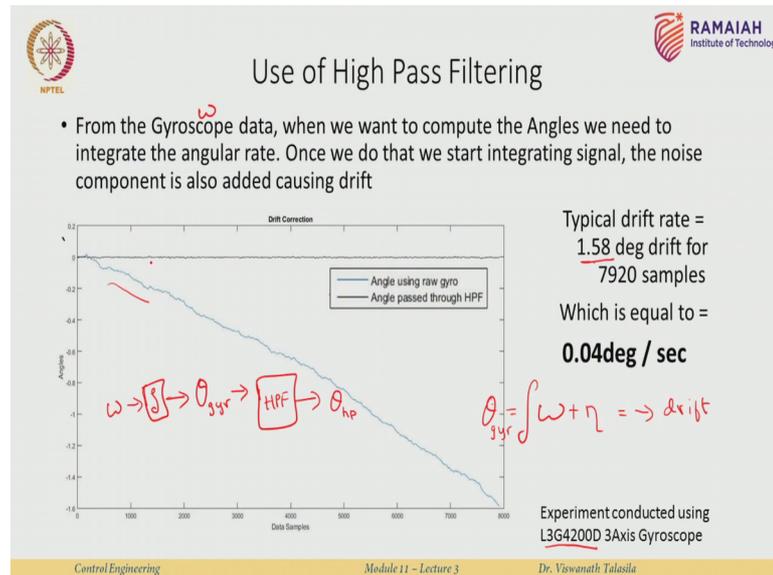
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So, why do we use a low pass filter for the angle calculated from the accelerometer? So, known that we know that angle can be calculated from the accelerometer and this has to be passed through the low pass filter, I will tell you why the low pass filtering is required if we considered this to be the accelerometer, we keep it stationary on the ground, we can get data which is very similar to this. So, we get a very stationary data which is around 83 degrees. So, the data we receive over here is automeen of 83 degrees, but it has a lot of jitter. So, it varies from around 82 degrees to around 85 degrees, 84 degrees. So, we need to pass it through low pass filter to remove the high frequency components. So, the red graph which is over here is the high frequency component jitter that we get when we calculate the angle from the accelerometer, we pass the high frequency component through a low pass filter with certain cut off frequency.

So, this cut off frequency can be calculated in such a way that the jitter which we see in the red graph can be removed and we calculate in this particular experiment to be 3 hertz. So, this frequency that the cut off frequency that we calculate, gives us a response which is in the yellow form, so, yellow form is the filtered out data and the red is the raw data. So, now, we can see that when the accelerometer is kept stationary on the ground we get a better curve which is centered at 83 degrees.

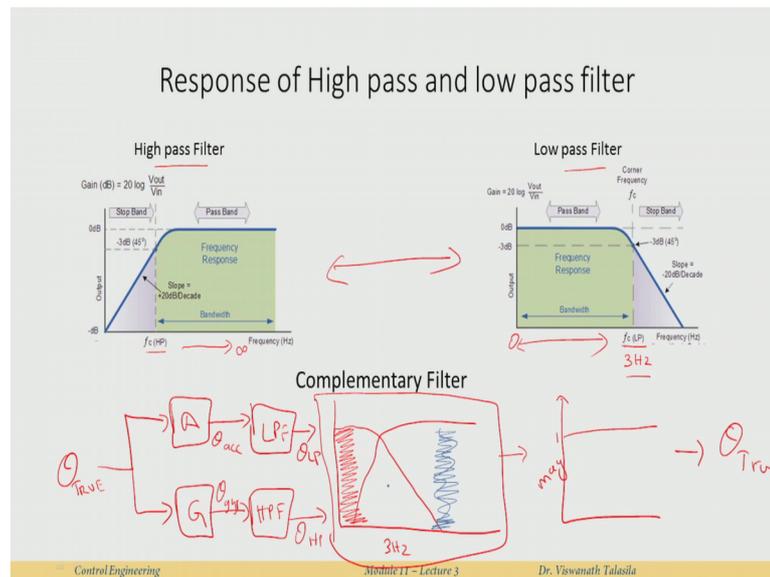
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So, now I will be showing you why we use the high pass filter. So, the gyroscope data that we have which is denoted by ω is in degrees per second. So, once this ω is passing through the numerical integration, we get the value of angle which is θ of the gyroscope.

So, this value is then pass through the high pass filter and now we get θ_{hp} . So, after getting the value of θ_{hp} , we see the experiments which is conducted using the L3G4200D which is a 3 axis gyroscope, if we consider this to be all gyroscope and keep it stationary on the ground we see that we get a drift. So, this drift is basically the drift where if we need to integrate are $\omega + \eta$. So, the noise also gets integrated when we trying to integrate the gyroscope value to find out θ and hence this causes are drift. So, this drift can be seen in this particular experiment where we use a 3 axis gyroscope and the blue line is the angle from the raw gyroscope and the black line which is above over here is the angle pass through the high pass filter.

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So, now we see the response of the high pass and the low pass filter the high pass filter which is on the left side has a response which is like this where the F_c is the cut off frequency of the high pass filter and you have a pass band from F_c to infinity. So, now, we have we look at the low pass filter where the low pass filter has frequency response which is similar to this there is a d_k after the F_c which is your cut off for the low pass filter. So, if I consider frequency for example, as 3 hertz. So, after 3 hertz there is a 20 db slop d_k in this response. So, the pass band is basically from 0 to 3 hertz. So, now, what is the complementary filter the complimentary filter is the combination of the high pass filter and the low pass filter.

So, let us consider the angle θ through this is measured from both your sensors as we know from the accelerometer and from the gyroscope. So, this angle which is the truth angle is measured from your accelerometer and the gyroscope using their computations the angle which is θ_{Accel} is passed through the low pass filter and the angle that is θ_{gyro} is passed through the high pass filter. So, we have seen already, why we pass the accelerometer angle and the gyroscope angle through the respective high pass and the low pass filters this angle which is low passed and high passed can be called as $\theta_{low\ pass}$ and $\theta_{high\ pass}$ is passed through a combination of the high pass filter and the low pass filter.

