

Control Engineering
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Module – 11
Lecture – 02
Inertial Sensors and their characteristics

Hi there and welcome to the next series of lectures in control systems. So, in this lecture, we going to be talking about little bit more detail about inertial sensors and their characterisation. This is usually a topic that students in instrumentation engineering take up, but this is becoming far more relevant for people across various disciplines because the measurement and analysis of movement; right is now spanning across multiple fields, while initially this was treated as a purely instrumentation issue which was mostly concerned with aircrafts and ship like navigation, ever since movement began started to become of great interest in robotics in human movement analysis for medical purposes and so on.

The overall usage of these inertial sensors has become far more critical, right. So, now, we have people from sports who are worried about movement analytics, people from even dance and theatre who were worried about movement and so on and so forth. So, it is not just core engineering, it is also lot of allied subjects as well, which is why we are going to be looking at some characterisation of inertial sensors, how they affect the computation of position and attitude, attitude means roll pitch and your angles and why they are important.

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The slide features the logos of NPTEL and RAMAIAH Institute of Technology. The main content is a list of topics under the heading 'Contents'. Handwritten red notes include 'navigation techniques (DR.) numerically integrate' next to the first item, the equation $\theta(k) = \theta(k-\Delta t) + \omega(k) \Delta t$, 'constant bias → consumer → medical/militar' with a circled question mark, and 'reduce navigational errors' underlined. A red arrow points from 'drift' in the gyroscope section to the underlined 'reduce navigational errors'.

Contents

- Story so far *navigation techniques (DR.) numerically integrate* $\theta(k) = \theta(k-\Delta t) + \omega(k) \Delta t$
- Accelerometer: ✓
 - Compute velocity and position
 - Noise characteristics
 - Drift
- Gyroscope:
 - Compute angular position
 - Noise characteristics and drift
- Brief introduction to filtering *reduce navigational errors*

constant bias → consumer → medical/militar

?

reduce navigational errors

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So, the story so far, we have seen various types of navigational techniques and we specifically said that we are going to be focusing primarily on dead reckoning, right and in the previous lecture, we discussed how to numerically integrate; we discussed how to numerically integrate, the measurements which are coming in from the accelerometer and the gyroscope in order to compute angular position and angular and translatory position. So, just to refresh your memory, to compute the angular position we did something like the angle at time k . At the sample k is the previously computed angle at the previous time sample plus the current measurement of the angular rate at the k th sample times delta t , right.

So, this was the numerical integration equation we used in order to compute angles and we did a similar thing for positions as well. What we are going to do today is to look at accelerometer we going to we already seen, how to compute velocity position, we specifically going to be looking at noise characterisation. More specifically, we are going to looking at a problem of drift which really comes about mainly in the gyroscope thing, it is actually not part of accelerometer, the reason I put this here is, it is very common mistake that people think that drift is a problem with IMUs, it is not. It is a problem which comes only with gyroscopes not with accelerometer. So, you should you should remember that.

Similarly, we going to look at the gyroscope, various noise characterisation of that and then what kind of conditions it has; noise characterisation and then we are finally, going to conclude with a very brief description of what kind of filtering techniques we may use in order to reduce navigational errors, right, good. So let us go ahead and quite a few of my students have contributed to various works in navigation that are been involved over the past few years, we have done projects with hospitals and autonomous vehicles and these people have actually contributed a lot and specifically Vinay Sridhar, he is going to be showing some experimental results over the next few lectures, alright.

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The slide is titled "Story so far" and "This lecture". It features the NPTEL logo on the top left and the RAMAIAH Institute of Technology logo on the top right. The "Story so far" section contains four bullet points: "The basic concept of navigation has changed very little over centuries", "The technologies and computing has, of course, changed dramatically", "Dead reckoning was one of the standard techniques used for navigation", and "Reference frames are crucial so as to provide standardized (positional) information". The "This lecture" section contains three bullet points: "We will study in some detail two inertial sensors (accelerometers and gyroscopes)", "We will study the effect of sensor noise on computing the position – drift !", and "While it is beyond the scope to go into some of the really important details of these sensors and their characterization – we will present enough information to have your first navigational system by end of this course !". The bottom of the slide has a yellow footer with the text "Control Engineering", "Module 11 – Lecture 2", and "Dr. Viswanath Talasila".

So, in this lecture; well, this is what I have basically described while you need to remember one thing, there are some extraordinarily important details of sensor characterisation which we will be brushing aside really quickly, without going too much into the details of that is actually very important. If you want to have extraordinarily good navigational precision accuracy, you need to really worry about a lot of details, but the objective for second or third year under grad student is really to just have enough information to develop your own navigational system by the end of this course, alright, that is really the objective of this set of lectures.

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General Introduction

- The sensors commonly used in dead reckoning are termed inertial sensors, and they comprise accelerometers, gyroscopes and magnetometers. We will not deal with magnetometers in this chapter.
- Sensor fabrication:
 - mechanical, optical, MEMS etc. We shall exclusively deal only with MEMS inertial sensors here.
- Pros and Cons of each sensor type:
 - ✓ The main disadvantage of mechanical sensors is that they contain moving parts; resulting in both wear and tear as well as friction induced measurement errors.
 - ✓ Optical sensors are highly accurate but they are very expensive.
 - MEMS sensors are really cheap (a simple 3 axis accelerometer can be easily purchased for a few hundred rupees), and highly versatile (as they can be easily and directly interfaced to microcontrollers).
- MEMS sensors cons
 - Measurements by MEMS sensors are significantly corrupted by noise. *filtering techniques (C.F., KF)*
- To counter this problem, researchers have developed really cool algorithms which are able to filter out this noise.

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So, general introduction; right, there are various types of inertial sensors and these basically dependent on how you actually fabricate and design them. So, you have mechanical optical MEMS and so on, we are going to be exclusively focusing on the use of MEM sensors in these series of lectures, I will explain this while later, the main disadvantage of mechanical sensors is they contain moving parts, right. So, there is a significant amount of wear and tear, fairly accurate, but they suffer from this problem optical sensors are extraordinarily accurate, but they are very very expensive MEM sensors are the kind of sensors that you know, you have in your phones, when you do the portrait to the landscape mode, you know, it shifts, you also have these sensors in various drones and UAVs, small robots which you keep playing around and so on. So, MEM sensors are really cheap. In fact, as I have written here you can buy simple MEMS 3 axis accelerometer for just a couple of 100 rupees and yeah and the really important reason why they are so useful is that they are highly versatile, I can actually just take a MEM sensor of the market and directly interface it to my accelerometer and start to work with that to do that with optical sensors mechanical sensors require a lot of additional electronics.

The problem with MEM sensors is that they are significantly corrupted by noise as compared to the optical or the mechanical sensors and we will see a few of these noise characterisations, but to make sure that you still continue to use these MEM sensors, there are really nice algorithms or primarily you can actually call them as filters or

filtering techniques which had been developed to compensate for this noise and 2 of the most commonly used techniques one is the complementary filter.

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Effect of sensor noise on dead reckoning computations

- Every single measurement by any sensor is corrupted by noise
- Since we compute position using these measurements – the final result should be erroneous. But how bad is it really? (remember, we are using MEMS sensors here)

Remember that the gyroscope measures the angular rate, and integrating that will give the angular position

$$\int \omega_x = \theta_x \quad \theta_x(k) = \theta_x(k-1) + \omega_x(k) * \Delta t$$

Handwritten note: first order difference

Since measurements are always noisy, we can write the basic measurement equation (for a gyroscope) as

$$\omega_{meas}(t) = \omega_{true}(t) + \eta(t)$$

Handwritten notes:
 measured ang velocity = true ang velocity + noise
 ⊕ noise additive
 * multiplier

the actual (true) angular velocity the noise parameter

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Which we will be discussing in the next series of lectures and other one is a Kalman filter, right well. So, let us go ahead, effect of sensor noise on dead reckoning computations. So, we have seen the first order difference equation for computing the angle angular displacement once you measure the angular rate, right. So, this is the first order differential equation, this is actually what gets implemented in your computers. Now it is important to remember that every sensor not just inertial sensors every sensor which is actually going to use this; the measurements are highly corrupted by noise. So, you can actually see this very often when you try to take you know, your selfies if you do 10, slightly poor lighting, you get lot of grainy images, if even if you do it in a extremely good lighting conditions and you try to zoom into the camera, into the image, you start to get lot of blurring effect and grainy images and so on this is true for all sensors.

You measure the temperature of this room by putting a very accurate digital thermometer costing you know 100s and 1000s of rupees, but still not going to get the perfect temperature because always going to be noise and of course, the same thing happens for inertial sensors as well. So, every measurement you make is actually a combination of 2 things, one is the actually dynamics which is happening. So, for example, you are measuring temperature, it is actually temperature in the room plus the noise. So, this can

be written for a gyroscope as follows. So, the measured angular velocity or the measured angular rate is basically the true angular velocity plus and I am going to write this with a circle, I will explain why plus noise now.

So, this is actually what you measure every time and you are never going to directly get the true angular velocity its always corrupted by noise the meaning of this circle over the plus sign is some noise are additive in nature. So, you actually simply add it up some noise can actually be multiplicative in nature; there are various kinds of noise as well. So, what we are going to consider here is primarily the additive noise. So, the angular velocity measured by a gyroscope is a true angular velocity plus noise that is a basic kind of equations we are going to be dealing with or behaviour.

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Angular position using dead reckoning

- Noise is a time dependent variable and this can sometimes really complicate our computations
- There are various factors which contribute to the noise, we will discuss them later
 - For now we will look at one factor: the constant bias

Suppose you are holding a stick in your hand, which for now we will assume is at a rest position. Let us connect a gyroscope to this stick and obtain real time measurements; we would expect to have $\omega_x = 0$, i.e. the angular velocity (along the axis X direction) is 0. Specifically, lets assume that the bias error is 10^{-3} deg/s. In the presence of noise

initial condition $\rightarrow \theta(0) = 0^\circ$ $k=0$
 $\omega_{meas}(t) = \omega_{true}(t) + \eta(t)$
 $\theta(1) = \theta(0) + \omega(1) \Delta t$ $k=1$
 $\theta(2) = \theta(1) + \omega(2) \Delta t$ $k=2$
 $\theta(3) = \theta(2) + \omega(3) \Delta t$

Handwritten notes: $\omega_{meas} = 0 + 0.001$, $k = 1: 16,000$, $\Delta t = \frac{1}{100 \text{ Hz}}$, 100 Hz , $100 \text{ samples every second}$, 16000

Graph: Angular displacement vs. samples (at 100Hz). Shows a linear drift over time.

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So, let us see one of the consequences of this. So, let us suppose and we look at a very specific example of one noise parameter which is a constant bias what is a meaning of a bias let us again take the example of the thermometer, let us say the temperature in this room, you know god or somebody has told us it is actually 23.1 degrees centigrade exactly know the decimal points 23.1.

The thermometer is going to show always 23.12. So, the exact temperature in the room was 23.1 degrees, god or someone has actually told you, this your thermometer shows 23.12, this is by the thermometer, let us go to another room, a different room where the temperature in the new room is 43 degrees and your thermometer will show 43.02. Go to

another place, there will always be a offset from the actual true temperature. In our particular example by 0.02 degree Celsius, this constant offset; this constant offset is called a bias is there in every sensor. So, let us see and actually it turns out that this is one of the most dominant noise effects which you find in all sensors.

So, let us see now in the next 2-3 slides, what is the effect of this bias on our computations. Now remember, I am going to first start off with a example of compute in the angular position by using the angular velocity measurements. So, we look at what happens to the angular position. The angles when bias is present, let us take us even more specific example in a simulation, I will do a practical experiment, later on in a simulation, let us say the bias error or the bias is 10^{-3} degree per second, it means every second I have a bias of 0.001 degrees. Remember, I am measuring the angular velocity. So, every second, there is this factor added. So, this factor is added every second as basically what you have over here.

Now, let us take a even more specific example, let us say I have this laser pointer in my hand and think of it as a pendulum, right which oscillates like this. Now if it goes say from this vertical position to the horizontal one, we say that there is a 90 degree change, if it takes 1 second to go from the vertical position to the horizontal, we say that the angular velocity is 90 degrees per second, if it takes 5 seconds to go from this horizontal position to the vertical position, we would say the angular velocity is 90 divided by 5 degrees per second, right. So, that is the rate or the velocity at which it moves. Now let us forget about motion, let us do something far simpler, let just keep it as this, it is perfectly stationary, it is not moving.

So, the measurement which you get, we would expect the measurement to be 0 degrees per second, right. It is not moving, it is perfectly stationary; however, every measurement is corrected by this bias value of 0.001, what would happen if I include this bias value in my numerical integration, let us see, what would have happened. So, to understand that we need 2 important, we need an important thing which is the amount or rather the sampling rate. This is a gyroscope a typical MEMS gyroscope, you can sample it anywhere from 50 hertz, all the way I imagined to around 300 hertz or something you can even do it higher, but the accuracies are not so good.