So, I will show you the combination of the low pass filter by taking the cut off frequency as 3 hertz. So, if I considered 3 hertz cut off frequency I have a response of my low pass filter which is like this and the response of my high pass filter which is like this. So, essentially if I pass the theta low pass and the theta high pass through the complementary filter block, I will get an output which is having a response with a magnitude of 1. So, the y axis is the magnitude and the response is always one. So, at the output we get theta true. So, for example, if my theta true is a lower frequency signal we can see that the low pass filter actually ways more in this complementary filter. So, if my signal comes in this region.

My low pass frequency is the low pass filter basically dominates over the high pass filter to give the magnitude as one and if I get a higher dynamic frequency which is in this region, my high pass filter takes over and the output from the high pass filter ways more in the magnitude of one to measure the theta true. So, now, I will be taking you through the design steps of the complementary filter.

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Design Steps

- Datasheet of the gyroscope and the accelerometer has to be seen to get the values of the bias and random noise present while data acquisition. We need to carefully then choose the "Cut-off Frequency" F_c .
We know the formula of F_c and Time constant " τ "

$$F_c = \frac{1}{2\pi RC}$$

F_c
 τ

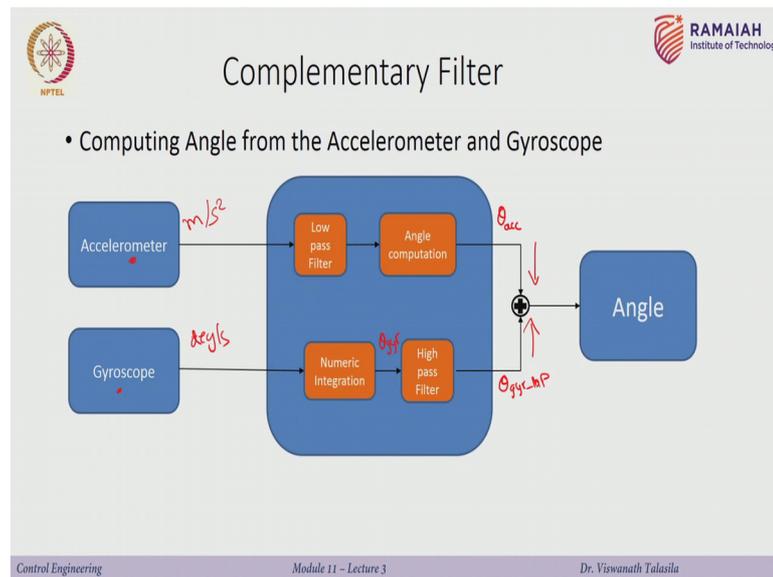
- We get the value of τ , $\tau = RC = \frac{1}{2\pi F_c}$

Note:
Cut-Off frequencies are chosen depending on the frequency of movement, noise- jitter in the accelerometer and the frequency of the drift present in the angle computed from gyroscope value

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So, we basically know that the complementary filter algorithm helps to get us better accurate angles and accurate competitions. So, I will be taking you through the design steps of the complementary filter. So, these to revise back the block diagram of the complementary filter.

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This is the complementary filter where the accelerometer which is present over here it gives you a value in meters per second square the gyroscope which is present over here it gives you values in degrees per second.

So, right now that the accelerometer is giving acceleration data the gyroscope is giving me angular velocity in degrees per second, we pass the accelerometer data through the low pass filter, here is the low pass filter why we have already discussed what low pass filter does and then we passed through the angle computation block where we get theta of your accelerometer. Now we have already know that from the gyroscope, we can also get the angle by numerically integrating the values of the gyroscope we get the angle theta gyroscope over here and we get theta gyro high pass. So, these 2 angles that we have that is your theta acceleration and theta gyro high pass has to be added together in the complementary filter block with the respective its and we compute the angles from that.

So, going to the design steps of how we design the complementary filter. So, we know that the gyroscope and the accelerometer are the sensor which you have using. So, it is very essential to see you the data sheets of these sensors because the sensors are the main part in the acquisition of the data. So, here once we see the data sheets of the accelerometer and the gyroscope, we get the values of the bias and also we get the random noise which is present while we are doing data acquisition. So, then we carefully

choose the cut off frequency the cut off frequency F_c which is mentioned over here is to be chosen based on basically 3 parameters. If you see the note below, the cut off frequencies are chosen depending on the frequency of movement that is how fast your actual movement is happening and the noise that is the jitter present in the accelerometer are shown. Previously, there is a lot of jitter present in the accelerometer and then also the frequency of the drift which is present in the angle competition of when we compute the angles from the gyroscope.

So, now, that we have chosen the cut off frequency, we know the formula for a cut off frequency and the time constant. So, the cut off frequency over here has the formula which is like this F_c is equal to $1 / (2\pi RC)$. Now depending on what cut off frequency, we choose we can find the value of tau. So, tau is basically R into C that is if you reframe the first equation which is above RC is equal to $1 / (2\pi F_c)$. So, now, we have the values of F_c and we have the value of tau. So, next we see; how we put these values of your tau and your cut off frequency to design a low pass filter.

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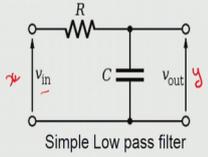


Low Pass filter Design



• Once we know the value of RC, we calculate the alpha values for the low pass and high pass filters, we can put the filter coefficients α into the filter equations

Low pass filter equation:



Equation of low-pass filter

$$y_i = x_i \left(\frac{\Delta t}{RC + \Delta t} \right) + y_{i-1} \left(\frac{RC}{RC + \Delta t} \right)$$

Δt = Sampling rate

$$\alpha_{lpf} = \left(\frac{\Delta t}{RC + \Delta t} \right)$$

Where α is the filter coefficient

F_c

τ

α_{lpf}

Simple Low pass filter

Simplified Equation for LPF

$$y_i = \alpha_{lpf} x_i + (1 - \alpha_{lpf}) y_{i-1}$$

So, as you see on the left side, there is a circuit which is the low pass filter you have the V_{in} over here we have the R and the C circuit. So, this is the passive low pass filter.