What this means is that my gyroscope is throwing out let us in a suppose we 100 hertz, just for the sake of discussion, it means my gyroscope is throwing out 100 samples of data every second, in 1 second, I get 100 samples of data from the gyroscope. So, if I take 0 to 1 second, then 2 seconds over here, I will have 100 samples of data, what do we do? So, this is basically the omega the angular velocity, what do we do with angular velocity; we integrate it. So, we basically say that θ_k is $\theta_{k-1} + \omega \Delta t$, what happens when you are integrating, remember, you are integrating the data at 100 hertz.

So, you are 100 samples per seconds each sample is being integrated. So, let us do this following example, let us suppose the initial value of the gyro which is perfectly addressed theta, the angle at times 0 is 0 degrees, this is the initial condition now. So, this is known, now I want to compute theta at the next sample. So, in this case ω was equal to 0, now let us see what happens when ω equal to 1. So, that is theta of 1 is nothing, but the previous angle which I had; the previous angular position which I had which was theta of 0 plus omega of 1, the current angular velocity which is measured times delta t whatever is the time interval, you are taking this delta t, in our case will be equal to 1 over 100 hertz because we are sampling at that rate.

So, we are going to look at this expression now. So, was theta of 0 that was 0, what is omega of 1? If it is a gyroscope made by god, it will show that its omega measured is exactly equal to 0, it is not made by god. So, you always have a small bias; well let us see what theta 1 becomes. So, theta will become 0.001, this is at k equal to 1, what happens at k equal to 2 at k equal to 2 theta of 2 is basically the previous angle which I have measured plus the current gyroscope value times delta t. Let us for the moment ignore the affect of delta t over here, we will just assume it to be 1 without any loss of generality, you will get the same kind of results when you take the correct delta t.

So, we will wrongly assume it to be equal to 1, but you get exactly the same kind of behaviour even if you take the correct delta t. So, theta of 2 is equal to theta of 1 which is 0.001 and again I have a bias, the true measurement is 0, but I have a bias, again I had 0.001, alright and this is nothing, but 0.002. Let us do it one last time. So, theta of 3 will be equal to theta of 2 plus omega of 3, sorry, this should be here 2 omega of 3 times delta t add it up again. So, you get 0.002, the previous value plus 0.001 and this becomes 0.003, what is happening? Every single time, I am integrating the gyroscope

measurement, the angular velocity, the actual angle computation, the angular displacement is drifting away, it should be exactly at 0, right because my measurement is actually 0, there should not be any angular velocity measured, but every single time we are integrating noise, the bias value and its drifting of a c 0.01, 0.02, 0.3 and so on.

You simulate this; I have done this for 16000 samples over here. So, k was equal to 1 to 16000 in my code, I will be sharing the code with you later and you do this for 16000 samples and you will see that the drift actually goes linearly like this. So, perfect linear curve whereas, true value of the angle. So, this is the angular position in degrees, it should have been 0 n for all time and what is the computed angle, it drifts away like this. This is always going to happen and a better quality sensor will reduce the amount of drift; may be it will be somewhere over here really expensive sensor may keep it somewhere over here, otherwise in every single case, you are never ever going to get a perfectly computed angle with a gyroscope thing, this is the phenomena of drift and drift here basically means you are drifting away from your from your true angular position well that is true for the gyroscope.

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Translation position using dead reckoning

- Double integrating the accelerometer measurements will result in translation position
- Since we are double integrating we can expect the final position calculation to be much worse (than when we single integrate)

Consider a vehicle which is stationary on a straight road. Let us connect an accelerometer to this vehicle and obtain real time measurements; we would expect to have $a_x = 0$, i.e. the acceleration (along the axis X direction) is 0. Specifically, lets assume that the bias error is 10^{-3} m/s^2 . In the presence of noise

$a_{meas}(t) = a_{true}(t) + \eta(t)$
 $= 0 + 0.001$

$a_{meas}(t) = 0 + 0.001 = 0.001$

$\int a_{meas} \rightarrow \text{Vel}_{comp}$

$\int \text{Vel}_{comp} \rightarrow \text{pos}_n$

$\Delta t = X \frac{1}{F_s}$

drift

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Now, what happens for the accelerometer; it is we will see, it is even worse and why. So, let us assume the same kind of noise profile. So, the bias is 10 power minus 3 meters per second, again, we will wrongly just assume delta t equal to 1, but that is not really important, you need to take care of it in your actual code to make sure that delta t is

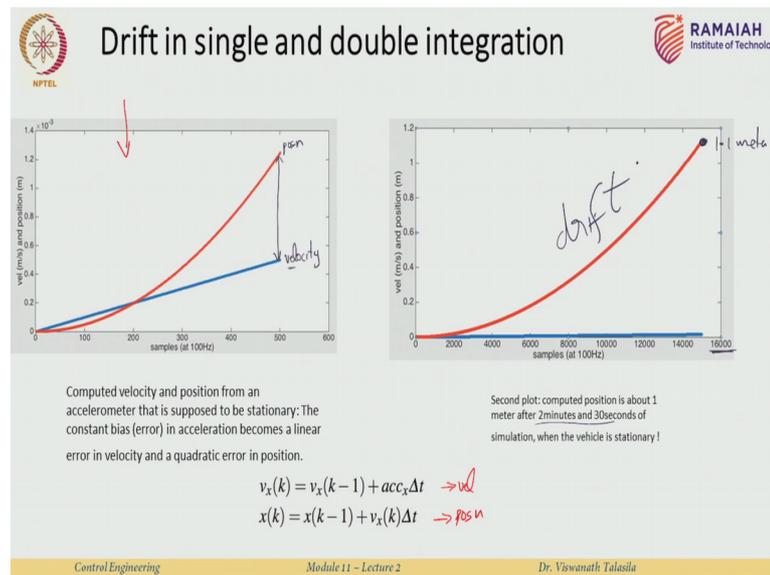
actually 1 over the sample rate that you should take care of that in any case, you are going to get exactly the same behaviour as we will show in the actual experiments, alright.

So, let us take the bias error for an accelerometer again as 0.001 meter per second square because that is what an accelerometer measures. Let us again assume this the same condition, the laser pointer is perfectly stationary. So, what would I measure 0, right? So, we would expect this to be 0 always; however, there this bias which comes in. So, this is what you are actually going to measure at every instant of time. So, for all t you are going to get 0.001 and we will now see that in order to compute the angular position from the angular velocity, we had to do a single integration for an accelerometer. To compute the translatory position, we need to a double integration; you can already imagine; what is going to happen.

So, you are going to from your accelerometer measured value, you are going to compute a velocity by a single integration from the computed velocity, I will call it velocity computed from the computed velocity integrated it a second time and you get your position which means you are; if you go to the previous slide, in every single integration the value of this bias; the 0.001 was added 1 time each integration process, right. So, here we added at once here, we added at the second time here, we added that the third time you know, what is going to happen here, well it is going to add this bias twice because you are double integrating the accelerometer.

So, if we know from the previous slide that if you plot the velocity as a function of time which was obtained by a single integration with a bias of 0.001, we going to get something like this, a straight line this is the drift when the true velocity should have been 0 because even the acceleration was 0. Now remember that we are double integrating the acceleration. So, it means that we take this linear drift over here integrated one more time and what you are going to get is actually a Parabola single integration results in the linear drift, a double integration results in a parabolic drift that is what you are going to see over here. So, this is the simulation which are done in a MATLAB, I will happily share the code with you and these are the 2 equations for one is for the velocity, one is for the position and you see that in the first plot over here you see in the first plot over here, right.

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So, what you see is this curve is the velocity curve which has which you got by single integrating the accelerometer data, this curve is actually a position which you got by once again integrating the velocity and you can see the error between the 2 because of the double integration has grown so much, right. Let us just run the simulation for a sufficiently long time, again all of this is synthetic data I have created this in MATLAB software, this is not true data the real experiment will be conducted a little bit later in the next lecture.

So, let us run this for 16000 samples, this is approximately 2 minutes and 30 seconds where the sensor was stationary perfectly station and you can now see after 2 minutes 30 seconds, my position which are supposed to be where 0 has now gone to almost 1.1 meters. So, it means the laser pointer which was supposed to be here, has gone even farther away from that more than 1.1 meters away, right, in just 2 minutes and 30 seconds while it is really not moving that is the affect of drift with a double integration. So, this is actually the problem with dead reckoning, you will always get drift in dead reckoning, if you remember the desert ant example which I gave before the poor guys is actually trying to use his legs, the ant computes the number of movements of the legs in order to calculate how much distance it has travelled.

Now let us say that he is counting you know 1, 2, 3, 4, 5, 6 and it by mistake, it forgets to count a few of the leg movements that error now starts to add up because there is an;

right a small error, it may forgotten 2 leg movements that gets integrated every single time like and it is going to produce this kind of a drift. So, it may have walked 10 meters and it thinks it has walked may be just 5 meters of 50 meters, it does not know and it is true for all navigational processes. So, drift is bas it is even worse in double integration. So, now, before we head with techniques of how to remove this drift, let us see in some detail what are the characteristics of these sensors?

Let us now let us look at the gyroscope, first the gyroscope is an interesting sensor, let us say you have a spinning ball or a gear as what you can see in your slide over here.

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The slide is titled "MEMS Gyroscope" and features the NPTEL and RAMAIAH logos. It contains a list of bullet points, a diagram of two interlocking gears, and handwritten notes in red ink.

- A gyroscope basically measures the rotation rate (angular velocity) of an object it is fixed to
- The measurements are done in an inertial frame of reference; even the rotation of earth (through the inertial frame) can be picked up in the gyroscope measurements.
- Difficult to measure Earth's rotation rate – as it is around 15 degrees per hour
- A gyroscope located at the equator and pointed exactly north (or south) will measure Earth's rotation rate (in the vertical axis)
- Placing the gyroscope at the poles will not measure Earth's rotation
- Luckily we are not using very sensitive gyroscopes, otherwise we would have to filter out the earth rotation measurements as well !

Handwritten notes in red ink include:

- $15^\circ - 3600 \text{ seconds}$
- $? - 1 \text{ sec}$
- $\frac{15}{3600} = 0.004^\circ/\text{sec}$ rotation rate of earth
- bias of sensor
- \gg rotation rate of earth.

At the bottom of the slide, it says: "Thanks to <https://gyrocraft.wordpress.com/about-3/> for the cool gear animations!" and "Dr. Viswanath Talasila".

Now, remember the gyroscope is a 3 axis sensor, you can also have; you can also have a single axis gyroscope, but we will be dealing only with 3 axis here. So, let us say, you have single axis gyroscope which is mounted like this and you have a ball which is spinning; spinning in a clockwise direction. Let us fix the gyroscope on the ball. So, the ball is spinning and gyroscope is; obviously, spins along with the ball with respect to the vertical axis. So, this fix up the rotation rate of the spinning ball, right. So, there is a small illustration over here.

So, these are the gears which are moving and you put a gyroscope in that and the gyroscope measures the rotation rate with respect to its vertical axis and it will say may be it is going at 10 degrees per second or whatever the actually velocity is; right. So, that is what you see. Now the cool thing about the gyros is the following where is this, yes,

the cool thing about the gyros is that where if you have a sufficiently sensitive gyroscope, you can even pick up the rotation rate of earth, right that is that is really amazing. So, you are on earth, earth is rotating all over you, I have absolutely no, I mean I cannot sense the rotation rate of earth, I mean I am sitting everything looks static to me, you place a sufficiently sensitive gyroscope and it can actually measure the rotation rate of earth. The problem with the rotation rate of earth is that it is very slow, it is 15 degrees per hour, you calculate this per second. So, really small number and what is 15 degrees per hour well let us see. So, that is 15 degrees, every 3000 sic 100 seconds, right.

So, what is the rotation rate for every second, well it is 15 divided by 3600 that is approximately 0.004 degrees per second. So, this is the rotation rate of earth and most gyros are not sensitive enough to pick up this rotation rate which is a good thing because every time you pick up such rotation rates you need to now compensate for this, right. So, it acts like a bias, it is not the true motion of your vehicle, it is not the true rotation motion of a vehicle, there is also an extra component of earth rotation rate, luckily our MEMS gyroscopes are not so accurate and they do not really pick up this thing and one the reasons why they do not pick up is that our noise component.

So, the actual bias of the sensors itself is usually much larger than the rotation rate of earth and because of that your noise dominates the earth rotation signal and you can never actually see it in our in these relatively low cost MEMS gyroscopes, the other thing is if you and I think you need to do this thought process yourself or thought experiment placing the gyro at the equator pointing exactly at north will give you the rotation rate of earth place. The same gyroscope, sorry, place, the same gyroscope exactly at the poles north pole or the south pole you will not measure the rotation rate of earth I really want you to think of that experiment, good.