So, now, we know that from previous lecture that if V_{in} is considered as x and V_{out} is considered as y we get this equation of the low pass filter where y_i is equal to x_i which is your input into Δt Δt is basically your sampling right . So, Δt is your

sampling rate and we know the value of RC which is equal to τ again plus Δt $\tau + \Delta t$ plus y of i minus one. So, this y of i minus 1 is the output of the previous function. So, y of i minus 1 into RC divided by RC plus Δt . So, now, with this equation we can get a simplified equation of the low pass filter which is in the corner below here y of i is equal to α low pass into x of i plus 1 minus α low pass into y of i minus 1.

So, this is the simplified equation of a low pass filter where α low pass is given by the equation Δt divided by RC plus Δt . So, with this we get from the previous slide we have seen that we get F_c we get the value of τ .

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The slide contains the following content:

- High pass filter equations:**
 - Simple High pass filter:** A circuit diagram showing an input voltage V_i connected to a capacitor C in series with a resistor R . The output voltage is V_o .
 - Equation of high-pass filter:**

$$y_i = y_{i-1} \left(\frac{RC}{RC + \Delta t} \right) + (x_i - x_{i-1}) \left(\frac{RC}{RC + \Delta t} \right)$$
 - Where α is the filter coefficient:**

$$\alpha_{\text{hpf}} = \left(\frac{RC}{RC + \Delta t} \right)$$
 - Simplified Equation for LPF:**

$$y_i = (\alpha_{\text{hpf}}) y_{i-1} + (\alpha_{\text{hpf}})(x_i - x_{i-1})$$

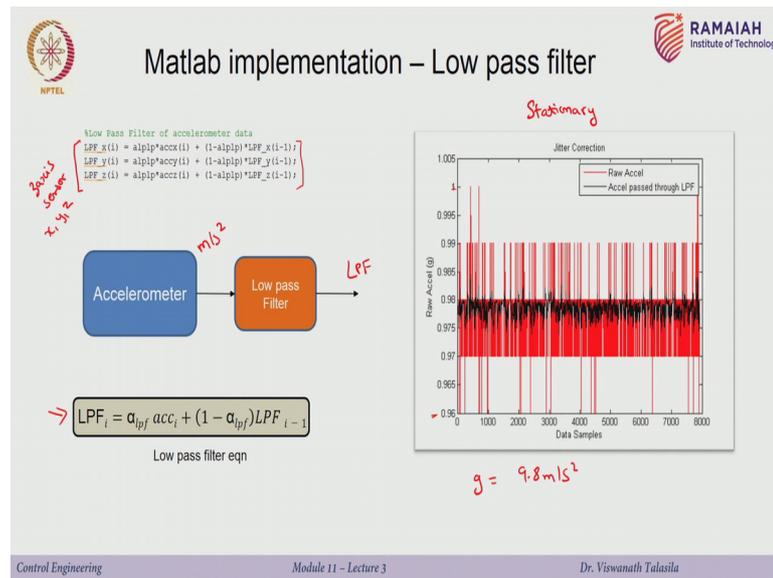
Handwritten notes on the slide include a bracket around F_c and Z , and α_{hpf} .

And now we have the value of the α , let us move on to designing the high pass filter. So, the high pass filter circuit is shown over here where V_i is here we are capacitor and your resistor and this is a passive simple first order high pass filter. So, previously, we have seen the equations of this high pass filter to be y of i is equal to y of i minus 1 into RC divided by RC plus Δt plus x of i minus x of i minus 1 into RC divided by RC plus Δt .

So, this equation can also be simplified and written as y of i is equal to α high pass into y of i minus 1 plus α high pass into x of i minus x of i minus 1. So, here we know that the input again is considered as x and the output is considered as y . So, with this equation we get the filter coefficient that is α high pass is equal to RC divided by RC plus Δt . So, now, for the high pass filter we have the cut off frequency we have the

value of the time constant and we have the value of the alpha high pass filter. So, now, we have seen that for the low pass filter and the high pass filter, we have the parameters which are stated here we have to combine these 2 filters together to make something called a complimentary filter.

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So, I will be showing you MATLAB implementation of the low pass filter. So, if you see away here, if you remember from the block diagram the accelerometer values which is coming as meters per second square is pass through the low pass filter and we get the value of LPF over here. So, we know that we were using 3 axis sensor which is x, y and z. So, hence we get LPF underscore x which is your low pass filter of the x axis LPF underscore y which is the low pass filter of your y axis and LPF z which is the low pass filter of your z axis. So, we pass this through the equation which is shown here I will be giving you the MATLAB implementation the complete file which includes the low pass filter application which also includes the high pass filter applications and also the complementary filter imp implementation.

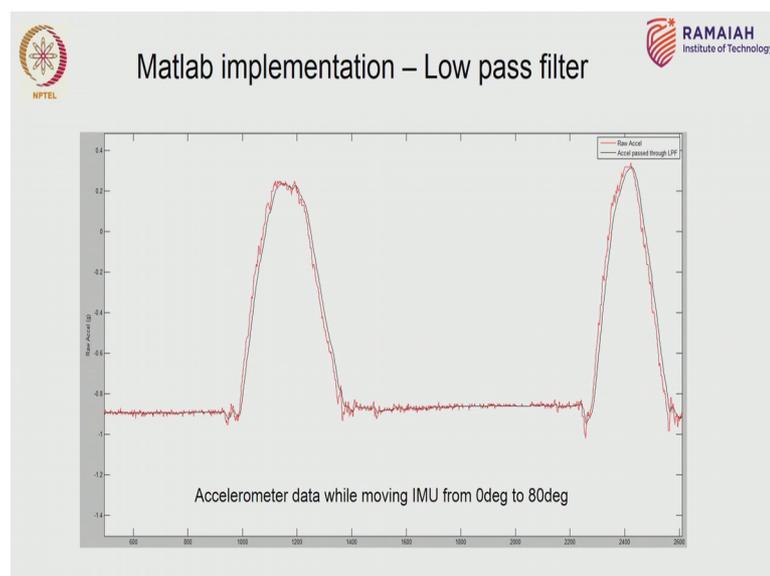
So, now we see that after we pass the values that we get from the accelerometer through the low pass filter with the equation which is shown away here we can see that the values are more stable, there is less amount of jitter present in your accelerometer if you see the graph on the right side the red graph is basically a raw acceleration value which is again

in meters per second square or you can considered to be g where g is equal to 9.8 meters per second square.