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Error types in a gyroscope

- So far we have seen how errors (bias error) affect the computation of position (translation and angular)
- The bias error considered was a constant offset.
- In general inertial sensors are affected by other errors sources as well.

Some of the standard errors considered in gyroscope modeling are

- Fixed bias
- Bias stability
- Bias instability
- Scale Factor
- Scale factor stability
- Misalignment
- Cross axis response
- White noise (ARW)
- Flicker noise etc

Handwritten notes on slide:
- Calibration: $\omega_{meas} - b_{bias} = \omega_{true}$
- Graph: ω vs t showing noise around 0. Mean values μ_1, μ_2, μ_3 are marked.
- Equation: $\omega_{meas} = \omega_{true} + b_{bias}$

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So, that is what the MEMS gyroscope does and well what are the errors which we can encounter in a gyroscope. So, far we have seen only the constant bias error and it has a constant offset, but there are lots of other errors. So, one which we saw was the constant bias then we have something called the biased stability or the bias instability both mean relatively similar things expressed differently and what this basically means is that we always assume our bias is constant, but that is not really true, you turn on your gyroscope or MEMS gyroscope and you calculate you keep it in the perfectly stationary condition.

So, let us say, we are looking at one particular axis it can be any axis as a function of time in a perfectly stationary condition. So, it may have may be some noise like that right this is the 0 value, it may have some slight deviation from 0 over a function of time and when you take this entire data and compute the mean of it over say about 5 minutes you will get a particular value. So, we will call it noise one you turn off the gyroscope and may be after couple of minutes turn it back on again and then what you are going to get is a slightly different noise signal again it will around 0, but it will be slightly different from the previous one.

And now when you compute the mean, it will be a different value slightly different, but it will be different you can repeat the experiment again if you like if you are sufficiently interested or bored in life and you compute the mean the third time, you are going to get a third keep repeating experiment every single rime turn it off turn it on wait for 5

minutes collect the data calculate the mean every time you are going to get a different answer which means your bias is not really stable it is not a fixed constant bias. So, that is another problem the reason why fixed constant bias would be amazing is because you can actually before the start of your experiment navigation robotic navigation whatever you can just turn on the gyroscope wait for 5 minutes calculate the mean of that that is your offset right.

Now, every time in the future, if it is a same offset; what I can do is the following. So, W measured is W true plus the noise, right, plus the noise and let us assume only bias a fixed constant, if I am able to model this one, if I am able to calculate this from all my previous experiments, all I need to do is to subtract from the W measurement, my bias constant offset and I will really get my true value this is actually a standard procedure which is called as calibration and we do this all the time with gyroscopes or even with accelerometers, well it would be nice except for the fact that your bias is also time varying your bias changes with time, it even has what is called the turn on turn off values you turn it on it changed turn it off, it changes, you take the same gyroscope and you run an experiment for 1 hour, for the first 5 minutes, it will be 1 bias value maybe, the next 10 minutes will be a slightly different bias value, after another half hour may be another slightly different bias value. So, bias is time varying there is also a change from turn on to the next turn on condition, then we will see other parameters called the scale factor misalignment and white noise these are the 3 things which we are going to see in the little bit of detail, right.

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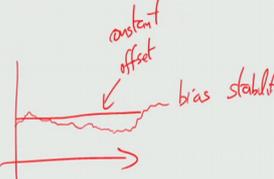


Error types in a gyroscope - 1



- **Fixed bias (deg/hr)**
 - Sensor gives a measurement which is always offset by the bias.
 - This is often the biggest source of error and also the easiest to eliminate (or reduce)
 - We have seen, through our simulations, that the drift due to a fixed bias is linear with a single integration
 - Error grows linearly with time (t)

- **Bias stability ((deg/hr)/hr)**
 - When the IMU is powered, the bias changed with time, and is affected by phenomena such as temperature, mechanical stress etc.
 - Removing this error source is quite complex and relies on external aiding sensors
 - It has been experimentally demonstrated that this error creates a random walk in velocity



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So, the fixed bias, we have already seen, we have also talked about bias stability. So, while we assume the bias is perfectly constant, it can actually be changing as a function of time like it will not go too much away from the actual value, but it could be changing, this is the bias stability and it is a function of time, this is the assumed constant offset bias, this is not always true. Now what is the bias measured in typically in degrees per hour because degrees per second is a very small number. So, typically they measure it in degrees per hour, the bias stability is measured in degree per hour per hour.

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Error types in a gyroscope - 2



- **Angle Random Walk (deg/√hr)**
 - This is an additive noise term which behaves like white noise
 - Integrating white noise (the velocity) produces a drift whose standard deviation grows proportionally to \sqrt{t}

- **Scale Factor (%)**
 - This expresses the relation between the input and output.
 - Ideally we want i/p and o/p to be exactly the same
 - Due to scale factor error, the output may appear to be a scaled version of the input.
 - So if input is 1 deg/s, and we have a 5% scaled factor error, then output will be 1.05 deg/s

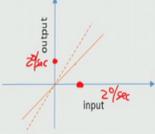
$W_{\text{meas}} = W_{\text{true}} + \eta$

$\theta = \eta_{\text{bias}} + \eta_{\text{time var bias}} + \text{white noise}$

θ varies \sqrt{t}

θ almost with t

η x scaling



Random walk is a random process that describes a sequence of successive random steps. Random walk modeling has been used in many fields to describe the path traced by a molecule in a gas, the search path of a foraging animal etc

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Because it is a variation of your bias, yes then you come to actually a very interesting noise parameter called the angle random walk and basically, what happens is every measurement of your gyroscope as we have seen we have the true value plus a corruption the noise, the noise is a combination of your bias your fixed bias, let us call it f bias, then you also have the time varying bias and you also have white noise for the interested students, white noise has some very nice statistical properties its 0 mean process for those who are hearing this for the first time, I encourage you to just go in Google and read a little bit white noise, you may have seen it in your television screen when the signal goes off and you get that kind of sound, right. So, that is a white noise.

So, the interesting thing with white noise is that it corrupts your measurements as we have seen over here and the way in which it corrupts, it is a fairly complicated mathematical way of deriving it, but basically, it corrupts it in such a way that the error grows as a function of square root of time. So, you actually say degree per square root hour whereas, for the constant bias, it was degree per square root hour, sorry, it was degree per hour, right, whereas, for the angle random walk is degree per square root hour.

So, the basic if you look at time and you look at the computed angle from the measured gyroscope, this would be the plot for the constant bias and now I would like you to think on your own; what would be the plot if theta varies. So, in this case, theta is varying linearly with time, right, it is varying linearly with time, I want you to think and plot and these will also be set of assignments later on for you how would theta look if theta varies as a function of the square root of time, it is a very simple MATLAB code which you can write then you have something called the scale factor.

The scale factor basically says the following if the input the real angular velocity is let us say 2 degree per second in an ideal world the sensor which measures this should also say that the output of the sensor is also 2 degree per second, what happens is in reality, you have a scale factor. So, the input is multiplied with some scaling parameter. So, and this is typically expressed as percentages. So, if you say that you have a 5 percent scaling error and the input is one degree per second the actual measurement will show it to be 1.05 this is not the bias it is a scale factor, alright.

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Story so far

NPTEL

RAMAIAH
Institute of Technology

- Misalignment and Cross Axis Sensitivity (in %)
 - A 3-axis gyroscope has 3 single axis gyroscopes mounted orthogonal to each other.
 - While manufacturing it is very difficult to ensure perfect orthogonality between the axes
 - This results in a correlation between two axis – ideally, the correlation should be zero
- Bias repeatability (deg/hr)
 - Each time the gyroscope is turned on, the initial bias is different !
 - Ideally we would like high bias repeatability
 - To keep things safe, we must always include a calibration routine at the start of each experiment

Use a rate table as a reference sensor to calibrate!

$w_y > 0$

$w_x \neq 0$

$w_x = 0$

$w_{meas} = w_{true} + ?$

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And then you have misalignment and cross axis sensitivity misalignment basically is the following we always talk about 3 axis measurements, right. So, the forward axis, the vertical axis and the axis which completes the right angle coordinate system. Now these axis system has to be perfectly 90 degrees offset from each other otherwise what is going to happen is the measurement; let us say you are rotating about this axis. So, the pitch angle; so, you are actually doing the pitch rotation over here. Now if the axis on are perfectly aligned, there will be a 0 measurement of angular velocity along this axis, right which I am holding, you will only see a angular velocity in the other 2 axis; however, because of manufacturing tolerances and errors and manufacturing these are not perfectly orthogonal to each other there is a slight deviation. So, when I hold this and I rotate I do a pitch up and a pitch down even this axis starts to measure a non 0 angular velocity.

So, to see this in a little bit more detail we would ideally want these to be perfectly orthogonal. So, this should be perfectly at 90 degrees, let us take the 2 axis case the same story applies for 3 axis. Now instead suppose you manufacture it in such a way that this axis was slightly off and this axis stays well almost the same. Now when you do a rotation about this axis all right you would expect. So, we will call this as x. So, we this as y x prime y prime, this would be x and this would be y when you rotate. Let us actually rotate about the other axis. So, when you rotate about say this axis you which means that you hold this constant and you rotate about that; right, it means the angular velocity along x must be equal to 0 and the angular velocity along y must be greater than

0; however, because this axis is slightly misaligned and they are not perfectly orthogonal. So, this may be like eighty nine point five degrees or something you will find that ω_x is not equal to 0 is slightly larger than 0 and that is not true, right. So, this is again a noise.

So, all these noise terms, they come together and they basically corrupt your measurement true plus noise and noise as you can see we have seen about 5 different types of noise terms, each one of them affects the overall computation in a slightly different way the most dominant one is a bias that is a largest value and for most applications, it is generally enough to compensate for the bias, we do not need to really worry about say this misalignment or angle random walks scale factor and all these things; if you can really compensate for this bias which is not too difficult, you can really solve many applications which do not require very high precision, fine, a similar story holds for accelerometer except that every noise term in accelerometer gets integrated 2 times. So, we need to be a little bit more careful.

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Introduction to the accelerometer

- An accelerometer basically measures the acceleration of an object it is fixed to
- The measurements are done in an inertial frame of reference; the earth's gravitational field can be easily measured
- Gravity acts in the inertial frame. The strong effect of gravity's acceleration ($\sim 9.8 \text{ m/s}^2$) can be measured by the accelerometers and is always significant when operating near the Earth's surface

Earth's irregular gravitational maps!

Gal=1cm/s²

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Let us see what an accelerometer actually measures you know an accelerometer measures acceleration right 3 axis accelerometer I am posing a question for all of you 3 axis accelerometer its perfectly stationary, let us say the I am inside a vehicle and I am and I am holding a 3 axis accelerometer, it is not moving the vehicle is perfectly stationary will this measure anything apart from noise, it will always have some noise.

Let us ignore noise say the god has made this accelerometer its absolutely no noise does it measure any non 0 component well the picture on the right this beautiful colour picture which you see over here, these 2 are basically the earth's gravitational field every time you place an accelerometer at a perfectly levelled position. So, the vertical axis is pointing directly towards the earth's vertical axis, what you are actually going to measure is the non 0 quantity along the vertical axis and that non 0 quantity is basically 9.8 meter per seconds square your gravitational force or the gravitational acceleration.

So, even if you are perfectly stationary place an accelerometer anywhere on earth anywhere on earth you are going to measure earth's gravity, this would be fairly if earth's gravity were constant. So, everywhere on earth, if earth's gravity is 9.8 meter per second square I know that I can compensate for that unwanted measurement because I do not really want to measure earth's gravity I want to measure how fast my vehicle is moving not my vehicle motion plus earth's gravity, I do not want to measure that. So, if it were perfectly 9 point meter per second square, life would have actually be fairly simple BCE from every measurement of my accelerometer, I would simply subtract 9.8 meter per second square and then I will actually get say a much better measurement with just a noise terms being available and your this one compensated the problem with that is that earth's gravitational field is irregular and you can see this picture.

And surprisingly even I dint know this it is very fascinating to see this, India actually has extremely well relatively speaking India has fairly low gravitational field over here. So, all the colours in dark are where the gravitational field is much are large and the blue colours. So, the bright red colour is where the field is very large and the blue colour is where the gravitational field is really low and interestingly India and literally where I am sitting right now. In Chennai the gravitational field is really low. So, it will be less than our expected 9.8 meter per second square.

So, now, this is another problem right you need to compensate for this, but you cannot do it directly unless you have this earth's gravitational field delta with you, right. So, really professional navigators they always they compute the latitude longitude altitude of where you are on earth all right and then they subtract out the corresponding gravitational acceleration from that particular place, right, but otherwise basically if you ignore of this an accelerometer basically measures the acceleration and it will always measures the earth's acceleration as well here also what we discussed before with a gyroscope the

misalignment becomes very important. So, if the axis were perfectly aligned. So, this vertical axis, let me re draw that that nicely.