So, your raw acceleration value which is the graph which is in red is more jitteree compared to the Accel pass through the low pass filter which is in black. So, we see over here that once we pass it through the low pass filter the mean of this whole line is very close to 9.8 meters 9.98 g, but otherwise when we do not passed through the low pass filter as we see in the red graph it varies from 120.96. So, we see that we get more accurate data when we passed through the low pass filter.

So, this is the data when the IMU which is present which is your sensor is kept stationery now we will see an implementation where your IMU is actually moving.

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So, we here again we have the same color combinations where the red graph is your raw acceleration value consider this to be your accelerometer, we have an accelerometer which is kept on my hand and I move the accelerometer from 0 degrees to 90 degrees and back to 0 degrees. So, this is the graph that I do get when I move the IMU which is my accelerometer over here from 0 degrees to around 80 degrees for example, and the red graph if you see is has a lot of jitter. So, we pass it through the low pass filter where the Accel values are pass through the low pass filter and is marked in black. So, we see that the black line is a more smooth a line compared to the red line.

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Matlab implementation – High pass filter

```

%Calculate angle from Gyro
-> Angle_gyro(i) = Angle_gyro(i-1) + Gyro_y*(time(i)-time(i-1));
%High pass the angle from Gyro
Angle_gyro_HPF(i) = (alpha*Angle_gyro_HPF(i-1) + alpha*(Angle_gyro(i)-Angle_gyro(i-1)));
    
```

Angle = $\int(\omega + n)$

Stationary data

Angle computed Gyroscope → High pass Filter → HPF

$$HPF_i = \alpha_{HPF} HPF_{i-1} + (\alpha_{HPF})(Ang_Gyro_i - Ang_Gyro_{i-1})$$

High pass filter eqn

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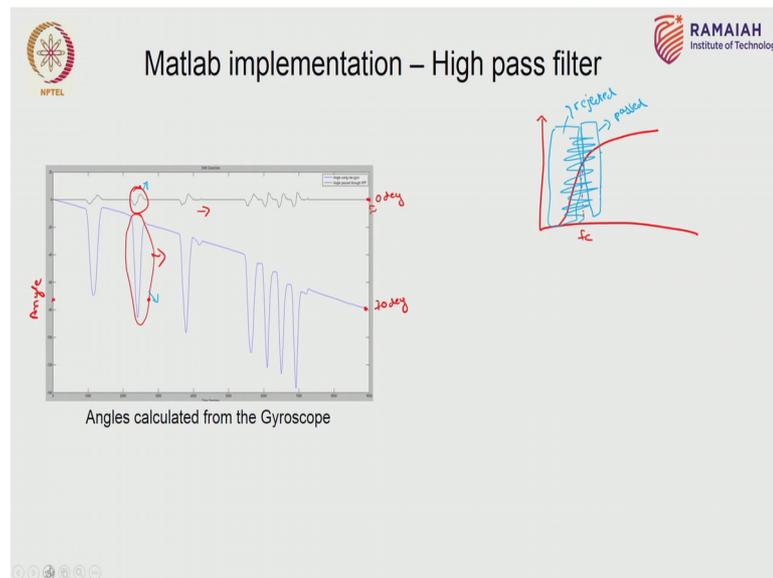
So, now we will see the MATLAB implementation of the high pass filter. So, we know that when we calculate angles from your gyroscope where the angle underscore gyro is be computed angle. So, here we have angle underscore gyro which we then pass it through the high pass filter. So, we know the equation of the high pass filter which is previously shown. So, we pass the angle underscore gyro through the high pass filter where the equation is shown over here as well and we get HPF. So, the value of HPF. So, this HPF value is the value which we are getting after we pass the angle computed from the gyro through the high pass filter here on the right side we can see a graph which shows the component of drift. So, if you see over here the IMU is kept stationary on the ground you see that this is the stationary data we always want the angle to be constant.

So, we keep it at around 0 degrees because of this integration there is a buildup of noise and then we see that the angle which we are computing this; the y axis is the angle the angle which we are computing from the gyroscope is always drifting. So, at around 8000 samples we are getting around 90 degrees of drift. So, we see that there is the angle which is not stationary, but the noise which is building up causing the angle to be drifted. So, now, we when we pass this angle through the high pass filter it does not allow changes as spoken previously and gives you a line which is the black line.

So, this black line is the line where when we have a stationary value it when it pass through the high pass filter we get almost a straight line. So, so from this we get to know

that we need to remove the component of the drift in your angle computed from the gyroscope and your component of jitter from the accelerometer. So, this can always be done only by the good selection of your F_c which is your cutoff frequencies. So, if your frequencies are not chosen properly your angle which maybe that your angle which you are calculating is drifting away or your jitter which is the component present which is the component of noise present in your accelerometer does not get removed properly.

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So, here is another MATLAB implementation of the high pass filter. So, on the y axis we have angles again which is computed from your gyroscope and your x axis is your data samples. So, we see over here again we conduct an experiment where the IMU is present on the hand we moved from 0 degrees to 80 degrees and back and we continuously repeated for 7 times. So, we see that there is a graph which is coming which is your blue line the raw angle which is calculated from your gyroscope. So, we see that the blue line over here has a lot of drift if you can see I am doing an experiment where I am going from 0 degrees to 80 degrees and coming back to 0 degrees.

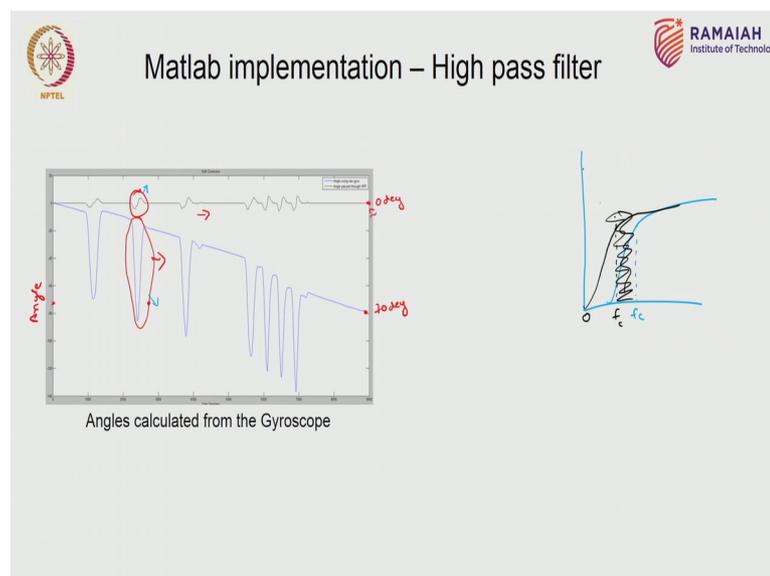
So, this experiment which I am doing has a lot of drift where for example, after 9000 samples, I am getting it to be around 70 degrees. So, which is this value of around 70 degrees is not correct because I have stopped the experiment at 0 degrees. So, hence I pass this through the high pass filter which is your black line again over here and I get a data where at 9000 samples I am almost at 0 degrees. So, you will say that there is an