So, let us say you have a vertical axis and the other 2 axis over here. So, this vertical axis would measure 9.8 meter per second square, if it were perfectly vertical, it is never the case because of manufacturing defects. So, maybe it is something like this now this would be the real vertical axis. Now because all the 3 axis are now slightly misaligned whereas, they should have 0 meter per second square, the other axis should also have been 0 meter per second square. Now each of the axis will measure a quantity greater than 0 meter per second square and of course, the vertical one will be the largest you know depending on how much misalignment has happened it may be 9.75 meter per second square or whatever this is the problem of misalignment and all the axis start to get corrupted by values which they should not be measuring. So, the same thing with the accelerometers as well you have a fixed bias you are doing a double integration and the drift grows really large and we seen that it goes linearly with single integration and the position it grows as a quadratic term.

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Noise characteristics of an accelerometer

- Fixed bias also called the O_g bias level (mg or $mg / ^\circ C$)**
 - O_g level refers to the voltages when the sensor is completely stationary (after removing the 1g earth gravitational component)
 - Sensor gives a measurement which is always offset by the bias.
 - This is often the biggest source of error and also the easiest to eliminate (or reduce)
 - We have seen, through our simulations, that the drift due to a fixed bias is linear with a single integration and parabolic with double integration
 - Drift velocity error grows linearly with time (t) and quadratic in position (t^2)
- Bias stability ((deg/hr)/hr)**
 - When the IMU is powered, the bias changed with time, and is affected by phenomena such as temperature, mechanical stress etc.
 - Removing this error source is quite complex and relies on external aiding sensors
 - It has been experimentally demonstrated that this error creates a random walk in velocity and the drift grows proportionally to $t^{3/2}$ in the position which grows proportionally to $t^{5/2}$
- Bandwidth**
 - Human body movement is at fairly low frequencies
 - Vibrations in some industrial applications can go from 100s of Hz to KHz.
- Level Arm Effect**
 - A body moves w.r.t. its center of gravity (c.g.). If an accel is mounted away from the c.g. of the body, then it does not measure the true acceleration

Note: Bandwidth is relevant for gyroscopes as well

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That parabolic shape the bias stability is similar to the angle random walk the behaviour is very is very simple and the drift when you do the single integration it grows as a power of $t^{3/2}$ you do a double integration it grows as $t^{5/2}$ whereas, the other one was the linear and quadratic this grows at these rates we will not really go into these 2

details except to say that the bandwidth of a sensor and not just accelerometer even for gyroscope is extremely important if you have let us take my arm let us say the arm is moving at the rate of one at a rate of 1 hertz right it means I am able to do this motion in a 1 hertz cycle which means it takes me one second to do this motion one seconds 2 seconds 3 seconds and so on.

If this is the motion which I always expect there is absolutely no need for me to use a very high bandwidth gyroscope or accelerometer right there is no need for me to use like a 1000 hertz bandwidth accelerometer because I do not do anything with all the remaining frequencies it really just the one hertz component which I am interested in if I am a missile and I am moving extremely fast, right, I do not know how fast this missile actually change their parts, but you could really be looking at fairly high frequency of say 50 hertz or something like if you have these what beam these cantilever beams which are attached to walls and you tap on them and you have these vibratory motion these can go really very high frequencies and for that you would use a different accelerometer as well, right.

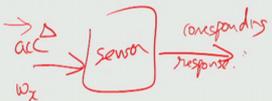
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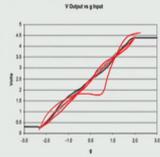


Other sensor issues to consider



- **Input Range ($\pm g$)**
 - This is the maximum angular rate that the sensor can measure
 - Inputs outside this range result are usually erroneous
 - For e.g. if we mount an accelerometer on a vehicle and the goal is to track the vehicle motion
 - Vehicle vibrations can dominate the actual signal of the vehicle movement
 - If we set the gyroscope to a low bandwidth, then the high frequency data cannot be properly measured
 - In such cases vibration isolation is preferred, or we need another way to estimate the vibrations
- Accuracy ✓
- Resolution ✓
- Sensitivity ✓
 - This is the ratio of the change in input acceleration to change in the output signal
 - Sensitivity is usually specified at a supply voltage and is expressed in units of mV/g (or LSB/g or mg/LSB)
 - It is usually given in a range - min and max - or in form of percentage deviation
- Nonlinearity ✓
 - The deviation from the straight line fit between input and output





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So, the bandwidth is an important property finally, while I will not spend too much time on these, it is important for everyone to realise this, the input range basically what I have just been talking to you the input range is very important for high speed dynamic motion, you would require a high bandwidth gyroscope or accelerometer for a low speed or low

dynamics motion, you would require a low bandwidth accelerometer or a auto gyroscope and the range actually determines that accuracy of course, is important the more accurate your sensor is the more expensive it is, but the more better it is resolution and sensitivity again are important.

Sensitivity basically says it is defined as a ratio of the change in the input to the actual measured value, right, sorry, is defined as the change in the input to the change in the output signal. So, if you take let us say your sensor over here, let us say, we give it an acceleration or we give it angular velocity for a delta change in the acceleration means for a very small change in the acceleration, if there is a corresponding change in the voltage or in the response over here that is very good it means that we can measure really small acceleration.

However, if it takes a large input to have some amount of data coming in over here then it means our accelerometer or gyroscope are not sensitive enough all right that is a basic meaning we will ignore this non-linearity you can look up data sheets we expect behaviours of sensors to be fairly linear around certain operating regions in real life as we have seen in the previous lectures they can be slightly non-linear not as much as I have drawn, but slightly non-linear. So, all these effects start to become important when you really start to worry about accuracy, if you are comfortable that your application should have reasonable accuracy reasonable accuracy is what you should define for most home automation or even most robotics application.

I mean a MEM sensor is good enough and even more even more specifically you do not need to really compute angle random walk you do not need to worry about misalignment and so on and. So, forth unless you want your robot which travels for say 2 kilometres to know exactly where it is with very high degree of precision if you are not too worried about that focus on the constant bias remove that through a calibration process which we will be showing you in the experiments and we are pretty much done.