issue with this graph where for example, the data which we are computing in the blue line which is your angle computation from the gyroscope has a bigger magnitude than the magnitude present over here in the black line.

So, your magnitude present over here is larger than the magnitude which is present in your high pass filter. So, now, I will explain why we get this value I will just draw a filter response the frequency response of this high pass filter taking the cutoff of F_c . So, in your blue graph we have some data which is being removed because the high pass filter is designed in such a way that the F_c is also removing some part of the signal. So, for example, the signal which we are getting is around this range. So, anything which is below the F_c is being rejected and only the signals which is on the right side of the F_c is being passed hence we see that there is a dropped in the magnitude in this graph as compare to this graph.

So, we need this is very important when we chose the cut off frequencies of your high pass filter we do not want the signal also to be removed we just want the drift to be removed over here. So, what do we do? So, what do we do over here?

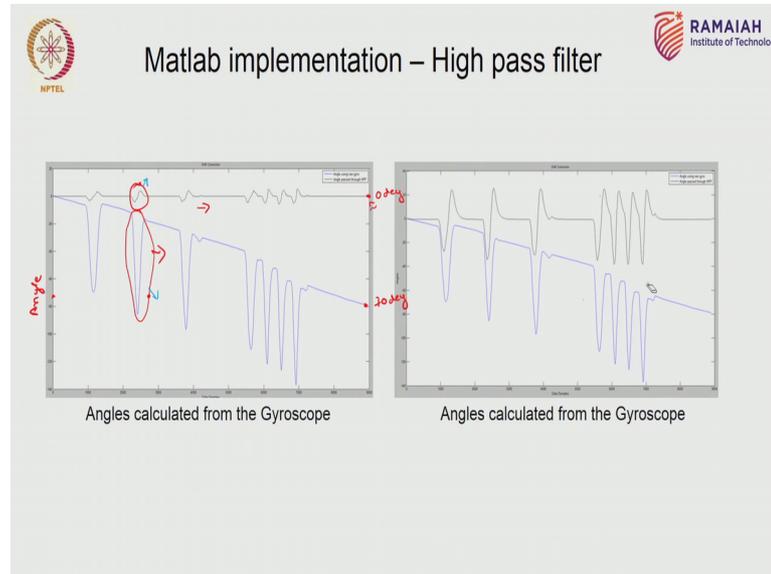
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So, now that we know that this is my F_c which is chosen in this graph I want to chose an F_c which is much lower which come somewhere to the left towards 0. So, that my filter response looks like this and the signal which is present in this area which is present in

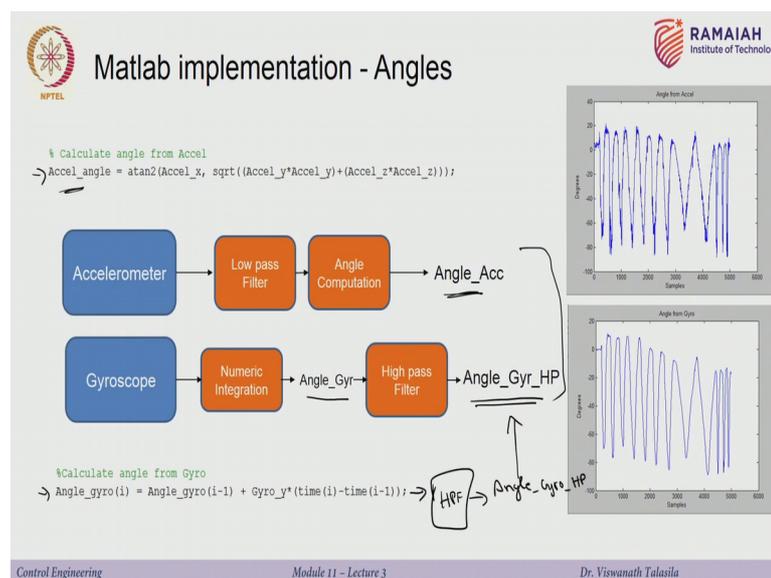
this area is also pass through. So, I will be showing you an implementation where we have actually changed the F c we have made it a lower F c.

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So, that we get better a values of the signal. So, on the right side FVC over here on the right side FVC over here the magnitude of the blue line and the black line is very similar only that the component of drift is being removed. So, here we have chosen an appropriate F c such that we do not reduce the magnitude of my response.

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We have already seen from the previous examples as to how to calculate the angle this is the MATLAB implementation of how we calculate the angle from your IMU from the accelerometer we calculate the angles in this manner. So, we take this equation and we get the value of the Accel. So, this Accel angle is got from this equation which is above from the gyroscope we pass it through the numerical integration and then with from the gyroscope we pass it through the numeric integration we get the angle gyro and the pass it through the angle gyro high pass to get the angle in the output.

So, here is how we calculate the angle from the gyroscope this is also shown previously. So, this is a reputation of this and now we pass this through the block of the high pass filter to get angle underscore gyro underscore hp which is this complementary filter is a filter in which we combined these 2 angles that is your angle accelerometer and angle gyro hp with certain weights to get an output response which compresses of the true signal.

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Complementary Filter

A mathematical form of this complementary filter can be written as:

$$\text{Angle} = (\alpha_{CF})(\text{Angle from accelerometer}) + (1 - \alpha_{CF})(\text{Angle from gyroscope})$$

$0 \leq \alpha_{CF} \leq 1$ $\alpha_{CF} = 0$

Weights are given depending on the frequency of movement:

- Lower frequency – Angle from Accelerometer higher weightage
- Higher Frequency – Angle from Gyroscope higher weightage

$\alpha_{CF} = 1$
Angle gyro → dominant

$\alpha_{CF} = 0$
Angle acc → dominant





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So, the complementary filter as we know follows this equation where the angle from your complementary filter angle from your complementary filter is equal to alpha C F which is your filter coefficient of the complimentary filter in to the angle that we get from your accelerometer plus one minus alpha C F which is again your filter coefficient of the complementary filter in to the angle we compute from the gyroscope after passing through the high pass filter

So, now, we need to choose the value of alpha C F. So, how do we choose the value of alpha C F. So, if you can say that the frequency of motion of for example, you are trying to measure a certain movement, for example, I am just rotating moving my hands from 0 degree to 90 degrees again and back. So, in you are you actually want to see what is the frequency of motion over here and from the frequency of motion we decide to give weightage to either the accelerometer or gave weightage to either to the gyroscope.

So, we see that we are moving the IMU from 0 to degree 0 degrees to 90 degrees and back to 0 degrees, we see the frequency of movement of this hand which is measure by your sensor and if the frequency is lower, we give more preference to our accelerometer if your frequency of motion is higher, we give more preference to the gyroscope here alpha C F is in between 0 to 1 where if we give the value of alpha C F is equal to 0 the angle from the gyroscope is dominant or if we give alpha C F is equal to 1 angle computed from your accelerometer is dominant.

So, we need a chose a value of alpha C F depending on how fast or how slow we are moving. So, after we chose the filter coefficients of the complementary filter we see that your low pass filter will give a response which is similar to this and you can say this is my F c and my high pass filter will give a response which is similar to this. So, now, that we are getting a response like this with the same F c we know that we will get a response of magnitude of one when we pass the signal through the complementary filter.

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Problem-1



Problem: Consider an induction motor rotating at 1100rpm, this is equal to 6600deg/sec and is approximately 18Hz, Let is strap on the IMU to calculate the angle of the motor shaft. Consider a Cutoff frequency of 20Hz and design the low-pass, high-pass filter and combine them to get a complementary filter. Assume the value C = 1μF and sampling rate of 50Hz → f_s

$$\begin{aligned}
 \text{at } & \frac{1}{f_s} \quad | \quad \text{rpm} \rightarrow \frac{1}{60} \text{ Hz} \quad f_m \\
 & = \frac{1}{50} = 0.02 \\
 & 1100 \text{ rpm} \rightarrow \frac{1}{60} \times 1100 \\
 & \approx 18 \text{ Hz} \\
 & f_c = 20 \text{ Hz}
 \end{aligned}$$

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So, now, we will be discussing a small problem on this design of the complementary filter here consider and induction motor which is rotating at 1100 rpm, 1100 rpm is equal to 66 double 0 degree per second. So, 1100 rpm is equal to 6600 degree per second which is approximately equal to eighteen hertz. So, we know that the equation for calculating from rpm to hertz, we get 1 rpm is equal to 1 by 60 hertz.

So, if we get 1100 rpm, then 1 by 60 in to 1100 we approximately get eighteen hertz. So, we considered the value of eighteen hertz which is your frequency of your message sigma and we strap on and IMU to calculate the angle of this motor shaft. So, we have an accelerometer and gyroscope which is present on the motor shaft which is computing the angles in the motor shaft. So, we know that in this problem we have already been given the value of F c which is F c is equal to 20 hertz.

So, here we need to design a low pass filter and a high pass filter to and combine them together to get a complementary filter we all do also given the value which is an assumption that C is equal 1 micro farad and your sampling rate which is F of S is equal to 50 hertz. So, we know the equation of F of S and delta t where delta t is equal to 1 by F of S. So, now, F of S the 1 by 50 which is equal to 0.02. So, we here if I have to just mark my parameter that I have I get F c is equal to 20 hertz your value of delta t is equal to 0.02 and the value of C is equal to 1 microfarad.

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Problem-1



Problem: Consider an induction motor rotating at 1100rpm, this is equal to 6600deg/sec and is approximately 18Hz, Let is strap on the IMU to calculate the angle of the motor shaft. Consider a Cutoff frequency of 20Hz and design the low-pass, high-pass filter and combine them to get a complementary filter. Assume the value $C = 1\mu\text{F}$ and sampling rate of 50Hz

$$F_c = 20\text{Hz}$$

$$\Delta t = 0.02$$

$$C = 1\mu\text{F}$$

$$Z = 0.00795$$

Solution: We know the cut-off frequency $F_c = 20\text{Hz}$, so we need to design a high pass and low-pass filter for the same cut-off frequency. We need the value of τ which is the time-constant. Sampling rate = 50hz ,

We know,

$$\tau = R \cdot C = \frac{1}{2 \cdot \pi \cdot F_c} = \frac{1}{2 \cdot \pi \cdot 20}$$

$$\tau = 0.0079579$$

$$Z = R \cdot C$$

$$R = \frac{Z}{C}$$

$$= \frac{0.00795}{1\mu\text{F}}$$

$$R = 7.95\text{K}\Omega$$

Now after we know the value of the time constant, we can assume a value of C to get R, So we assume $C = 1\mu\text{F}$ and we calculate R as,

$$R = \frac{\tau}{C} = 7.95\text{K}\Omega$$

$$F_c, \Delta t, C, Z, R$$

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So, now we see how to design the low pass filter. So, we know that your value of F_c is equal to 20 hertz. So, we need to design a high pass filter and the low pass filter for the same cut off frequency and we also we need to find the value of tau which is your time constant where tau is equal to R into C where your sampling rate is 50 hertz, we know the equation of tau where tau is equal to R into C which is again equal to 1 by $2\pi F_c$ which is equal to 1 by 2π in to π in to F_c which is 20, we get the value of tau to be 0.079579.

So, now we know the value of tau which is equal to 0.00795 after getting the value of tau we calculate the value of r where tau is equal to R into C R is equal to C by tau and equal to 1 microfarad divided by 0.00795. So, your value of r comes to 7.95 kilo ohms which is given over here. So, from this problem we have find out a few parameters which is your F_c which is your delta t your C your tau and R with this parameters.