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ADXL Datasheet



The ADXL345 is a small, thin, low power, 3-axis accelerometer with high resolution (13-bit) measurement at up to ± 16 g. Digital output data is formatted as 16-bit two's complement and is accessible through either a SPI (3- or 4-wire) or I²C digital interface. The ADXL345 is well suited for mobile device applications. It measures the static acceleration of gravity in tilt-sensing applications, as well as dynamic acceleration resulting from motion or shock. Its high resolution (4 mg/LSB) enables measurement of inclination changes less than 1.0°.

Measurement Range	User selectable	$\pm 2, \pm 4, \pm 8, \pm 16$	g
Nonlinearity	Percentage of full scale	± 0.5	%
Inter-Axis Alignment Error		± 0.1	Degrees
Cross-Axis Sensitivity ¹		± 1	%
OUTPUT RESOLUTION			
16 g Range	Each axis	10	Bits
± 2 g Range	10-bit resolution	10	Bits
± 4 g Range	Full resolution	11	Bits
± 8 g Range	Full resolution	12	Bits
± 16 g Range	Full resolution	13	Bits
SENSITIVITY			
Sensitivity at X _{acc} , Y _{acc} , Z _{acc}	Each axis	232	256
Scale Factor at X _{acc} , Y _{acc} , Z _{acc}	± 2 g, 10-bit or full resolution	15	3.9
Sensitivity at X _{acc} , Y _{acc} , Z _{acc}	± 2 g, 10-bit or full resolution	196	128
Scale Factor at X _{acc} , Y _{acc} , Z _{acc}	± 4 g, 10-bit resolution	7.0	7.8
Sensitivity at X _{acc} , Y _{acc} , Z _{acc}	± 4 g, 10-bit resolution	58	64
Scale Factor at X _{acc} , Y _{acc} , Z _{acc}	± 8 g, 10-bit resolution	14.0	15.6
Sensitivity at X _{acc} , Y _{acc} , Z _{acc}	± 8 g, 10-bit resolution	29	32
Scale Factor at X _{acc} , Y _{acc} , Z _{acc}	± 16 g, 10-bit resolution	28.1	31.2
Sensitivity Change Due to Temperature	± 16 g, 10-bit resolution		± 0.01
Bias (LSB)			
g Output for X _{acc} , Y _{acc}	Each axis	± 150	± 40
g Output for Z _{acc}		± 80	± 250
g Offset vs. Temperature for x, y-axes		± 0.8	mg/°C
g Offset vs. Temperature for z-axis		± 4.5	mg/°C

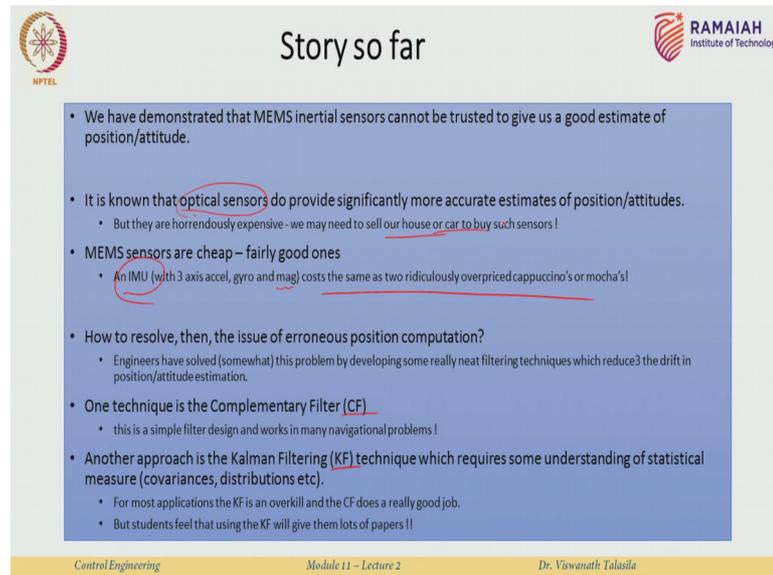
$1\text{ g} = 9.8\text{ m/s}^2$

bias can be really high !!

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So, this is accelerometer data sheet from analogue electronics and there is quite some interesting information which they provide in the in the data sheet. So, you can see that the bias values fairly high it is about 150 milligram; 150 mg; the g basically 1 g would basically be 9.8 meter per second square. This is actually the acceleration, I am experiencing right now without any motion, if I add an extra motion its 9.8 added to that extra acceleration which my body will be providing at stationary positions, it is 9.8 meter per second square and as you can; that the bias is 150 mg that is fairly large which is why it is important to compensate for that; yes. So, we are almost towards the end of this lecture and what we have seen is that MEM sensor are fairly cheap for a few 100 rupees; you can buy accelerometers and gyroscope. In fact, I encourage you to go ahead and buy these sensors.

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Story so far

- We have demonstrated that MEMS inertial sensors cannot be trusted to give us a good estimate of position/attitude.
- It is known that optical sensors do provide significantly more accurate estimates of position/attitudes.
 - But they are horrendously expensive - we may need to sell our house or car to buy such sensors!
- MEMS sensors are cheap - fairly good ones
 - An IMU (with 3 axis accel, gyro and mag) costs the same as two ridiculously overpriced cappuccino's or mocha's!
- How to resolve, then, the issue of erroneous position computation?
 - Engineers have solved (somewhat) this problem by developing some really neat filtering techniques which reduce the drift in position/attitude estimation.
- One technique is the Complementary Filter (CF)
 - this is a simple filter design and works in many navigational problems!
- Another approach is the Kalman Filtering (KF) technique which requires some understanding of statistical measure (covariances, distributions etc).
 - For most applications the KF is an overkill and the CF does a really good job.
 - But students feel that using the KF will give them lots of papers!!

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Because 2 of these sensors cost the same as 2 ridiculously over priced cappuccinos or I do not know what you guys have in these fancy coffee shops, right. So, a complete IMU with a accelerometer gyroscope and this something called a magnetometer is like a compass complete IMU costs less than 2 cappuccinos, well, there are various techniques which we will be seeing in the next lecture, on how to compensate for these noise problems. Finally, if you are really desperate and you want incredibly good accuracy maybe, you are an engineer working in aircrafts or in missile developments or even in autonomous land vehicles or air vehicles where precision accuracy is extraordinarily important for whatever reason that you have you would really try to go in for optical sensors. They are of course, very expensive, you may need to sell your house or car to buy one of these sensors, but that is how it goes the more expensive the better the accuracy.

So, thank you very much. In the next set of slides, we will actually move on to filtering to see, how we can reduce navigation errors specifically we will be looking at example of complementary filter we will conclude that with a set of experiments.

Thank you very much.