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Problem-1

Now that we know the value of R and C , we can draw the passive Low-pass and High-pass filter circuits as,

Low pass filter Circuit

$\alpha_{lpf} = \left(\frac{\Delta t}{RC + \Delta t} \right) = \left(\frac{0.02}{0.00795 + 0.02} \right)$
 $\alpha_{lpf} = 0.7155$

LPF

$y_i = (0.71)x_i + (0.29)y_{i-1}$

$y \rightarrow$ output
 $x \rightarrow$ input

High pass filter Circuit

$\alpha_{hpf} = \left(\frac{RC}{RC + \Delta t} \right) = \left(\frac{0.00795}{0.00795 + 0.02} \right)$
 $\alpha_{hpf} = 0.2844$

HPF

$y_i = (0.28)y_{i-1} + (0.28)(x_i - x_{i-1})$

$y \rightarrow$ output
 $x \rightarrow$ input

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We start to design, the filter on the left side if you see it is the filter circuit of the low pass filter and on the right side we see that it is a filter circuit of the high pass filter.

So, we know the values of R which is 7.95 kilo ohms, I mark it on the right side as well 7.95 kilo ohms and see which is 1 microfarad and C which is 1 microfarad. So, we actually completed the first question where we have designed the low pass filter circuit and we also design the high pass filter circuit now we need the values of alpha. So, now, on the left hand side, we want to find the all a value of alpha low pass where we can a

put it in to the equation. So, the value of alpha low pass if you remember the equation is equal to delta t divided by R C plus delta t, if I have to re write again 0.02 divided by 0.00795 plus 0.02, this is your value of alpha low pass where you get alpha low pass is equal to 0.7155.

So, this value of alpha low pass you can put it in to the equation of your low pass filter where why is your output and x is your input. So, from this, we have actually got a, the equation of your low pass filter. Now we move on to the high pass filter where the equation for calculating the alpha high pass which we want to put it in to the filter equation the equation, the formula for alpha high pass is equal to R C divided by R C plus delta t which is 0.00795 divided by 0.00795 plus 0.02. So, if we calculate the value of alpha high pass we get the value 0.2844, we put this equation in to the equation of the high pass filter where y of i y is your output and x is your input.

So, now we have design a basically 2 circuits which is your passive low pass filter circuit and your passive high pass filter circuit next.

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Problem-1

We know the transfer functions of the low pass filter and high pass filter as

Low pass filter Circuit

$H(s)_{lpf} = \frac{1}{1 + RCs}$ $Z = RC = 0.00795s$

$H(s)_{lpf} = \frac{1}{1 + (0.00795)s}$

High pass filter Circuit

$H(s)_{hpf} = \frac{RCs}{1 + RCs}$

$H(s)_{hpf} = \frac{(0.00795)s}{1 + (0.00795)s}$

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We have also calculated the value of alpha low pass and the value of alpha high pass and also put it in to the equations of your low pass filter and the high pass filter. Now we move on to finding out the transfer function of these 2 filters. So, now, we know the value of R is equal to point, sorry, we know the value we know the value of R is equal to 7.95 kilo ohm and value of the capacitor is 1 micro farad.

Similarly, over here, 1 micro farad and 7.95 kilo ohms. So, we already have studied Professor Vishwanath has a taught you about the transfer function of these filters. So, we know the transfer function of the low pass filter which is given over here H of S low pass filter is equal to one divided by 1 plus RCs. So, we know the value of R C where tau is equal to R C is equal to 0.00795. So, we get the transfer function of your low pass filter to be 1 divided by 0.00795 S.

So, now we know the transfer function of the high pass filter where H of S HPF is equal to RCs divided by one plus RCs where again R C is your tau which is equal to 0.00795. So, we get the value of H of S high pass filter is equal to 0.00795 into S divided by 1 plus 0.00795 into S. So, this is the transfer function of your low pass filter of your high pass filter and on the left side is transfer function of your low pass filter.

So, now we have come from the cut off frequency you have calculated your R and C values and your tau values and we also calculated your alpha values put it into the equation of the low pass filter and the high pass filter and we have come to get the transfer function of the low pass filter and the high pass filter. After we have the transfer functions of the high pass filter and the low pass filter, we now find out what is the corner frequency cause with the corner frequency we can get the bode plot. So, we can draw the bode plot by using the corner frequency which is omega C which is in terms of radiums per second.

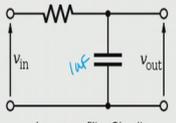
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Problem-1

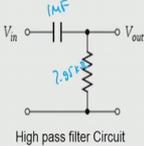


Now we can get the corner frequencies ω_c of each of the filters



Low pass filter Circuit

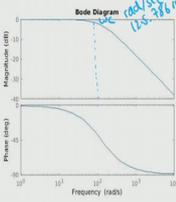
$\omega_c = \frac{1}{RC} = 125.786 \text{ rad/sec}$

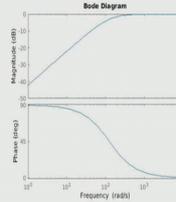


High pass filter Circuit

$\omega_c = \frac{1}{RC} = 125.786 \text{ rad/sec}$

Bode plots:





Handwritten notes:

- $\omega_c = 1 \rightarrow 0.00795$ (with $\rightarrow \text{rad/sec}$)
- $\omega_c = 125.786 \text{ rad/sec}$ (with $\rightarrow \text{rad/sec}$)
- steps for design $\rightarrow F_c$, calculate Z
- $Z = R + C$
- $R, C \rightarrow$ circuit LFP & HPF
- $\rightarrow \alpha_{LFP}, \alpha_{HPF}$
- \rightarrow Eqns of LFP, HPF
- \rightarrow Transfer Fns
- \rightarrow Bode plots

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So, now that we got the equation, I will again input the values of R and C is equal to 7.95 kilo ohms and C is equal to 1 micro farad 1 micro farad and 7.95 kilo ohms. So, these are the 2 equations where we calculate the corner frequency which is ω_c , ω_c is equal to 1 divided by R C is equal to which is ω_c is equal to 1 divided by 0.00795 when we do this equation this computation without ω_c is equal to 125.786 radium per second which is the same for your high pass filter as well.

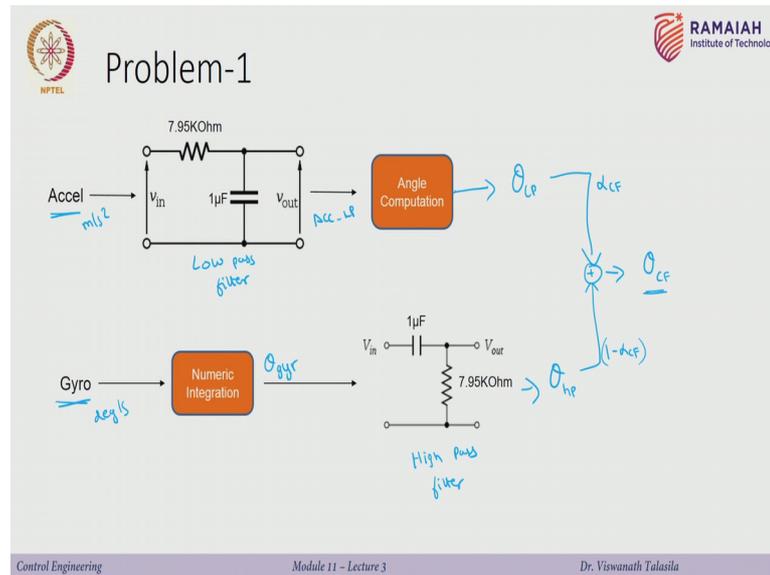
So, your ω_c for your high pass filter and your ω_c for your low pass filter are the same where both ω_c is equal to 125.786 radiums per second. So, now, with transfer function, there is a in the implementation of the MATLAB gold that we have. We will be given it you we also shown the bode plot. So, with that transfer function you can actually draw the bode plot or in a free handle drawing you can draw the bode plot of this by in this manner. So, 125.786 is your 3 dB margin which comes over here. So, over here you get the value of ω_c in radiums per second.

So, we know the corner frequency of this bode plot where ω_c is equal to 125.786 radiums per second. So, here are the bode plots of the low pass filter and your high pass filter it follows the curve as shown here and it follows the curve as shown over here. So, with this, we have actually designed the low pass filter and the high pass filter and also got your bode plots. So, just to revise, just to summarize on what steps we have taken steps for design can be retain like this. So, we choose the values of F c, then calculate the tau where tau is equal to R into C.

Next we get the values of R and C and input into the circuit respectively for the low pass filter and the high pass filter after we get the circuit, we calculate the values of alpha low pass and the value of alpha high pass filter. So, these are the filter coefficients, we put it into the equations of low pass filter and high pass filter. After we get the equations of the high pass filter and the low pass filter we get the transfer functions of these 2 filters once we get the transfer functions of these 2 filters we get the bode plots of these 2 filter.

So, we actually completed the design of the low pass filter and the high pass filter now we will see how to combine these 2 filters to get the complementary filter.

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If you remember the block diagram, we had Accel values which is meter per second square your gyro values which are getting in degrees per second. So, this Accel value is passed through the low pass filter and you get the value of Accel low pass from the value of Accel low pass, we pass it through the angle computation block which is shown previously and we get the value of theta low pass.

So, this is the theta which is the angle got from the Accel value pass through the low pass filter and gone through the computational block for calculating the angle. Now we go to the gyroscope where the gyroscope is passed through the numerical integration and we get the angle theta gyroscope, this gyroscope theta gyroscope is pass through the high pass filter and we get the value of theta high pass.

Now, that we have the value of theta high pass and theta low pass we combine these together with weights of α C F and $1 - \alpha$ C F to get theta C F which is your complementary filter. So, here is complete design of the complementary filter, you can actually implement it if you have an accelerometer which is interface and a gyroscope which is interfaced you can re-gap a circuit which is similar to this on the top where you have low pass filter with resistor and capacitor you passing the input from the input end and you get the output from the output end you can pass this block through and angle computation block which is shown previously to get theta low pass and you use the value of α C F to pass it into the complementary filter.

So, from the gyroscope you can actually do something like a numeric integration you will get the value of theta gyroscope you can take this value and re-gap a circuit for the high pass filter where your capacitor is one micro farad and your resistor is 7.95 kilo ohms approximately 7.95 kilo ohms and you will get the value of theta high pass you can use this theta high pass which is multiplied by this weight which is $1 - \alpha C F$ and also put it into the complementary filter. So, this is how you can implement a complementary filter in real time.

So, now we have shown the implementation of the complementary filter and how to actually design it from end to end from choosing your cut off frequency and coming to your bode plots and also integrating these to filters which is your low pass filter and high pass filter to get a complementary filter in the forum we will be having some reference material as some of you all would like.

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Reference material

* The reference material is found in the forum

- An Arduino code, which interfaces an IMU- GY80 which gives:
 - Raw Accelerometer and Gyroscope data (Serial Printing)
- A Matlab code, which reads data stored in a .csv file giving:
 - Plot of raw data \rightarrow *accd gyro*
 - Filtering techniques (HPF and LPF) and plotting
 - Complementary filter implementation and plotting \rightarrow *mfc*

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We will going to give you an Arduino code where which interfaces an IMU which is basically considering Gy 80, this is the sensor that we have chosen this Gy 80 consist of an accelerometer gyroscope and magneto meter will be giving you the codes where the Gy 80 will be computing Accel values and your gyroscope values.

So, this Accel values and the gyroscope values will be printed on serial monitor you can actually inter face it to MATLAB or something like that where in our MATLAB code we will also have an option where this serial data that we get from an actual sensor which is

your Gy 80 will be taken into MATLAB which will read the data from these Accel or also read a data from the CSV file, the choice will be given where 2 different MATLAB codes for either reading the data or directly receiving the data taking the data from CSV file will be given will be able to plot the raw data which is your Accel values your gyroscope values will be showing you the filtering techniques which is your high pass filter and the low pass filter and also plotting it and we will be showing you in another file which is the complementary filter equation for implementation of the complementary filter and also plotting the angles which we get from the complementary filter.

I hope you guys have enjoyed this course. This was a practical implementation of the complementary filter as I said the reference material will be given and if you require any help please a feel free to bring me.

Thank you.